17. (2 points) Let X be a random variable with a beta(.5, .8) distribution. Use a suitable simulation to approximate $\mathcal{E}(X)$, $s(X)$, $\mathcal{E}(X^{-1/3})$.

18. (2 points) Let X be a random variable with a beta(.5, .8) distribution. Use a suitable simulation to approximate $\mathcal{E}(\sqrt{X})$ with an error of less than 10^{-3} and explain why you think you have achieved this accuracy. *1 point will be subtracted if you use more than 100 times as many simulations as is necessary .*

19. (2 points) Suppose X and Y are independent random variables that both have a uniform $U(0,1)$ distribution. Define the new random variable

$$Z = X | Y \leq \sin^2(2\pi X)\,.$$

Use **R** to generate a random sample of size at least 10000 for Z, make a histogram, and describe what you see.

20. (4 points) A breathalyzer that is used by the police to detect drunk drivers correctly identifies a drunk driver with probability 0.99 and falsely identifies a sober driver as drunk with probability 0.02. Then answer the following questions:
a) What is the sensitivity of this breathalyzer? What is its specificity? *Look up the terms "specificity" and "sensitivity".*
b) About one in 500 drivers are drunk. Somebody gets pulled over and fails the breathalyzer test. What is the probability that she is drunk?

21. (5 points) Consider a binomial distribution, $X \sim Binom(n = 50, p = 0.2)$

(1) Find $\Pr(X < 20)$ and $\Pr(X > 10|X < 20)$ with an exact computation.
(2) Consider a binomial distribution $X \sim Binom(n = 500, p = 0.1)$.
(3) Find $\Pr(X < 60), \Pr(X < 60|X > 30), \Pr(X > 30|X < 60)$ with a simulation.

22. (5 points) The Cauchy distribution has the property that the mean $\frac{1}{n}\sum_{i=1}^{n} X_i$ of n independent copies $X_1, \ldots, X_n$ has the same distribution as each individual X_i. Demonstrate this as best as you can with simulations for different values of n, using suitable plots. Include one such plot in your solution and explain it.

23. (5 points) Given an exponentially distributed random variable X with intensity λ and some number $A > 0$, defined the new random variable

$$Y = (X|X > A) - A\,.$$

Intuitively, think of X as the unknown time when a bomb goes off after you push a button. You push the button and watch a timer. If the bomb hasn't gone off by the time A, you reset the timer to 0 and keep it running. Then Y is displayed on the timer when the bomb goes off.
Show that Y has the same cumulative distribution function as X. If you're unsure how to do this, I suggest you use the following steps and explain them.
a) Write down the distribution function of X.
b) Find a formula for $\mathcal{P}(X > A)$.
c) Given z, find a formula for $\mathcal{P}(X > z$ and $X > A)$.
d) Find a formula for $\mathcal{P}(X > z|X > A)$.
e) Explain why $\mathcal{P}(Y > y) = \mathcal{P}(X > y + A|X > A)$.
f) Find the formula for $\mathcal{P}(Y \leq y)$.