Università degli Studi di Padova
Department of Information Engineering

Master's Degree Course in
ICT for Internet and Multimedia

Satellite Communication System

Project: Orbital Propagator

Gauri Pravishi - 2041448
Yelyzaveta Pervysheva - 2039398
Seyedali Hosseinishamoushaki - 2041399
Mir Mohammad Reza Heidarzad Namin - 2041430
Sanaz Baradaran Rowhani

Academic Year 2022/2023

Contents

Overview

This report presents a comprehensive analysis of the design and performance of a satellite orbit desighn around the Moon for Position, Velocity, and Time (PVT) purposes. The report begins by introducing the key parameters and initial conditions necessary for the satellite orbit design. It covers the initialization of both the satellite and receiver positions based on provided geodetic coordinates and Keplerian parameters.

The objective is to achieve accurate PVT calculations on the lunar surface by ensuring full coverage and visibility of at least four satellites throughout the orbital period. The initial geodetic coordinates of the receiver, along with the satellite's Keplerian parameters, including semi-major axis, eccentricity, and orbit inclinations, are provided as input. The report discusses the key steps involved in the design process.

A graphical visualization of the satellite orbit and the Moon's surface is generated to provide a clear representation of the scenario. To ensure effective PVT calculations, coverage requirements are discussed, including full coverage of the entire Moon's surface and full coverage of the Moon's south pole. These requirements guarantee that at least four satellites are in view at all times, enabling accurate positioning calculations. The motion of the satellite and receiver is analyzed in detail. The satellite's position is calculated at each time step using its initial position and orbital speed, considering the Moon's spherical shape and the effect of position errors.

The receiver's position is updated along the Moon's surface, accounting for the Moon's rotation around the Zaxis. Observables and pseudorange generation are crucial for PVT calculations. The program determines the line of sight between the satellite and receiver based on masking angles. The distance between them is calculated, and pseudoranges, representing the measured distances, are generated by accounting for transmission time delays.

Pseudorange rates, indicating the rate of change of pseudorange over time, are also computed for precise positioning applications. The report presents the simulation results, including distance, pseudorange, and pseudorange rate plots over time. These results allow for a comprehensive analysis of the satellite orbit's performance and its impact on PVT calculations.

In conclusion, this report provides a detailed overview of the satellite orbit design around the Moon for PVT purposes. It emphasizes the importance of full coverage and visibility of satellites for accurate positioning calculations. The report suggests areas for future work, such as incorporating more accurate lunar surface models and advanced orbit determination algorithms. Overall, this analysis serves as a valuable resource for understanding the design considerations and challenges involved in achieving precise PVT calculations on the lunar surface.

Chapter 1

Design Considerations and Assumptions

1.1 Review of Keplerian parameters

Keplerian parameters are a set of orbital elements used to describe the motion of an object in space, particularly in the context of celestial mechanics.

The six main Keplerian parameters are as follows:

- Semi-major axis (a) It represents the average size of the orbit and is defined as half of the longest diameter of the elliptical orbit.
- Eccentricity (e) It measures the deviation of an orbit from a perfect circle.
- Inclination (ß) It specifies the tilt of the orbital plane with respect to a reference plane. The inclination is measured in degrees and ranges from 0° to 180°.
- Argument of periapsis (ω) It defines the orientation of the

orbit within the orbital plane. Periapsis is given in degrees.

- Longitude of the ascending node (Ω) It denotes the angle between the reference direction (such as a fixed point in space) and the ascending node. The ascending node is the point where the orbit crosses the reference plane from below and is measured in degrees.
- Mean anomaly (M) It represents the fraction of the orbital period that has elapsed since the object last passed through periapsis. The mean anomaly is an angular parameter measured in degrees.

By knowing these Keplerian parameters, along with the gravitational constant and the mass of the central body, it is possible to accurately calculate the position and velocity of an object at any given time using orbital mechanics equations, such as Kepler's equations or numerical integration methods.

1.2 Orbit Design

1.2.1 Satellite and Receiver Initialization and Assumptions

The satellite and receiver positions are initialized based on user-provided parameters such as the receiver's initial latitude, longitude (but not altitude because it is equal to the radius of the Moon), and the satellite's initial true anomaly. The Moon's radius and the speed of light are also considered in the calculations. We have also mentioned below, some of the assumptions based on which orbits were designed.

1.2.2 The Initial Position of the Receiver

To establish the starting location of the receiver on the moon, it is necessary to convert the latitude, and longitude into Cartesian coordinates. Below in the equation, we introduce formulas where B is latitude C is longitude

$$X = r_{\text{moon}} \cos(B) \cos(C) \tag{1.1}$$

$$Y = r_{\text{moon}} \cos(B) \sin(C) \tag{1.2}$$

$$Z = r_{\text{moon}} \sin(B) \tag{1.3}$$

In order to calculate the subsequent position of the receiver at each time step, it is necessary to convert the Cartesian coordinates of the receiver to spherical coordinates. This conversion is required to determine the θ angle and then the ϕ angle.

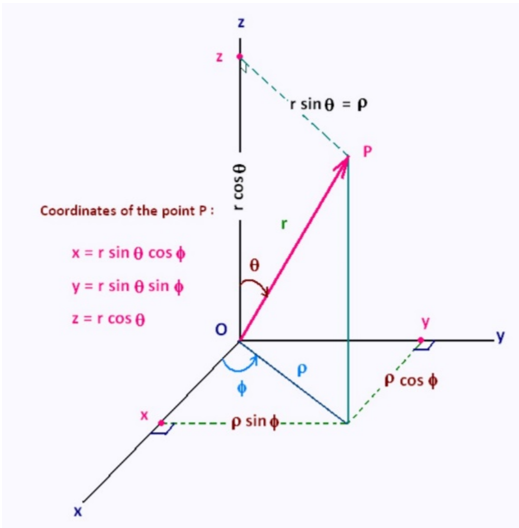


Figure 1.1: Spherical Co-ordinates Systems Representation

$$z = rcos(\theta) \longrightarrow \theta = cos^{-1}(\frac{z}{r}) \tag{1.4}$$

$$x = rsin(\theta)cos(\phi) \longrightarrow \phi = cos^{-1}(\frac{x}{rsin(\theta)}) \tag{1.5}$$

$$y = rsin(\theta)sin(\phi) \longrightarrow \phi = sin^{-1}(\frac{y}{rsin(\theta)}) \tag{1.6}$$

1.2.3 The initial position of the Satellite

To establish the initial position of the satellite orbiting the moon, we must first determine the polar coordinates. Given the Keplerian parameters, such as the semi-major axis (a), eccentricity (e), and true anomaly (θ), we can calculate the magnitude of the vector between the center of the moon and the satellite (r) using the following formula:

$$r = \frac{a(1-e^2)}{1+ecos(\theta)} \tag{1.7}$$

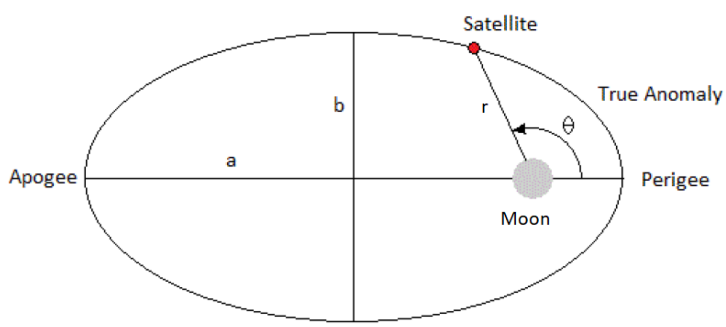


Figure 1.2: Satellite in a Keplerian Orbit and its Parameter

Then, it is necessary to convert the polar coordinates into Cartesian coordinates. To accomplish this, we visualize the vector (r) and assign its components to the X and Y coordinates based on the value θ:

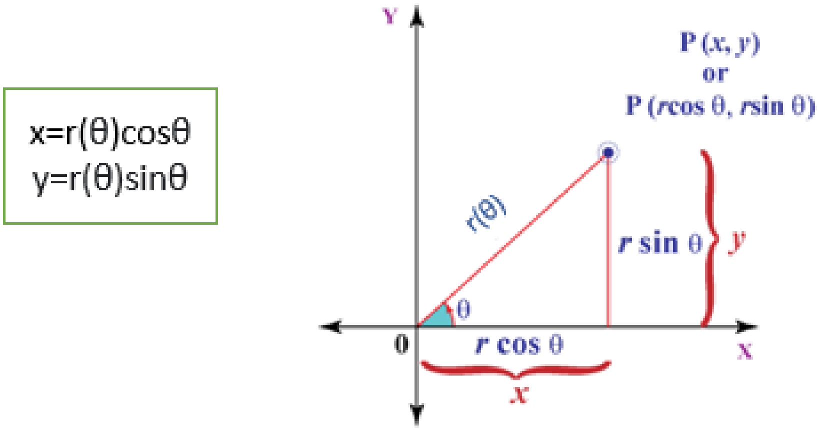


Figure 1.3: Polar Co-ordinates to Cartesian Co-ordinates conversion

To ensure that our orbit lies on the X-Y plane, we set the Z coordinate to zero. If the user specifies a different inclination, we can easily adjust by rotating the plane we generate using the parameters ω, Ω, and i.

To accomplish this, we establish the rotation matrix around the Z-axis, X-axis, and Y-axis using the Rotation matrices based on Keplerian parameters.

Rotation matrix around the x-axis:

$$R_x(\omega) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega) & -\sin(\omega) \\ 0 & \sin(\omega) & \cos(\omega) \end{bmatrix}$$

Rotation matrix around the y-axis:

$$R_y(i) = \begin{bmatrix} cos(i) & 0 & sin(i) \\ 0 & 1 & 0 \\ -\sin(i) & \sin(i) & \cos(i) \end{bmatrix}$$

Rotation matrix around the z-axis:

$$R_z(\Omega) = \begin{bmatrix} \cos(\Omega) & -\sin(\Omega) & 0 \\ \sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Combining the rotation matrices:

$$R(i, \omega, \Omega) = R_z(\Omega) \cdot R_y(i) \cdot R_x(\omega)$$

1.2.4 Plot of the moon

We make the assumption that the moon is a spherical object, and every point on its surface completes a full rotation of 2π radians in a period of 27 days. By dividing 2π by the rotational period of 27 days (converted to seconds), we can calculate the angular speed of the moon.

$$\omega = \frac{2\pi}{T} \tag{1.8}$$

1.2.5 Plot of the orbit

Once we have computed the rotated orbit, we proceed to plot it around the moon. Additionally, it is important to determine the period of the orbit. To achieve this, we utilize the following formula:

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \tag{1.9}$$

Here, G represents the gravitational constant of the moon, and M corresponds to the mass of the moon which are given as follows:

$$G = 6.67428 \times 10^{-11}(m^3 kg^{-1} s^{-2}) \tag{1.10}$$

$$M = 7.34767309 \times 10^{22}(kg) \tag{1.11}$$

1.2.6 Plot of Receiver and Satellite

Having already configured the settings for the receiver and satellite, our task now is to specify the size and color for visualizing the satellite, as well as the size and color for the receiver.

We combine all the plot settings and generate a graphical representation of the Moon, Receiver, and Satellite.

1.3 Satellite and Receiver Motion

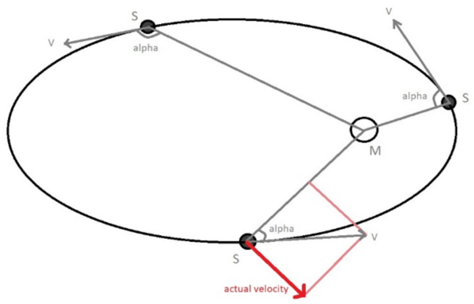
1.3.1 Satellite Motion

To calculate the subsequent position of the satellite at each time step, we begin by determining its linear velocity using the following formula (r is magnitude of position vector ($t = 0$)):

$$v = \sqrt{(G\ M\ (\frac{2}{r} - \frac{1}{a})} \qquad \text{(m/s)} \tag{1.12}$$

Next, we need to calculate the actual linear velocity of the satellite, which represents the magnitude of the velocity vector in the direction of the orbit. This step is necessary due to the elliptical nature of the orbit, where the angle between the velocity vector and the vector connecting the satellite to the center of the moon varies.

Actual velocity is a vector, so it has magnitude and also direction. To calculate the magnitude of the velocity vector and also its direction we need to calculate α which is angle between the vector which connects satellite to the center of the moon and the velocity vector at start time. To calculate this angle, we computed derivative of the position vector at the start time numerically . That determines the direction of instantaneous velocity at start time



$$|\overrightarrow{v_a}| = |\vec{v}|\, sin\, \alpha$$

Figure 1.4: Actual Velocity of revolving Satellite with respect to the Moon

$$\vec{v} = \lim_{\substack{\Delta t \\ \to 0}} \frac{\Delta \vec{r}}{\Delta t} \tag{1.13}$$

Then we create a vector which connects the satellite to the center of the moon and the velocity vector at start time and we compute the angle α :

By taking the inner product of two vectors, we obtain the following result:

$$\overrightarrow{\vartheta_{IV}} \cdot \overrightarrow{\vartheta_{SO}} = |\overrightarrow{\vartheta_{IV}}|.|\overrightarrow{\vartheta_{SO}}|.\cos\alpha \tag{1.14}$$

Then, the value of α, can be computed using the following equation:

$$\alpha = \cos^{-1}\left(\frac{\overrightarrow{\vartheta_{IV}} \cdot \overrightarrow{\vartheta_{SO}}}{|\overrightarrow{\vartheta_{IV}}|.|\overrightarrow{\vartheta_{SO}}|}\right) \tag{1.15}$$

So now we can calculate the following :

$$|\vec{\mathbf{va}}| = |\vec{\mathbf{v}}| \cdot \sin(\alpha) \tag{1.16}$$

Afterwards, we proceed to compute the angular velocity:

$$\omega = \frac{\overrightarrow{|v_a|}}{R} \tag{1.17}$$

Where R represents the radius of the Moon.

We assumed that the magnitude of the satellite's angular velocity is constant during the timestep. Subsequently, we determine the new true anomaly by adding the product of the angular velocity and the time step to the previous true anomaly:

$$\theta_{new} = (\omega \times timestep) + \theta_{old} \tag{1.18}$$

Lastly, we compute the new magnitude of the position vector (r) by using the new true anomaly and its formula mentioned before. We also update α with the method that we already mentioned. Alternatively, we convert the polar coordinates of the new position into Cartesian coordinates and subsequently rotate it based on the provided inclination.
Finally, we repeat this process to obtain the positions of the satellite at different time intervals throughout the duration of the orbit.

1.3.2 Receiver Motion

We calculate the step duration for the receiver by multiplying the angular velocity of the moon by the time step.

$$d = \omega_M \times timestep \tag{1.19}$$

Afterwards, we add the duration step to the previous ϕ angle to obtain the new ϕ angle for the receiver.

$$\phi_{new} = d + \phi_{old} \tag{1.20}$$

Following that, we convert the spherical coordinates to Cartesian coordinates in order to determine the new position of the receiver.

1.4 Line of Sight

The purpose of this section is to calculate the angle between the position vectors of the satellite and the receiver and determine if the satellite is within the line of sight of the receiver. The line of sight is determined based on a threshold angle called in our approach an insight threshold angle (τ) and masking angle (μ)

First, the insight threshold angle is calculated by adding 90 degrees to the masking angle and converting it to radians. The masking angle represents the minimum elevation angle required for the satellite to be considered in sight.

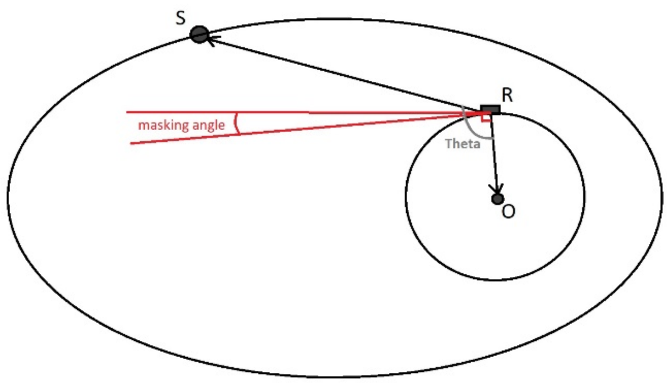


Figure 1.5: Masking Angle Representation

$$\tau = \mu + 90^{\circ} \tag{1.21}$$

By taking the inner product of two vectors, we obtain the following result:

$$\overrightarrow{\vartheta_{RO}} \cdot \overrightarrow{\vartheta_{RS}} = |\overrightarrow{\vartheta_{RO}}|.|\overrightarrow{\vartheta_{RS}}|.\cos\theta \tag{1.22}$$

Then, the value of theta(θ), can be computed using the following equation:

$$\theta = \cos^{-1}\left(\frac{\overrightarrow{\vartheta_{RO}} \cdot \overrightarrow{\vartheta_{RS}}}{|\overrightarrow{\vartheta_{RO}}|.|\overrightarrow{\vartheta_{RS}}|}\right) \tag{1.23}$$

Finally, we check if the calculated angle between the satellite and the receiver is greater than the specified insight threshold angle. If the angle is greater, it indicates that the satellite is within the line of sight of the receiver. In this scenario, the receiver will be displayed in green. Conversely, if the angle is not greater than the insight threshold angle, it signifies that the satellite is not within the line of sight of the receiver. In this case, the receiver will be displayed in red.

$$\theta > \tau \longrightarrow \text{within the LoS} \tag{1.24}$$

$$\theta < \tau \longrightarrow \text{not within the LoS} \tag{1.25}$$

1.5 Pseudorange and Pseudorange Rate

1.5.1 Pseudorange

Pseudorange is an estimated distance between a satellite and a receiver in a satellite navigation system. It is derived by measuring the time it takes for the satellite's signal to reach the receiver and multiplying it by the speed of light. The pseudorange (PR) formula can be expressed in terms of the time duration that it takes for the signal to travel the distance between satellite and receiver (t_{rec}), offset of the receiver clock from the system time (t_u), offset of satellite clock from system time (advance is positive, delay is negative) (δt), and the speed of light (c) as follows:

$$PR = c(t_{rec} + t_u - \delta t) \tag{1.26}$$

In this formula:

- t_{rec} can be calculated by dividing the distance between the satellite and receiver by the speed of light

$$t_{rec} = \frac{D}{c} \tag{1.27}$$

To determine the distance, we should compute the following equation:

$$D = \sqrt{(x, y, z)^2_{rec} - (x, y, z)^2_{sat}} \tag{1.28}$$

1.5.2 Pseudorange Rate

The pseudorange rate is the rate at which the measured distance between a satellite and a receiver in a navigation system is changing over time. It provides information about the relative velocity or speed of the object being tracked. By calculating the change in the pseudorange over a specific time interval, the receiver can determine how fast the object is moving. In our approach, we didn't consider the Doppler effect.

$$\text{Pseudorange Rate} = \frac{\text{Change in Pseudorange}}{\text{Time step}} \tag{1.29}$$

1.6 Coverage and Constellation

The purpose of this part of the report is to evaluate the coverage and constellation of satellites orbiting the Moon. The evaluation aims to achieve two types of coverage: full coverage of the entire Moon's surface and full coverage of the Moon's south pole. This part presents an overview of the methodology used for satellite coverage evaluation, including the concept of coverage, satellite positions, receiver positions, and visibility calculations.

1.6.1 Coverage Concept

The coverage concept refers to the ability to have a sufficient number of satellites in view at all times during the orbital period to ensure accurate Position, Velocity, and Time (PVT) calculations. Full coverage of the entire Moon's surface and the Moon's south pole is desired to enable comprehensive monitoring and communication capabilities.

1.6.2 Satellite Positions

By calculating the satellite positions over time, taking into account the rotational motion of the Moon, an accurate depiction of satellite coverage can be achieved. To create a map of the Receiver's positions on the surface of the moon, we can define specific ranges of latitude and longitude. By specifying a starting degree and an ending degree for both latitude and longitude, we can generate a grid-like structure similar to Figure [1.8]. Next, we can compute the Cartesian product of these latitude and longitude matrices, resulting in a set of coordinates that represent the intersection points. These intersection points correspond to the positions of the Receiver on the moon's surface.

1.6.3 Receiver Positions

The accurate positioning of receivers on the Moon's surface is essential for assessing satellite visibility. In determining receiver positions, factors such as latitude and longitude ranges are considered. These positions are computed over time, accounting for the rotational speed of the Moon. Receiver positions enable the identification of satellites in view from specific locations on the lunar surface. It is important to note that when designing orbits around the moon for this particular task, extensive research [1]. After analyzing the article, we concluded that we need to make modifications to the program so that we can position any desired number of orbits, as well as accommodate multiple satellites and receivers.

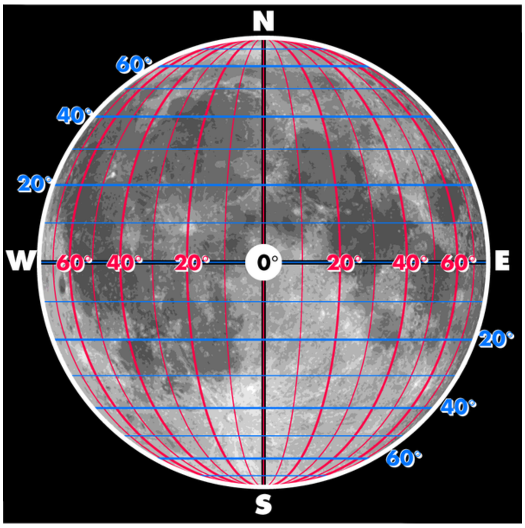


Figure 1.6: Receiver Position on the Moon

1.7 Satellite Visibility Calculation

To determine the satellite visibility by the receiver, we aim to ascertain when the receiver will fall within the propagation cone of a satellite's antenna.

In order to calculate this, we need to first determine the value of ζ zeta, which represents the angle formed between the slant height of the cone and the radius of the moon that is connected to it. The computation of ζ is accomplished by employing the Law of Sines.

Once ζ has been determined, we can proceed to assess whether the receiver falls within the satellite's antenna cone. This evaluation is based on the β angle refers to the angle created by the connected vector between the receiver and satellite, and the connected vector between the receiver and the center of the Moon. For satellite visibility to occur, two conditions must be met. Firstly, the β angle should be smaller than 180 degrees but greater than ζ. This indicates that the satellite and receiver are within each other's line of sight.

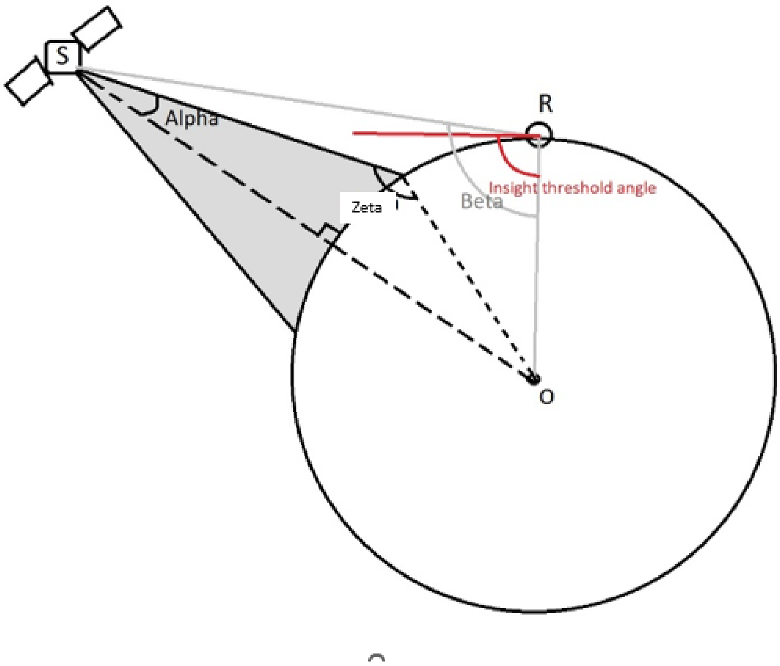


Figure 1.7: Satellite and Receiver Positions on the Moon

$$\theta < \beta < 180 \tag{1.30}$$

Additionally, as previously explained, there is another angle known as the "Insight threshold angle." This angle is the sum of the masking angle and 90 degrees. In order for satellite visibility to be achieved, the β angle must also exceed this Insight threshold angle.

$$\tau < \beta \tag{1.31}$$

To summarize, in order for the receiver to have satellite visibility, the β angle must be less than 180 degrees and greater than the maximum value between θ and the Insight threshold angle. By considering these conditions, we can determine whether the receiver is positioned favorably within the satellite's propagation cone for effective communication and data transmission.

$$\max(\theta, \tau) < \beta < 180 \tag{1.32}$$

Chapter 2

Design results

2.1 General orbit design

During our project, we utilized MATLAB software to efficiently design the motion of receiver orbits and the rotation of the Moon.Through numerical modeling functions, we explored various orbital parameters and their impact on the Moon's rotation.This approach enhanced our understanding of the system's behavior and aided in determining optimal parameters for receiver orbit motion, ensuring effective communication. To implementation and analysis, we operated under specific assumptions that provided a framework for our work, defining project parameters and conditions.

Here, we present a brief introduction to the key assumptions guiding our project:

- We assume that the center of the moon is at the origin of the Cartesian Coordinate System, and the receiver is fixed on the moon's surface.
- The starting time is set to zero (START_TIME = 0).
- The radius of the moon is 1,737,500 meters.
- The moon rotates around the Z-axis at a constant speed.
- The initial orbit is on the X-Y plane.

- The running process will finish after one orbital period, but you can modify the ending time as needed.
- With these assumptions, you need to provide the program with the following inputs:
  - The initial geodetic coordinates of the receiver in degrees.
  - The initial true anomaly of the satellite in radians.
  - Keplerian parameters 'a' (in kilometers) and 'e'.
  - Satellite orbit inclination around the X-axis, Y-axis, and Z-axis in degrees.
  - Time step, transmission time, delta time (all in seconds), and the masking angle in degrees.
- Note that the receiver color will turn green when the satellite is in sight and red when it is not in sight of the receiver.

When using our orbital propagator, we visualized the results in the form of a graphical interface. Each point on the graph represented a certain position of the receiver relative to the satellite in space.

When the dot turned red, it indicated that the recipient was out of the satellite's field of view, whereas the green dot meant that the recipient was in the satellite's field of view.

Below is a table with three cases of input values, which contains information about the parameters of three different orbits that were considered in our study.

The graphical representation below vividly demonstrates how changing parameters such as the shape and angle of an orbit directly impact various orbital characteristics. These changes result in variations in the orbital period, orbital speed, and the distance

Table 2.1: Input values from user side for orbit propagator

Lat_0_R	Long_0_R	true_anomaly_0	a (m)
0	0	3.14	6000000
15	15	0	6000000
60	75	4.7	7000000

e	W	I	Omega
0.6	30	45	0
0.5	10	20	30
0.7	30	45	30

time_step	t_u	δt	Masking_Angle
60	0	0	10
60	0	0	10
60	0	0	10

from the central body. By altering these parameters, the shape of the orbit can be transformed, leading to adjustments in the periapsis distance (the point of closest approach to the central body) and the apoapsis distance (the point of farthest distance from the central body). Furthermore, modifying the orbit's shape and angle affects the amount of time spent in different parts of the orbit, creating dynamic patterns of orbital motion. Thus, through visual analysis, we can observe the intricate relationship between parameter adjustments and their consequential effects on fundamental aspects of orbital dynamics.

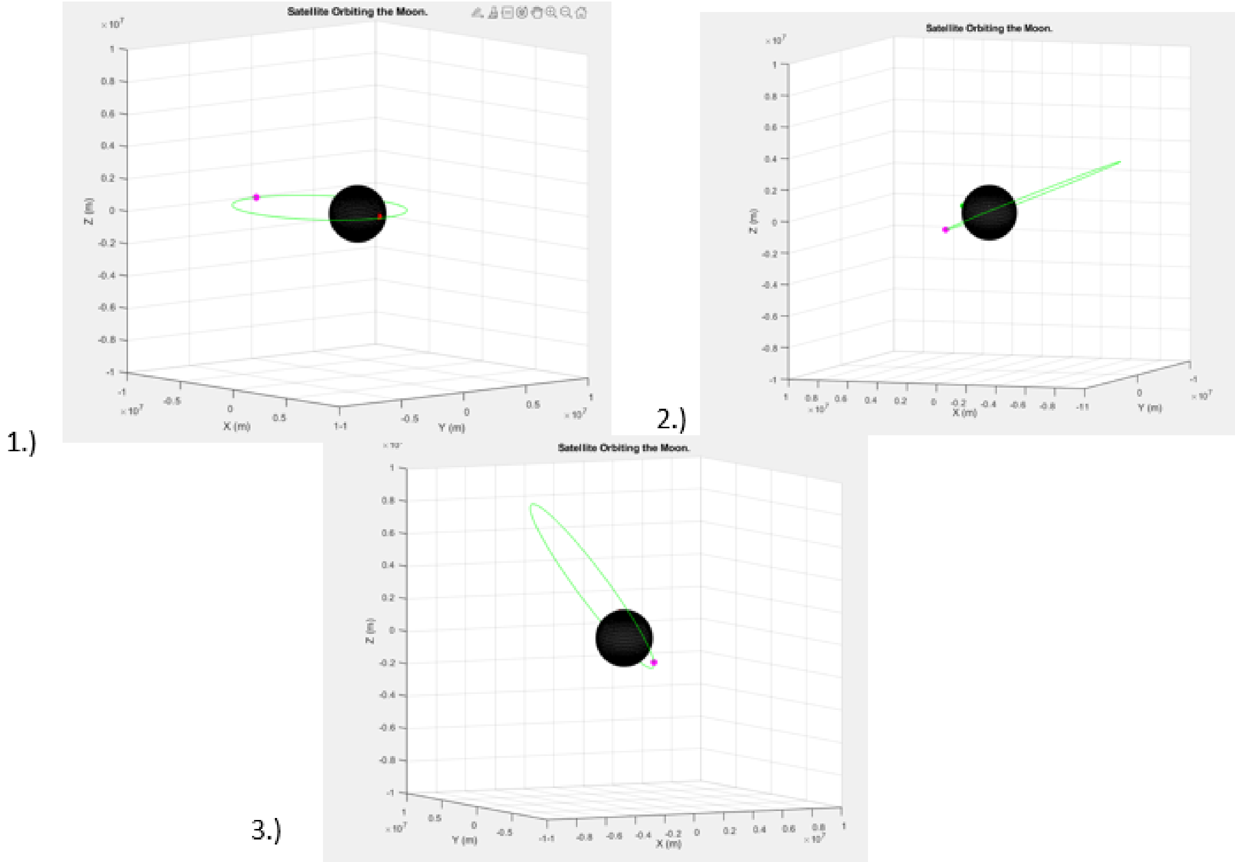
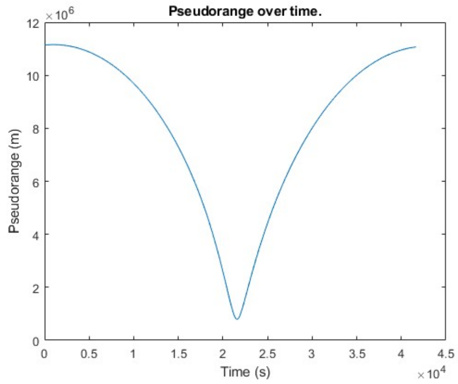


Figure 2.1: Graphical representation of different orbits

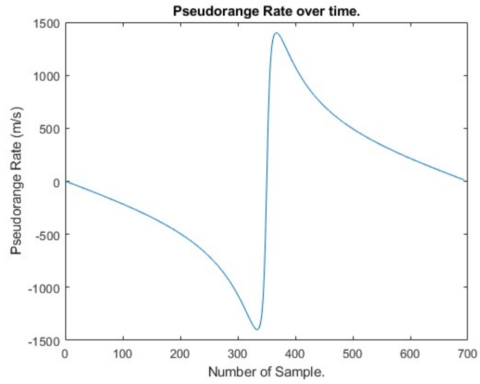
By measuring the pseudorange from multiple satellites, a receiver can perform trilateration to determine its position on Earth. By analyzing the pseudorange rate from multiple satellites, the receiver can calculate its velocity and direction of movement.

Below you can find plots of orbit design and constellation geometry influence the shape and distribution of pseudorange measurements.

The inclination of an orbit influences the behavior of satellites within that orbit. It refers to the angle between the orbital plane of a satellite and the Moon's equator. The inclination determines the path of the satellite, its coverage of different regions, visibility from specific locations, orbital period, and constellation design.



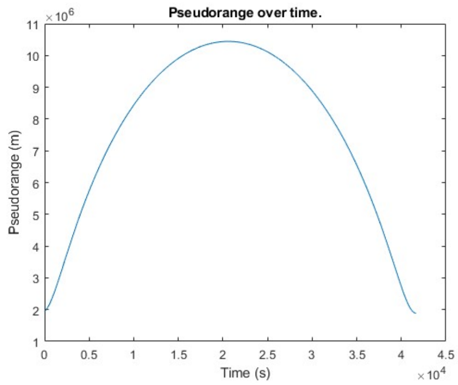
(a) Pseudorange over time for orbit 1



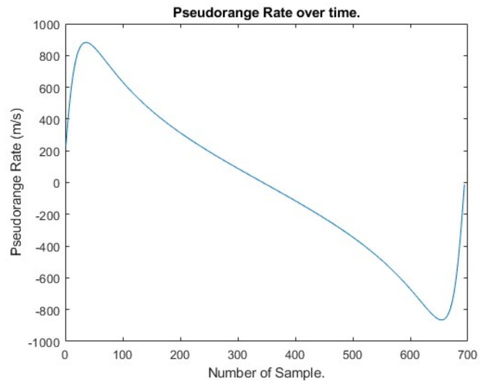
(b) Pseudorange rate over time for orbit 1

Figure 2.2: Pseudorange and Pseudorange rate over time for orbit 1

Understanding the inclination is crucial for optimizing satellite operations and applications.



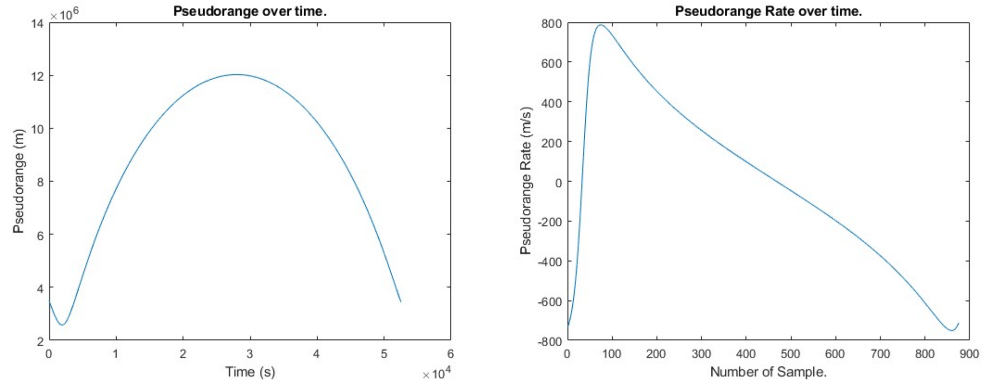
(a) Pseudorange over time for orbit 2



(b) Pseudorange rate over time for orbit 2

Figure 2.3: Pseudorange and Pseudorange Rate over Time for orbit 2

We created a simulation of a lunar satellite and receiver system, where the receiver is located on the Moon's surface and rotates with the Moon's rotation. Additionally, we implemented a dynamic receiver model, allowing for more realistic tracking and positioning. To provide a visual representation, a graphical plot of the scenario was generated, depicting the satellite's orbit and the receiver's position on the Moon's surface.



(a) Pseudorange over time for orbit 3 **(b)** Pseudorange rate over time for orbit 3

Figure 2.4: Pseudorange and pseudorange rate over time for orbit 3

In the next step, before discussing constellations, it is important to highlight that our designs have successfully passed our goal of taking coverage close to 100 percent. This validation ensures that every user within a specific area of the mobile satellite system will always have a minimum visibility of at least four satellites. With south pole we achieve 95 and 100 % respectfululy.

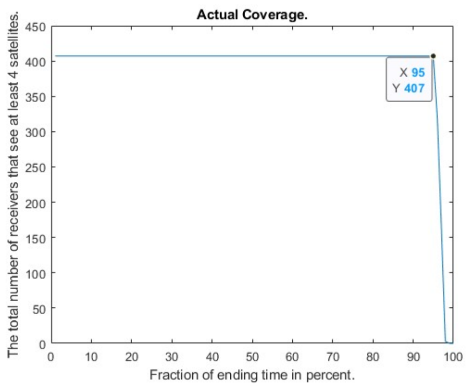


Figure 2.5: South pole

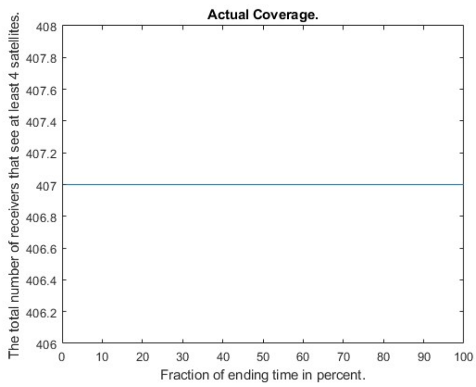


Figure 2.6: South pole rectangular configuration

Figure 2.7: Actual coverage test South pole

Plus for full Moon we reached 100 and 100 % respectfully coverage

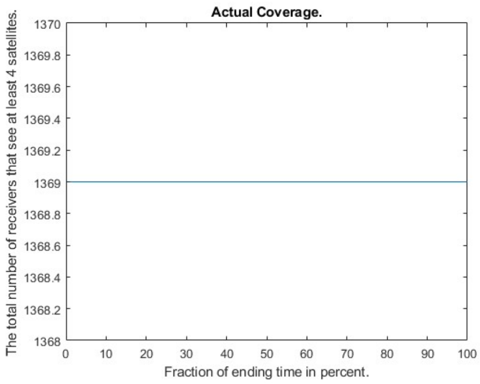


Figure 2.8: Whole moon

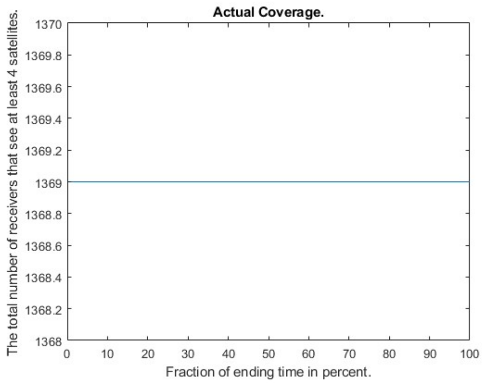


Figure 2.9: Moon rectangular configuration

Figure 2.10: Actual coverage Moon test

2.2 South pole coverage configuration ELFO

It is important to clarify that in our research, we approached the South Pole of the Moon as a region rather than a specific point. This region spans from 80 to 90 degrees south latitude. While conducting our analysis, we took into account the unique characteristics and challenges associated with this particular area. This approach allowed us to assess the feasibility and potential implications of our satellite system design across a range of latitudes, including the order if we were interested in a particular area present at the South Pole.

For our first constellation design, we implemented the Elfo constellation, which utilizes a specific orbit design to achieve optimal coverage and performance. The ELFO constellation consists of satellites arranged in a carefully planned orbital configuration. Elliptical Lunar Frozen Orbits (ELFO) refer to a specialized type of orbit configuration around the Moon that combines characteristics of elliptical orbits and frozen orbits. In an ELFO, a spacecraft follows an elliptical path with varying distances from the lunar surface while maintaining a relatively fixed orientation with respect to the Moon. This constellation design has been previously studied and proposed by researchers in the field [1] , further validating its effectiveness.

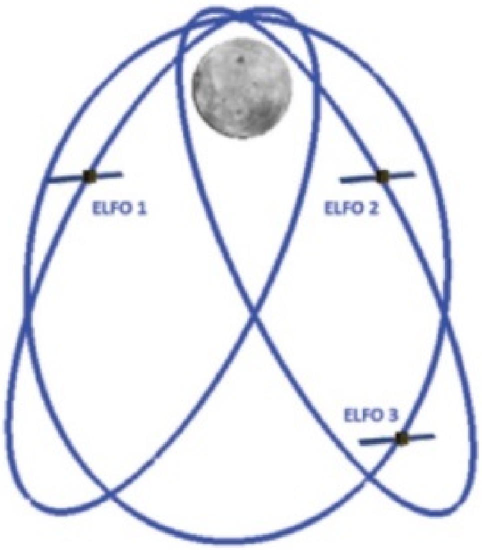


Figure 2.11: Elliptical Lunar Frozen Orbits (ELFO) with one satellite in each orbital plane [1]

In our constellation data, we have designed a satellite constellation consisting of 12 satellites and a network of 407 receivers. The constellation operates with a time step of 300 seconds, covering a total duration of 172,650 seconds. The data provided includes various parameters and configurations for the constellation design and receiver network.

2.2.1 Constellation and Receiver Data

- Time Step: 300 seconds
- Ending Time: 172,650 seconds

- Masking Angle: 0 degrees
- Satellite Antenna Cone Angle: 90 degrees
- t_u : 0 seconds
- δt: 0 seconds
- Number of Satellites: 12
- Number of Receivers: 407
- Receiver Latitude Start Point: -90 degrees
- Receiver Latitude End Point: -80 degrees
- Receiver Latitude Resolution: 1 degree
- Receiver Longitude Start Point: -180 degrees
- Receiver Longitude End Point: 180 degrees
- Receiver Longitude Resolution: 10 degrees

In our constellation design, we implemented a configuration that utilizes three orbits for positioning the satellites symmetrically on each side of the orbit. To achieve this symmetrical arrangement, we made use of a parameter called TA0 (in radians) to adjust the satellite positions within their respective orbits.

Below, you can find a detailed table with data for each satellite in the constellation. Each row represents a specific satellite and provides the corresponding values for the semi-major axis (a), eccentricity (e), inclination (I), the argument of perigee (ω), right ascension of the ascending node (Ω), and initial true anomaly (TA_0). This comprehensive information enables a thorough understanding of the orbital characteristics and positioning of each satellite, essential for accurate analysis and simulations of their trajectories for running our Matlab program.

Table 2.2: ELFO constellation satellite input

a	e	I	ω	Ω	TA_0
9750500	0.7	63.5	90	0	0
9750500	0.7	63.5	90	0	1.57
9750500	0.7	63.5	90	0	3.14
9750500	0.7	63.5	90	0	4.71
9750500	0.7	63.5	90	120	0
9750500	0.7	63.5	90	120	1.57
9750500	0.7	63.5	90	120	3.14
9750500	0.7	63.5	90	120	4.71
9750500	0.7	63.5	90	240	0
9750500	0.7	63.5	90	240	1.57
9750500	0.7	63.5	90	240	3.14
9750500	0.7	63.5	90	240	4.71

As observed in the figure below, the constellation configuration ensures that the majority of the time, each receiver has visibility to at least three or four satellites. In plots below colours represents receivers.

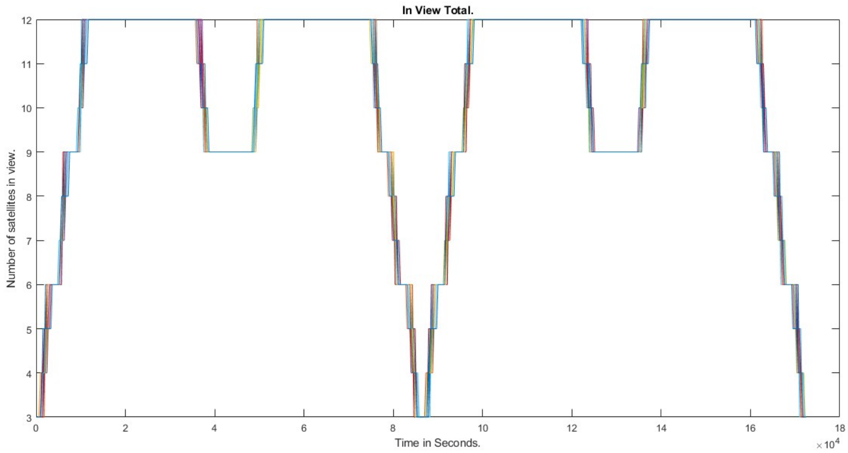


Figure 2.12: Elliptical Lunar Frozen Orbits (ELFO) visibility

2.3 South pole rectangular configuration

In our pursuit of designing an optimized constellation, we explored various configurations for the satellites on each orbit. One particular configuration we investigated is known as the rectangular configuration. This arrangement involves placing four satellites evenly spaced on each orbit, creating a symmetrical pattern. To achieve this configuration, we utilized Keplerian parameters and calculated the initial true anomaly of the satellites accordingly.

After determining the initial true anomalies based on the rectangular configuration, we conducted extensive testing to evaluate the performance and effectiveness of the constellation. This involved simulations and analysis to assess factors such as coverage, connectivity, and signal strength. By iteratively adjusting and refining the initial true anomalies, we aimed to find a constellation arrangement that maximizes the overall performance and minimizes potential interference or coverage gaps.

Table 2.3: Rectangular configuration constellations satellite input

a	e	I	ω	Ω	TA_0
9750500	0.7	63.5	90	0	0
9750500	0.7	63.5	90	0	1.57
9750500	0.7	63.5	90	0	3.14
9750500	0.7	63.5	90	0	4.71
9750500	0.7	63.5	90	120	2.0944
9750500	0.7	63.5	90	120	3.6652
9750500	0.7	63.5	90	120	5.2359
9750500	0.7	63.5	90	120	0.5236
9750500	0.7	63.5	90	240	4.1888
9750500	0.7	63.5	90	240	5.7596
9750500	0.7	63.5	90	240	1.0472
9750500	0.7	63.5	90	240	2.6179

Through process of development , we strived to find a better constellation configuration that provides optimal coverage and connectivity for our intended applications. .

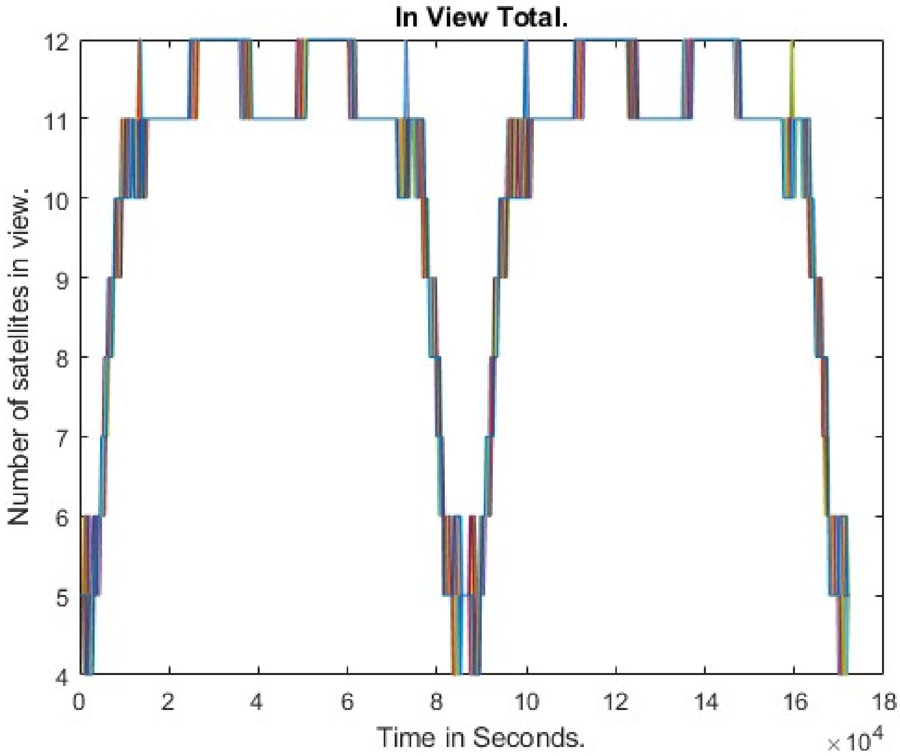


Figure 2.13: Elliptical Lunar Frozen Orbits (ELFO) rectangular visibility

The rectangular configuration of satellites was created to ensure that each receiver always has a view of at least four satellites. By evenly spacing four satellites on each orbit, we aimed to achieve good coverage and connectivity for our constellation.

2.4 Full moon coverage

To achieve full moon coverage with our constellation, we have implemented the same ELFO (Elliptical Lunar Frozen Orbits) configuration in both the northern and southern hemispheres, consisting of 3 orbits each. The main modification is adjusting the argument of perigee to -90 degrees. Additionally, we have included NRHO (Near-Rectilinear Halo Orbit) in both the north and south regions.

In the study, a DRO (Distant Retrograde Orbit) at a distance of

45,000 km from the moon was considered, as shown in Figure 3.9. The orbit has an inclination of 0 degrees in the XYZ coordinate system.

Furthermore, the NRHO is positioned 70,000 km above the South Pole area. It has an eccentricity of 0.8 and a perigee range of ±90 degrees. The NRHO orbits are rotated only in the Y matrix.

In summary, our constellation comprises a total of 9 orbits, including the ELFO orbits in the north and south (6 orbits) and the NRHO orbits in the north and south regions and dro.

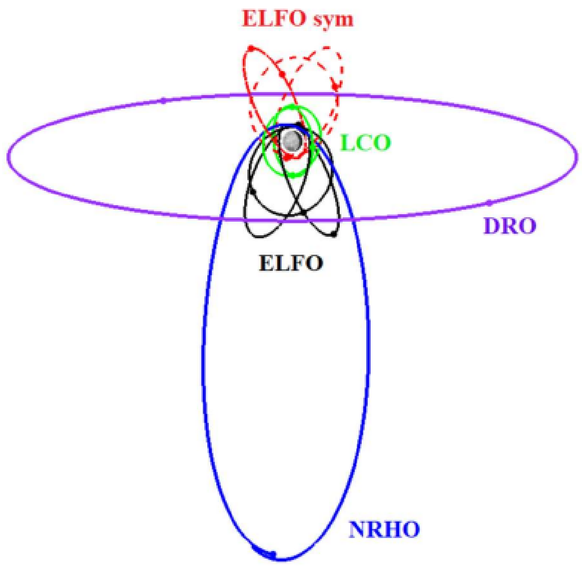


Figure 2.14: Different Lunar Orbits which are considered as potential orbits for a Lunar Navigation Satellite System (Citation 1)

In our constellation data, we have designed a satellite constellation consisting of 36 satellites and a network of 1369 receivers. The constellation operates with a time step of 300 seconds, covering a total duration of 172,650 seconds. The data provided includes various parameters and configurations for the constellation design and receiver network.

2.4.1 Constellation and Receiver Data

- Time Step: 300 seconds
- Ending Time: 172,650 seconds
- Masking Angle: 0 degrees
- Satellite Antenna Cone Angle: 90 degrees
- t_u : 0 seconds
- δt: 0 seconds
- Number of Satellites: 36
- Number of Receivers: 1369
- Receiver Latitude Start Point: -90 degrees
- Receiver Latitude End Point: 90 degrees
- Receiver Latitude Resolution: 5 degrees
- Receiver Longitude Start Point: -180 degrees
- Receiver Longitude End Point: 180 degrees
- Receiver Longitude Resolution: 10 degrees

Below, you will find a table summarizing the data for each satellite in our constellation, including information on the NRHO, DRO. For ELFO, please refer to the previous subsections for a detailed explanation of each orbit's parameters and configurations.

As depicted in the figure below, it is evident that there are a minimum of six satellites within the line of sight for each receiver in our constellation. This observation leads us to assume that we have successfully passed the validation test. The presence of six or more satellites ensures that there is adequate coverage and connectivity for our communication network. With such a

Table 2.4: Full moon coverage

a	e	I	ω	Ω	TA_0
45000000	0.7	0	0	0	0
45000000	0.7	0	0	0	1.57
45000000	0.7	0	0	0	3.14
45000000	0.7	0	0	0	4.71
70000000	0.7	0	90	0	0
70000000	0.7	0	90	0	1.57
70000000	0.7	0	90	0	3.14
70000000	0.7	0	90	0	4.71
70000000	0.7	0	-90	0	0
70000000	0.7	0	-90	0	1.57
70000000	0.7	0	-90	0	3.14
70000000	0.7	0	-90	0	4.71

configuration, we can expect reliable and continuous communication services for all receivers across the designated coverage area. But we can assume that we can reduce the cost of optimizing that design.

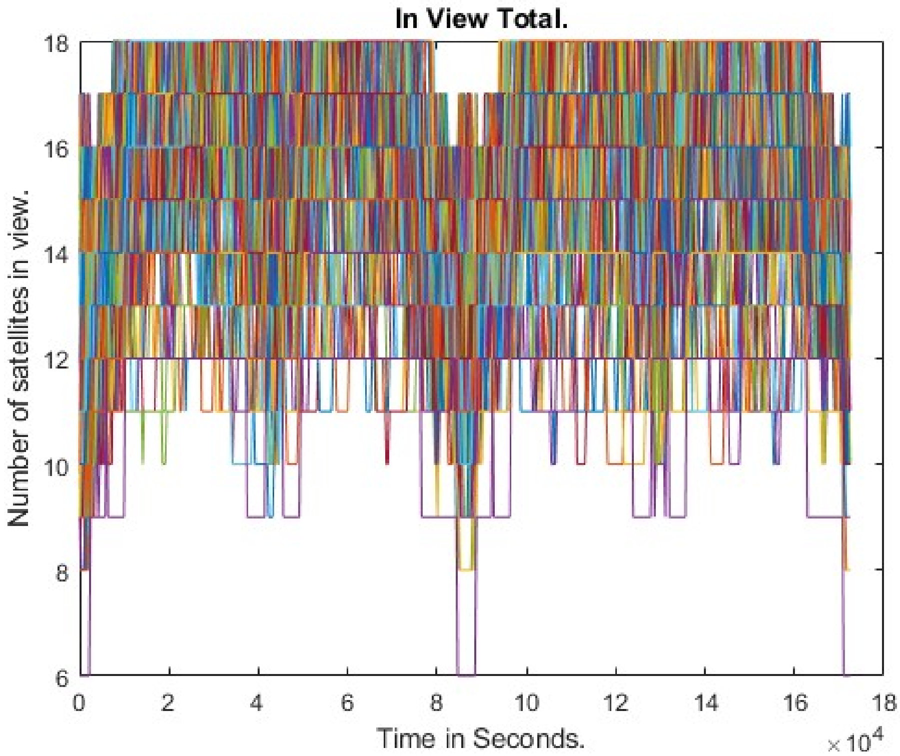


Figure 2.15: Full moon coverage visibility

We deeply believe that using different natures of orbits can offer significant benefits for various navigation purposes. The utilization of diverse orbital configurations, such as the ELFO, NRHO, and DRO, allows us to cater to specific requirements and optimize the performance of our satellite constellation. Each orbit type possesses unique characteristics that make it well-suited for specific applications. By leveraging these different orbits, we can enhance navigation capabilities, ensure global coverage, and address the diverse needs of users in different regions.

2.5 Full moon coverage rectangular configuration

In addition to the ELFO, NRHO, and DRO orbits, we also implemented a rectangular configuration for achieving full moon coverage. The table below provides detailed information about the constellations, including the orbital parameters and configurations utilized for achieving full moon coverage.

Table 2.5: Recatngular Configuration

a	e	I	ω	Ω	TA_0
45000000	0.7	0	0	0	0
45000000	0.7	0	0	0	1.57
45000000	0.7	0	0	0	3.14
45000000	0.7	0	0	0	4.71
70000000	0.7	0	90	0	0
70000000	0.7	0	90	0	1.57
70000000	0.7	0	90	0	3.14
70000000	0.7	0	90	0	4.71
70000000	0.7	0	-90	0	0
70000000	0.7	0	-90	0	1.57
70000000	0.7	0	-90	0	3.14
70000000	0.7	0	-90	0	4.71

In the figure below, we observe that the minimum number of satellites in view for each receiver has increased from 6 to 7 when using the rectangular configuration. This notable improvement suggests that the implementation of the rectangular configuration has significantly enhanced the performance of our constellation.

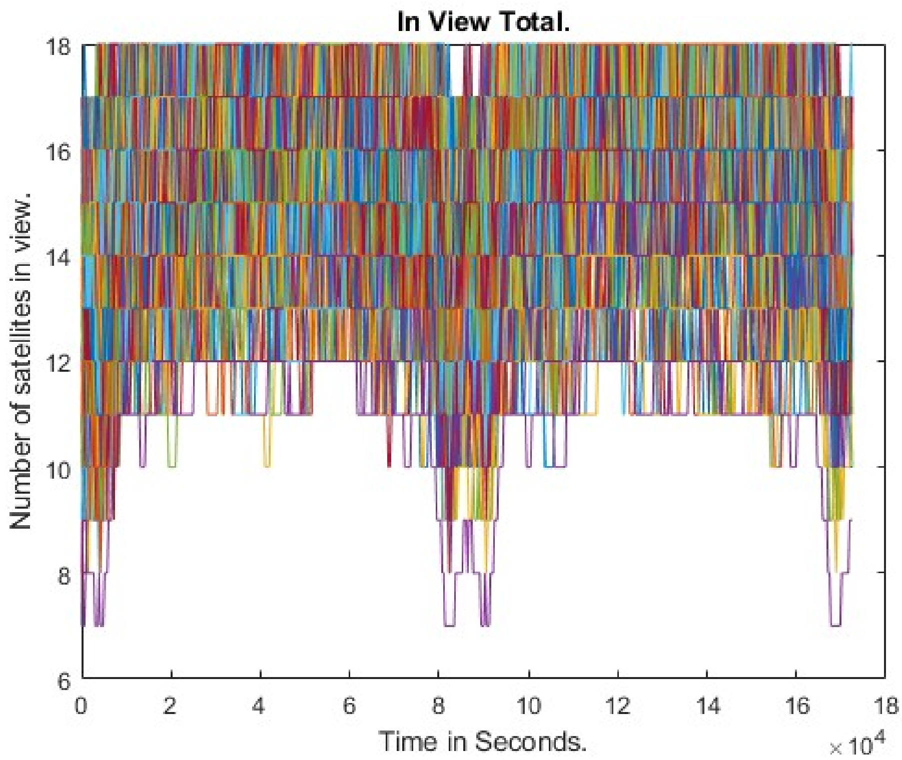


Figure 2.16: Full moon coverage with rectangular configuration visibility

Although the minimum number of satellites in view for each receiver has increased to 7 with the rectangular configuration, we acknowledge that this may still be considered a relatively high number. However, it is important to emphasize that the primary goal of our constellation design was to ensure functionality and reliable coverage.

2.6 Future Work

Moving forward, there are several important areas to focus on in order to enhance satellite constellation design. These efforts aim to overcome challenges, improve performance, and expand the capabilities of constellations. The following areas should be considered for future work:

1. **Exploring Advanced Constellation Configurations:** We want to highlight that in our aproach we have the way to many sattelites and we want to find another constelation with less ammount of sattelites.To improve performance and coverage, it's important to investigate different ways of arranging satellites. Plus we need to consider that our moon is not a sphere its geoid, so our model need to be appreoved .

2. **Elimination of any possible collisions** A modification to the original Elliptical Lunar Frozen Orbits (ELFO) by altering the eccentricity, initial true anomaly, and other parmeters of the orbits can be introduced. This adjustment can be made to eliminate any collision points between the orbits and minimize the chances of disturbances caused by overlapping orbits.

2.7 Conclusion

In conclusion, this analysis focuses on the important aspects of designing a navigation satellite system for the Moon, which is valuable for both, space agencies and commercial purposes.The concept of lunar navigation is introduced, involving three phases with the ultimate goal of a global navigation system for the Moon. Even with a small number of satellites, a significant portion of the Moon's surface can be covered effectively, serving various users in different locations. Additionally, we determined the high-level requirements for ranging satellites, allowing for the identification of antenna needs in future analyses and defining the navigation payload.

2.8 References

[1] Schonfeldt, M., Grenier, A., Delépaut, A., Giordano, P., Swinden, R., Ventura-Traveset, J., Blonski, D. and Hahn, J., 2020, November. *"A system study about a lunar navigation satellite transmitter system"*. In 2020 European Navigation Conference (ENC) (pp. 1-10). IEEE.

2.9 Work Distribution

Name	**Responsibilities**
Gauri Pravishi	Theoretical Concepts , Overleaf Design
Yelyzaveta Pervysheva	Developing Matlab Orbit Propagator, Orbit Design, Data Standardization, Result chapter of the Report Overleaf
Sanaz Baradaran Rowhani	General Conceptualization
Seyedali Hosseinishamoushaki	Summarizing Report, Mathematical Concepts, Plots of Concept
Mir Mohammad Reza Heidarzad Namin	Development of orbit propagator on Matlab, Mathematical Calculations, Measurement of Design Performance