HW5

Yigao Li

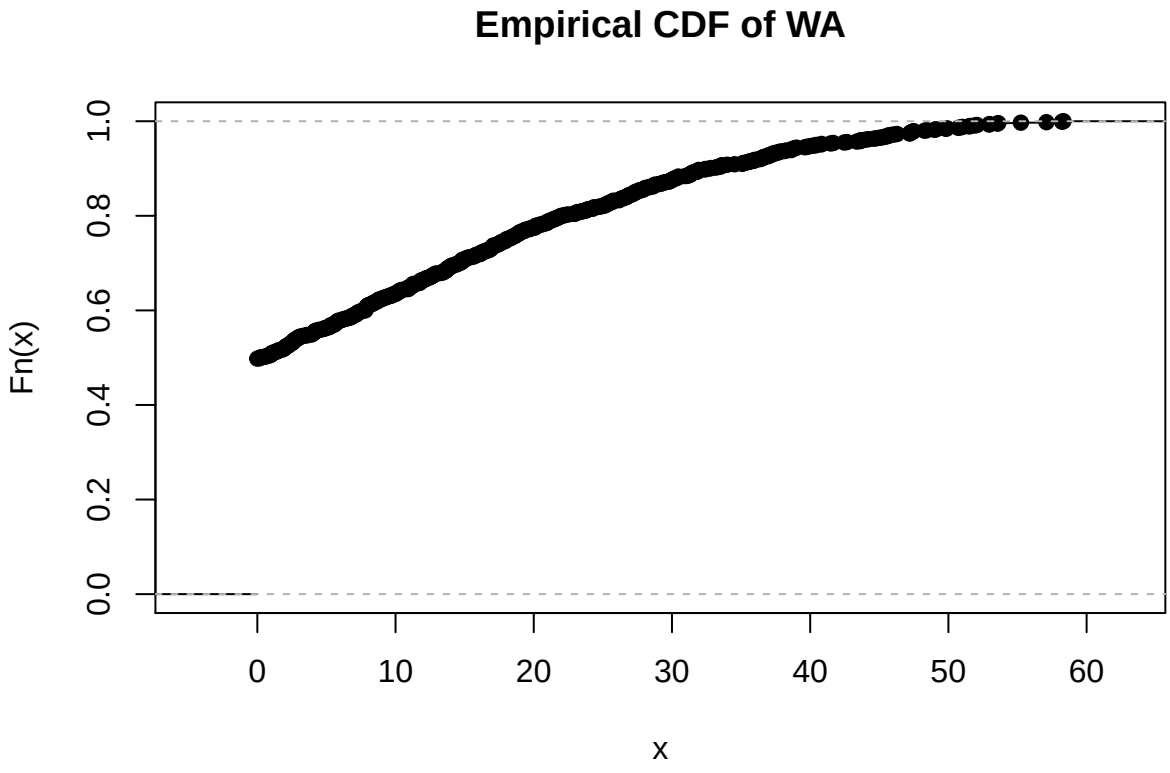
October 8, 2017

Problem 33

```
xa <- runif(1000, min = 0, max = 60)
xb <- runif(1000, min = 0, max = 60)
wa <- xa - xb
for (i in c(1:1000)){
  if (wa[i] < 0){
    wa[i] <- 0
  }
}
cat("Mean:", mean(wa))
```

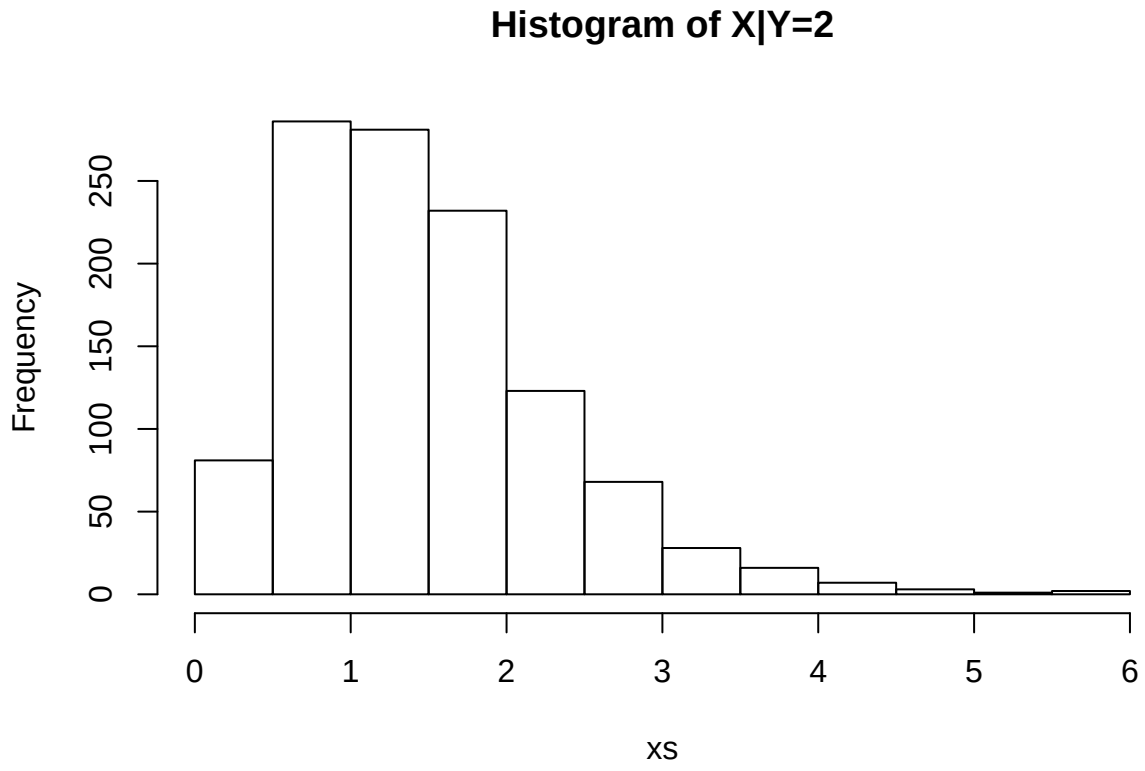
```
## Mean: 10.30798
```

```
plot(ecdf(wa), main = "Empirical CDF of WA")
```



Problem 34

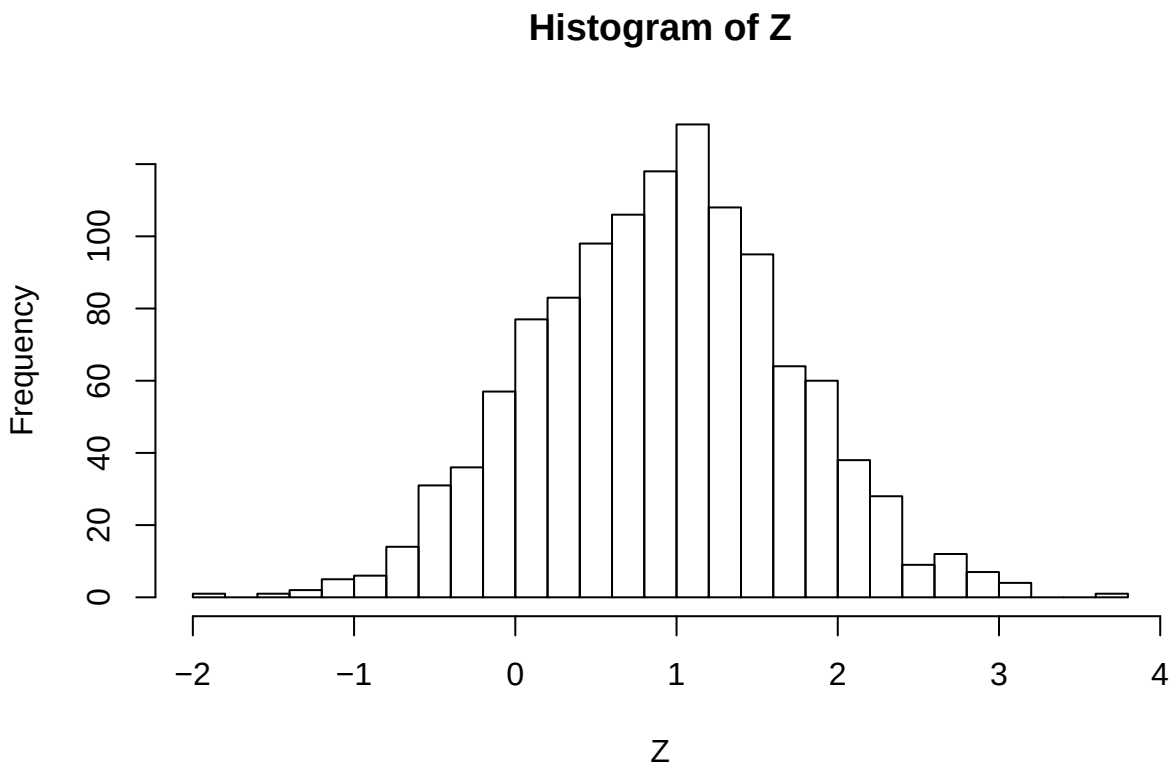
```
xl <- c()
yl <- c()
xs <- c()
for (i in c(1:9000)){
  x <- rexp(1, rate = 1)
  xl <- c(xl, x)
  y <- rpois(1, lambda = x)
  yl <- c(yl, y)
  if (y == 2){
    xs <- c(xs, x)
  }
}
hist(xs, main = "Histogram of X|Y=2")
```



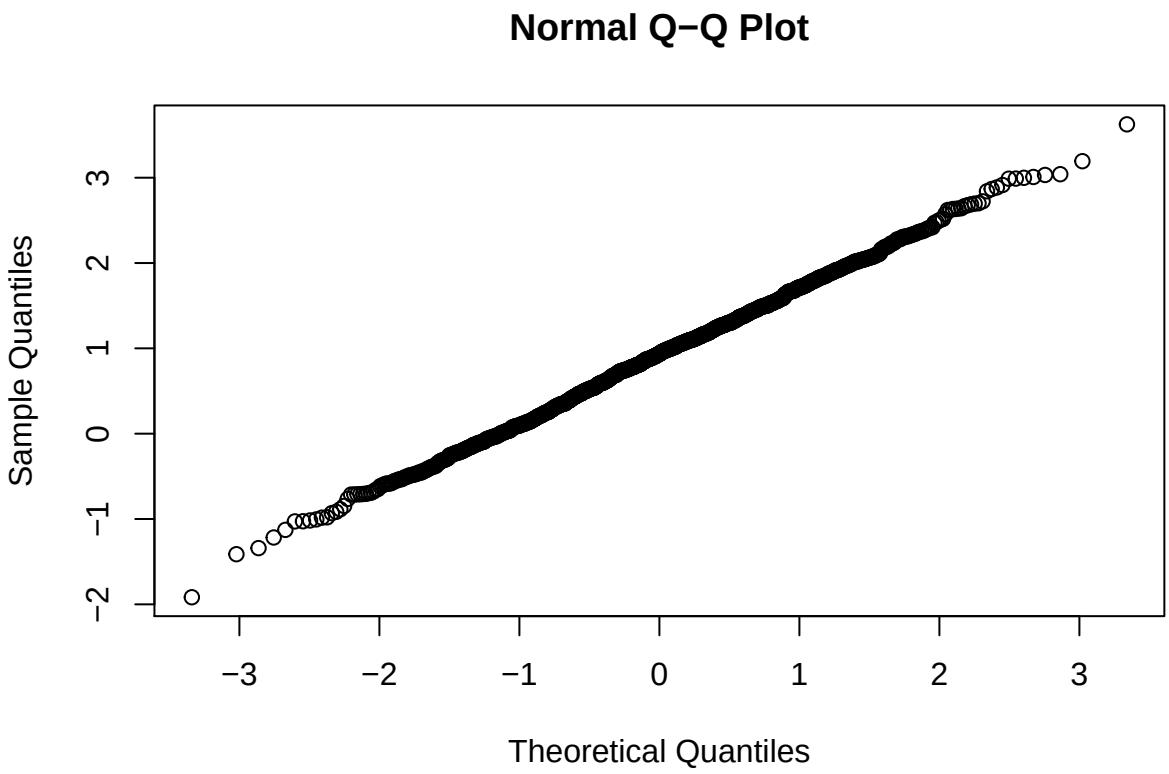
Problem 35

```
Z <- c()
for (i in c(1:5000)){
  x <- rnorm(1)
  y <- rnorm(1)
  if (x + y >= 1){
    Z <- c(Z, y)
```

```
  }
}
hist(Z, breaks = 24)
```




```
qqnorm(Z)
```




```
cat("According to QQ Plot, Z may have a normal distribution.\n
Mean:", mean(Z), "\n
Standard deviation:", sd(Z))
```

```
## According to QQ Plot, Z may have a normal distribution.
##
## Mean: 0.914984
##
## Standard deviation: 0.7908942
```

Problem 36

$$\begin{aligned} E[Y|X] &= E[aX + b + Z|X] \\ &= E[aX|X] + E[b|X] + E[Z|X] \\ &= aE[X|X] + b + E[Z] \\ &= aX + b + 0 = aX + b \end{aligned}$$

Problem 37

a) $Y|\mathcal{A} = X$

b) $Pr(\mathcal{A}|Y = x) = Pr(U < \frac{\ell(x)}{M}) = \frac{\ell(x)}{M}$

c)

$$\begin{aligned} Pr(\mathcal{A}) &= \sum_{x\in\mathcal{R}} Pr(\mathcal{A}|Y=x)Pr(Y=x) \\ &= \sum_{x\in\mathcal{R}} \frac{\ell(x)}{M}\frac{1}{N} \\ &= \sum_{x\in\mathcal{R}} \frac{cp(x)}{M}\frac{1}{N} \\ (\sum_{x\in\mathcal{R}} p(x) = 1) &= \frac{c}{MN} \end{aligned}$$

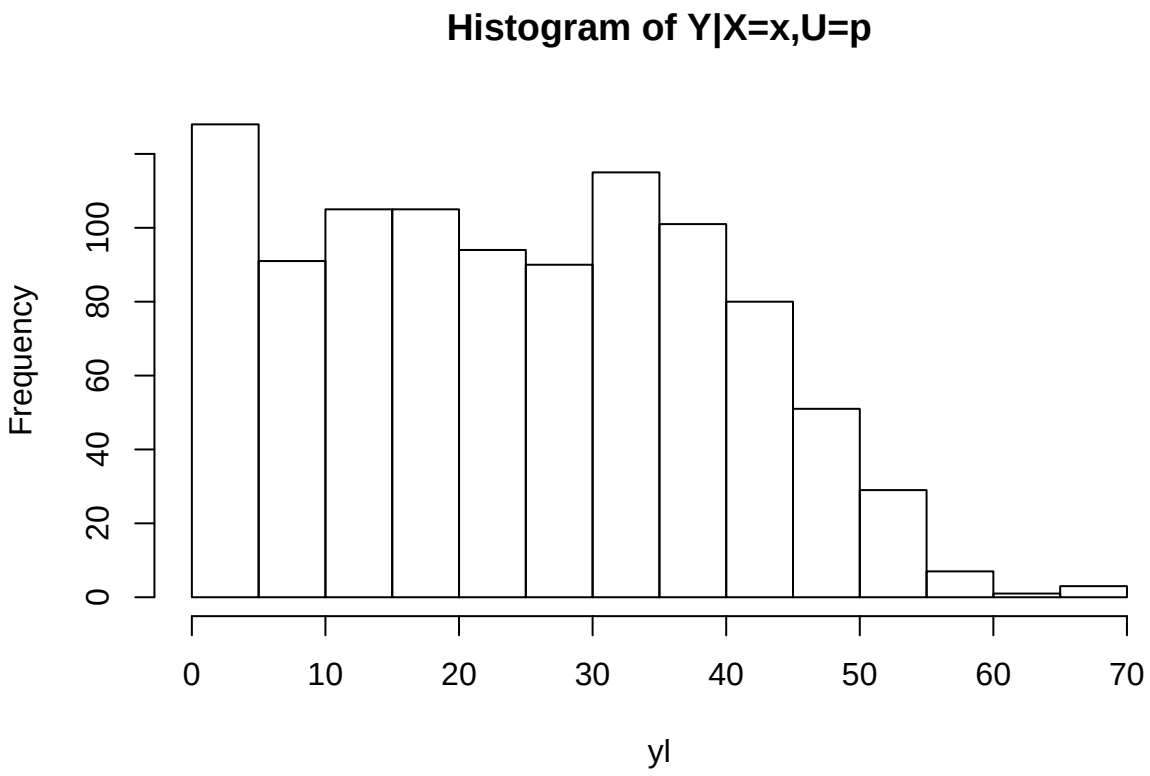
d)

$$\begin{aligned} Pr(X=x) &= Pr(Y=x|\mathcal{A}) \\ &= \frac{Pr(\mathcal{A}|Y=x)Pr(Y=x)}{Pr(\mathcal{A})} \\ &= \frac{\frac{\ell(x)}{M}\frac{1}{N}}{\frac{c}{MN}} \\ &= \frac{\ell(x)}{c} \\ &= p(x) \end{aligned}$$

Problem 38

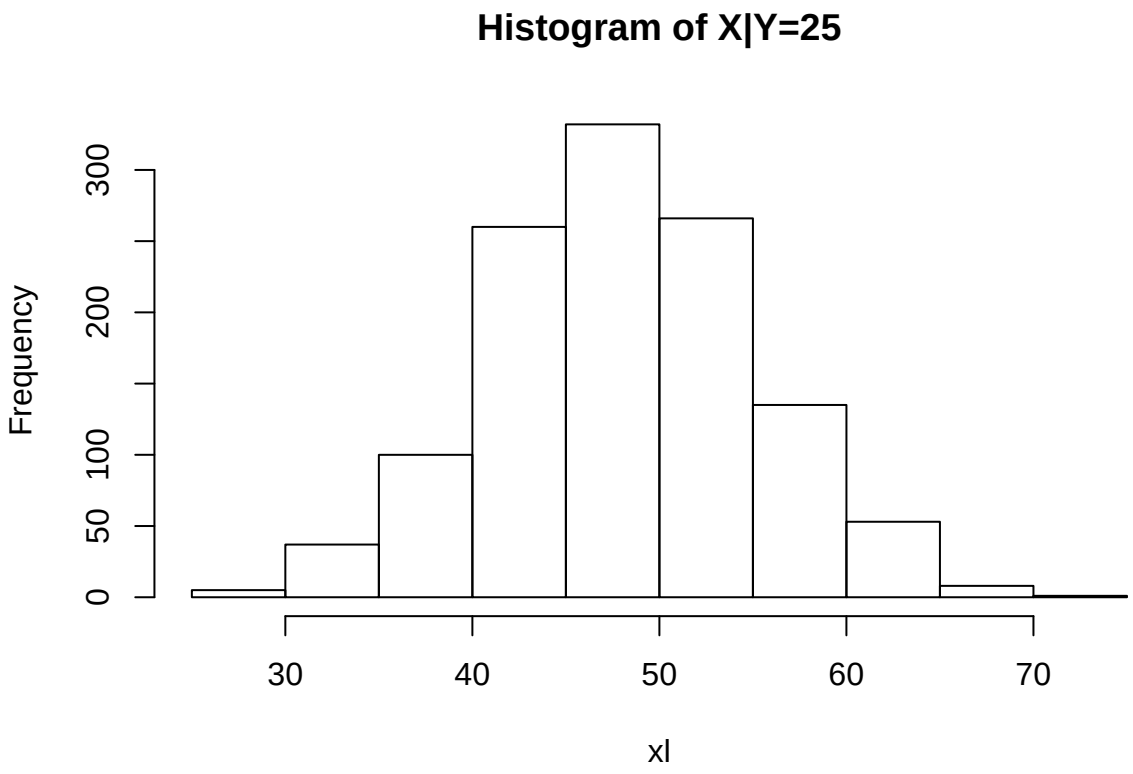
a)

```
yl <- c()
for (i in c(1:1000)){
  x <- rpois(1, lambda = 50)
  u <- runif(1)
  y <- rbinom(1, size = x, prob = u)
  yl <- c(yl, y)
}
hist(yl, main = "Histogram of Y|X=x,U=p")
```



b)

```r
xl <- c()
for (j in c(1:60000)){
  x <- rpois(1, lambda = 50)
  u <- runif(1)
  y <- rbinom(1, size = x, prob = u)
  if (y == 25){
    xl <- c(xl, x)
  }
}
hist(xl, main = "Histogram of X|Y=25")
```

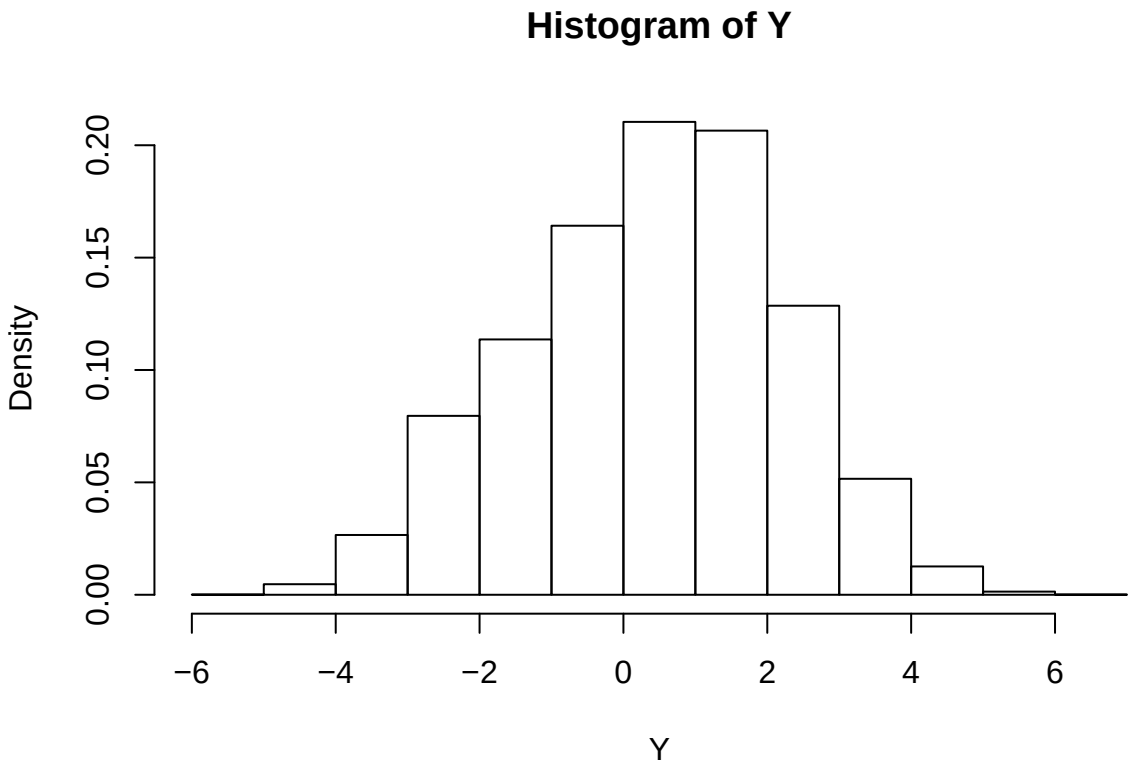


Problem 39

1. $E[Y|X=1] = E[Y_1] = \mu_1$, $E[Y|X=2] = E[Y_2] = \mu_2$,
   $E[Y] = E[Y|X=1]Pr(X=1) + E[Y|X=2]Pr(X=2) = \mu_1 w_1 + \mu_2 w_2$.
2. $E[Y^2|X=1] = Var(Y|X=1) + E^2[Y|X=1] = Var(Y_1) + E^2[Y_1] = \sigma_1^2 + \mu_1^2$, $E[Y^2|X=2] = Var(Y|X=2) + E^2[Y|X=2] = Var(Y_2) + E^2[Y_2] = \sigma_2^2 + \mu_2^2$,
   $Var(Y) = E[Var(Y|X)] + Var(E[Y|X])$
   $= Var(Y|X=1)Pr(X=1) + Var(Y|X=2)Pr(X=2) + E[E^2[Y|X]] - E^2[E[Y|X]]$
   $= \sigma_1^2 w_1 + \sigma_2^2 w_2 + \mu_1^2 w_1 + \mu_2^2 w_2 - (\mu_1 w_1 + \mu_2 w_2)^2$
3.

```
Y <- c()
y1 <- rnorm(10000, mean = -2, sd = 1)
y2 <- rnorm(10000, mean = 1, sd = sqrt(2))
w1 <- 0.2
w2 <- 0.8
x <- sample.int(2, 10000, replace = TRUE, prob = c(w1, w2))
for (i in c(1:10000)){
  if (x[i] == 1){
    Y <- c(Y, y1[i])
  } else{
    Y <- c(Y, y2[i])
  }
}
```

```r
hist(Y, freq = FALSE)
```



Problem 40

Find $Pr(\bar{x} > 51)$
By CLT, $\frac{S_{30}-30\times 48}{\sqrt{30}} \sim N(0, 9^2)$

$$\begin{aligned} Pr(\bar{x} > 51) &= Pr(30\bar{x} > 30 \times 51) \\ &= Pr(30\bar{x} - 30 \times 48 > 30 \times 51 - 30 \times 48) \\ &= Pr(\frac{30\bar{x} - 30 \times 48}{\sqrt{30}} > \frac{30 \times 51 - 30 \times 48}{\sqrt{30}}) \\ &= Pr(\frac{30\bar{x} - 30 \times 48}{\sqrt{30}} > 3\sqrt{30}) \end{aligned}$$

```r
cat("Therefore, the estimated probability the mean height of a random sample of 30 boys is",
    pnorm(3*sqrt(30), sd = 9, lower.tail = FALSE))
```

```
## Therefore, the estimated probability the mean height of a random sample of 30 boys is 0.03394458
```