High Quality Problems from Jon Scieszka's *Math Curse*

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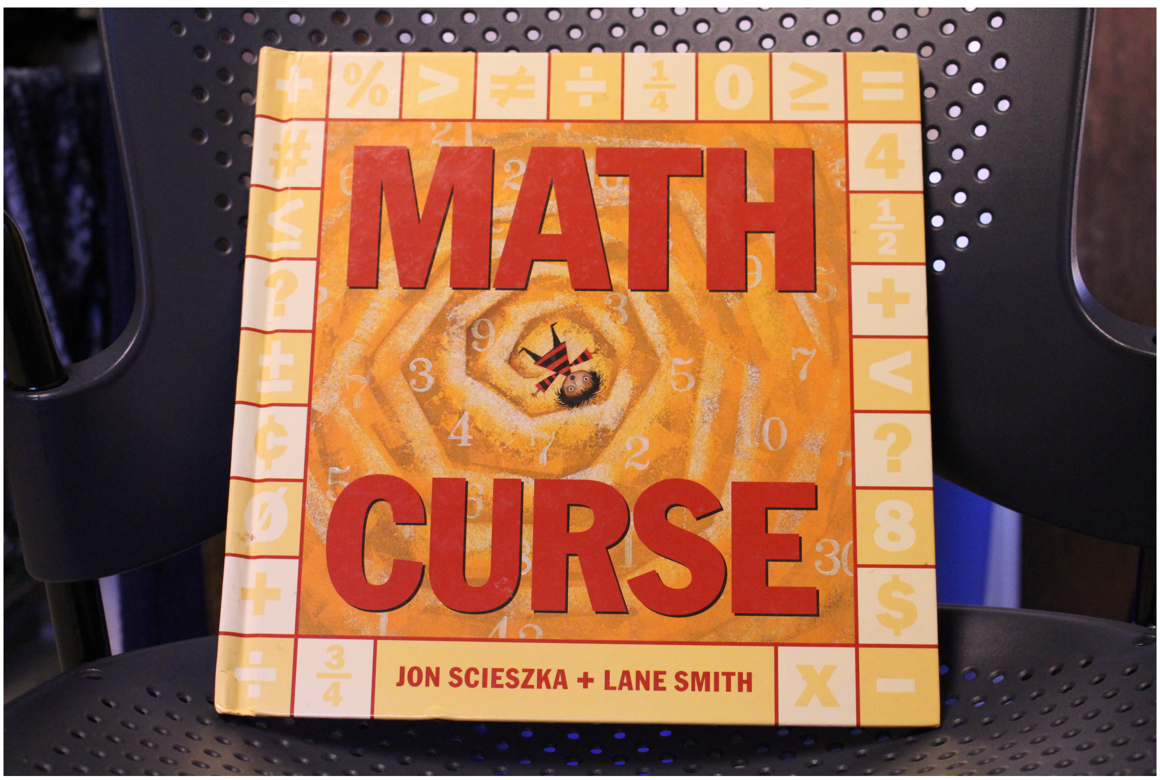

Overview

While taking Principles and Methods for Teaching Mathematics (EDUC 463), our class discussed the qualities that make for engaging mathematical problems and activities. We determined that an activity might be considered "high-quality" if it promotes both cognitive demand as well as sociocultural and constructivist theories of learning. Our class decided that an instructional practice that utilizes high-quality mathematical problems is most conducive to student learning.

My project explores the children's book, *Math Curse*, by Jon Scieszka. In *Math Curse*, the main character is troubled by the prevalence of math throughout his daily life. Throughout the text, he encounters a series of quantitative problems about his routine experiences. My project analyses 3 of those problems in order to explain that they are high quality. Using the framework we developed in EDUC 463, I ask the following questions to determine the quality of each problem:

1. Is the problem cognitively demanding?
2. Can the problem be connected to sociocultural and constructivist theories of learning?

A problem may be described as high quality if it meets both of the above criteria. Additionally, for each problem I will provide a rationale for its choice, a description of potential manipulatives to be used for solving the problem, and a curricular context.

Problem #1: Unit and Measure

Description:

"I take the milk out for my cereal and wonder:

1. How many quarts in a gallon?
2. How many pints in a quart?
3. How many inches in a foot?
4. How many feet in a yard?
5. How many yards in a neighborhood? How many inches in a pint? How many feet in my shoes?" (Scieszka unpaged)

Rationale:

The problem entails finding equalities between different units of measure. Students must determine the mathematical meaning of equality in order to reason about how much of one unit is "in" another unit. This idea of equality between units is an important aspect of primary grade mathematics and, as such, is widely discussed throughout the Common Core State Standards for Mathematics. The problem also maintains that students use units appropriately. For example, the problem erroneously asks "How many inches in a pint?" Because a pint measures the volume of a liquid while an inch measures distance, equality between the two is impossible. Furthermore, the problem explicitly confuses polysemous mathematical language ("How many feet in a yard?/How many yards in a neighborhood? […] How many feet in my shoes?") Students must decide whether the language of the problem refers to yards and feet in a way that is mathematical, commonplace, or both.

Useful manipulatives:

Students might use actual pints, quarts, and gallons to compare the amount of liquid each one holds. Similarly, students might use rulers and yard sticks to compare distance. A physical comparison between distance and volume would solidify students' understanding of what different units of measure mean and how to use them appropriately.

Mathematical context:

This problem is relevant to the study of unit and measure. It engages students in the comparison of similar and dissimilar units. As it's presented in the problem, these dissimilarities prompt students to think about the relationship between dimension and units of measure. For instance, students should realize that quarts, which measure a 3-dimensional attribute (volume) couldn't be compared to inches, which measure a 1-dimensional attribute (distance). Students might also explore this problem within the context of mathematical language. Questions like "How many feet in a yard?" can be interpreted two ways; one could determine the number of feet in one yard (the linear unit of measure) or one back yard.

Cognitive demand:

The problem is cognitively demanding because it requires students to compare different units of measure in an appropriate way. Students must reason proportionally about the size of one unit of measure compared to another. Students must also differentiate between types of units and determine which to use. Furthermore, the exploration of these relationships may occur before students are introduced to formal notation. This establishes a conceptual background for students' later exploration of unit conversion and dimensional analysis.

Associated tasks will have multiple entry points. Students might pour a liquid between a pint, quart, and gallon to see how many times the small measurements "go into" the bigger ones. Such an activity is appropriate for ELL students as well as students with disabilities because it is highly physical. Advanced students might explore less distinct units of measure, such as pints to gallons or inches to feet.

Sociocultural & constructivist implications:

An activity involving this problem could be constructivist. Students might be asked to form ideas about relationships between units, using physical objects such as liquid and containers. Students create knowledge out of their own authentic mathematical experience, thereby concretely establishing fundamental concepts. In this way, students learn what measurement really means.

A sociocultural activity might involve students' discussion of why certain units of measure are appropriate or inappropriate for a given situation. In this case, groups of students might work together to determine and explain why comparing inches and pints is not possible. Students may also be working in their ZPD, assisted by a teacher. The teacher's job in this scenario is to guide students' thinking about relationships between units of measure.

Problem #2: Multiplication

Description:

"There are 24 kids in my class [...] We sit in 4 rows with 6 desks in each row. What if Mrs. Fibonacci rearranges the desks to make 6 rows? 8 rows? 3 rows? 2 rows? I count the 24 kids in our class again, this time by 2s. Jake scratches his paper with one finger. How many fingers are in our class? Casey pulls Eric's ear. How many ears are in our class? This new girl, Kelly, sticks her tongue out at me? How many tongues in our class?" (Scieszka unpaged)

Rationale:

The problem explicitly introduces multiplicative thinking. Using a familiar context (rows of desks and body parts), the problem enables students to take a "groups of" approach to multiplication. Additionally, the problem encourages students to visualize multiplication in two ways. The questions about desks inspire students to visualize an array model of multiplication (rows and columns), while the questions about body parts foster the idea of objects per group. The body part questions empower students to move from counting the total number of body parts by ones to counting by groups. For example, if one student has 2 ears (objects per 1 group) then 24 students must have 24 X 2 or 48 ears (objects per total number of groups). The desk questions also foster ideas about mathematical composition and arrangement. Students notice that the number of desks (24) does not change, despite the number of rows or columns.

Useful manipulatives:

Students might use tiles or Unifix cubes to model the arrays of desks in Mrs. Fibonacci's classroom. Physically manipulating the array reinforces mathematical ideas about quantity and arrangement, thereby contributing to students' developing number sense. Students might also use Unifix cubes to model groups in the questions about body parts. (However, a more organic way to do this might be to have students model the problem with their own fingers, ears, and so on.)

Mathematical context:

This problem might be explored within the context of developing students' multiplicative thinking. The problem uses real-life examples to describe multiplication in terms of "groups of." The concept of "group of" is critical to students' understanding of multiplication. The problem then works to foster students understanding about of the meaning of multiplication, by providing a relevant example from students' classroom literature.

Cognitive demand:

The problem is cognitively demanding because it encourages students to think about multiplication in terms of "groups of." It explicitly guides students to consider multiple visuals for multiplication; in this case, groups (questions about body parts) and arrays (questions about desks). Students must reason about what multiplication means in order to solve the problem. Likewise students must decide which model of multiplication is most appropriate given the context of the problem. As they are challenged by the problem, students form both number sense and multiplicative thinking.

A task with multiple entry points could be constructed around this problem. ELLs might count their fingers and ears in their native language. By verbalizing ideas in their native language, ELLs are able to devote more cognitive energy to the mathematics of the problem. Students with disabilities might benefit from starting the problem with smaller numbers. Instead of beginning the problems with a class of 24 students, students with disabilities might explore the problem with a class of 6 students.

Sociocultural & constructivist implications:

The problem is constructivist because students must actively participate in multiplicative thinking while simultaneously defining multiplication in terms of "groups of." Because students are involved in the physical transition from counting by 1s to counting by groups and using arrays, they build a tangible relationship to multiplication. Students also expand their exiting schema about counting. They assimilate new information about counting by groups (2s, 3s, etc.) into a schema which previously included only counting by ones. This process of assimilation too makes the problem constructivist.

The problem is inherently sociocultural. The literal context of the problem deals with groups of students. A corresponding activity must then include groups of students within its framework. Students work together, using body parts such as ears and fingers to promote a kinesthetic understanding of multiplication and multiplicative thinking. This physical activity supports mathematical understanding of multiplication, on both the order of the individual and the group. Such an activity would also promote discussion about how students reasoned about each problem. This differentiated reasoning helps students to see one problem from multiple viewpoints.

Problem #3: Counting

Description:

"Mrs. Fibonacci says there are many ways to count. She asks us to give some examples. Russell counts on his fingers: '1,2,3,4,5,6,7,8,9,10.' Molly says: '2,4,6,8,10…' Mrs. Fibonacci says: 'I always count 1,1,2,3,4,5,8,13… But on the planet Tetra, kids have only 2 fingers on each hand. They count 1,2,3,10… And on the planet Binary kids have only 1 finger on each hand. They count 1,10…'

1. What are the next five numbers in each sequence above?" (Scieszka unpaged)

Rationale:

This problem requires students to count in and reason about bases other than base-10. In order to do this, students must investigate what a "base" is. They must also reason about why our base-10 system works. In its approach to bases, the problem fosters multiplicative thinking. It contrasts the difference between using fingers to count by ones versus by twos, fours, etc. Furthermore, the problem introduces the algebraic idea of "sequence." The problem also correctly uses mathematical language, such as "Tetra" and "Binary."

Useful manipulatives:

To investigate different bases, students may use counters that can be grouped distinctly. For example, coffee stirrers can be tied into bundles, using rubber bands.

Students may use these counters to explore ideas about different bases, by changing the amount of counters per bundle. A bundle of 10 counters would represent our base-10 system, whereas a bundle of 4 counters would represent counting "on the planet Tetra" or in base-4.

Mathematical context:

Students might investigate this problem while refining complex ideas about multiplication. Although the problem requires multiplicative thinking as a foundation, it advances students' understanding of multiplication through the idea of "groups of." Students might also work through this problem within the context of algebraic sequences. Using both physical objects and symbols to model mathematical ideas about sequences is a hallmark of algebraic thought.

Cognitive demand:

The problem is cognitively demanding because it asks students to reflect on our base-10 number system while simultaneously challenging them to see counting patterns in other bases. Counting in other bases, particularly while using manipulatives, explicitly reinforces ideas about how we can group numbers. This idea about "grouping numbers" in turn reinforces multiplicative thinking.

An activity could be structured so that the problem has multiple entry points. Because an activity about counting in different bases would likely involve physical manipulatives, such an activity would be appropriate for ELLs. Additionally, *Math Curse* provides relevant visuals. These help students see how each character in the story uses their hands to count. (See above image.)

Sociocultural & constructivist implications:

The problem is constructivist in that students must physically create mathematical "bases" before working with numerical representations. Moreover, students use authentic literary experiences to reason about complex mathematical ideas. They construct their knowledge about bases out of their own personal interaction with the text.

The problem could be used to create a sociocultural activity. Students might be asked to work in groups to model different bases with manipulatives. Within these groups, students would discuss mathematical ideas about grouping numbers within different bases. The context of the problem is also sociocultural in and of itself. In *Math Curse*, Mrs. Fibonacci asks students to describe their individual methods for counting. The text shows how some students count by ones, while others by twos, and so on. The concept of sharing mathematical idea, specifically counting in different bases, promotes a sociocultural approach to learning.

Summary

The need for interesting and engaging mathematics activities is ubiquitous throughout the field of education. Teachers are responsible for posing students with high-quality mathematical tasks in order to forward their learning in a meaningful way. Nebulous definitions of “high quality” can often make it difficult for teachers to judge the caliber of a problem or activity. In Principles and Methods for Teaching Mathematics (EDUC 463), we determined that high quality mathematical tasks are both cognitively demanding and speak to the sociocultural and constructivist theories of learning. My project applies this concise definition of a high quality task within the context of Jon Scieszka’s *Math Curse*, a piece of children’s literature about mathematics. My project seeks to explain why three of the problems presented in *Math Curse* are high quality, exploring their cognitive demand and their relationship to theories of learning. Throughout my project I contend that the problems presented in *Math Curse* are high quality, making this book a valuable resource for the elementary and middle school math classroom. This paper serves to reassert that claim by describing the specifics of cognitively demanding tasks and theories of learning as they relate to *Math Curse*.

An engaging mathematical problem or activity must employ students in a productive struggle, thereby creating a high level of cognitive demand. According to Clark, cognitively demanding tasks “challenge students to develop and apply strategies, serve as a means to introduce new concepts, and offer a context for using skills” (Clark 67). In other words, for a problem to be cognitively demanding, students’ reasoning about that problem must help create significant mathematical understanding. Additionally, for a task to be cognitively demanding, it must have multiple entry points. Van de Walle describes the idea of "multiple entry points" as meaning that "…the task has varying degrees of challenge within it or it can be approached in a variety of ways" (Van de Walle 37). Students from various levels of mathematical understanding should be able to engage with cognitively demanding tasks in a meaningful way. Multiple entry points are especially important for ELL students as well as students with disabilities. Cognitively demanding problems are a hallmark of *Math Curse*. For example, Scieszka poses a series of questions about measurement: "How many pints in a quart? How many inches in a foot?" (Scieszka unpaged) He then asks, "How many inches in a pint?" (Scieszka unpaged) This erroneous question requires that students explore the true meaning of measurement, thereby challenging them to make mathematical connections. A related activity could be designed with multiple entry points. As I describe in my project, "Students might pour a liquid between a pint, quart, and a gallon to see how many times the small measurements 'go into' the bigger ones. Such an activity is appropriate for ELL students as well as students with disabilities because it is highly physical" (see Problem #1: Cognitive Demand).

An engaging problem or activity must also speak to sociocultural and constructivist theories of learning. Sociocultural theory suggests that learning is a product social exchange. As Van de Walle describes, one feature of sociocultural theory is that "…mental processes exist between and among people in social learning settings, and from these social learning settings the learner moves ideas into his or her own psychological realm" (Van de Walle 20). An additional feature of sociocultural theory is the Zone of Proximal Development. Van de Walle defines the ZPD as a "'range' of knowledge that may be out of reach for a person to learn on his or her own, but is accessible if the learner has support from peers or more knowledgeable others" (Van de Walle 20). In a sociocultural model, students work with a more knowledgeable other (usually the teacher) to achieve higher-level understanding. Several problems featured in *Math Curse* can be adapted for sociocultural activities. Scieszka asks multiplication questions within the context of the total number of a body parts present in a classroom. Because "the literal context of the problem deals with groups of students" an associated activity would require students to work in groups. While in groups, students reason collectively and discuss mathematical ideas, thus promoting sociocultural theory (see Problem #2: Sociocultural & constructivist implications).

On the other hand, constructivist theory of learning proposes that students create knowledge through their own authentic experiences. By engaging with problems in a productive way, students can draw conclusions about mathematics. As Van de Walle proposes, "At the heart of constructivism is the notion that learners are not blank slates but rather creators (constructors) of their own learning" (Van de Walle 10). Constructivist theory suggests two distinct processes for this construction of knowledge: assimilation and accommodation. According to Van de Walle, "Assimilation occurs when a new concept 'fits' with prior knowledge and the new information expands an existing network" (Van de Walle 20). On the other hand, "Accommodation takes place when the new concept does not 'fit' with the existing network (causing what Piaget called *disequilibrium*), so the brain revamps or replaces the existing schema" (Van de Walle 20). *Math Curse* presents numerous problems that translate easily to constructivist activities. One such problem interrogates the idea of bases other than base ten. Related activities might require students to "physically create mathematical 'bases' before working with numerical representations." Students must accommodate knowledge about different bases into their existing base-10 schema, thereby participating in a constructivist task (see Problem #3: Sociocultural and constructivist implications).

Cognitive demand along with sociocultural and constructivist theories of learning are essential attributes to any engaging mathematical problem or activity. A problem or activity may only be considered "high quality" if it possesses the aforementioned components. Jon Scieszka's *Math Curse* offers teachers and students an assortment of high-quality problems, which can be adapted to create engaging classroom activities. Because of its use of cognitively demanding problems and its connection to theories of learning, *Math Curse* is a worthwhile classroom text.

Works Cited

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Van de Walle, J. A. (2013). *Elementary and middle school mathematics*. Boston, MA: Pearson.