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Stochastic inversion of fire test data for the T-dependant thermal diffusivity of SA pine

by

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Project(Civil Engineering)458

Final Draft

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Nomenclature

Constants

g = $9.81\,\mathrm{m/s^2}$

Variables

κ	Thermal conductivity	[W/m·K]
α	Thermal diffusivity	[$\mathrm{m^2/s}$]

Chapter 1

Introduction

This chapter will introduce the problem addressed in this project. Previous similar projects as well as the value of this research will also be addressed.

1.1 Background and Motivation

Traditionally the thermal conductivity, otherwise referred to as the κ-value, of timber is based simply off the EN 1995:1-1-2004 or similar standards. This research project will aim to obtain the thermal diffusivity of cross laminated SA-Pine timber by further analysing data obtained by S. van der Westhuyzen for his study of the samples' charring rate.

The thermal diffusivity of timber is a unobservable quantity that cannot be measured directly. Instead, it is related to measurements of temperature and time through differential models. When heat diffusion is calculated using Finite Element methods(TODO:choose which FEM), the process is usually simplified to a linear problem (Fish, 2007). Due to the changes in thermal diffusivity of timber with temperature, as can be seen in EN 1995:1-1-2004(pg number TODO), the diffusivity cannot be linearly modelled. Therefore, the problem lends itself to being analysed by inversion techniques. The aforementioned approach will allow us to obtain information about the diffusivity based on the combination of the information assumed prior to measuring, further referred to as the prior, and the measured data. Using statistical inversion leads to a probability distribution that provides us with a collection of diffusivity estimates and their corresponding probabilities.

Currently the fire rating of specific timber samples are based on fire tests conducted in a furnace. The furnace is kept at increasing temperatures corresponding with the Standard or ISO 834 fire curve as specified in ISO 834 ISO (1999). This process becomes very costly if it has to be repeated every time that timber is used for construction, as timber usage for multiple story construction projects have increased over the past decades. This increase is partially due to the sustainability of timber as a construction material: not only is it renewable but it also has a small carbon footprint (Salvadori, 2017)

1.2 Aim and objectives

During the course of the project, the student will aim to meet the following objectives:

1. Modify a Finite Element Model into an accurate and effective function;
2. Compare the model data to the actual acquired data;
3. Solve for the thermal diffusivity using Bayes' theorem of inverse problems; and
4. Evaluate and explore the posterior probability distribution using the following methods:
   (a) Maximum a Posteriori
   (b) Markov-Chain Monte Carlo

1.3 Current knowledge

The current κ-values used for the design of timber elements are taken from the EURO code (ref TODO (CEN, 2004)) and are show in figure 1.3.

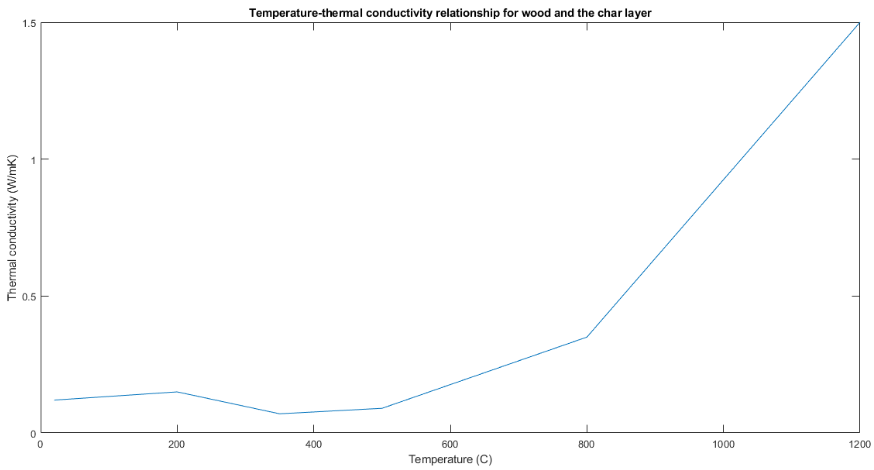


Figure 1.3.1: Standard temperature-thermal conductivity relationship for timber from (CEN, 2004)

1.4 Program

Chapter 2

Technical Foundation

2.1 Finite Element Model

Finite element methods (or finite element analysis) is used when the behaviour of an element cannot be accurately depicted by a simple mathematical equation.

2.1.1 History/Origin

2.1.2 method of FEM

A larger element is broken into smaller elements. Assumptions made on the smaller scale have a lesser effect on the final answer than the same assumptions made on a large scale would have had.

2.1.3 heat eq

In its simplest form, the one-dimensional heat diffusion equation is a partial differential equation 2.1.1 dependant on the temperature and thickness of the element. The heat diffusion equation is based on Fourier's Law...(TODO)

$$q = -k\frac{dT}{dx} \tag{2.1.1}$$

2.2 Bayes' theorem of inverse problems

The method of statistical inversion is dependant on a fundamental understanding of the Bayes' theorem of inverse problems. The student obtained this understanding through studying Chapter 3 of statistical and Computational Inverse problems by Kaipio and Somersalo (2005), further referred to merely as Kaipio. There are four principles of Statistical inversion that is essential to the thorough understanding of these models. Firstly, it is the principle that any variable in the model needs to be modelled as a random variable. This randomness is based on the extent of information that is available. To ensure that the extent of knowledge is accurately portrayed in the model, the extent of knowledge will be coded into the probability distributions assigned to the different variables. Finally, it needs to be understood that the solution of a statistical inversion is a posterior probability distribution. A generalized equation of Bayes' theorem can be seen in 2.2.1 taken from Kaipio.

$$\pi_{\text{post}}(x) = \pi(x|y_{\text{observed}}) = \frac{\pi_{\text{pr}}(x)\pi(y_{\text{observed}}|x)}{\pi(y_{\text{observed}})} \tag{2.2.1}$$

2.3 Markov Chain Monte Carlo

Markov Chain Monte Carlo (MCMC) is a method of integration. This will be used to determine the mean of the κ-values at specific temperatures. Markov Chain Monte Carlo is a method that was created by combining the concept of Monte Carlo sampling and a Markov Chain. To fully understand MCMC, the methods that it was created from need to be further investigated.

2.3.1 Markov Chains

2.3.2 Monte Carlo Integration

Monte Carlo integration is used to evaluate a probability distributuion. The evaluation is done by drawing a collection of random numbers from the distribution. These numbers are then used as the sample and a sample mean is taken. The arithmetic sample mean can be used to approximate the population mean in accordance with the law of large numbers (Gilks *et al.*, 1996)

Chapter 3

Implementation

This chapter will elaborate on the test data used as well as the process that was followed to achieve the results in Chapter 4.

3.1 Existing data

The data used was acquired by van der Westhuyzen *et al.* (2020) for an article assessing the charring rate of both SA-Pine and Eucalyptus. For the purpose of this project, only the data obtained from the SA-Pine test was considered and analysed.

3.1.1 Summary of test

The test was conducted on a sample of 100mm by 0.9m x 0.9m panel of cross-laminated SA-pine. This sample was then divided into nine cubes of 100 mm x 100 mm x 100 mm. Each cube was fitted with seven Type K-thermocouples placed at consecutive 16.5 mm drilled holes, as can be seen in Figure 3.1.1. The test panel was tested in a furnace and was exposed to the standard ISO 834 Fire curve 3.1.2 on one side and room temperature on the other. The panel was exposed to the fire curve for 50 minutes, at which stage near complete de-lamination was observed and the test ended.

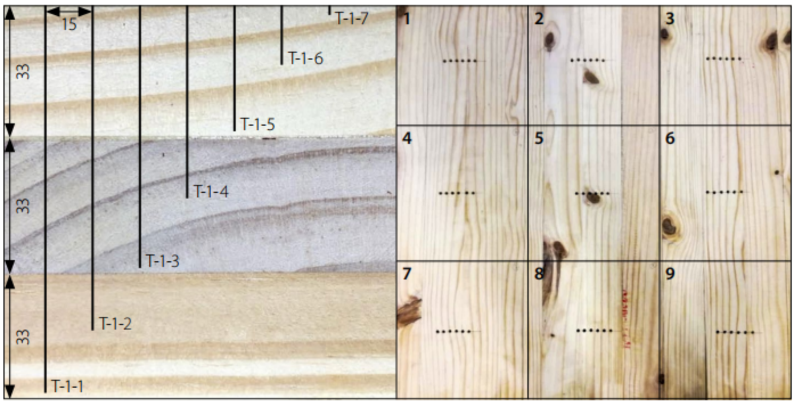


Figure 3.1.1: Thermocouple layout in test conducted by van der Westhuyzen *et al.* (2020) cross-section (left) and overall layout (right)

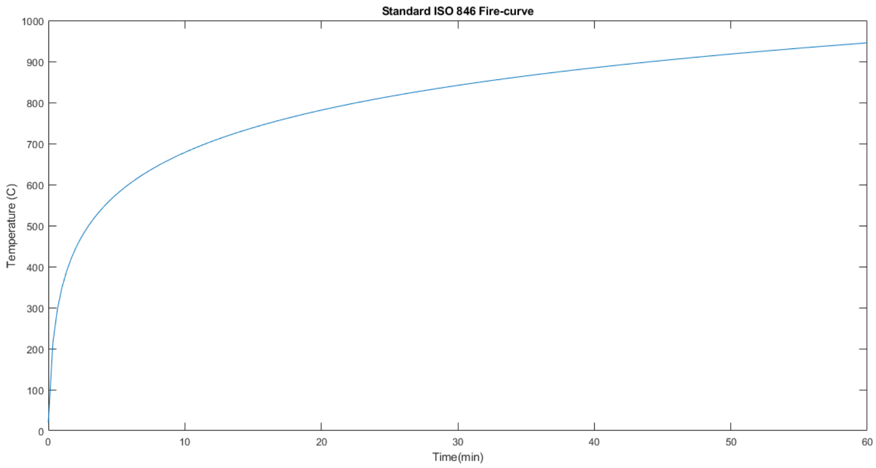


Figure 3.1.2: Standard ISO fire curve TODO

3.1.2 Potential inaccuracies

As with most tests, everything is not always perfect. The potential inaccuracies are discussed below.

In the data, it was observed that two of the thermocouples broke during testing, this resulted in temperature with a magnitude of 10^{13}. That temperature is not possible as the highest ever recorded temperature reached was $4\text{x}10^{12}$ and that only occurred in a atomic explosion. This malfunction required that two of the depth measurements were no longer the average between nine samples but instead the average between eight. Another inaccuracy that could potentially influence the accuracy of the final result is the accuracy of the depth of the holes in which the thermocouples were placed.

There is also debate about the significance of the contribution of the timber burning to the temperature inside the furnace. For the purposes of this project, it will be assumed that the timber burning does not contribute to the temperature inside the furnace.

3.2 Finite Element Modelling

A one-dimensional finite element model that simulates what we expect to obtain from the fire tests based on the simplified K-values provided in EN 1995:1-1-2004 is modified into a function. This function should provide the temperature of the modelled element based on a specified location and thermal conductivity. The derivation and adaptation of the model are expanded on below.

3.2.1 Derivation

3.2.2 Existing Model

For this project an existing finite element model of heat diffusion by Prof. N de Koker is modified for usage in the Bayes' theorem 2.2.1. This model is used to determine the likelihood function. The current model uses the standard Euro code k-values as well as the specific heat specified in the (CEN, 2004).

The model discretises the wooden element into 32 different elements. For finite element analysis, there are always more elements used to generate the model than usually evaluated. This is done to improve the accuracy of said model. The model is a one dimensional finite element model that takes time differentiation into account.

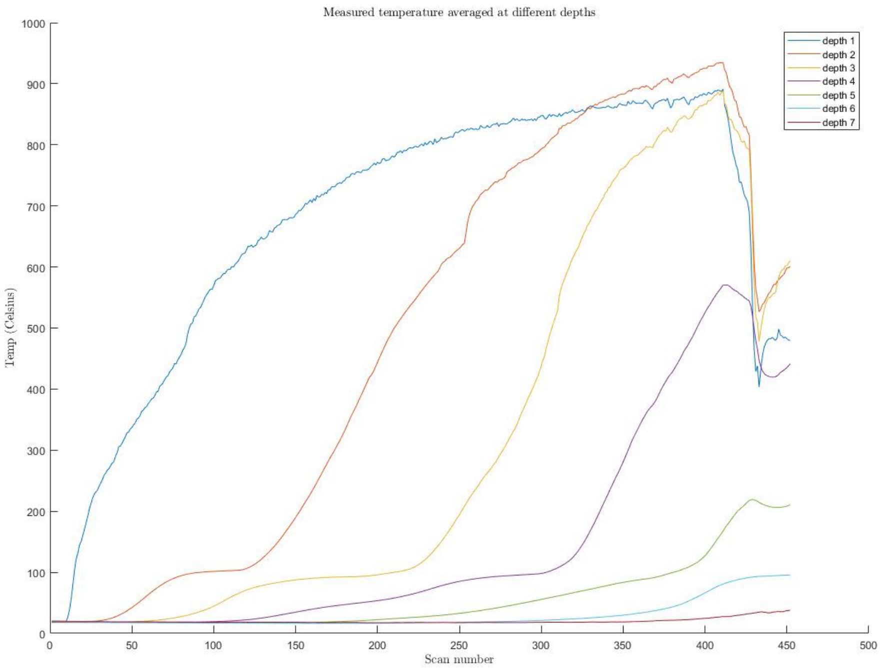
3.2.3 Adapted Model

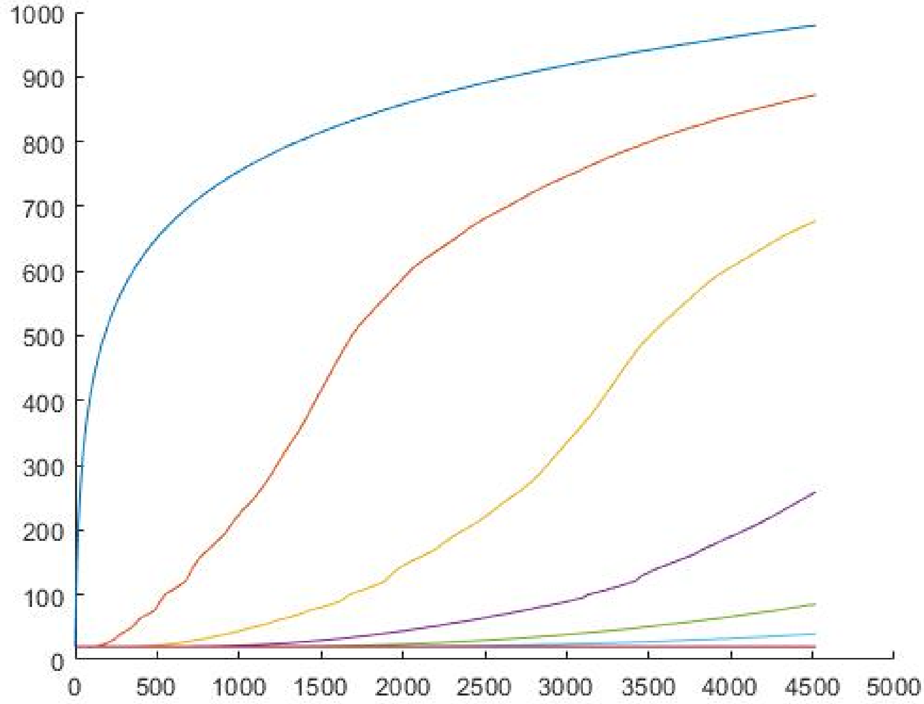
The model was changed into a function that takes κ-values and provides a new temperature distribution over the elements for the different κ-values. This function is used in the posterior calculation to determine the likelihood function.

3.3 Inversion method

Chapter 4

Results

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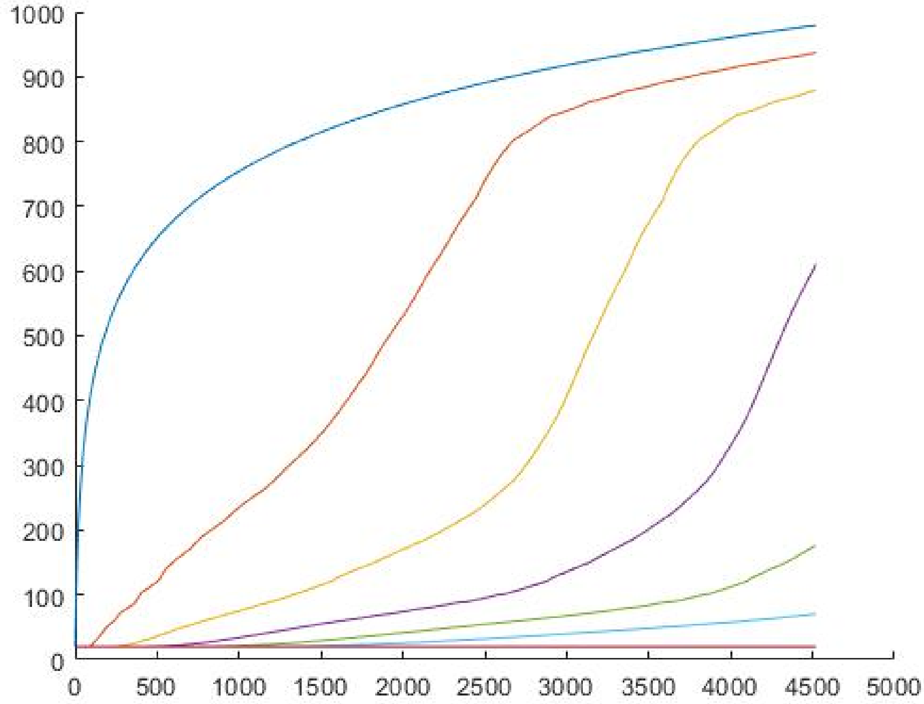
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Chapter 5

Discussion

Chapter 6

Summary and Conclusion

i conclude I am stupid.

Appendix A

Program

Appendix B

GA outcomes

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