

作业12解答:

1. 用深度优先搜索策略设计一个 $O(e)$ 时间的算法, 判定一个有向图是否无圈(无有向圈)。

解: 一个有向图 D 没有有向圈当且仅当 D 的每一个顶点 v 就是一个强连通分支, 且 v 就是包含 v 的强连通分支的根。因此, 可用求 D 的所有强连通分支的算法, 判定 D 的强连通分支数是否等于 D 的顶点数即可。

算法:

BEGIN

COUNT := 1;

number := 1; 强连通分支的个数

FOR all v in V DO mark v "new";

initialize STACK to empty;

WHILE there exists a vertex v marked "new" DO

 SEARCHC(v);

 IF number = COUNT THEN RETURN(true)

 ELSE RETURN(false);

END;

PROCEDURE SEARCHC(v);

BEGIN

 mark v "old"; 求强连通分支

 DFNUMBER[v] := COUNT;

 COUNT := COUNT + 1;

 LOWLINK[v] := DFNUMBER[v];

 push v on STACK;

FOR each vertex w on $L[v]$ DO

IF w is marked "new" THEN

BEGIN

SEARCHC(w);

$LOWLINK[v] := \min(LOWLINK[v], LOWLINK[w]);$

END

ELSE

IF ($DFNUMBER[w] < DFNUMBER[v]$) AND

w is on STACK THEN

$LOWLINK[v] := \min(DFNUMBER[w], LOWLINK[v]);$

IF $LOWLINK[v] = DFNUMBER[v]$ THEN

BEGIN

number := number + 1;

REPEAT

pop x from top of STACK;

print x ;

UNTIL $x = v$;

print "end of strongly connected component";

END

END;