CS 520 Final: Question 1 – Decision Making

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1） It is obvious that if one of the dogs is on the right side of the sheep and one of the dogs is below the sheep, the sheep can only go up or left. Therefore, if the dogs can follow the sheep in the following round and keep the condition of one is on the right side and one is below, finally the sheep will be cornered in the upper left corner of the field. According to this strategy, we only need to focus on how to make the two dogs arrive at the right side and the lower side. Since the dogs can only move one cell per round, the expected rounds to arrives the target position will equal to the Manhattan distance to the target. In general condition, there are two situations, if Dog1 go to the right side and Dog2 go to the lower side, then the number of rounds we need is $\mathrm{S1} = \max\,(Manhattan(Dog1, right\ side), Manhattan(Dog2, lower\ side))$; if Dog1 go to the right side and Dog2 go to the lower side, then the number of rounds we need is $\mathrm{S2} = \max\,(Manhattan(Dog2, right\ side), Manhattan(Dog1, lower\ side))$. Then we compute $\min\,(S1, S2)$, and choose this situation as our strategy. We just need to let the two dogs go to the expected positions now. There are a few special situations we need to concern about, which is if the sheep appears in the rightmost or the bottom of the field, we cannot let one of our dogs go to the right side or the lower side. In this situation, if the sheep appears in the rightmost of the field, we let one of our dogs go to the lower side and the other one goes to the lower left side, then the sheep can only go up or left. If it goes up, the two dogs go up and follow it, when it arrives the top of the field, it has to go left; if it goes left, the dog below it in the previous round goes up and arrives the right side of the sheep, we can just perform as it in general situation now. In situation of the sheep appears in the bottom of the field, we can let one of our dogs go to the right side and the other one goes to the upper right side, then just perform as what I said before. The most extreme situation is that the sheep appear in the lower right corner of the field, at this time, we just compare the distance of $\max\,(Manhattan(Dog1, upper\ side), Manhattan(Dog2, upperleft\ side))$, $\max\big(Manhattan(Dog2, upper\ side), Manhattan(Dog1, upperleft\ side)\big)$, $\max\,(Manhattan(Dog1, left\ side), Manhattan(Dog2, upperleft\ side))$ and $\max\big(Manhattan(Dog2, left\ side), Manhattan(Dog1, upperleft\ side)\big)$, find the smallest value and follow that path. Since if there is only one way for the sheep, it must go out of the corner at last. However, the sheep will often go out of the corner by itself before our dogs arrive. We need to re-compute the distance every round since the position of the sheep would be different at every round.

2） I need 14.50 rounds on average to corner the sheep.

3） I think the worst possible initial state is that the sheep appears in the lower right corner and the dogs appear in the upper left corner, which means one dog is in the upper left corner and the other one is on its right side or lower side. In this case, the value of the Manhattan distance between the sheep and the dogs is maximum, while the value of the Manhattan distance between the sheep and the upper left corner is also maximum. Therefore, it has the largest expectation of the number of rounds to make the dog arrive the correct side of the sheep and drive the sheep to the upper left corner. For this reason, I consider this case as the worst possible initial state.

4） We can use expectation to compute the optimal position where we can place our dog.

Assume that Dog1 will go to the right side of the sheep. We can have the formulation that:

$$E(x)=\frac{7\cdot 7}{8\cdot 8}\cdot\frac{7\cdot(2+3+4+5+6+7+8)}{7\cdot 7}+\frac{7}{64}\cdot\frac{8+8+8+8+8+8+8}{7\cdot 1}+\frac{7}{64}\cdot\frac{2+3+4+5+6+7+8}{7\cdot 1}+\frac{1}{64}\cdot\frac{8}{1}=5.375$$

$$E(y)=\frac{7\cdot 7}{8\cdot 8}\cdot\frac{7\cdot(1+2+3+4+5+6+7)}{7\cdot 7}+\frac{7}{64}\cdot\frac{2+3+4+5+6+7+8}{7\cdot 1}+\frac{7}{64}\cdot\frac{7+7+7+7+7+7+7}{7\cdot 1}+\frac{1}{64}\cdot\frac{7}{1}=4.484375$$

Assume that Dog2 will go to the lower side of the sheep. We can have the formulation that:

$$E(x)=\frac{7\cdot 7}{8\cdot 8}\cdot\frac{7\cdot(1+2+3+4+5+6+7)}{7\cdot 7}+\frac{7}{64}\cdot\frac{2+3+4+5+6+7+8}{7\cdot 1}+\frac{7}{64}\cdot\frac{7+7+7+7+7+7+7}{7\cdot 1}+\frac{1}{64}\cdot\frac{7}{1}=4.484375$$

$$E(y)=\frac{7\cdot 7}{8\cdot 8}\cdot\frac{7\cdot(2+3+4+5+6+7+8)}{7\cdot 7}+\frac{7}{64}\cdot\frac{8+8+8+8+8+8+8}{7\cdot 1}+\frac{7}{64}\cdot\frac{2+3+4+5+6+7+8}{7\cdot 1}+\frac{1}{64}\cdot\frac{7}{1}=5.25$$

As we can only place our dogs in the integer cell and the upper left corner is (0,0), the optimal place of Dog1 is (4,3) and the optimal place of Dog2 is (3,4).

5） There might exist a better strategy. Since at each time we make our decision of how our dogs go after the sheep go before, it makes that there might be some ‘waste of time’ before the dogs arrive the correct side of sheep. If we can predict where the sheep most likely to go in the next round, we can let our dogs arrive at the location, which can prevent the sheep go to right side or lower side in advance.