HW4

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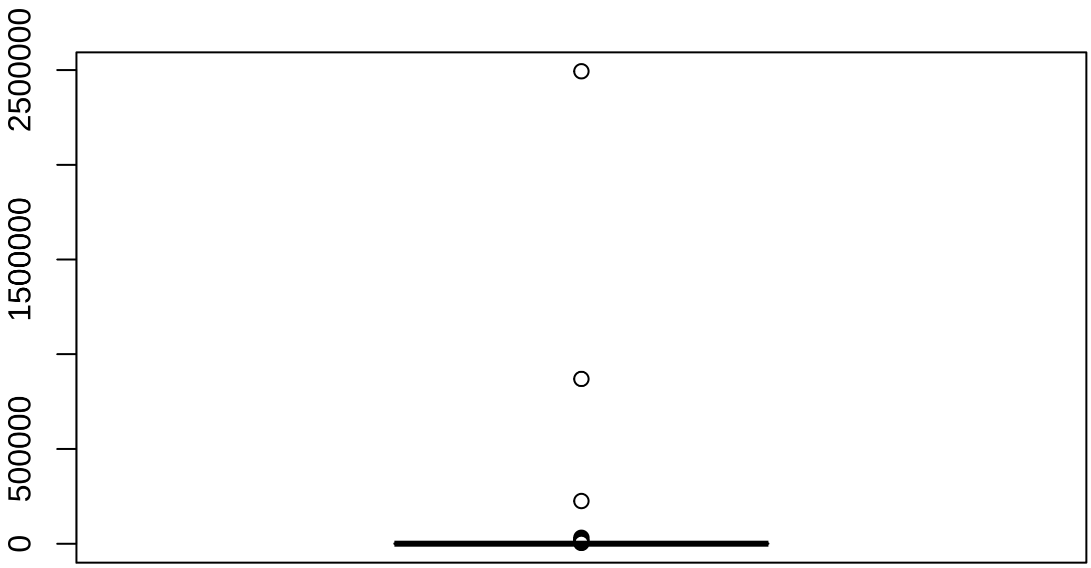
Problem 25

```
v <- c()
for (i in c(1:100)){
  x <- 0
  count <- 0
  while (TRUE){
    if (runif(1) > 0.5){
      x <- x + 1
    } else{
      x <- x - 1
    }
    count <- count + 1
    if (x == 15){
      break
    }
  }
  v = append(v, count)
}
boxplot(v)
```




```r
summary(v)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
##      45.0     155.5     350.0   37933.1    1770.0 2493307.0
```

Problem 26

St. Petersburg strategy betting on a block of 6 numbers.
- Bet 1 dollar
- After a loss, six times your last bet
- After a win, bet 1 dollar again
- Stop when you have lost all your money

Problem 27

Let X be the amount of dollars won or lost after a single game.
$Pr(X = 3) = \frac{10}{38} = \frac{5}{19}$
$Pr(X = 1) = \frac{8}{38} = \frac{4}{19}$
$Pr(X = -1) = \frac{8}{38} = \frac{4}{19}$
$Pr(X = -3) = \frac{12}{38} = \frac{6}{19}$
$E[X] = 3 \times \frac{5}{19} + \frac{4}{19} - \frac{4}{19} - 3 \times \frac{6}{19} = -\frac{3}{19}$

Problem 28

False. Assume j location is one of the corner tiles and k is the adjacent tile along the longer side. The probability that the frog will jump from j to k is $\frac{1}{2}$ but the probability from k to j is $\frac{1}{3}$. Therefore, this random walk is not symmetric.

Problem 29

X follows discrete uniform distribution $X \sim U(1, n)$ and Y follows binomial distribution $Y \sim Binom(k, p)$.

$$Pr(X = k) = \frac{1}{n}$$
$$Pr(Y = y|X = k) = \binom{k}{y} p^y (1-p)^{1-y}$$
$$\begin{aligned} Pr(X = k, Y = y) &= Pr(Y = y|X = k)Pr(X = k) \\ &= \frac{1}{n}\binom{k}{y} p^y (1-p)^{1-y} \end{aligned}$$

Problem 30

a)

```
m <- c(1/2, 1/4, 0, 1/4, 0, 0, 0, 0, 0,
       1/6, 1/2, 1/6, 0, 1/6, 0, 0, 0, 0,
       0, 1/4, 1/2, 0, 0, 1/4, 0, 0, 0,
       1/6, 0, 0, 1/2, 1/6, 0, 1/6, 0, 0,
       0, 1/6, 0, 1/6, 1/2, 1/6, 0, 0, 0,
       0, 0, 1/4, 0, 1/4, 1/2, 0, 0, 0,
       0, 0, 0, 1/4, 0, 0, 1/2, 1/4, 0,
       0, 0, 0, 0, 0, 0, 1/4, 1/2, 1/4,
       0, 0, 0, 0, 0, 0, 0, 0, 1)
zog_transition_matrix <- matrix(m, ncol=9, nrow=9, byrow=T)
zog_matrix <- zog_transition_matrix %*% zog_transition_matrix %*% zog_transition_matrix
cat("P{ X3 = 3 | X0 = 1 } =", zog_matrix[1,3])
```

```
## P{ X3 = 3 | X0 = 1 } = 0.0625
```

$$X_0 = 1 \begin{cases} X_1 = 1(p = \frac{1}{2}) \begin{cases} X_2 = 1(p = \frac{1}{2}) \\ X_2 = 2(p = \frac{1}{4}) \implies X_3 = 3(p = \frac{1}{6}) \\ X_3 = 4(p = \frac{1}{4}) \end{cases} \\ X_1 = 2(p = \frac{1}{4}) \begin{cases} X_2 = 1(p = \frac{1}{6}) \\ X_2 = 2(p = \frac{1}{2}) \implies X_3 = 3(p = \frac{1}{6}) \\ X_2 = 3(p = \frac{1}{6}) \implies X_3 = 3(p = \frac{1}{2}) \\ X_2 = 5(p = \frac{1}{6}) \end{cases} \\ X_1 = 4(p = \frac{1}{4}) \end{cases}$$

$$Pr(X_3 = 3|x_0 = 1) = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{1}{2} \times \frac{1}{6} + \frac{1}{4} \times \frac{1}{6} \times \frac{1}{2} = \frac{1}{16} = 0.0625$$

b)

```r
m <- c(1/2, 1/4, 0, 1/4, 0, 0, 0, 0, 0,
       1/6, 1/2, 1/6, 0, 1/6, 0, 0, 0, 0,
       0, 1/4, 1/2, 0, 0, 1/4, 0, 0, 0,
       1/6, 0, 0, 1/2, 1/6, 0, 1/6, 0, 0,
       0, 1/6, 0, 1/6, 1/2, 1/6, 0, 0, 0,
       0, 0, 1/4, 0, 1/4, 1/2, 0, 0, 0,
       0, 0, 0, 1/4, 0, 0, 1/2, 1/4, 0,
       0, 0, 0, 0, 0, 0, 1/4, 1/2, 1/4,
       0, 0, 0, 0, 0, 0, 0, 0, 1)
zog_transition_matrix <- matrix(m, ncol=9, nrow=9, byrow=T)

a <- T
count <- 0
while (a){
  zog_last_matrix <- zog_transition_matrix
  zog_transition_matrix <- zog_transition_matrix %*% zog_transition_matrix
  count <- count + 1
  for (i in c(1:8)){
    a <- F
    if (zog_transition_matrix[i,9] < 0.5){
      a <- T
      break
    }
  }
}

b <- T
while (b){
  for (i in c(1:8)){
    if (zog_last_matrix[i,9] < 0.5){
      b <- F
    }
  }
  if (b == T){
    zog_last_matrix <- zog_transition_matrix
    zog_transition_matrix <- zog_transition_matrix %*% zog_transition_matrix
    count <- count + 1
  }
}

cat("The chance of the caveman's survival for more than", count,
    "steps is less 50% no matter where he starts.")
```

```
## The chance of the caveman's survival for more than 7 steps is less 50% no matter where he starts.
```

Problem 31

a) $N \sim Pois(\lambda)$, $Pr(N = n) = \frac{\lambda^n e^{-\lambda}}{n!}$

$X \sim Binom(n, p)$, $Pr(X = k|N = n) = \binom{n}{k} p^k (1-p)^{n-k}$

$Pr(X = k, N = n) = \frac{\lambda^n e^{-\lambda}}{n!} \binom{n}{k} p^k (1-p)^{n-k}$

b)

```r
simX <- function(lambda, p, k){
  n <- rpois(k, lambda = lambda)
  b <- rbinom(k, size = n, prob = p)
}
```

c) $\lambda = 44$, $p = 0.5$, $E[X] = \lambda p$

```r
mean(replicate(100, simX(44, 0.5, 100)))
```

```
## [1] 22.0413
```

Problem 32

a) $Pr(X < 12|X < 18) = \frac{Pr(X<12, X<18)}{Pr(X<18)} = \frac{Pr(X<12)}{Pr(X<18)}$

```r
pbinom(11, 80, 0.2)
```

```
## [1] 0.100598
```

```r
pbinom(17, 80, 0.2)
```

```
## [1] 0.6707507
```

```r
pbinom(11, 80, 0.2)/pbinom(17, 80, 0.2)
```

```
## [1] 0.1499783
```

$Pr(X < 12|X < 18) \approx 0.1499783$

$E[X|X < 18]$

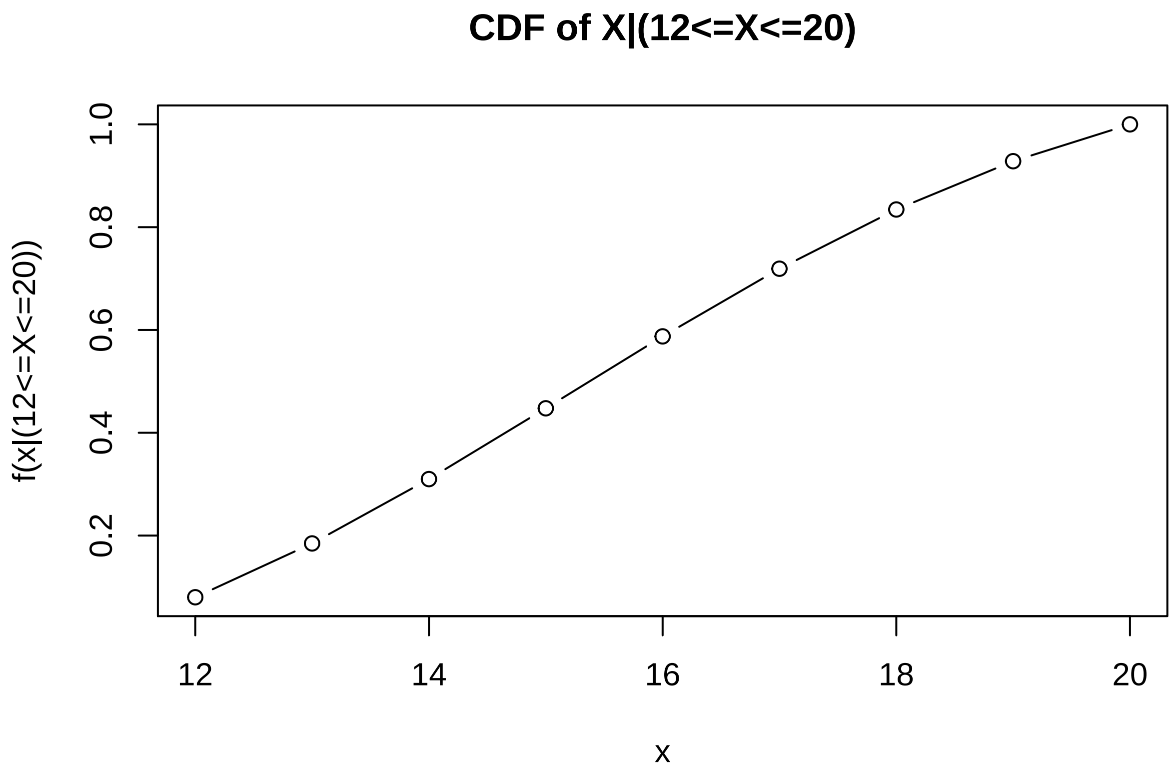
```r
E <- 0
for (i in c(0:17)){
  E <- E + i*dbinom(i, 80, 0.2)/pbinom(17, 80, 0.2)
}
print(E)
```

```
## [1] 14.0393
```

$E[X|X < 18] \approx 14.0393$

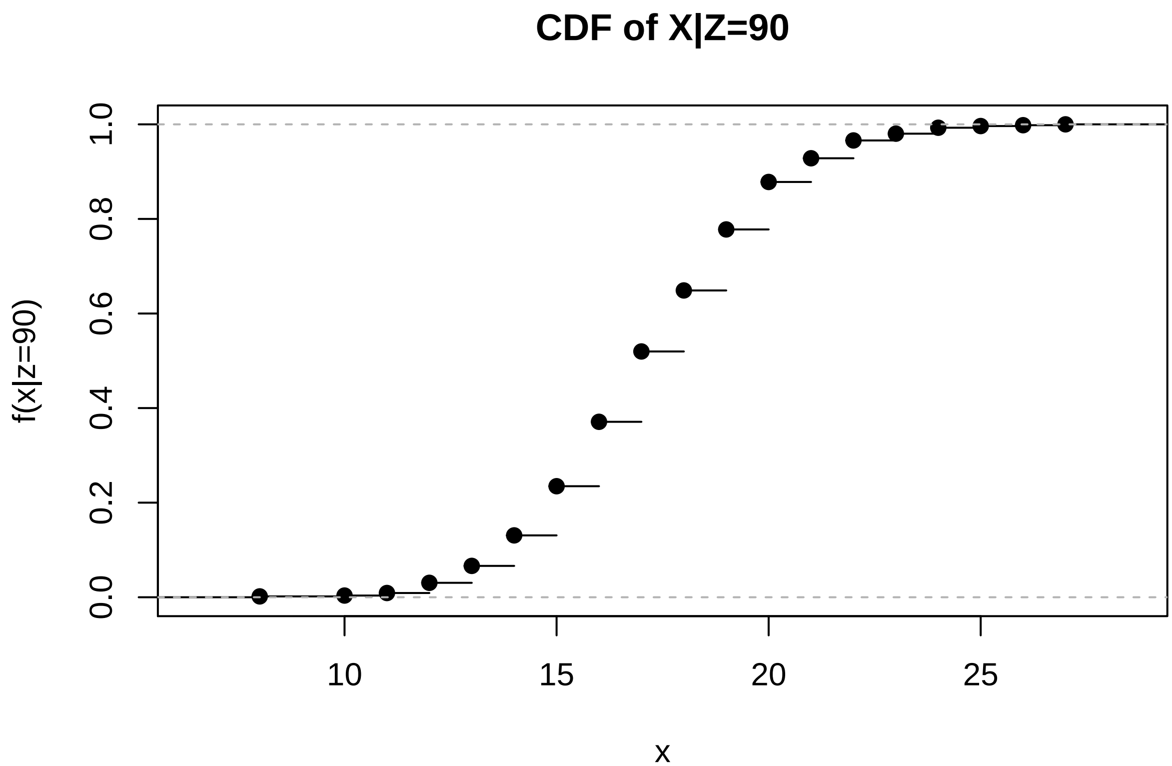
b)

```r
x = 12:20
denominator = pbinom(20, 80, 0.2) - pbinom(11, 80, 0.2)
y = sapply(x, function (x) (pbinom(x, 80, 0.2) - pbinom(11, 80, 0.2))/denominator)
plot(x, y, type = 'b', main = 'CDF of X|(12<=X<=20)', ylab = 'f(x|(12<=X<=20))')
```



c)

```
n = 1e4
xcz <- c()
x <- rbinom(n, 80, 0.2)
y <- rbinom(n, 100, 0.7)
for (i in c(1:n)){
  if (x[i] + y[i] == 90){
    xcz = c(xcz, x[i])
  }
}
plot(ecdf(xcz), main = 'CDF of X|Z=90', ylab = 'f(x|z=90)')
```



d)

```r
n = 1e4
x <- rbinom(n, 80, 0.2)
y <- rbinom(n, 100, 0.7)
z <- c()
k <- c(10, 15, 20)
for (kvalue in k){
  for (i in c(1:n)){
    if (x[i] == kvalue){
      z <- c(z, x[i]+y[i])
    }
  }
  cat("E[Z|X=", kvalue, "]=", mean(z), "\n")
}
```

```
## E[Z|X= 10 ]= 80.07051
## E[Z|X= 15 ]= 83.73634
## E[Z|X= 20 ]= 85.45374
```

e)

```r
n = 1e4
x <- rbinom(n, 80, 0.2)
y <- rbinom(n, 100, 0.7)
xczk <- c()
k <- c(80, 90, 100)
for (kvalue in k){
```

```r
  for (i in c(1:n)){
    if (x[i]+y[i] == kvalue){
      xczk <- c(xczk, x[i])
    }
  }
  cat("E[X|Z=", kvalue, "]=", mean(xczk), "\n")
}
```

```
## E[X|Z= 80 ]= 13.79843
## E[X|Z= 90 ]= 16.00859
## E[X|Z= 100 ]= 16.23893
```