Let ground truth causal graph be $G^{*}=(V^{*}, E^{*})$, the global causal graph of round t is $G^{t}=(V^{t}, E^{t})$, the local causal graph is $G_{k}=\left(V^{k}, E^{k}\right)$, and the number of star graph is M.

$$\text{SHD } =\left|G^{*} \oplus G^{t}\right|$$

Assumption1 (Local Existence Skeleton Faithfulness): For any $e_{ij} \in E^{*}$ and $e_{ij}^{\prime} \notin E^{*}$, the existence probability satisfy:

$$\begin{aligned} & \mathrm{P}\left(e_{ij} \in E^{k}\right) \geq 1-\delta_{1} \\ & \mathrm{P}\left(e_{ij}^{\prime} \notin E^{k}\right) \geq 1-\delta_{2} \end{aligned}$$

Thus, for the global existence probability:

$$\begin{aligned} & \mathrm{P}\left(e_{ij} \in E^{t}\right) \geq \sum_{k=1}^{K} \frac{\left|D_{k}\right|}{|D|}\left(1-\delta_{1}\right) \\ & \mathrm{P}\left(e_{ij}^{\prime} \notin E^{t}\right) \geq \sum_{k=1}^{K} \frac{\left|D_{k}\right|}{|D|}\left(1-\delta_{2}\right) \end{aligned}$$

Assumption2 (Star graph Faithfulness): Let the direction judgment of e_{ij} by E^{k} be $\mathrm{d}_{\mathrm{k}}\left(e_{ij}\right)$, the direction judgment of e_{ij} by E^{t} be $\mathrm{d}_{\mathrm{t}}\left(e_{ij}\right)$, and the true direction is $\mathrm{d}_{*}\left(e_{ij}\right))$:

$$\mathrm{P}\left(\mathrm{~d}_{\mathrm{k}}\left(e_{ij}\right)=\mathrm{d}_{*}\left(e_{ij}\right)\right) \geq 1-\delta_{3}$$

Such that, for G_{t}

$$\mathrm{P}\left(\mathrm{~d}_{\mathrm{t}}\left(e_{ij}\right)=\mathrm{d}_{*}\left(e_{ij}\right)\right) \geq \sum_{j=1}^{M} \frac{\operatorname{deg}\left(\mathrm{v}_{\mathrm{j}}\right) \times\left(1-\delta_{3}\right)}{\sum_{\mathrm{h}=1}^{\mathrm{M}} \operatorname{deg}\left(\mathrm{v}_{\mathrm{h}}\right)}$$

The total error satisfy:

$$\begin{aligned} & \left(2-\sum_{\mathrm{k}=1}^{\mathrm{K}} \frac{\left|\mathrm{D}_{\mathrm{k}}\right|}{|\mathrm{D}|}\left(1-\delta_{1}\right)-\sum_{\mathrm{k}=1}^{\mathrm{K}} \frac{\left|\mathrm{D}_{\mathrm{k}}\right|}{|\mathrm{D}|}\left(1-\delta_{2}\right)\right)+\left(1-\sum_{\mathrm{j}=1}^{\mathrm{M}} \frac{\operatorname{deg}\left(\mathrm{v}_{\mathrm{j}}\right) \times\left(1-\delta_{3}\right)}{\sum_{\mathrm{h}=1}^{\mathrm{M}} \operatorname{deg}\left(\mathrm{v}_{\mathrm{h}}\right)}\right) \\ & =\sum_{\mathrm{k}=1}^{\mathrm{K}}\left(\delta_{1}+\delta_{2}\right)+\left(1-\sum_{\mathrm{j}=1}^{\mathrm{M}} \frac{\operatorname{deg}\left(\mathrm{v}_{\mathrm{j}}\right) \times\left(1-\delta_{3}\right)}{\sum_{\mathrm{h}=1}^{\mathrm{d}} \operatorname{deg}\left(\mathrm{v}_{\mathrm{h}}\right)}\right) \end{aligned}$$

As δ_{1}, δ_{2} and δ_{3} approach 0 , the global error converges to 0 , demonstrating the theoretical soundness of our approach.