A PROOFS FOR TECHNICAL RESULTS

Lemma 4.2. The function $\mathcal{T}$ is a bijection. Moreover, for any infinite path π under $\mathcal{C}$, π is non-zeno iff $\mathcal{T}(\pi)$ is non-zeno.

Proof. The first claim follows directly from the determinism and totality of DTAs. The second claim follows from the fact that $\mathcal{T}$ preserves time elapses in the transformation. ☐

Proposition 4.4. For any scheduler σ for $\mathcal{C}$ and any initial mode q for $\mathcal{A}$, we have $\mathcal{T}\left(AccPaths^{\mathcal{A},q}_{\mathcal{C},\sigma}\right) = RPaths_{\theta(\sigma)}$.

Proof. By definition, the set $AccPaths^{\mathcal{A},q}_{\mathcal{C},\sigma}$ equals

$$\left\{\pi \in Paths^{\omega}_{\mathcal{C},\sigma} \mid \mathbf{ACC}\left(inf(traj(\xi_\pi)), \mathcal{F}\right)\right\}$$

where ξ_π is the unique run of $\mathcal{A}$ on $\mathcal{L}(\pi)$ with initial configuration $(q^*, \mathbf{0})$ for which q^* is the unique location such that $(q, \mathbf{0}) \xRightarrow{\mathcal{L}(\ell^*)} (q^*, \mathbf{0})$. Let $\pi = (\ell_0, \nu_0)a_0(\ell_1, \nu_1)a_1 \dots$ be any infinite path under $\mathcal{C}$. By the definition of $\mathcal{T}$ we have

$$\mathcal{T}(\pi) = ((\ell_0, q_0), \nu_0 \cup \mu_0)a'_0((\ell_1, q_1), \nu_1 \cup \mu_1)a'_1 \dots$$

in the form (5) such that $\xi_\pi = \{(q_n, \mu_n, b_n)\}_{n \in \mathbb{N}_0}$ is the unique run on $\mathcal{L}(\pi) = b_0 b_1 \dots$. Moreover, π follows σ iff $\mathcal{T}(\pi)$ follows $\theta(\sigma)$ by definition. Then it is obvious that

$$trace(\mathcal{T}(\pi)) = q_0 q_1 \cdots = traj(\xi_\pi).$$

It follows that $inf(trace(\mathcal{T}(\pi)))$ is Rabin accepting by $\mathcal{F}$ iff $inf(traj(\xi_\pi))$ is Rabin accepting by $\mathcal{F}$. Hence the result follows from Lemma 4.2. ☐

Proposition 4.5. For any scheduler σ for $\mathcal{C}$ and mode q, the followings hold:

- $\mathfrak{p}^{\sigma}_{q} = \mathbb{P}^{\mathcal{C},\sigma}\left(AccPaths^{\mathcal{A},q}_{\mathcal{C},\sigma}\right) = \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}_q,\theta(\sigma)}\left(RPaths_{\theta(\sigma)}\right)$;
- $\mathbb{P}^{\mathcal{C},\sigma}\left(\{\pi \mid \pi \text{ is zeno}\}\right) = \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}_q,\theta(\sigma)}\left(\{\pi' \mid \pi' \text{ is zeno}\}\right)$.

Proof. Define the probability measure $\mathbb{P}'$ by: $\mathbb{P}'(A) = \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}_q,\theta(\sigma)}(\mathcal{T}(A))$ for $A \in \mathcal{F}^{\mathcal{C},\sigma}$. We show that $\mathbb{P}' = \mathbb{P}^{\mathcal{C},\sigma}$. By [8, Theorem 3.3], it suffices to consider cylinder sets as they form a pi-system (cf. [8, Page 43]). Let $\rho = (\ell_0, \nu_0)a_0 \dots a_{n-1}(\ell_n, \nu_n)$ be any finite path under $\mathcal{C}$. By definition, we have that

$$\begin{aligned}\mathbb{P}^{\mathcal{C},\sigma}(Cyl(\rho)) &= \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}_q,\theta(\sigma)}(Cyl(\mathcal{T}(\rho))) \\ &= \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}_q,\theta(\sigma)}(\mathcal{T}(Cyl(\rho))) \\ &= \mathbb{P}'(Cyl(\rho)) \ .\end{aligned}$$

The first equality comes from the fact that the product construction preserves transition probabilities. The second equality is due to $Cyl(\mathcal{T}(\rho)) = \mathcal{T}(Cyl(\rho))$. The final equality follows from the definition. Hence $\mathbb{P}^{\mathcal{C},\sigma} = \mathbb{P}'$. Then the first claim follows from Proposition 4.4 and the second claim follows from Lemma 4.2. ☐

B THE HARDNESS RESULT

Below we prove the hardness of the PTA-DTRA problem. It is proved in [25] that the reachablity-probability problem for arbitrary PTAs is *EXPTIME*-complete. We show a polynomial-time reduction from the PTA reachibility problem to the PTA-DTRA problem as follows. For an arbitrary PTA $\mathcal{C}$ in the form (1) and a set $F \subseteq L$ of final locations, let $\mathcal{C}' = \left(L, \ell^*, \mathcal{X}, Act, inv, enab, prob, AP', \mathcal{L}'\right)$ where $AP' := AP \cup \{acc\}$ and $\mathcal{L}'$ is defined by

$$\mathcal{L}'(l) := \begin{cases} \mathcal{L}(l) & \text{if } \ell \notin F \\ \mathcal{L}(l) \cup \{acc\} & \ell \in F \end{cases}$$

for which acc is a fresh atomic proposition. We also construct the DTRA $\mathcal{A}'$ by

$$\mathcal{A}' := (\{q_0, q_1\}, \Sigma, \emptyset, \Delta, \{(\emptyset, \{q_1\})\})$$

where $\Sigma := \{\mathcal{L}'(\ell) \mid \ell \in L\}$ and Δ contains exactly the following rules:

- $(q_0, U, \mathbf{true}, \emptyset, q_1) \in \Delta$ for all $U \in \Sigma$ such that $\mathrm{acc} \in U$;
- $(q_0, U, \mathbf{true}, \emptyset, q_0) \in \Delta$ for all $U \in \Sigma$ such that $\mathrm{acc} \notin U$;
- $(q_1, U, \mathbf{true}, \emptyset, q_1) \in \Delta$ for all $U \in \Sigma$.

It is then straightforward from definition that an infinite path under $\mathcal{C}$ visits some location in F iff the infinite path (under $\mathcal{C}'$) is accepted by $\mathcal{A}'$ under initial mode q_0. Hence, under any scheduler (for both $\mathcal{C}$ and $\mathcal{C}'$), the probability to reach F in $\mathcal{C}$ equals the probability that $\mathcal{C}'$ observes $\mathcal{A}'$ under initial mode q_0. It follows that the problem to compute the maximum/minimum probability to reach F can be polynomially reduced to the PTA-DTRA problem. Hence the problem PTA-DTRA is EXPTIME-hard.

C PROOF FOR PTA-TRA UNDECIDABILITY

Theorem 5.2. Given a PTA $\mathcal{C}$ and a TRA $\mathcal{A}$, the problem to decide whether the minimal probability that $\mathcal{C}$ *observes* $\mathcal{A}$ (under a given initial mode) is equal to 1 is undecidable.

Proof. Let $\mathcal{A} = (Q, \Sigma, \mathcal{Y}, \Delta, \mathcal{F})$ be any TRA where the alphabet $\Sigma = \{b_1, b_2, \cdots, b_k\}$ and the initial mode is q_{start}. W.l.o.g, we consider that $\Sigma \subseteq 2^{AP}$ for some finite set AP. This assumption is not restrictive since what b_i's concretely are is irrelevant, while the only thing that matters is that Σ has k different symbols. We first construct the TRA $\mathcal{A}' = (Q', \Sigma', \mathcal{Y}, \Delta', \mathcal{F})$ where:

- $Q' = Q \cup \{q_{init}\}$ for which q_{init} is a fresh mode;
- $\Sigma' = \Sigma \cup \{b_0\}$ for which b_0 is a fresh symbol;

- $\Delta' = \Delta \cup \{\langle q_{init}, b_0, \mathbf{true}, \mathcal{Y}, q_{start}\rangle\}$.

Then we construct the PTA

$$\mathcal{C}' = (L, \ell^*, \mathcal{X}, Act, inv, enab, prob, AP, \mathcal{L})$$

where:

- $L := \Sigma'$, $\ell^* := b_0$, $\mathcal{X} := \emptyset$ and $Act := \Sigma$;
- $inv(b_i) := \mathbf{true}$ for $b_i \in L$;
- $enab(b_i, b_j) := \mathbf{true}$ for $b_i \in L$ and $b_j \in Act$;
- $prob(b_i, b_j)$ is the Dirac distribution at $(\emptyset, b_j)$ (i.e., $prob(b_i, b_j)(\emptyset, b_j) = 1$ and $prob(b_i, b_j)(X, b) = 0$ whenever $(X, b) \neq (\emptyset, b_j)$), for $b_i \in L$ and $b_j \in Act$;
- $\mathcal{L}(b_i) := b_i$ for $b_i \in L$.

Note that we allow no clocks in the construction since clocks are irrelevant for our result. Since we omit clocks, we also treat states (of $\mathcal{C}'$) as single locations. Below we prove that $\mathcal{A}$ accepts all time-divergent timed words over Σ with initial mode q_{start} iff the minimal probability that $\mathcal{C}'$ observes $\mathcal{A}'$ with initial mode q_{init} equals 1.

Consider any time-divergent infinite timed word $w = t_0 b'_0 t_1 b'_1 \cdots$ over Σ (where $t_i \in \mathbb{R}$ and $b'_i \in \Sigma$). We define an infinite sequence $\{\rho_n\}_{n\in\mathbb{N}_0}$ of finite paths (of $\mathcal{C}'$) inductively as follows:

- $\rho_0 := b_0 (= \ell^*)$; (Note that we treat states as locations since clocks are irrelevant.)
- for $m \geq 0$, $\rho_{2m+1} := \langle s_0, a_0, s_1, \dots, a_{k-1}, s_k, t_m, s_k\rangle$ if $\rho_{2m} = \langle s_0, a_0, s_1, \dots, a_{k-1}, s_k\rangle$;
- for $m \geq 0$, $\rho_{2m+2} := \langle s_0, a_0, s_1, \dots, a_{k-1}, s_k, b'_m, b'_m\rangle$ if $\rho_{2m+1} = \langle s_0, a_0, s_1, \dots, a_{k-1}, s_k\rangle$.

Intuitively, the sequence $\{\rho_n\}_{n\in\mathbb{N}_0}$ is constructed by letting the PTA $\mathcal{C}'$ read the timed word w in a stepwise fashion, while adjusting the next location upon reading a symbol (as an action) from Σ. Then one can define a scheduler σ_w by:

- $\sigma_w(\rho_{2m}) := t_m$ for $m \geq 0$;
- $\sigma_w(\rho_{2m+1}) := b'_m$ for $m \geq 0$;
- $\sigma_w(\rho)$ is arbitrarily defined if ρ is not from the sequence $\{\rho_n\}_{n\in\mathbb{N}_0}$.

Intuitively, σ_w always chooses time-delays and actions from w. From definition, $\mathbb{P}^{\mathcal{C}',\sigma_w}(\{\pi \mid \mathcal{L}(\pi) = w\}) = 1$. Note that σ_w is time divergent since w is time divergent. Hence

$$\mathfrak{p}^{\sigma_w}_{q_{init}} = \begin{cases} 1 & \text{if } \mathcal{A} \text{ accepts } w \text{ with } (q_{start}, \mathbf{0}) \ , \\ 0 & \text{if } \mathcal{A} \text{ rejects } w \text{ with } (q_{start}, \mathbf{0}) \ . \end{cases}$$

where the underlying PTA (resp. TRA) is $\mathcal{C}'$ (resp. $\mathcal{A}'$). Then we have that $\inf_\sigma \mathbb{P}^{\mathcal{C},\sigma}\left(AccPaths^{\mathcal{A}',q_{init}}_{\mathcal{C}',\sigma}\right) = 1$ iff $\mathcal{A}$ accepts all time-divergent timed words w.r.t. $(q_{start}, \mathbf{0})$. $\square$