# HW2

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## Problem 9

```r
cat("P{X<=10} =", pgamma(10, shape = 2.5, scale = 5))
```

```
## P{X<=10} = 0.450584
```

```r
cat("P{X>5} =", pgamma(5, shape = 2.5, scale = 5, lower.tail = FALSE))
```

```
## P{X>5} = 0.849145
```

```r
cat("P{|X-8|<3} =", pgamma(11, shape = 2.5, scale = 5) - pgamma(5, shape = 2.5, scale = 5))
```

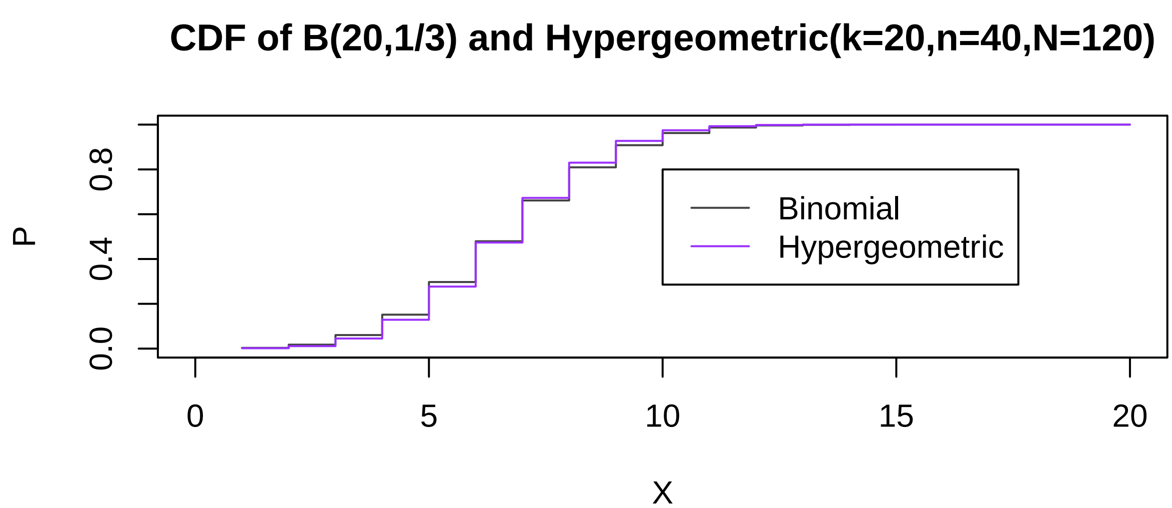
```
## P{|X-8|<3} = 0.3557715
```

```r
cat("Quantile z for P{X<z} = .1 is", qgamma(0.1, shape = 2.5, scale = 5))
```

```
## Quantile z for P{X<z} = .1 is 4.02577
```

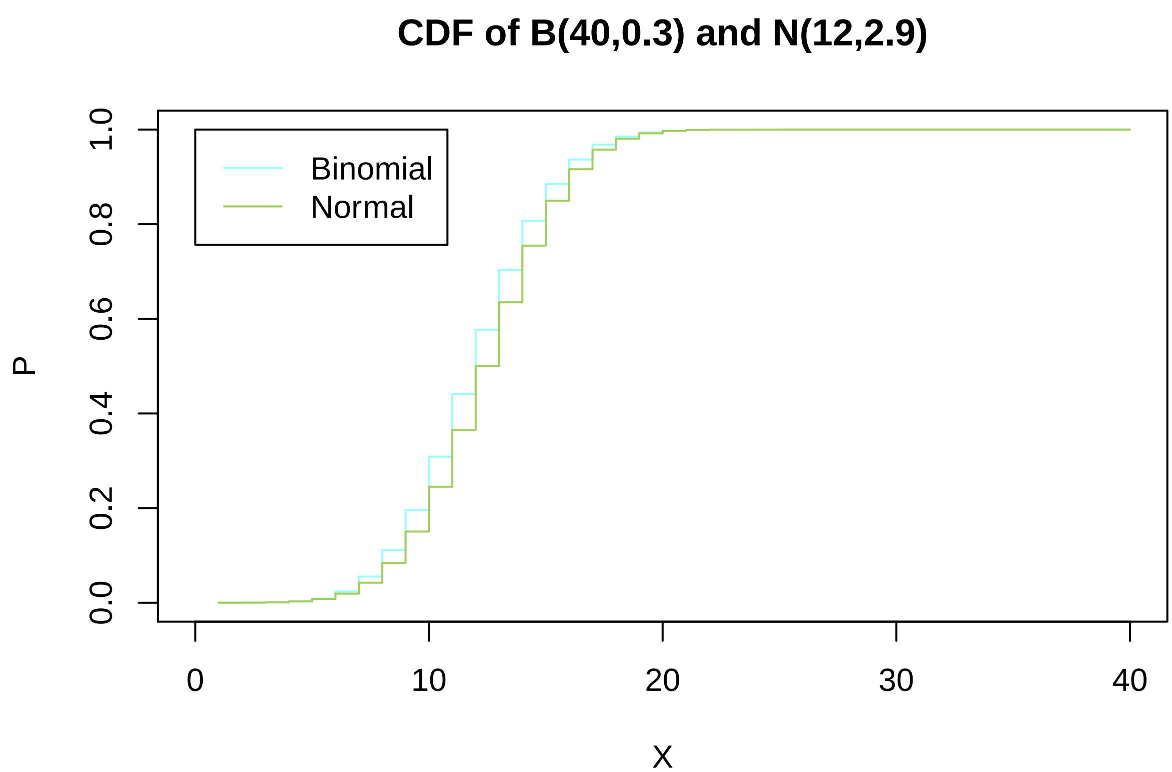
## Problem 10

```r
x1 <- 1:20
yb1 <- pbinom(x1, size = 20, prob = 1/3)
yh <- phyper(x1, m = 40, n = 80, k = 20)
plot(x1, yb1, type = 's', col = 'gray26', xlim = c(0,20), ylim = c(0,1),
     main = "CDF of B(20,1/3) and Hypergeometric(k=20,n=40,N=120)", xlab = 'X', ylab = 'P')
lines(x1, yh, type = 's', col = 'purple1')
legend(10, 0.8, legend = c("Binomial", "Hypergeometric"), col = c("grey26", "purple1"),
       lty = 1:1)
```



## Problem 11

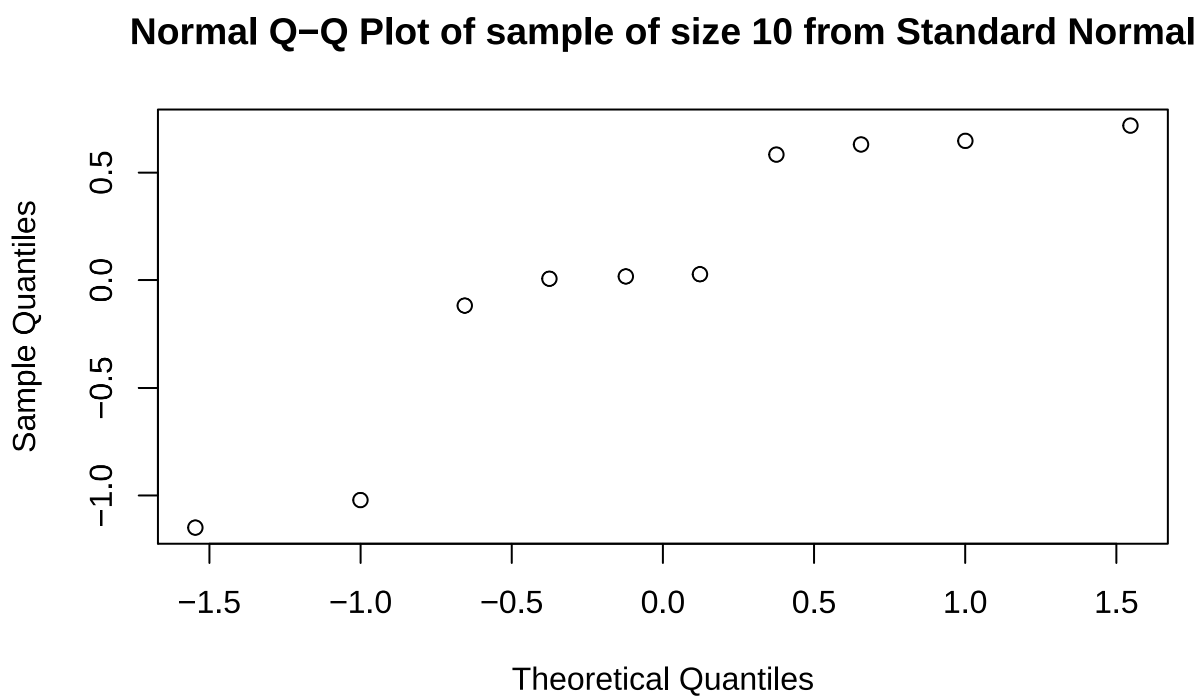
```
x2 <- 1:40
yb2 <- pbinom(x2, size = 40, prob = 0.3)
yn <- pnorm(x2, mean = 12, sd = 2.9)
plot(x2, yb2, type = 's', col = 'darkslategray1', xlim = c(0,40), ylim = c(0,1),
     main = "CDF of B(40,0.3) and N(12,2.9)", xlab = 'X', ylab = 'P')
lines(x2, yn, type = 's', col = 'darkolivegreen3')
legend(0, 1, legend = c("Binomial", "Normal"), col = c("darkslategray1", "darkolivegreen3"),
       lty = 1:1)
```



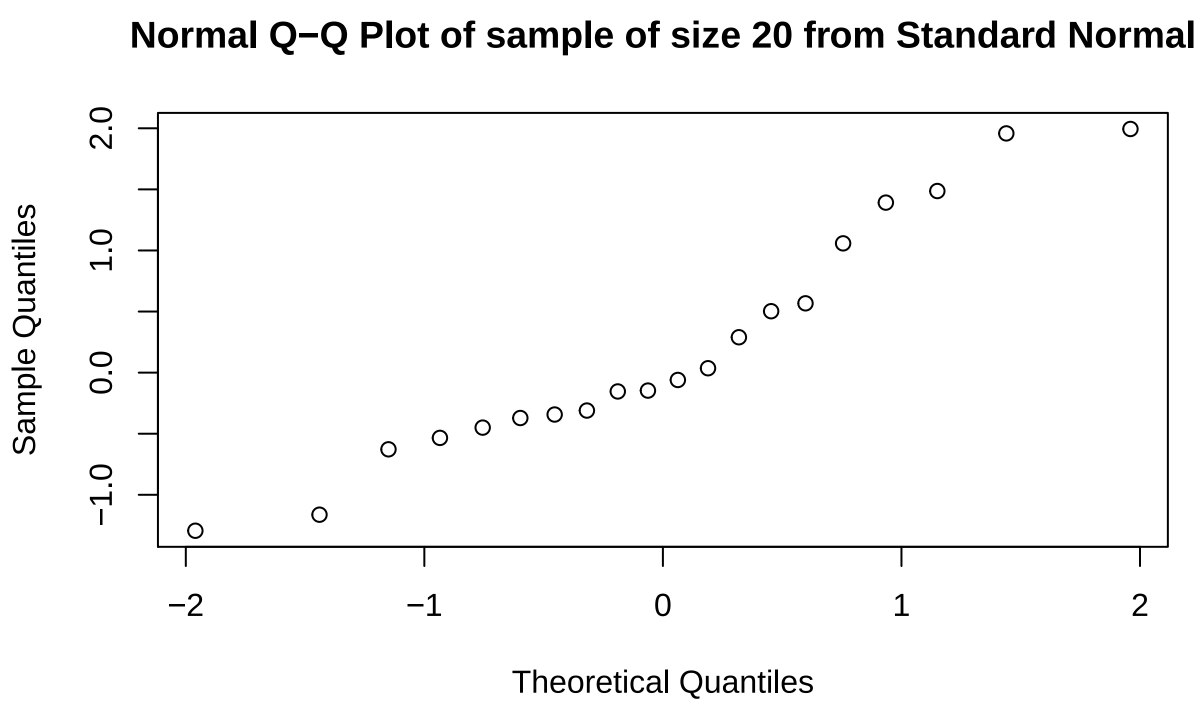

Although these two distributions are close, for each integer value of $X$, probability of binomial distribution is always larger than that of normal distribution.

## Problem 12

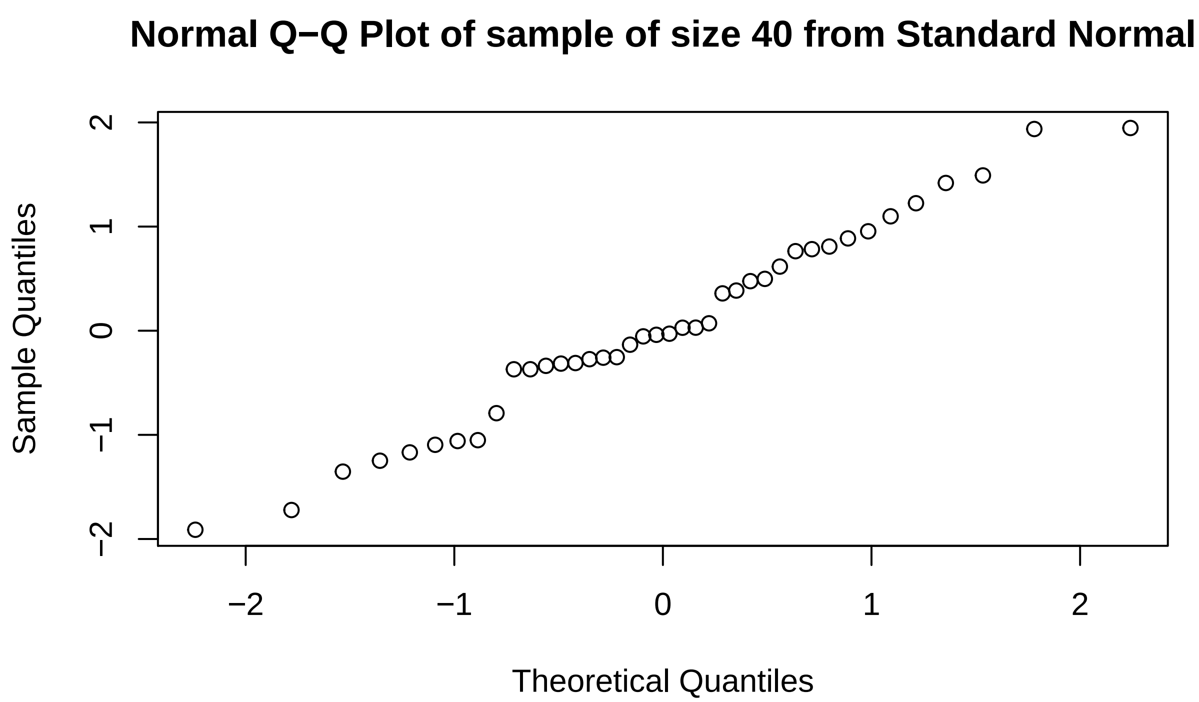
```
sample10 <- rnorm(10)
sample20 <- rnorm(20)
sample40 <- rnorm(40)
sample100 <- rnorm(100)
sample1000 <- rnorm(1000)
qqnorm(sample10, main = "Normal Q-Q Plot of sample of size 10 from Standard Normal")
```



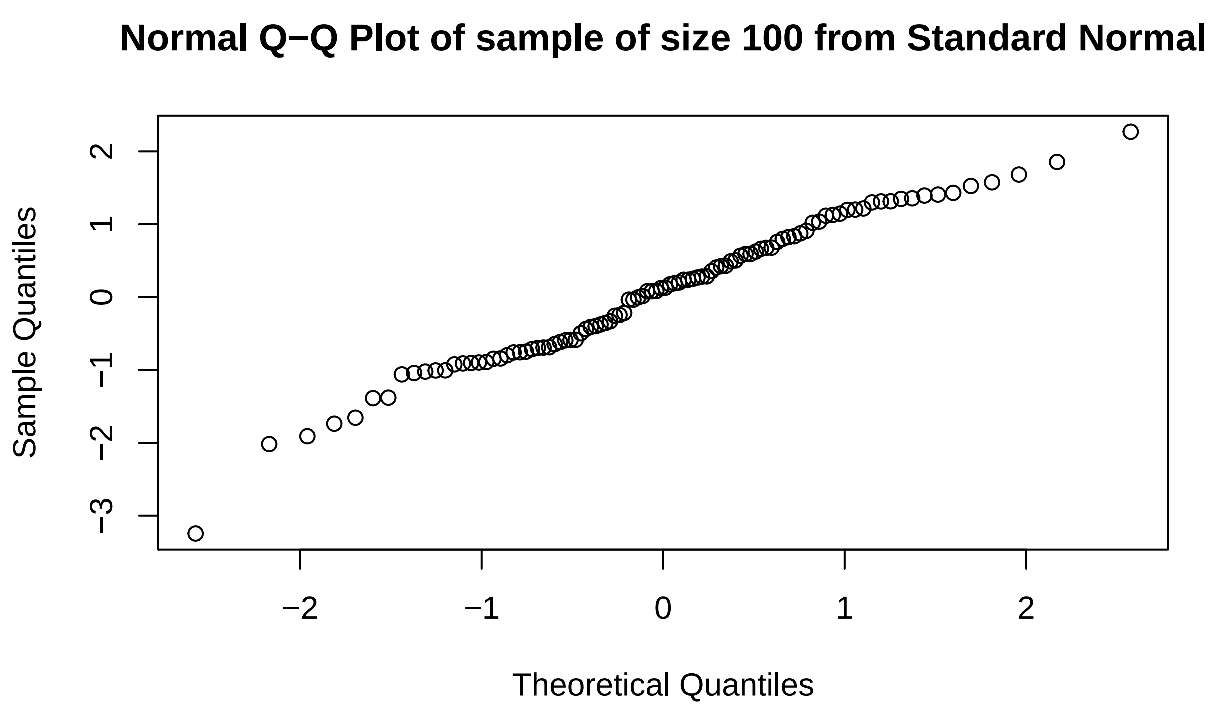

```r
qqnorm(sample20, main = "Normal Q-Q Plot of sample of size 20 from Standard Normal")
```



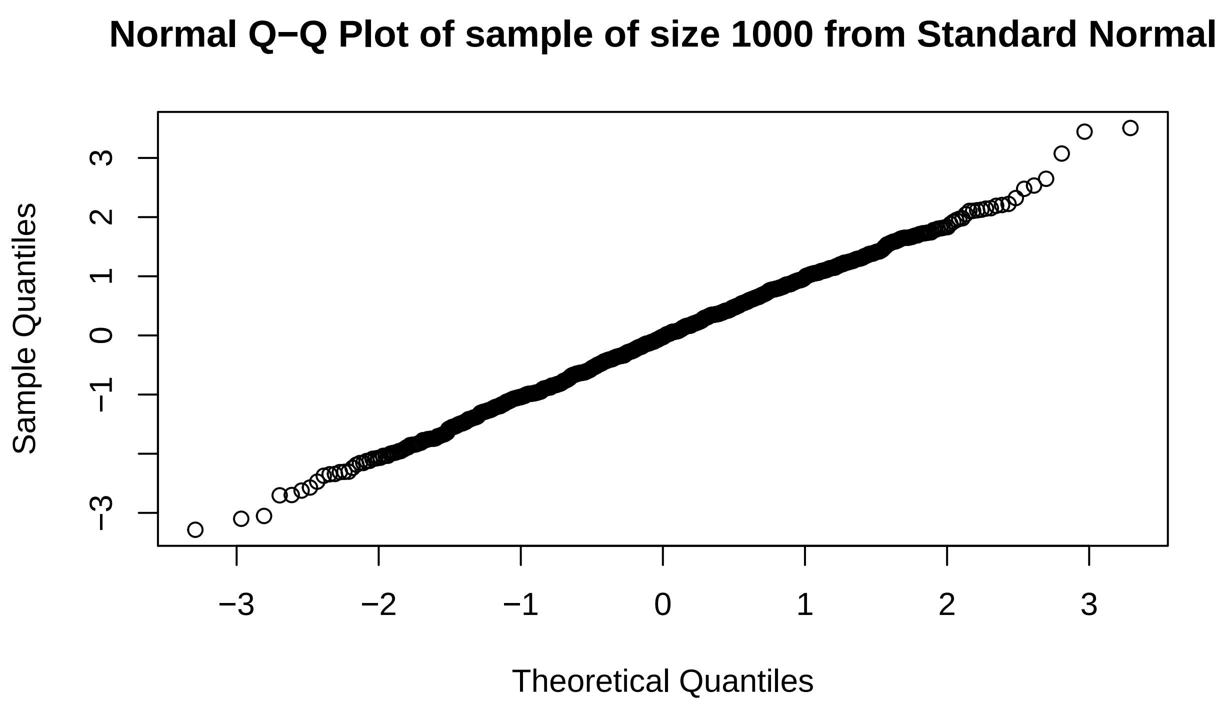
```r
qqnorm(sample40, main = "Normal Q-Q Plot of sample of size 40 from Standard Normal")
```




```r
qqnorm(sample100, main = "Normal Q-Q Plot of sample of size 100 from Standard Normal")
```



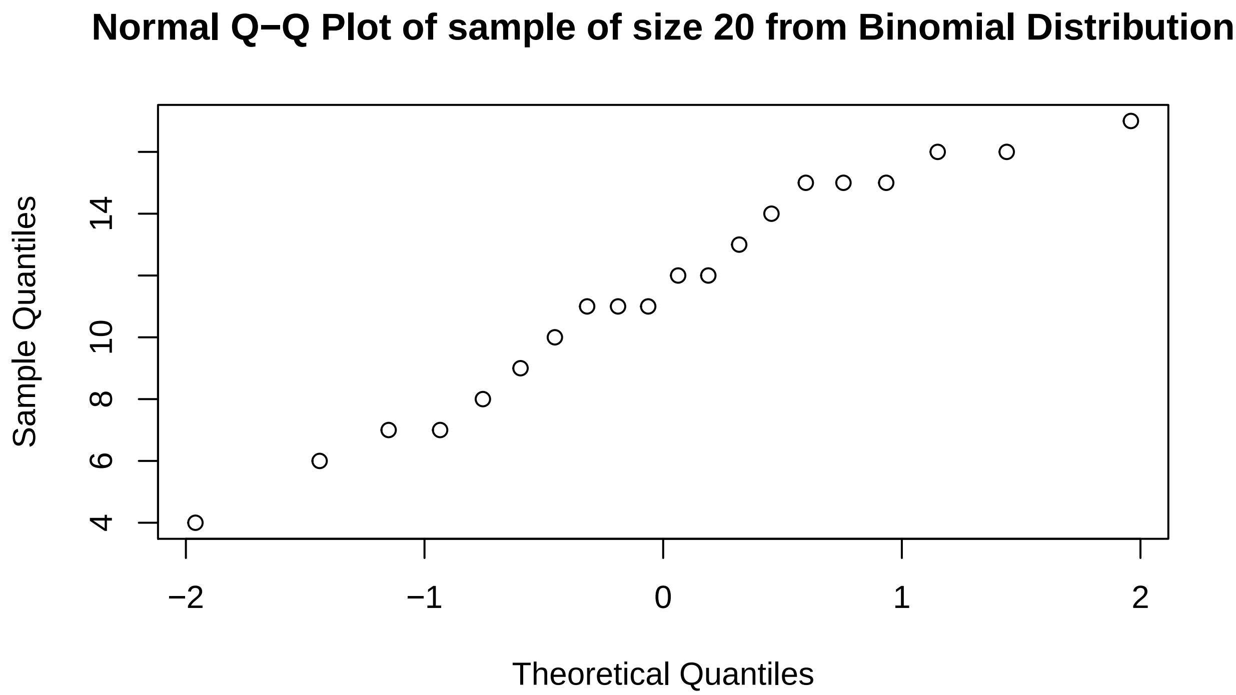
```r
qqnorm(sample1000, main = "Normal Q-Q Plot of sample of size 1000 from Standard Normal")
```



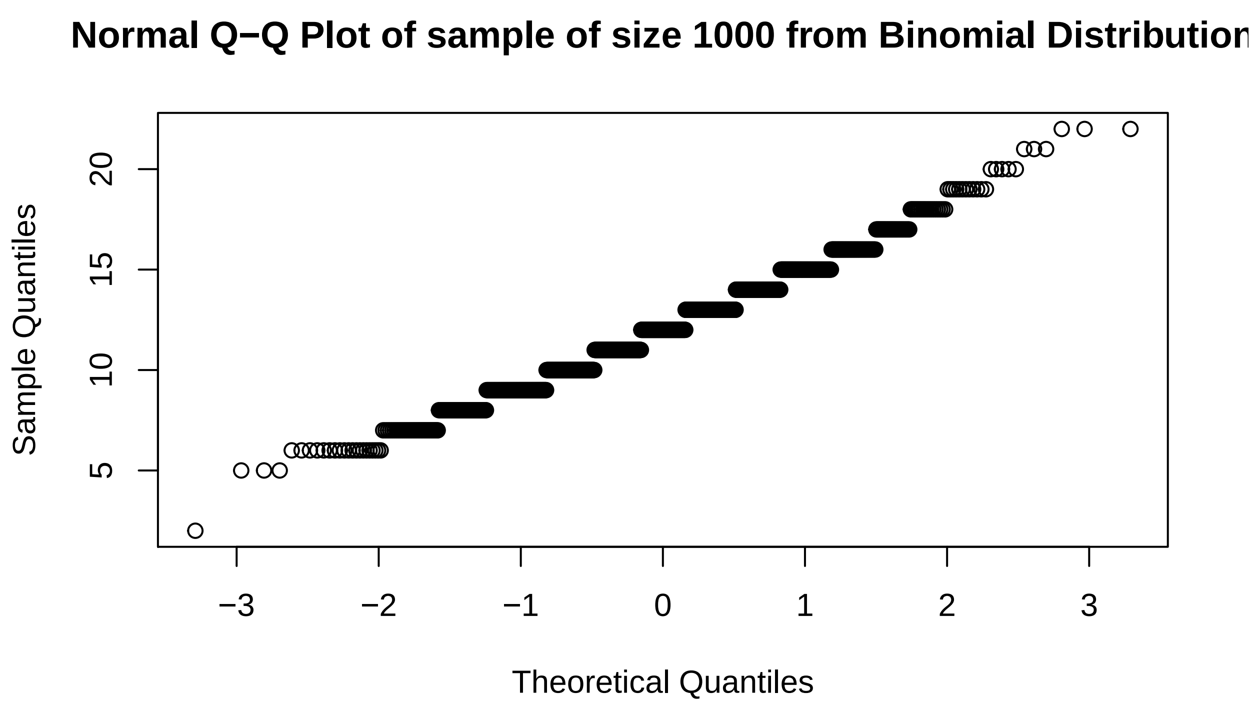

When sample size gets larger, QQ plot is closer to a straight line. Quantiles not in $[-1, 1]$ are the main deviations from being a straight line.

### Problem 13

```r
sampleB20 <- rbinom(20, size = 40, prob = 0.3)
sampleB1000 <- rbinom(1000, size = 40, prob = 0.3)
qqnorm(sampleB20, main = "Normal Q-Q Plot of sample of size 20 from Binomial Distribution")
```




```r
qqnorm(sampleB1000, main = "Normal Q-Q Plot of sample of size 1000 from Binomial Distribution")
```




Different from a straight line, this QQplot is a staircase plot because binomial distribution is discrete.

## Problem 14

(a)

```
cat("The probability that a standard normally distributed variable is larger than 3 is",
    pnorm(3, lower.tail = FALSE))
```

```
## The probability that a standard normally distributed variable is larger than 3 is 0.001349898
```

(b)

```
cat("The probability that a normally distributed variable with mean 35 and standard
    deviation 6 is larger than 42 is", pnorm(42, mean = 35, sd = 6, lower.tail = FALSE))
```

```
## The probability that a normally distributed variable with mean 35 and standard
##     deviation 6 is larger than 42 is 0.1216725
```

(c)

```
cat("The probability of getting 10 out of 10 successes in a binomial distribution with
    probability 0.8 is", dbinom(10, size = 10, prob = 0.8))
```

```
## The probability of getting 10 out of 10 successes in a binomial distribution with
##     probability 0.8 is 0.1073742
```

(d)

```
cat("The probability of X < 0.9 when X has the standard uniform distribution is", punif(0.9))
```

```
## The probability of X < 0.9 when X has the standard uniform distribution is 0.9
```

(e)

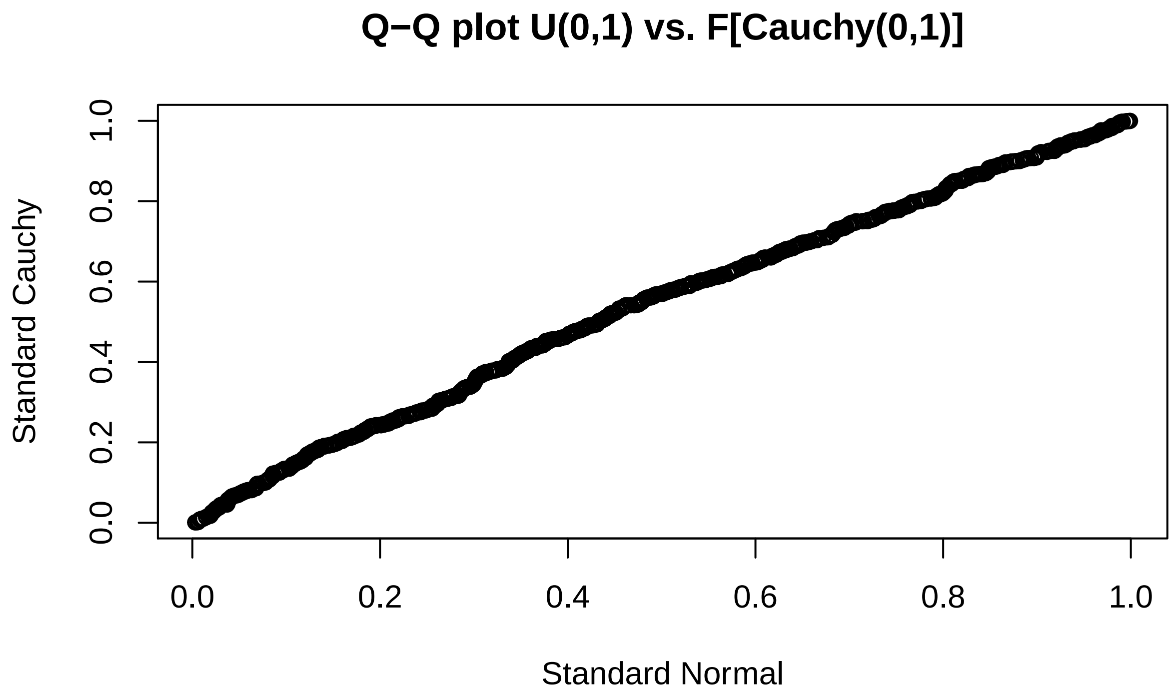
```
cat("The probability of X > 6.5 in a Chi-Squared distribution with 2 degrees of freedom is",
    pchisq(6.5, df = 2, lower.tail = FALSE))
```

```
## The probability of X > 6.5 in a Chi-Squared distribution with 2 degrees of freedom is 0.03877421
```

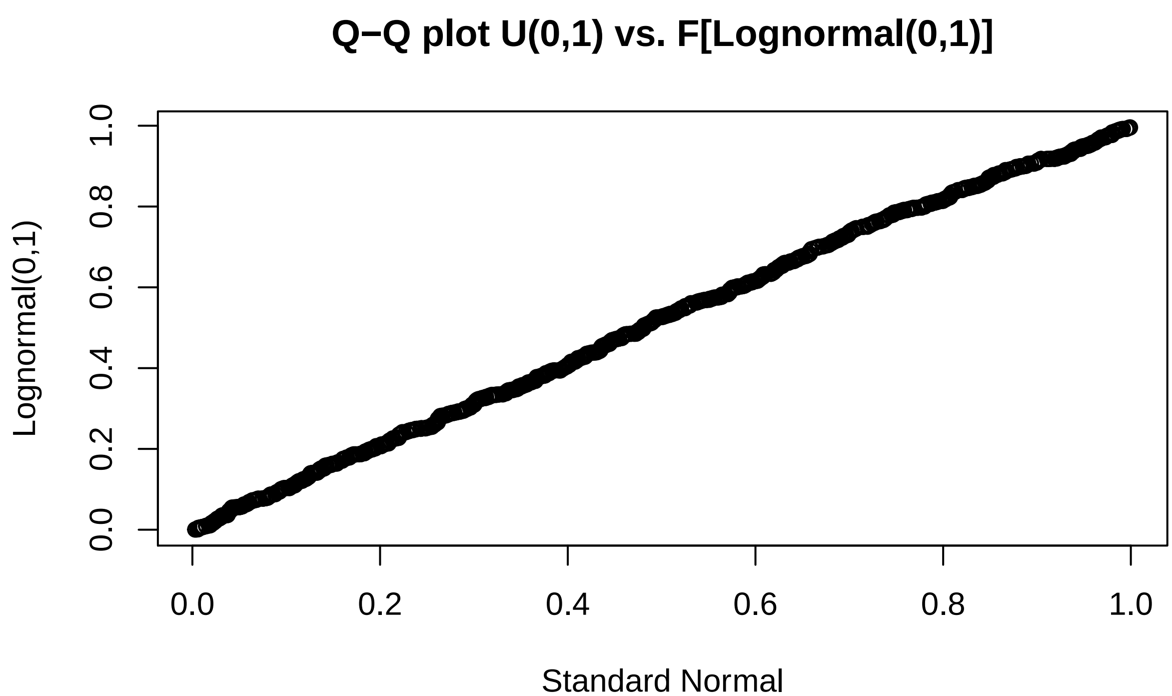
## Problem 15

3 choices of continuous distributions: Standard Cauchy, Log-normal($Lognormal(0, 1)$), Student's t (with 1 degree of freedom)

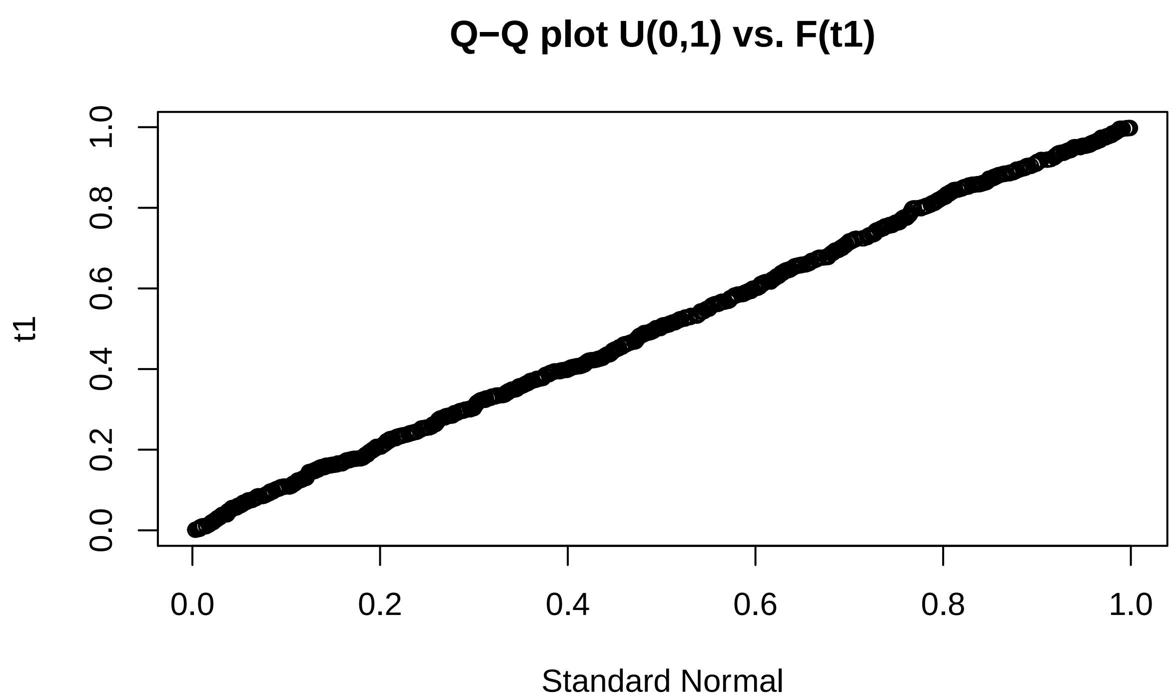
```
sampleC <- rcauchy(1000)
sampleL <- rlnorm(1000)
sampleT <- rt(1000, df = 1)
sampleU <- runif(1000)
UC <- pcauchy(sampleC)
UL <- plnorm(sampleL)
UT <- pt(sampleT, df = 1)
qqplot(sampleU, UC, main = "Q-Q plot U(0,1) vs. F[Cauchy(0,1)]", xlab = "Standard Normal",
      ylab = "Standard Cauchy")
```




```
qqplot(sampleU, UL, main = "Q-Q plot U(0,1) vs. F[Lognormal(0,1)]", xlab = "Standard Normal",
       ylab = "Lognormal(0,1)")
```



```r
qqplot(sampleU, UT, main = "Q-Q plot U(0,1) vs. F(t1)", xlab = "Standard Normal", ylab = "t1")
```




From QQplots above, CDF of random variable follows Standard Uniform Distribution. Assume that $X \sim exp(\lambda)$, prove that $U = F(x) \sim U(0,1)$, where $F$ is cumulative distribution function of $X$.

$$F(X) = 1 - e^{-\lambda X}$$

$$\begin{aligned}
F_U(u) = P\{U \leq u\} &= P\{F(X) \leq u\} \\
&= P\left\{1 - e^{-\lambda X} \leq u\right\} \\
&= P\left\{X \leq -\frac{log(1-u)}{\lambda}\right\} \\
&= 1 - e^{-\lambda[-\frac{log(1-u)}{\lambda}]} \\
&= u
\end{aligned}$$

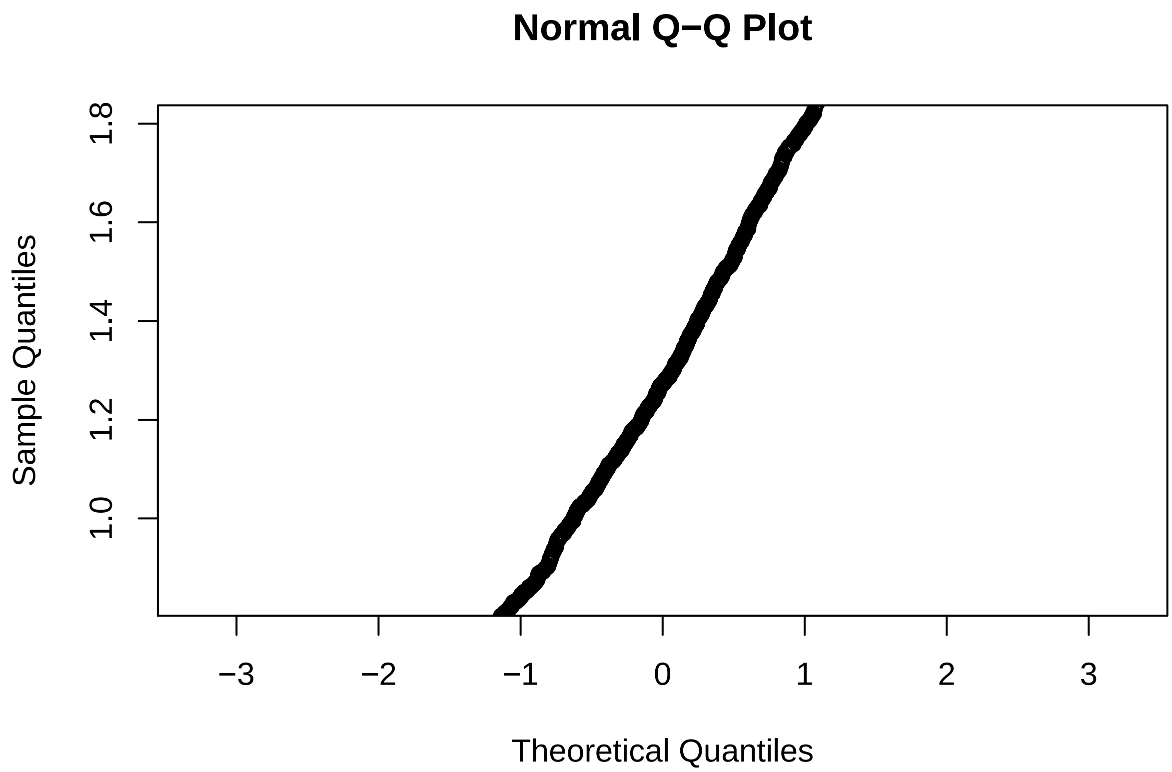
$$f_U(u) = F'_U(u) = 1$$

Therefore, $U$ has a standard uniform distribution.

## Problem 16

Take $\lambda = 1$ and $\alpha = \frac{1}{2}$

```r
y <- rexp(1000) + rexp(1000)
Y <- y^(1/2)
qqnorm(Y, ylim = c(mean(Y) - sd(Y), mean(Y) + sd(Y)))
```


Normal Q−Q Plot
Sample Quantiles
1.0
1.2
1.4
1.6
1.8
−3
−2
−1
0
1
2
3
Theoretical Quantiles