作业22解答：

1. 用FFT算法计算向量(0, 2, 3, -1, 4, 5, 7, 9)的DFT变换，然后对上述计算结果，求逆DFT变换。

1. 解：(1) DFT变换

$a=(0,2,3,-1,4,5,7,9)$，按树叶顺序排序放入A中：

$A=(0,4,3,7,2,5,-1,9)$。 $n=8$

$s=1,\ m=2^s=2,\ \omega_m=e^{2\pi i/m}=e^{\pi i}=-1,\ \omega=1$

$j=0,\ k=0,2,4,6$

$A[0]=4,\ A[1]=-4,\ A[2]=10,\ A[3]=-4,$

$A[4]=7,\ A[5]=-3,\ A[6]=8,\ A[7]=-10,$

$A=(4,-4,10,-4,7,-3,8,-10)$

$s=2,\ m=2^s=4,\ \omega_m=e^{2\pi i/m}=e^{\frac{\pi}{2}i}=i,\ \omega=1$

$j=0,\ k=0,4$

$A[0]=14,\ A[2]=-6,\ A[4]=15,\ A[6]=-1$

$j=1,\ k=1,5,\quad \omega=\omega*\omega_m=i$

$A[1]=-4-4i,\ A[3]=-4+4i,\ A[5]=-3-10i,\ A[7]=-3+10i$

$A=(14,-4-4i,-6,-4+4i,15,-3-10i,-1,-3+10i)$

$s=3,\ m=2^3=8,\ \omega_m=e^{2\pi i/m}=e^{\frac{\pi}{4}i},\ \omega=1$

$j=0,\ k=0$

$A[0]=29,\quad A[4]=-1$

$j=1,\ k=1,\ \omega=\omega*\omega_m=e^{\frac{\pi}{4}i}$

$A[1]=A[1]+\omega A[5]=-4-4i+(-3-10i)e^{\frac{\pi}{4}i}$

$A[5]=A[1]-\omega A[5]=-4-4i+(3+10i)e^{\frac{\pi}{4}i}$

$j=2,\ k=2,\ \omega=\omega*\omega_m=e^{\frac{\pi}{4}i}*e^{\frac{\pi}{4}i}=e^{\frac{\pi}{2}i}=i$

$A[2]=A[2]+\omega A[6]=-6+i*(-1)=-6-i$

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$A[6]=A[2]-\omega A[6]=-6-i*(-1)=-6+i$

$j=3,\ k=3,\ \omega=\omega*\omega_m=e^{\frac{\pi}{2}i}*e^{\frac{\pi}{4}i}=e^{\frac{3\pi}{4}i}$

$A[3]=A[3]+\omega A[7]=(-4+4i)+(-3+10i)e^{\frac{3\pi}{4}i}$

$A[7]=A[3]-\omega A[7]=(-4+4i)+(3-10i)e^{\frac{3\pi}{4}i}$

最后求得:

$A=(29,(-4-4i)-(3+10i)e^{\frac{\pi}{4}i},-6-i,(-4+4i)+(-3+10i)e^{\frac{3\pi}{4}i},$
$-1,(-4-4i)+(3+10i)e^{\frac{\pi}{4}i},-6+i,(-4+4i)+(3-10i)e^{\frac{3\pi}{4}i})$

再求逆DFT变换:

$y=(29,(-4-4i)-(3+10i)e^{\frac{\pi}{4}i},-6-i,(-4+4i)+(-3+10i)e^{\frac{3\pi}{4}i},$
$-1,(-4-4i)+(3+10i)e^{\frac{\pi}{4}i},-6+i,(-4+4i)+(3-10i)e^{\frac{3\pi}{4}i})$

按反转顺序排序放入A中:

$A=(29,-1,-6-i,-6+i,(-4-4i)-(3+10i)e^{\frac{\pi}{4}i},(-4-4i)+(3+10i)e^{\frac{\pi}{4}i},$
$(-4+4i)+(-3+10i)e^{\frac{3\pi}{4}i},(-4+4i)+(3-10i)e^{\frac{3\pi}{4}i})$, $n=8$

$s=1,\ m=2^s=2,\ \omega_m^{-1}=e^{-2\pi i/m}=e^{-\pi i}=-1,\ \omega^{-1}=1.$

$j=0,\ k=0,2,4,6$

$A[0]=28,\ A[1]=30,\ A[2]=-12,\ A[3]=-2i$

$A[4]=-8-8i,\ A[5]=(-6-20i)e^{\frac{\pi}{4}i},\ A[6]=-8+8i,$

$A[7]=(-6+20i)e^{\frac{3\pi}{4}i}$

$A=(28,30,-12,-2i,-8-8i,(-6-20i)e^{\frac{\pi}{4}i},-8+8i,(-6+20i)e^{\frac{3\pi}{4}i})$

$s=2,\ m=2^s=4,\ \omega_m^{-1}=e^{-2\pi i/m}=e^{-\frac{\pi}{2}i}=-i,\ \omega^{-1}=1.$

$j=0,\ k=0,4$

$A[0]=16,\ A[2]=40,\ A[4]=-16,\ A[6]=-16i$

$j=1,\ k=1,5;\quad \omega^{-1}=\omega^{-1}*\omega_m^{-1}=-i$

$A[1]=30+(-i)*(-2i)=28,\ A[3]=30-(-i)*(-2i)=32,$

$A[5]=(-6-20i)e^{\frac{\pi}{4}i}+(-i)*(-6+20i)e^{\frac{3\pi}{4}i}=-12e^{\frac{\pi}{4}i},$

$A[7]=(-6-20i)e^{\frac{\pi}{4}i}-(-i)*(-6+20i)e^{\frac{3\pi}{4}i}=-40ie^{\frac{\pi}{4}i}$

$A=(16,28,40,32,-16,-12e^{\frac{\pi}{4}i},-16i,-40ie^{\frac{\pi}{4}i})$

$s=3,\ m=2^s=8,\ \omega_m^{-1}=e^{-2\pi i/m}=e^{-\frac{\pi}{4}i},\ \omega^{-1}=1.$

$j=0,\ k=0$

$A[0]=0,\ A[4]=32$

$j=1,\ k=1,\ \omega^{-1}=\omega^{-1}*\omega_m^{-1}=e^{-\frac{\pi}{4}i}$

$A[1]=28+e^{-\frac{\pi}{4}i}*(-12e^{\frac{\pi}{4}i})=16$

$A[5]=20-e^{-\frac{\pi}{4}i}*(-12e^{\frac{\pi}{4}i})=40$

$j=2,\ k=2,\ \omega^{-1}=\omega^{-1}*\omega_m^{-1}=e^{-\frac{\pi}{4}i}*e^{-\frac{\pi}{4}i}=e^{-\frac{\pi}{2}i}=-i$

$A[2]=40+(-i)*(-16i)=24$

$A[6]=40-(-i)*(-16i)=56$

$j=3,\ k=3,\ \omega^{-1}=\omega^{-1}*\omega_m^{-1}=e^{-\frac{\pi}{2}i}*e^{-\frac{\pi}{4}i}=e^{-\frac{3\pi}{4}i}$

$A[3]=32+e^{-\frac{3\pi}{4}i}*(-40i)e^{\frac{\pi}{4}i}=-8$

$A[7]=32-e^{-\frac{3\pi}{4}i}*(-40i)e^{\frac{\pi}{4}i}=72$

最后得:

$A=(0,16,24,-8,32,40,56,72)$

而 $a=\frac{1}{8}A=(0,2,3,-1,4,5,7,9)$