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Stochastic inversion of fire test data for the T-dependant thermal diffusivity of SA pine

by

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Project(Civil Engineering)458

Project Proposal

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Nomenclature

Constants

$g = \quad 9.81\,\mathrm{m/s^2}$

Variables

κ	Thermal conductivity	[W/m·K]
α	Thermal diffusivity	[$\mathrm{m^2/s}$]

Chapter 1

Introduction

1.1 Background and Motivation

Traditionally the thermal diffusivity, otherwise referred to as the α-value, of timber is based simply off the EN 1995:1-1-2004 or similar standards. This research project will aim to obtain the thermal diffusivity of cross laminated SA-Pine timber by further analysing data obtained by S van der Westhuyzen for his study of the samples' charring rate.

The thermal diffusivity of timber is a unobservable quantity that cannot be measured by itself, instead it is related to measurements of temperature and time through differential models. When heat diffusion is calculated using Finite Element methods the process is usually simplified to a linear problem (Fish, 2007). Due to the changes in thermal diffusivity of timber with temperature, as can be seen in EN 1995:1-1-2004(pg number TODO), the diffusivity cannot be linearly modelled. Therefore the problem lends itself to being analysed by inversion techniques. The aforementioned approach will allow us to obtain information about the diffusivity based on the combination of the information assumed prior to measuring, further referred to as the prior, and the measured data. Using statistical inversion leads to a probability distribution that provides us with a collection of diffusivity estimates and their corresponding probabilities.

1.2 Aim and objectives

During the course of the project the student will aim to meet the following objectives:

1. Modify a Finite Element Model into an accurate and effective function.
2. Compare the model data to the actual acquired data.

3. Solve for the thermal diffusivity using Bayes' theorem of inverse problems
4. Evaluate and explore the posterior probability distribution using the following methods:
   (a) Maximum a Posteriori
   (b) Markov-Chain Monte Carlo

1.3 Program

Chapter 2

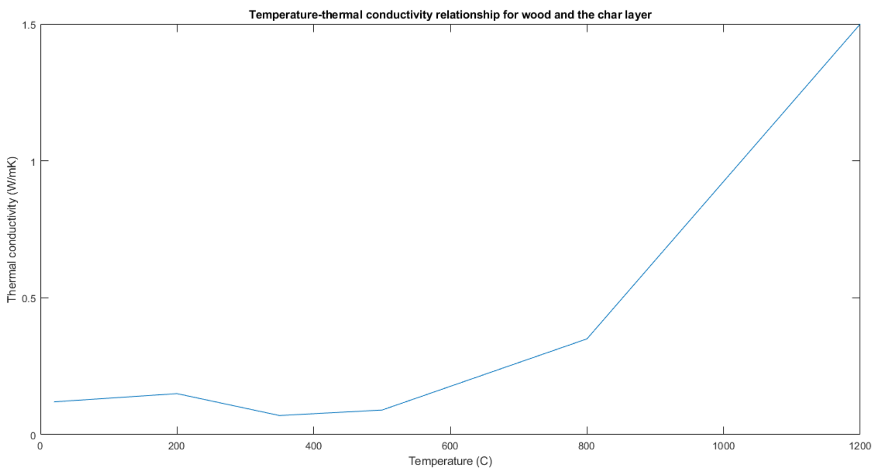
Literature Review

2.1 Introduction

This section aims to summarise the current body of knowledge available on the topic. The relevance to this information and the report will also be indicated.

2.2 K- value

The current K-values used for the design of timber elements are taken from the EURO code (ref TODO (CEN, 2004))



2.3 Bayes' theorem of inverse problems

The method of statistical inversion is dependant on a fundamental understanding of the Bayes' theorem of inverse problems. The student obtained this understanding through studying Chapter 3 of Statistical and Computational Inverse problems by Kaipio and Somersalo (2005), further referred to merely as Kaipio. There are four principles of Statistical inversion that is essential to the thorough understanding of these models. Firstly it is the principle that any variable in the model needs to be modelled as a random variable. This randomness is based on the extent of information that is available. To ensure that the extent of knowledge is accurately portrayed in the model the extent of knowledge will be coded into the probability distributions assigned to the different variables. Finally it needs to be understood that the solution of a statistical inversion is a posterior probability distribution. A generalized equation of Bayes' theorem can be seen in 2.3.1 taken from Kaipo.

$$\pi_{\text{post}}(x) = \pi(x|y_{\text{observed}}) = \frac{\pi_{\text{pr}}(x)\pi(y_{\text{observed}}|x)}{\pi(y_{\text{observed}})} \tag{2.3.1}$$

2.4 Heat diffusion equation

In it's simplest form the one dimensional heat diffusion equation is a partial differential equation 2.4.1 dependant on the temperature and thickness of the element. The heat diffusion equation is based on Fourier's Law

$$q = -k\frac{dT}{dx} \tag{2.4.1}$$

Chapter 3

Finite Element Modelling

3.1 Existing Model

For this project an existing finite element model of heat diffusion by Prof. N de Koker will be modified for usage in the Bayes' theorem 2.3.1. This model will be used to determine the likelihood function. The current model uses the standard euro code k-values as well as the specific heat specified in the (CEN, 2004).

The model descritizes the wooden element into 32 different elements. For finite element analysis there are always more elements used to generate the model than usually evaluated. This is done to improve the accuracy of said model. The model is a one dimensional finite element model that takes time differentiation into account.

3.2 Adapted Model

The model was changed into a function that takes k-values and provides a new temperature distribution over the elements for the different k-values. This function is used in the posterior calculation to determine the likelihood function.

Chapter 4

Measured Data

4.1 Measured Data

4.1.1 Summary of test

The data used was acquired by van der Westhuyzen *et al.* (2020) for an article assessing the charring rate of both SA-Pine and Eucalyptus. For the purpose of this project only the data obtained from the SA-Pine test was considered and analysed. The test sample was a 100 mm by 0.9m x 0.9m panel of cross-laminated SA-pine, this sample was then divided into nine cubes of 100 mm x 100 mm x 100 mm. Each cube was fitted with seven Type K-thermocouples placed at consecutive 16.5 mm drilled holes. The test panel was tested in a furnace and was exposed to the standard ISO 834 Fire curve 4.1.1 on one side and room temperature on the other. The panel was exposed to the fire curve for 50 minutes at which stage near complete de-lamination was observed and the test ended.

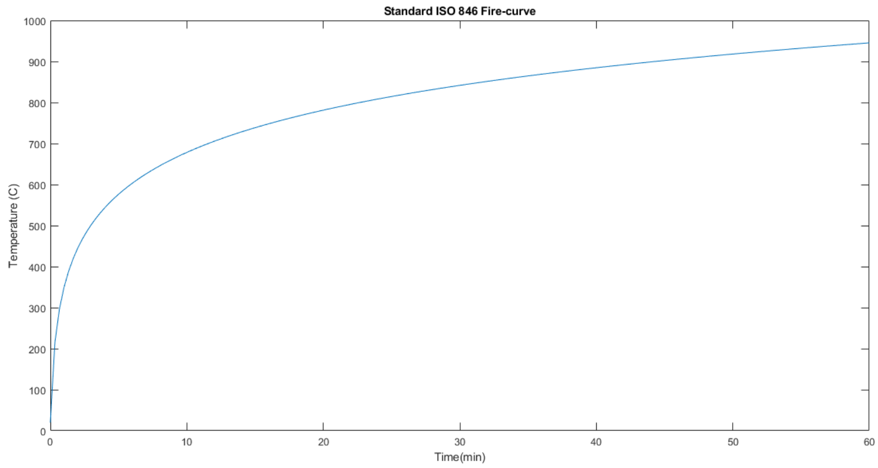


Figure 4.1.1: Standard ISO fire curve TODO

4.1.2 Potential inaccuracies

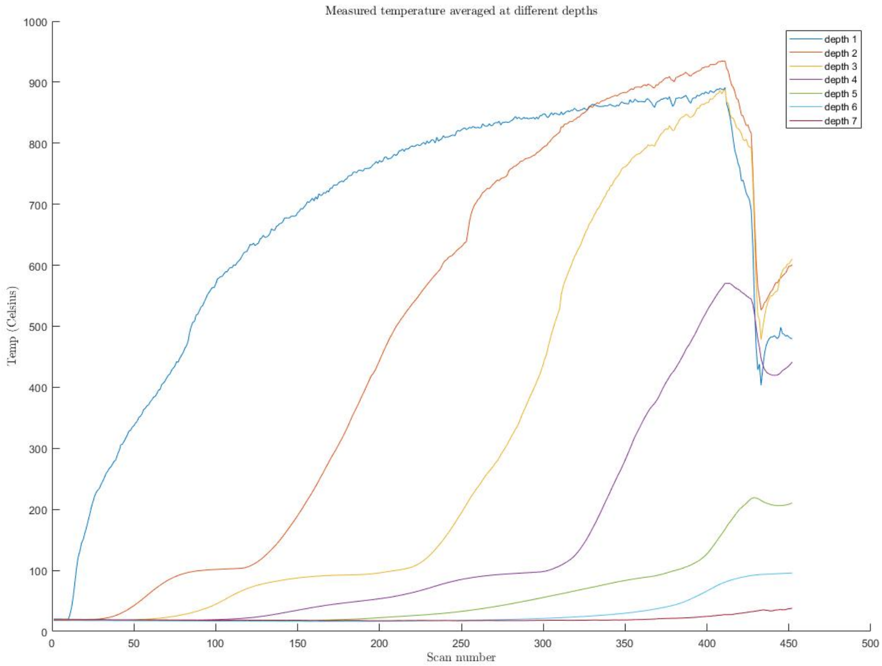
As with most test everything is not always perfect. In the data it was observed that two of the thermocouples broke during testing, this resulted in temperature with a magnitude of 10^{13}. That temperature is not possible as the highest ever recorded temperature reached was $4x10^{12}$ and that only occurred in a atomic explosion This malfunction required that two of the depth measurements were no longer the average between nine samples but instead the average between eight.

Another inaccuracy that could potentially influence the accuracy of the final result is the accuracy of the depth of the holes in which the thermocouples were placed. As this was done by hand.

There is also debate about the significance of the contribution of the timber burning to the temperature inside the furnace. For the purposes of this project it will be assumed that the timber burning does not contribute to the temperature inside the furnace.

The assumption that the panel is constantly at room temperature on the outside is also inaccurate as there is heat radiating from the panel that increases the temperature surrounding the panel.

4.1.3 Results


Measured temperature averaged at different depths
depth 1
depth 2
depth 3
depth 4
depth 5
depth 6
depth 7
Temp (Celsius)
Scan number
0
100
200
300
400
500
600
700
800
900
1000
0
50
100
150
200
250
300
350
400
450
500

Chapter 5

Methodology

I did things.

5.1 Proposed methodology

5.1.1 Finite Element Model

A one-dimensional finite element model that simulates what we expect to obtain from the fire tests based on the simplified K-values provided in EN 1995:1-1-2004 will be modified into a function. This function should provide the temperature of the modelled element based on a specified location and thermal conductivity.

5.1.2 Optimization

5.1.3 Markov Chain Monte Carlo

Markov Chain Monte Carlo(MCMC) is a method of integration. This will be used to determine the mean of the k-values at specific temperatures. Markov Chain Monte Carlo is a method that was created by combining the concept of Monte Carlo sampling and a Markov Chain. To fully understand MCMC the methods that it was created from need to be further investigated.

Markov Chains

Monte Carlo Integration

Monte Carlo integration is used to evaluate a probability distributuion. The evaluation is done by drawing a collection of random numbers from the distribution. These numbers are then used a the sample and a sample mean is taken. The arithmetic sample mean can be used to approximate the population mean in accordance with the law of large numbers (Gilks *et al.*, 1996)

Chapter 6

Summary and Conclusion

i conclude I am stupid.

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