组合数学：

$$\sum_{i=0}^{m} C_n^i * C_m^i = C_{n+m}^m (m <= n)$$

$$\sum_{i=0}^{k} C_n^i * C_m^{k-i} = C_{n+m}^k$$

$$\sum_{i=1}^{n} C_n^i * C_m^{i-1} = C_{2n}^{n-1}$$

$$\sum_{k=1}^{n} (2k-1)^2 = \frac{n(4n^2-1)}{3}$$

$$\sum_{k=1}^{n} (2k-1)^3 = n^2(2n^2-1)$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{k=1}^{n} k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{k=1}^{n} k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$$

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

$$\sum_{k=1}^{n} k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

$$\sum_{k=1}^{n} k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$$

数论公式：

$$\lim_{n\to+\infty} \frac{\pi(n)}{n/\ln n} = 1$$

$$\ln n - \frac{3}{2} \le \frac{n}{\pi(n)} \le \ln n - \frac{1}{2} (n \ge 67)$$

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

$$(a^m-1,a^n-1)=a^{(m,n)}-1\,(a>1,m,n>0)$$

$$(a^m-b^m,a^n-b^n)=a^{(m,n)}-b^{(m,n)}\,(a>b,\gcd(a,b)=1)$$

$$(F_n,F_m)=F_{(n,m)}\,(F_n=F_{n-1}+F_{n-2})$$

$$\sum_{i=1}^{N}\gcd(i,N)=\sum_{d|N}d\varphi(N/d)$$

$$A^x \bmod m=A^{(x \bmod \pi(m)+\pi(m)) \bmod m}\,(x\geq\pi(m))$$

$$\sum_{i=1}^{N}\frac{N}{\gcd(i,N)}=\sum_{d|N}d\varphi(d)=(\frac{{p_1}^{2a_1+1}+1}{p_1+1})(\frac{{p_2}^{2a_2+1}+1}{p_2+1})\times\ldots\times(\frac{{p_k}^{2a_k+1}+1}{p_k+1})\,(N={p_1}^{a_1}{p_2}^{a_2}\ldots{p_k}^{a_k})$$

$$(n+1)lcm(C_n^0,C_n^1,\ldots C_n^{n-1},C_n^n)=lcm(1,2,\ldots n+1)$$

$$\gcd(ab,m)=\gcd(a,m)\times\gcd(b,m)$$

> **常见递推：**

$$Catalan数：h(n)=h(0)*h(n-1)+h(1)*h(n-2)+\ldots+h(n-1)*h(0)$$

$$h(n)=\frac{4n-2}{n+1}h(n-1)$$

$$h(n)=\frac{C_{2n}^n}{n+1}$$

$$h(n)=C_{2n}^n-C_{2n}^{n-1}$$

将 n 个不同元素构成m个圆排列的数目：

$$s_u(n+1,m)=s_u(n,m-1)+ns_u(n,m)$$

$$s_s(n+1,m)=s_s(n,m-1)-ns_s(n,m)$$

将n个不同的元素拆分成m个集合的方案数：

$$S(n+1,m)=S(n,m-1)+mS(n,m)$$