MAP55640 Final Project

Li Yihai
liy35@tcd.ie
Mathematics Institute
High Performance Computing
Dublin, Ireland

ABSTRACT

KEYWORDS

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1 INTRODUCTION

Numerical methods of solving partial differential equations (PDE) have demonstrate far better performance than many other methods such as finite difference methods (FDM) [**<empty citation>**], finite element methods (FEM) [**<empty citation>**], Lattice Boltzmann Method (LBM) [**<empty citation>**] and Monte Carlo Method (MC) [**<empty citation>**]. In recent years, researchers in the field of deep learning have mainly focused on how to develop more powerful system architectures and learning methods such as convolution neural networks (CNNs) [**<empty citation>**], Transformers [**<empty citation>**] and Perceivers [**<empty citation>**] . In addition, more researchers have tried to develop more powerful models specifically for numerical simulations. Despite of the relentless progress, modeling and predicting the evolution of nonlinear multiscale systems which has inhomogeneous cascades-scales by using classical analytical or computational tools inevitably encounts severe challanges and comes with prohibitive cost and multiple sources of uncertainty.

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2 RELATED WORK

3 PROBLEM SETUPS

In this project, I chose to use various numerical approaches to approximate the solutions. It begins with Finite Difference Methods(FDM) and an other method employed by deep neural networks which leverage by their capability as universal function approximators [**DNN-HORNIK1989359**].

The Kormoglov PDEs are series of equations which describe the motions of Brownian Motions [**SolveKorPDE**]. In general, let $T \in (0, +\infty)$, $d \in \mathbb{N}$, let $\mu : \mathbb{R}^d \to \mathbb{R}^d$ and $\sigma : \mathbb{R}^d \to \mathbb{R}^{d\times d}$ be the Lipschitz continuous fucntions. Let $\varphi : \mathbb{R}^d \to \mathbb{R}$ be a function, and u be a function from Hilbert Space $[0, T] \times \mathbb{R}^d$ to $\mathbb{R}$

$$\begin{aligned} u : [0, T] \times \mathbb{R}^d &\longrightarrow \mathbb{R} \\ (t, x) &\longmapsto u \end{aligned}$$

with at most polynomially growing partial derivatives. The problem is that u satisfied a below system on $D = [0, T] \times [a, b]^d$ and $a, b \in \mathbb{R}^d$ with $a < b$,

$$\frac{\partial u}{\partial t} = \frac{1}{2}\mathrm{Trace}_{\mathbb{R}^d}\left[\sigma(x)\sigma^*(x)\left(\mathrm{Hess}_x u\right)\right] + \langle \mu, \nabla_x u\rangle_{\mathbb{R}^d} \tag{1}$$

$$u(0, x) = \varphi(x), \quad x \in [a, b]^d \tag{2}$$

$$\left.\left(\frac{\partial u}{\partial \vec{n}} + \lambda u\right)\right|_{x\in\Gamma} = g(t, x) \quad \forall t \in [0, T],\ x \in \mathbb{R}^d \tag{3}$$

The goal is to numerically appriximate the stablized state $u(T, x)$ of the system in the fureture time T, and in the hypercude space $[a, b] \in \mathbb{R}^d$ with various boundary conditions g.

Reprompt problem, this work aimed at solving this problem in a bigger picture which requires a more general form. In this project, I consider the parametrized and nonliner PDEs of the general form

$$\frac{\partial u}{\partial t} + \mathcal{N}\left[u; \lambda\right] = 0 \quad t \in [0.T], x \in D \tag{4}$$

where $\mathcal{N}[\cdot; \lambda]$ stands for a nonlinear operator parametrized by λ.

4 METHODOLOGY

5 EXPERIMENTS

6 CONCLUSION

7 ACKNOWLEDGEMENT