1 Stochastic Process

1.1 Geometirc Brownian Motion

The stochastic process X_t is said to follow GBM if it satisfying the following SDE

$$dX_t = \mu X_t dt + \sigma X_t dW_t \tag{1}$$

W is the Brownian motion which determine the process from beginning $S_{t=0}$ to $S_{t=T}$.

$$W_k = \sum_{t=1}^{k} b_t, \quad k = 1, \dots, m \tag{2}$$

where b is the added randomness to the model. which stores a random number coming from the standard normal distribution $N(0,1)$.

The solution of above SDE has the analytic solution

$$S_k = S_0 \prod_{i=1}^{k} e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)} \tag{3}$$

1.2 Ornstein-Uhlenbeck process

The Ornstein-Uhlenbeck process differential equation is given by

$$dX_t = aX_t dt + \sigma dW_t \tag{4}$$

An additional drift term is sometimes added:

$$dX_t = \sigma dW_t + a(\mu - x_t)dt \tag{5}$$

where σ and a is constants and $\{W_t, t \leq 0\}$ is a standard Brownian motion.

$$X_t = e^{at} X_0 + \sigma \int_0^t e^{a(s-t)} dW_s \tag{6}$$

To approximate the numerical solution, we use Euler-Maruyama method to estimate X_t.

Take (5) for example. First, partition the interval $[0,T]$ into N equidistance sunintervals. Then, recursively solving X_{n+1} by

$$X_{n+1} = X_n + a(\mu - x_t) \triangle t + \sigma \triangle dW_s \tag{7}$$

where $\triangle W_n$ is obtained by sampling a random number from normal distribution with expected value zero and variance $\triangle t$.

2 Model

2.1 Generative Adversarial Networks (GANs)

The optimizing approach for GAN is based on [1].

During the training, discriminator D and generator G play the following two-player minimax game during the training. In the first inner loop of training, we update the discriminator by ascending its stochastic gradient:

$$\bigtriangledown_{x_d} \frac{1}{m} \sum_{t=1}^{m} \left[\log D(x^{(t)}) + \log \left(1 - D\left(G\left(z^{(t)}\right)\right)\right)\right] \tag{8}$$

After finishing the first inner loop of training, we update the generator by descending its stochastic gradient with only one step:

$$\bigtriangledown_{x_g} \frac{1}{m} \sum_{t=1}^{m} \log \left(1 - D\left(G\left(z^{(t)}\right)\right)\right) \tag{9}$$

where x_d and x_g represents the variables for discriminator and generator; t indicates the time for each vector; $z^{(t)}$ is noise sample.

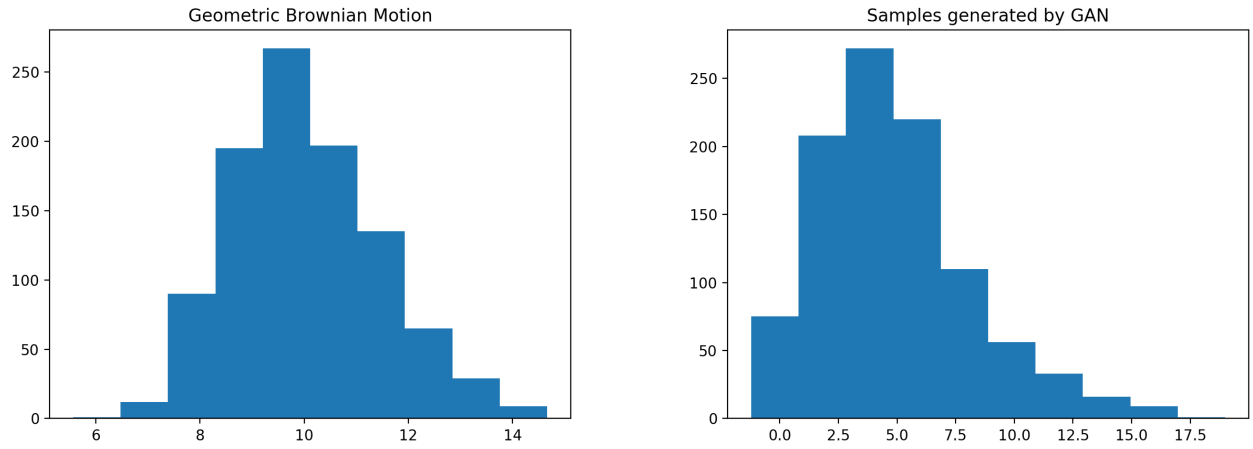

Figure 1: Distribution in specific day

3 Experiments

3.1 Geometric Brownian Motion

The distribution at $S(2)$ for geometric brownian motion and GAN is presented in Figure 1.

The path for geometric brownian motion and GAN is presented in Figure 2. In here, the input is the single vector. We expect GAN can generate the path of output similar to the input.

References

[1] Ian J. Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio. Generative adversarial nets. In *Proceedings of the 27th International Conference on Neural Information Processing Systems - Volume 2*, NIPS14, page 26722680, Cambridge, MA, USA, 2014. MIT Press.

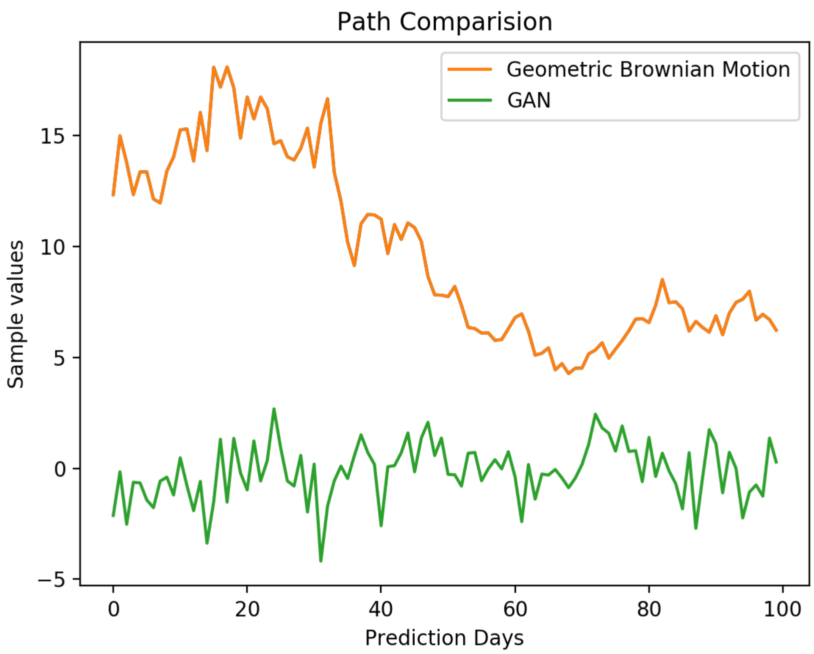

Figure 2: Path Simulation