CS 520 Final: Question 3 – GoatDiscoveryBot

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a) LogicalGoatDiscoveryBot:

$$P = (In\ A) \cup (In\ B) \cup (In\ C)$$

ProbabilisticGoatDiscoveryBot:

$$P(In\ A) + P(In\ B) + P(In\ C) = 1$$

$$P(In\ A) = P(In\ B) = P(In\ C) = \frac{1}{3}$$

b) Under the logical formulation, we cannot make a decision about selecting which one will be better in current condition. There is not an obvious choice of best action.

c) Under the probabilistic formulation, we can compare the value of $P(In\ A)$, $P(In\ B)$ and $P(In\ C)$ in current condition to make the choice. There is not an obvious choice of best action now since the three values are all $\frac{1}{3}$ now.

d) The logical formulation will be $P = (In\ A) \cup (In\ C)$ after updating.

e) First, we consider about three action. While I select A, if the goat is in A, the probability of that the goat is not in B is $\frac{1}{2}$; if the goat is in B, the probability of that the goat is not in B is 0; if the goat is in C, the probability of that the goat is not in B is 1. According to the total probability, the probability of that the goat is not in B while I select A is:

$$\begin{aligned} &P(Select\ A, CBMHBot\ tells\ not\ in\ B) \\ &\quad = P(Select\ A, CBMHBot\ tells\ not\ in\ B | In\ A) \cdot P(In\ A) \\ &\quad + P(Select\ A, CBMHBot\ tells\ not\ in\ B | In\ B) \cdot P(In\ B) \\ &\quad + P(Select\ A, CBMHBot\ tells\ not\ in\ B | In\ C) \cdot P(In\ C) \\ &\quad = \frac{1}{2} \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = \frac{1}{2} \end{aligned}$$

Then according to the Bayes' theorem, the probabilistic formulation will be:

$$P(In\ A|Select\ A, CBMHBot\ tells\ not\ in\ B)$$

$$= \frac{P(Select\ A, CBMHBot\ tells\ not\ in\ B|In\ A) \cdot P(In\ A)}{P(Select\ A, CBMHBot\ tells\ not\ in\ B)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(In\ B|Select\ A, CBMHBot\ tells\ not\ in\ B)$$

$$= \frac{P(Select\ A,\ \ CBMHBot\ tells\ not\ in\ B|In\ C) \cdot P(In\ B)}{P(Select\ A, CBMHBot\ tells\ not\ in\ B)} = \frac{0 \cdot \frac{1}{3}}{\frac{1}{2}} = 0$$

$$P(In\ C|Select\ A, CBMHBot\ tells\ not\ in\ B)$$

$$= \frac{P(Select\ A, CBMHBot\ tells\ not\ in\ B|In\ C) \cdot P(In\ C)}{P(Select\ A, CBMHBot\ tells\ not\ in\ B)} = \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

f) Under the logical formulation, we have known that the goat is not in B, so we will never select the result of 'Re-Select B'. However, we still cannot make a decision about choosing which one will be better between 'Re-Select A' and 'Re-Select C'. There is not an obvious choice of best action.

g) Under the probabilistic formulation, now we know the new values of $P(In\ A)$, $P(In\ B)$ and $P(In\ C)$ after CBMHBot told us that the goat is not in B. We can compare these values and select the action that has the higher probability. As we known that $P(In\ A) = \frac{1}{3}$, $P(In\ B) = 0$ and $P(In\ C) = \frac{2}{3}$, it is obvious that 'Re-Select C' is the best action.

h) Under the logical formulation, I will never select location B, but there is no difference between select location A and select location C. As a bot, I will not do any redundant action, so I will stick with location A.

i) Under the probabilistic formulation, I will change to select location C. Since it has higher probability that the goat is in location C, it is obvious that if I change to select location C, it is more likely to find the goat.

j) ProbabilisticGoatDiscoveryBot is more successful in this mission. LogicalGoatDiscoveryBot can only provide us the choices, which ones can be selected, and which ones cannot. We are not able to know the priorities of the ones which can be selected. ProbabilisticGoatDiscoveryBot not only updates the condition about whether one can be selected, but also tells us which one has the higher priority to be selected by using probability. In this case, after CBMHBot tells us that the goat is not in B, LogicalGoatDiscoveryBot cannot tell us which one is the better choice between A and C, but ProbabilisticGoatDiscoveryBot can provides us a better choice. So ProbabilisticGoatDiscoveryBot performs better in this mission.

Bonus: If CBMHBot is biased, according to the description, if the goat is in A, the probability of that CBMHBot tells me the goat is not in B is p, the probability of that CBMHBot tells me the goat is not in C is 1 – p, which means that:

$$P(Select\ A, Biased\ CBMHBot\ tells\ in\ B | In\ A) = p$$

Then we have that:

$$\begin{aligned} &P(Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B) \\ &\quad = P(Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B | In\ A) \cdot P(In\ A) \\ &\quad + P(Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B | In\ B) \cdot P(In\ B) \\ &\quad + P(Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B | In\ C) \cdot P(In\ C) \\ &\quad = p \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = \frac{1+p}{3} \end{aligned}$$

We can update of our probabilistic formulation as:

$$\begin{aligned} &P(In\ A | Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B) \\ &= \frac{P(Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B | In\ A) \cdot P(In\ A)}{P(Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B)} = \frac{p \cdot \frac{1}{3}}{\frac{1+p}{3}} \\ &= \frac{p}{1+p} \end{aligned}$$

$$\begin{aligned} &P(In\ B | Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B) \\ &= \frac{P(Select\ A,\ Biased\ CBMHBot\ tells\ not\ in\ B | In\ C) \cdot P(In\ B)}{P(Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B)} = \frac{0 \cdot \frac{1}{3}}{\frac{1}{2}} \\ &= 0 \end{aligned}$$

$$\begin{aligned} &P(In\ C | Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B) \\ &= \frac{P(Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B | In\ C) \cdot P(In\ C)}{P(Select\ A, Biased\ CBMHBot\ tells\ not\ in\ B)} = \frac{1 \cdot \frac{1}{3}}{\frac{1+p}{3}} \\ &= \frac{1}{1+p} \end{aligned}$$

It is obvious that we still will never select B. Since $p \leq 1$, if $p = 1$, the probability of the goat is in A and the probability of the goat is in C are the same, so I will stick with my initial selection. If $p < 1$, the probability of the goat is in C will larger than the probability of the goat is in A, so I will switch to C.