Mathematics problems

1 Elementary algebra

Problem 1.1. Simplify

$$\frac{x^{32}}{x^9 \cdot x^2} \cdot \frac{x^7}{x^2}$$

Problem 1.2. Solve for x:

$$8^2 \cdot 4^x \cdot 2^x = 8^4$$

Problem 1.3. Calculate the missing value. If $\frac{x}{y}$ is 3, then $x^{-4}y^4 = \ldots$

Problem 1.4. Calculate

$$\frac{\sqrt{4^{15}}}{\sqrt{16^7}}$$

Problem 1.5. True or False (x and y and z are real numbers):

(a) $x + (y + z) = (y + x) + z$

(b) $y(x + z) = xy + zy$

(c) $x^{y+z} = x^z + x^y$

(d) $\frac{x^z}{x^y} = x^{y-z}$

Problem 1.6. Find the solution set for the inequality below:

$$\ln(x) \geq e$$

2 Functions of one variable

Problem 2.1 (Based on SYD 2.5.6). The relationship between temperatures measured in Celsius and Fahrenheit is linear. $0°C$ is equivalent to $32°F$ and $100°C$ is the same as $212°F$. Which temperature is measured by the same number on both scales?

Problem 2.2. Take the following function $f(x) = 3x - 12$. Find y if $f(y) = 0$.

Problem 2.3. Find all values of x that satisfy:

$$9^{x^2 - 6x + 2} = 81$$

Problem 2.4. Solve the following problem. If the annual GDP growth of a country is 3%, how long does it take the economy to triple its GDP?

Problem 2.5. Calculate the following value

$$\log_\pi \left(\frac{1}{\pi^5} \right)$$

3 Calculus

Problem 3.1. Calculate the following sum

$$\sum_{i=0}^{\infty} \left(\frac{1}{5^i} + 0.3^i \right)$$

Problem 3.2. Find the following limit

$$\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$$

Problem 3.3. Find the slope of the function $f(x) = x^3 - 4$ at $(-2, -12)$.

Problem 3.4. Find the derivative of the following function:

$$f(x) = \frac{x^5 + 3}{x^2 - 1}$$

Problem 3.5. Find the second derivative of the following function:

$$f(x) = x^9 + 3$$

Problem 3.6. Is the function $f(x) = \frac{1}{x}$ continuous at 0? Why?

Problem 3.7. Consider the following function. Find all of its local minima, local maxima or inflection points.

$$f(x) = 4x^3 - 12x$$

Problem 3.8. Let $f(x, y) = x^3 - y^2$. Calculate $f(2, 3)$

Problem 3.9. Consider the following function: $f(x, y) = \ln(x - 3y)$. For what combinations of x and y is this function defined?

Problem 3.10. Find the following partial derivative:

$$\frac{\partial}{\partial x} \left(x^5 y^7 + \frac{x^2}{y^3} \right)$$

Problem 3.11. Find the local maxima or minima of the following function:

$$f(x, y) = \sqrt{xy} - x - y$$

Problem 3.12. Solve the following constrained optimization problem using Lagrange's method: $\max x^2 y^2$ s.t. $2x + y = 9$

4 Linear algebra

Problem 4.1. Take the following matrices:

$$A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix}$$

What is $B \cdot A$?

Problem 4.2. Take the following matrices:

$$A = \begin{bmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{bmatrix}$$

What is $A \cdot B$?

Problem 4.3. What is the transpose of the following matrix?

$$\begin{bmatrix} e & 93 & 4.7 \\ 2 & 6.1 & 4.22 \\ 4 & \pi & 0 \end{bmatrix}$$

Problem 4.4. Calculate the determinant of

$$\begin{bmatrix} 2 & 6 \\ 2 & 8 \end{bmatrix}$$

5 Probability theory

Problem 5.1. You run an experiment where you toss a dice two times. Each time you get either 1, 2, 3, 4, 5 or 6. What is the sample space of your experiment?

Problem 5.2. Assume that in a certain country 0.1% of the population uses a certain drug. You have a way to test drug use, which will give you a positive result in 98% of the cases where the individual is indeed a drug user and a negative result in 99.7% of the cases where the individual doesn't use the drug. What is the probability that someone with a positive drug test is indeed a drug user?

Problem 5.3. You run an experiment in which you toss a dice 20 times and record how many times you ended up with a 1, 2, 3, 4, 5 or 6. Your random variable is the number of times you ended up with a 5. What is expected value of this random variable?