

1. Elementary algebra.

$$1.1 \frac{z''}{z^4 \cdot z^5} = \frac{z''}{z^9} = z^2$$

$$1.2 6^2 \cdot 3^x \cdot 2^x = 6^9 \quad 6^2 \cdot (3 \cdot 2)^x = 6^9$$

$$6^2 \cdot 6^x = 6^9$$

$$6^{(2+x)} = 6^9$$

$$2+x = 9$$

$$x = 7$$

$$1.3 x^3 \cdot y^{-3} = (x \cdot y)^{-3}$$

$$\therefore x \cdot y = 5$$

$$\therefore (x \cdot y)^{-3} = 5^{-3}$$

$$= \frac{1}{125}$$

$$1.4 \frac{\sqrt{3^{10}}}{\sqrt{9^3}} = \frac{\sqrt{3^{10}}}{\sqrt{3^6}} = \sqrt{3^4} = 3^2 = 9$$

$$1.5 \quad a) T \quad b) T \quad c) F \quad d) T \quad (x \neq 0)$$

$$1.6 \quad \frac{4x-10}{4} \geq 4$$

$$4x-10 \geq 16$$

$$4x \geq 26$$

$$x \geq \frac{13}{2}$$

$$x \geq 6.5$$

2 Functions of one variable

$$2.1 \quad F = \left(\frac{212-32}{100}\right)C + 32$$

$$\text{if } F = C$$

$$\left(\frac{212-32}{100}\right)C + 32 = C$$

$$1.8C + 32 = C$$

$$C = -40$$

$$F = C = -40$$

$$2.2 \quad 7y+3=52 \rightarrow y=7$$

$$2.3 \quad x^2 - 2x + 2 = 2$$

$$x^2 - 2x = 0 \rightarrow x_1 = 0 \quad x_2 = 2$$

$$2.4 \quad a(1+2\%)^x = 2a \rightarrow x = \log_{1.02} 2$$

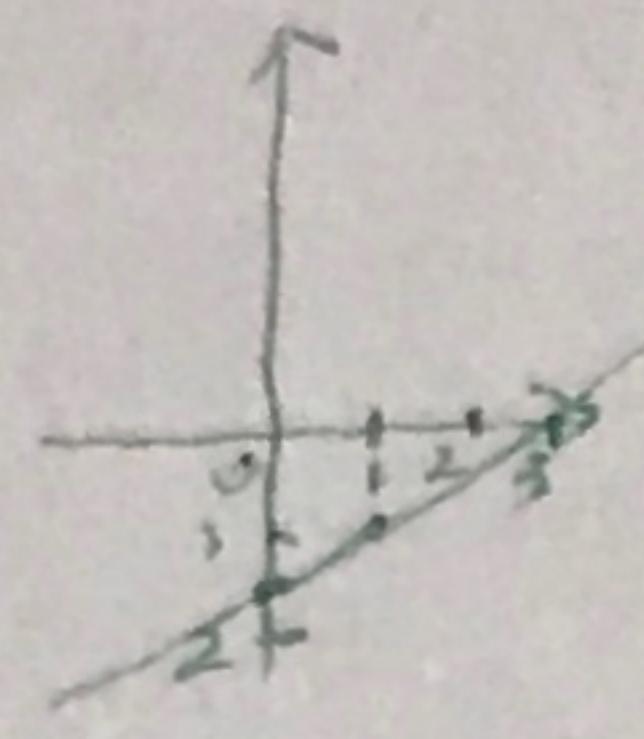
$$1.02^x = 2 \rightarrow x = 35.002$$

$$2.5 \quad \ln\left(\frac{1}{e^3}\right) = \log_e e^{-3} = -3$$

3. Calculus

$$3.1 \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^{3i} + \left(\frac{1}{2}\right)^i \\ = \sum_{i=0}^{\infty} \left(\frac{1}{8}\right)^i + \left(\frac{1}{2}\right)^i \\ = \frac{1}{1-\frac{1}{8}} + \frac{1}{1-\frac{1}{2}} \\ = \frac{22}{7}$$

$$3.2 \lim_{x \rightarrow 3} \frac{x-3}{2} = \frac{3-3}{2} = 0$$



$$3.3 f(x) = x^2 - 4 \quad f(x)' = 2x \\ f(-1)' = -2$$

$$3.4 \frac{d}{dx} \frac{x^2+3}{x+2} = \frac{2x(x+2)-(x^2+3)}{(x+2)^2} \\ = \frac{2x^2+4x-x^2-3}{(x+2)^2} \\ = \frac{x^2+4x-3}{(x+2)^2}$$

$$3.5 \frac{d^2}{dx^2} 4x^3+4 = (12x^2)' \\ = 24x$$

$$3.6 \text{ graph of } f(x) = \frac{1}{x}$$

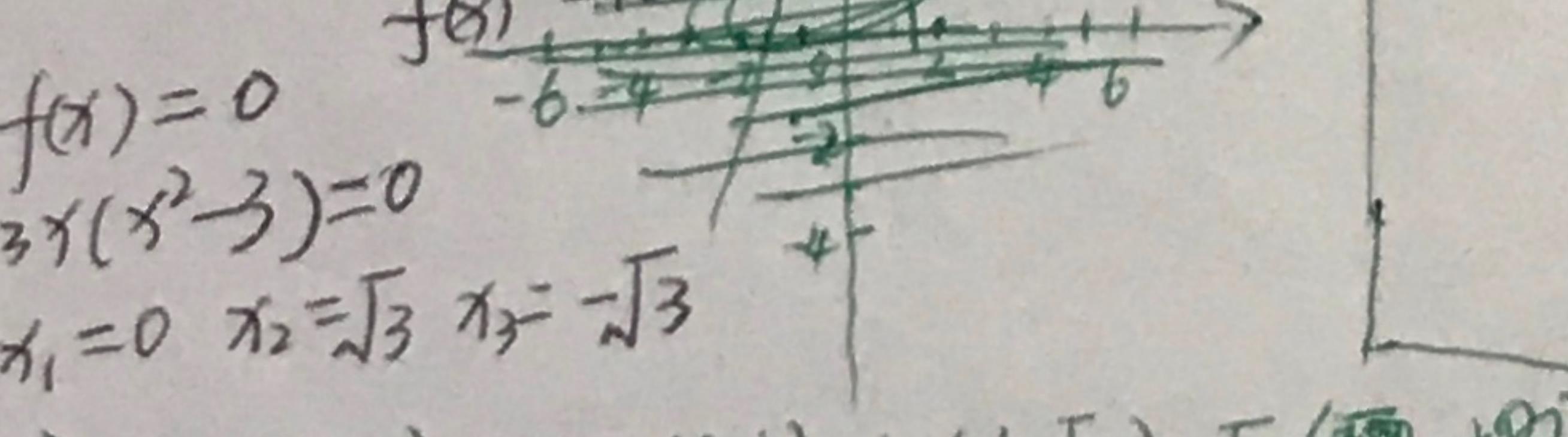
$\therefore (-\infty, 0)$ $f(x)$ approaching to $-\infty$

$(0, +\infty)$ $f(x)$ approaching to $+\infty$

\therefore function $f(x) = \frac{1}{x}$ does not continuous at 0.

$$3.7 f(x) = 3x^2 - 9x$$

$$\begin{aligned} f'(x) &= 9x^2 - 9 \\ f'(x) = 0 &\Rightarrow 9x^2 - 9 = 0 \\ x &= \pm 1 \end{aligned}$$



$$(-\infty, -\sqrt{3}) \cup \sqrt{3}, (\sqrt{3}, 1) \cup (1, 0) \cup (0, 1) \cup (1, \sqrt{3}) \cup \sqrt{3}, (\sqrt{3}, +\infty)$$

$f(x)$	-	0	+	+	+	0	-	-	-	0	+
$f'(x)$	+	+	+	0	-	-	0	+	+	+	+
slope	/	/	/	max	\	\	\	min	\	\	\
$f''(x)$	-	-	-	-	-	0	+	+	+	+	+
NU	\	\	\	\	\	\inf	V	V	V	V	V

$$3.8 f(2,3) = 2^3 \cdot 3^2 = 8 \cdot 9 = 72$$

$$3.9 2x-y > 0 \quad y < 2x$$

$$3.10 \frac{\partial^2}{\partial x^2} x^5 + x^2 y^3 \\ = (5x^4 + 2y^3 x)' \\ = 20x^3 + 2y^3$$

$$3.11 f(x, y) = \sqrt{xy} - \frac{1}{4}x - \frac{1}{4}y$$

\therefore forgot how to calculate
I'll review it later.

X

$$3.12 \max x^2 y^2 \text{ s.t. } x+y=5$$

$$\partial = x^2 y^2 - \lambda(x+y-5)$$

$$\frac{\partial}{\partial x} = 2xy^2 - \lambda$$

$$\frac{\partial}{\partial y} = 2x^2 y - \lambda$$

$$\frac{\partial}{\partial \lambda} = x+y-5$$

$$\therefore 2xy^2 - \lambda = 0 \quad 2x^2 y - \lambda = 0$$

$$2xy^2 = \lambda \quad 2x^2 y = \lambda$$

$$\therefore 2xy^2 = 2x^2 y$$

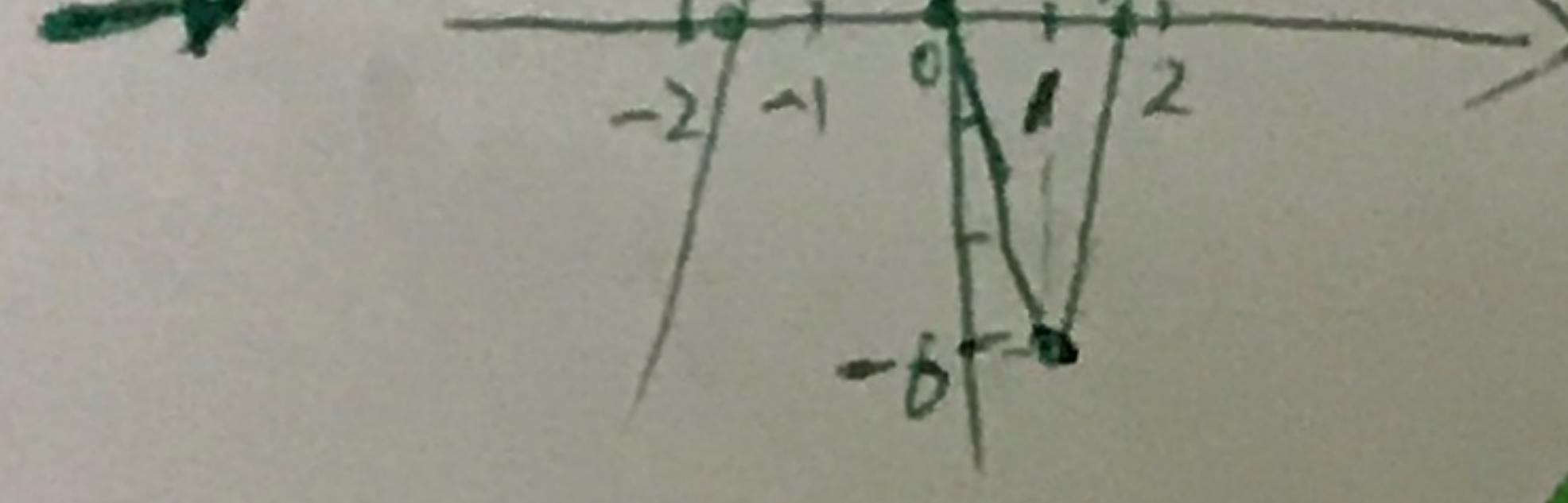
$$x=y$$

$$\therefore x+y=5$$

$$\therefore x=y=2.5$$

$$\max x^2 y^2 = (2.5)^2 \times (2.5)^2$$

$$= (\frac{5}{2})^4 = 39.0625$$



4 Linear algebra

4.1 $A \cdot B = \begin{bmatrix} 2 \times 1 + 3 \times 2 & 2 \times 4 + 3 \times 1 & 2 \times 1 + 3 \times 2 \\ 4 \times 1 + 1 \times 2 & 4 \times 4 + 1 \times 1 & 4 \times 1 + 1 \times 2 \\ 1 \times 1 + 2 \times 2 & 1 \times 4 + 2 \times 1 & 1 \times 1 + 2 \times 2 \end{bmatrix} = \begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 6 \\ 5 & 6 & 5 \end{bmatrix}$

4.2 $B \cdot A = \begin{bmatrix} 1 \times 2 + 4 \times 4 + 1 \times 1 & 1 \times 3 + 4 \times 1 + 1 \times 2 \\ 2 \times 2 + 1 \times 4 + 2 \times 1 & 2 \times 3 + 1 \times 1 + 2 \times 2 \end{bmatrix} = \begin{bmatrix} 19 & 9 \\ 10 & 11 \end{bmatrix}$

4.3 $A^T = \begin{bmatrix} 3.3 & 2 & 4 \\ 5.1 & 6.1 & 5.76 \\ 4.7 & 1.23 & 0 \end{bmatrix}$

4.4 $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = 2 \times 5 - 3 \times 4 \\ = -2$

5 Probability theory

5.1 $2^4 = 16$

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5.2 If drug = a
 If no drug = 99a

$$\frac{a \times 99\%}{a \times 99\% + 99a \times 0.5\%} = \frac{0.99}{0.99 + 0.495} = \frac{2}{3}$$

5.3 $E(P_{\text{first time}}) = \frac{1+2+3+4+5+6}{6} = \frac{7}{2}$

$E(P_{\text{second}}) = \frac{1+2+3+4+5+6}{6} = \frac{7}{2}$

\therefore First time & Second time are independent events

$\therefore E_P = \frac{7}{2} + \frac{7}{2} = 7$