# MAP55672 NOTEs

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## I. Lecture 1

### *1.1 Householder Transformation*

In the $n$-dim vector space $\mathbb{C}^n$, vector $x$ has: selected a vector $v$, such that specific entities of transformed vector $x'$ equal to $0$. It workers as, selected vector $h$ gives equation

$$x - 2\langle x, v\rangle v = x - 2v(v^* x) = [x - 2vv^*]\, x$$

where Transformation matrix $[x - 2vv^*]$ denoted with $H$, and $2$ stands for the reflection.

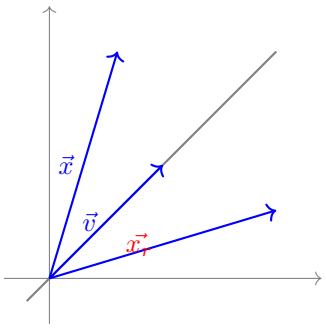


Fig 1. Demonstration of Householder Transformation

The $\vec{h}$ we discussed in lecture, is the orthogonal vector of $\vec{v}$ which follows

$$\vec{h} = \pm(\vec{x} - \vec{x_r})$$

and the sign is chosen for stability. Most importantly, the $H$ has such propositions 1.1.

$$H = H^* = H^{-1} \tag{1.1}$$

Thus, it gives us a shortcut for a sequential matrices series of $\{H_k\}_{k=1}^n$ which has

$$\prod_{k=n}^{1} H_k A = R$$

such that

$$A = \prod_{k=1}^{n} H_k^{-1} R = \overbrace{\prod_{k=1}^{n} H_k^*}^{Q} R$$