x variables

e	::=		expressions
	$\mid$	x	
	$\mid$	$\mathsf{0}$	
	$\mid$	$\mathsf{S}\ e$	
	$\mid$	$e_1\ e_2$	
	$\mid$	$e_1\ p\ e_2$	
	$\mid$	$\lambda x.e$	bind x in e
	$\mid$	$\mathsf{fix}\, x.e$	bind x in e
	$\mid$	$\mathsf{ifz}\ e\{\mathsf{0} \hookrightarrow e_0 \mid \mathsf{S}\ x \hookrightarrow e_1\}$	bind x in e_1
	$\mid$	$\mathsf{let}\ x = e\ \mathsf{in}\ e'$	bind x in e'
	$\mid$	$\mathsf{halt}\ e$	
	$\mid$	(e)	S

p	::=		primitives
	$\mid$	$+$	
	$\mid$	$-$	
	$\mid$	$\times$	
	$\mid$	$\div$	

Γ	::=		contexts
	$\mid$		
	$\mid$	$\Gamma, x\ \mathsf{ok}$	

$\boxed{e\ \mathsf{val}}$ values

$$\frac{}{\mathsf{0\ val}}\ \textsc{Val\_zero}$$

$$\frac{e\ \mathsf{val}}{\mathsf{S}\ e\ \mathsf{val}}\ \textsc{Val\_succ}$$

$\boxed{e_1\ p\ e_2 = e_3}$ arithmetics

$$\frac{e\ \mathsf{val}}{\mathsf{0}\ +\ e = e}\ \textsc{Arith\_plus0}$$

$$\frac{e_1\ +\ e_2 = e_3}{\mathsf{S}\ e_1\ +\ e_2 = \mathsf{S}\ e_3}\ \textsc{Arith\_plus1}$$

$$\frac{e\ \mathsf{val}}{e\ -\ \mathsf{0} = e}\ \textsc{Arith\_minus0}$$

$$\frac{e\ \mathsf{val}}{\mathsf{0}\ -\ e = \mathsf{0}}\ \textsc{Arith\_minus1}$$

$$\frac{e_1\ -\ e_2 = e_3}{\mathsf{S}\ e_1\ -\ \mathsf{S}\ e_2 = e_3}\ \textsc{Arith\_minus2}$$

$$\frac{e\ \mathsf{val}}{\mathsf{0}\ \times\ e = \mathsf{0}}\ \textsc{Arith\_mult0}$$

$$\frac{\begin{array}{c} e_1\ \times\ e_2 = e_3 \\ e_2\ +\ e_3 = e_4 \end{array}}{\mathsf{S}\ e_1\ \times\ e_2 = e_4}\ \textsc{Arith\_mult1}$$

$$\frac{e\ \mathsf{val}}{\mathsf{0}\ \div\ \mathsf{S}\ e = \mathsf{0}}\ \textsc{Arith\_div0}$$

$$\frac{e_1 \;-\; e_2 = e_3 \qquad e_3 \;\div\; e_2 = e_4}{e_1 \;\div\; e_2 = \mathsf{S}\; e_4}\;\textsc{Arith\_div1}$$

$\boxed{\Gamma \vdash_\Lambda e\,\mathsf{ok}}$ well-formedness

$$\frac{x \in \Gamma}{\Gamma \vdash_\Lambda x\,\mathsf{ok}}\;\textsc{L\_exp\_var}$$

$$\frac{}{\Gamma \vdash_\Lambda \mathsf{0}\,\mathsf{ok}}\;\textsc{L\_exp\_zero}$$

$$\frac{\Gamma \vdash_\Lambda e\,\mathsf{ok}}{\Gamma \vdash_\Lambda \mathsf{S}\; e\,\mathsf{ok}}\;\textsc{L\_exp\_succ}$$

$$\frac{\Gamma \vdash_\Lambda e_1\,\mathsf{ok} \qquad \Gamma \vdash_\Lambda e_2\,\mathsf{ok}}{\Gamma \vdash_\Lambda e_1\; e_2\,\mathsf{ok}}\;\textsc{L\_exp\_app}$$

$$\frac{\Gamma \vdash_\Lambda e_1\,\mathsf{ok} \qquad \Gamma \vdash_\Lambda e_2\,\mathsf{ok}}{\Gamma \vdash_\Lambda e_1\; p\; e_2\,\mathsf{ok}}\;\textsc{L\_exp\_prim}$$

$$\frac{\Gamma, x\,\mathsf{ok} \vdash_\Lambda e\,\mathsf{ok}}{\Gamma \vdash_\Lambda \lambda x.e\,\mathsf{ok}}\;\textsc{L\_exp\_lam}$$

$$\frac{\Gamma, x\,\mathsf{ok} \vdash_\Lambda e\,\mathsf{ok}}{\Gamma \vdash_\Lambda \mathsf{fix}\, x.e\,\mathsf{ok}}\;\textsc{L\_exp\_fix}$$

$$\frac{\Gamma \vdash_\Lambda e\,\mathsf{ok} \qquad \Gamma \vdash_\Lambda e_0\,\mathsf{ok} \qquad \Gamma, x\,\mathsf{ok} \vdash_\Lambda e_1\,\mathsf{ok}}{\Gamma \vdash_\Lambda \mathsf{ifz}\; e\{\mathsf{0} \hookrightarrow e_0 | \mathsf{S}\; x \hookrightarrow e_1\}\,\mathsf{ok}}\;\textsc{L\_exp\_ifz}$$

$\boxed{\Gamma \vdash_\Lambda e_1 \equiv e_2}$ equivalence

$$\frac{\Gamma \vdash_\Lambda e\,\mathsf{ok}}{\Gamma \vdash_\Lambda e \equiv e}\;\textsc{L\_eq\_refl}$$

$$\frac{\Gamma \vdash_\Lambda e \equiv e'}{\Gamma \vdash_\Lambda e' \equiv e}\;\textsc{L\_eq\_comm}$$

$$\frac{\Gamma \vdash_\Lambda e \equiv e' \qquad \Gamma \vdash_\Lambda e' \equiv e''}{\Gamma \vdash_\Lambda e \equiv e''}\;\textsc{L\_eq\_trans}$$

$$\frac{\Gamma \vdash_\Lambda e_1 \equiv e_1' \qquad \Gamma \vdash_\Lambda e_2 \equiv e_2'}{\Gamma \vdash_\Lambda e_1\; e_2 \equiv e_1'\; e_2'}\;\textsc{L\_eq\_app'}$$

$$\frac{\Gamma \vdash_\Lambda e_1 \equiv e_1' \qquad \Gamma \vdash_\Lambda e_2 \equiv e_2'}{\Gamma \vdash_\Lambda e_1\; p\; e_2 \equiv e_1'\; p\; e_2'}\;\textsc{L\_eq\_prim'}$$

$$\frac{\Gamma, x\,\mathsf{ok} \vdash_\Lambda e \equiv e'}{\Gamma \vdash_\Lambda \lambda x.e \equiv \lambda x.e'}\;\textsc{L\_eq\_lam'}$$

$$\frac{\Gamma, x\,\mathsf{ok} \vdash_\Lambda e \equiv e'}{\Gamma \vdash_\Lambda \mathsf{fix}\, x.e \equiv \mathsf{fix}\, x.e'}\;\textsc{L\_eq\_fix'}$$

$$\frac{\begin{array}{c}\Gamma \vdash_\Lambda e \equiv e' \\ \Gamma \vdash_\Lambda e_0 \equiv e_0' \\ \Gamma, x\,\mathsf{ok} \vdash_\Lambda e_1 \equiv e_1'\end{array}}{\Gamma \vdash_\Lambda \mathsf{ifz}\ e\{0 \hookrightarrow e_0 | \mathsf{S}\ x \hookrightarrow e_1\} \equiv \mathsf{ifz}\ e'\{0 \hookrightarrow e_0' | \mathsf{S}\ x \hookrightarrow e_1'\}}\ \text{L\_EQ\_IFZ'}$$

$$\frac{\begin{array}{c}\Gamma, x\,\mathsf{ok} \vdash_\Lambda e_2\,\mathsf{ok} \\ \Gamma \vdash_\Lambda e_1\,\mathsf{ok}\end{array}}{\Gamma \vdash_\Lambda (\lambda x.e_2)\ e_1 \equiv [e_1/x]e_2}\ \text{L\_EQ\_APP}$$

$$\frac{e_1\ p\ e_2 = e_3}{\Gamma \vdash_\Lambda e_1\ p\ e_2 \equiv e_3}\ \text{L\_EQ\_PRIM}$$

$$\frac{\Gamma, x\,\mathsf{ok} \vdash_\Lambda e\,\mathsf{ok}}{\Gamma \vdash_\Lambda \mathsf{fix}\,x.e \equiv [\mathsf{fix}\,x.e/x]e}\ \text{L\_EQ\_FIX}$$

$$\frac{\Gamma \vdash_\Lambda e_0\,\mathsf{ok}}{\Gamma \vdash_\Lambda \mathsf{ifz}\ 0\{0 \hookrightarrow e_0 | \mathsf{S}\ x \hookrightarrow e_1\} \equiv e_0}\ \text{L\_EQ\_IFZ0}$$

$$\frac{\begin{array}{c}\Gamma \vdash_\Lambda e\,\mathsf{ok} \\ \Gamma, x\,\mathsf{ok} \vdash_\Lambda e_1\,\mathsf{ok}\end{array}}{\Gamma \vdash_\Lambda \mathsf{ifz}\ \mathsf{S}\,e\{0 \hookrightarrow e_0 | \mathsf{S}\ x \hookrightarrow e_1\} \equiv [e/x]e_1}\ \text{L\_EQ\_IFZ1}$$

$\boxed{\Gamma \vdash_\mathrm{K} e\,\mathsf{ok}}$ well-formedness

$$\frac{x \in \Gamma}{\Gamma \vdash_\mathrm{K} x\,\mathsf{ok}}\ \text{K\_EXP\_VAR}$$

$$\frac{}{\Gamma \vdash_\mathrm{K} 0\,\mathsf{ok}}\ \text{K\_EXP\_ZERO}$$

$$\frac{\Gamma \vdash_\mathrm{K} e\,\mathsf{ok}}{\Gamma \vdash_\mathrm{K} \mathsf{S}\,e\,\mathsf{ok}}\ \text{K\_EXP\_SUCC}$$

$$\frac{\begin{array}{c}\Gamma \vdash_\mathrm{K} e_1\,\mathsf{ok} \\ \Gamma \vdash_\mathrm{K} e_2\,\mathsf{ok}\end{array}}{\Gamma \vdash_\mathrm{K} e_1\ e_2\,\mathsf{ok}}\ \text{K\_EXP\_APP}$$

$$\frac{\begin{array}{c}\Gamma \vdash_\mathrm{K} e_1\,\mathsf{ok} \\ \Gamma \vdash_\mathrm{K} e_2\,\mathsf{ok}\end{array}}{\Gamma \vdash_\mathrm{K} e_1\ p\ e_2\,\mathsf{ok}}\ \text{K\_EXP\_PRIM}$$

$$\frac{\Gamma, x\,\mathsf{ok} \vdash_\mathrm{K} e\,\mathsf{ok}}{\Gamma \vdash_\mathrm{K} \lambda x.e\,\mathsf{ok}}\ \text{K\_EXP\_LAM}$$

$$\frac{\Gamma, x\,\mathsf{ok} \vdash_\mathrm{K} e\,\mathsf{ok}}{\Gamma \vdash_\mathrm{K} \mathsf{fix}\,x.e\,\mathsf{ok}}\ \text{K\_EXP\_FIX}$$

$$\frac{\begin{array}{c}\Gamma \vdash_\mathrm{K} e\,\mathsf{ok} \\ \Gamma \vdash_\mathrm{K} e_0\,\mathsf{ok} \\ \Gamma, x\,\mathsf{ok} \vdash_\mathrm{K} e_1\,\mathsf{ok}\end{array}}{\Gamma \vdash_\mathrm{K} \mathsf{ifz}\ e\{0 \hookrightarrow e_0 | \mathsf{S}\ x \hookrightarrow e_1\}\,\mathsf{ok}}\ \text{K\_EXP\_IFZ}$$

$$\frac{\begin{array}{c}\Gamma \vdash_\mathrm{K} e\,\mathsf{ok} \\ \Gamma, x\,\mathsf{ok} \vdash_\mathrm{K} e'\,\mathsf{ok}\end{array}}{\Gamma \vdash_\mathrm{K} \mathsf{let}\ x = e\ \mathsf{in}\ e'\,\mathsf{ok}}\ \text{K\_EXP\_LET}$$

$$\frac{\Gamma \vdash_\mathrm{K} e\,\mathsf{ok}}{\Gamma \vdash_\mathrm{K} \mathsf{halt}\,e\,\mathsf{ok}}\ \text{K\_EXP\_HALT}$$

$\boxed{\Gamma \vdash_{\mathrm{K}} e_1 \mapsto e_2}$ reduction

$$\frac{\Gamma, x\ \mathsf{ok} \vdash_{\mathrm{K}} e_2\ \mathsf{ok} \quad \Gamma \vdash_{\mathrm{K}} e_1\ \mathsf{ok}}{\Gamma \vdash_{\mathrm{K}} (\lambda x.e_2)\ e_1 \mapsto [e_1/x]e_2}\ \text{K\_STEP\_APP}$$

$$\frac{e_1\ p\ e_2 = e_3}{\Gamma \vdash_{\mathrm{K}} e_1\ p\ e_2 \mapsto e_3}\ \text{K\_STEP\_PRIM}$$

$$\frac{\Gamma, x\ \mathsf{ok} \vdash_{\mathrm{K}} e\ \mathsf{ok}}{\Gamma \vdash_{\mathrm{K}} \mathsf{fix}\, x.e \mapsto [\mathsf{fix}\, x.e/x]e}\ \text{K\_STEP\_FIX}$$

$$\frac{\Gamma \vdash_{\mathrm{K}} e_0\ \mathsf{ok}}{\Gamma \vdash_{\mathrm{K}} \mathsf{ifz}\ 0\{0 \hookrightarrow e_0 | \mathsf{S}\ x \hookrightarrow e_1\} \mapsto e_0}\ \text{K\_STEP\_IFZ0}$$

$$\frac{\Gamma \vdash_{\mathrm{K}} e\ \mathsf{ok} \quad \Gamma, x\ \mathsf{ok} \vdash_{\mathrm{K}} e_1\ \mathsf{ok}}{\Gamma \vdash_{\mathrm{K}} \mathsf{ifz}\ \mathsf{S}\, e\{0 \hookrightarrow e_0 | \mathsf{S}\ x \hookrightarrow e_1\} \mapsto [e/x]e_1}\ \text{K\_STEP\_IFZ1}$$

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Definition rules:         47 good    0 bad
Definition rule clauses: 111 good    0 bad
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