Monte Carlo with Python

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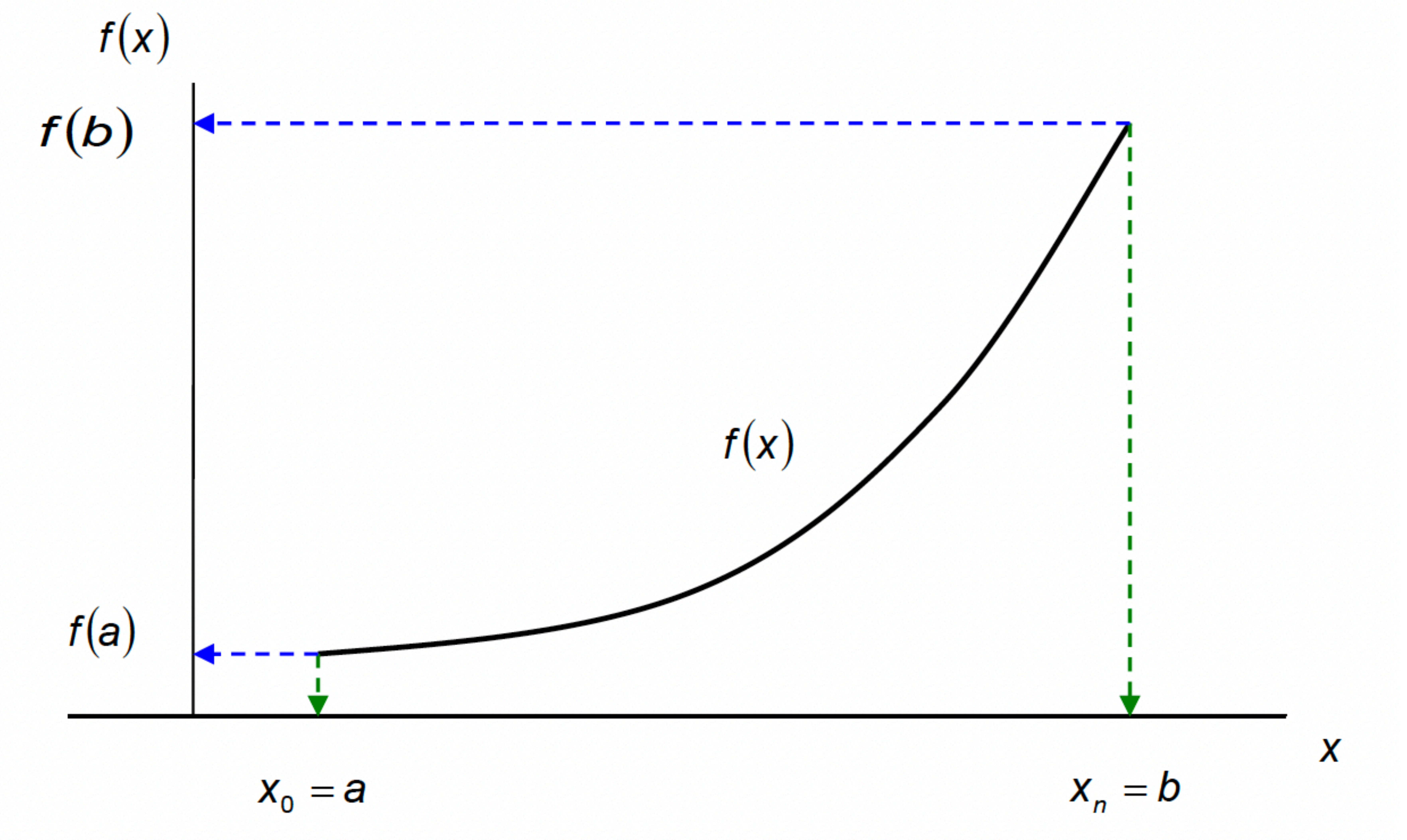
Introduction of numerical integration



Let's integrate a simple function!

- Given a function:

$$\int_0^{1.5\pi} \sin(x)dx$$

- We are going to integrate this using Monte Carlo using Python!

How to start?

- Integration in numerical methods/computational method meaning we take the sum under the curve $f(x)$ for the range given within the limit of integration (in this case between 0 to 1.5π).

- In normal numerical integration method, we can use several methods like mid-point method, trapezoid method or Simpson's rule to estimate the sum under the curve.

Various method in numerical integration

$$\boxed{I(f) = M(f) = (b-a)f\left(\frac{a+b}{2}\right)}.$$

Mid point rule

$$\boxed{T_1(f) = \frac{(b-a)}{2}\left[f(a) + f(b)\right]}.$$

Trapezoid rule

$$\boxed{\begin{aligned} S_2(f) &= \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right] \\ &= \frac{(b-a)}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right] \end{aligned}}.$$

Simpson's rule

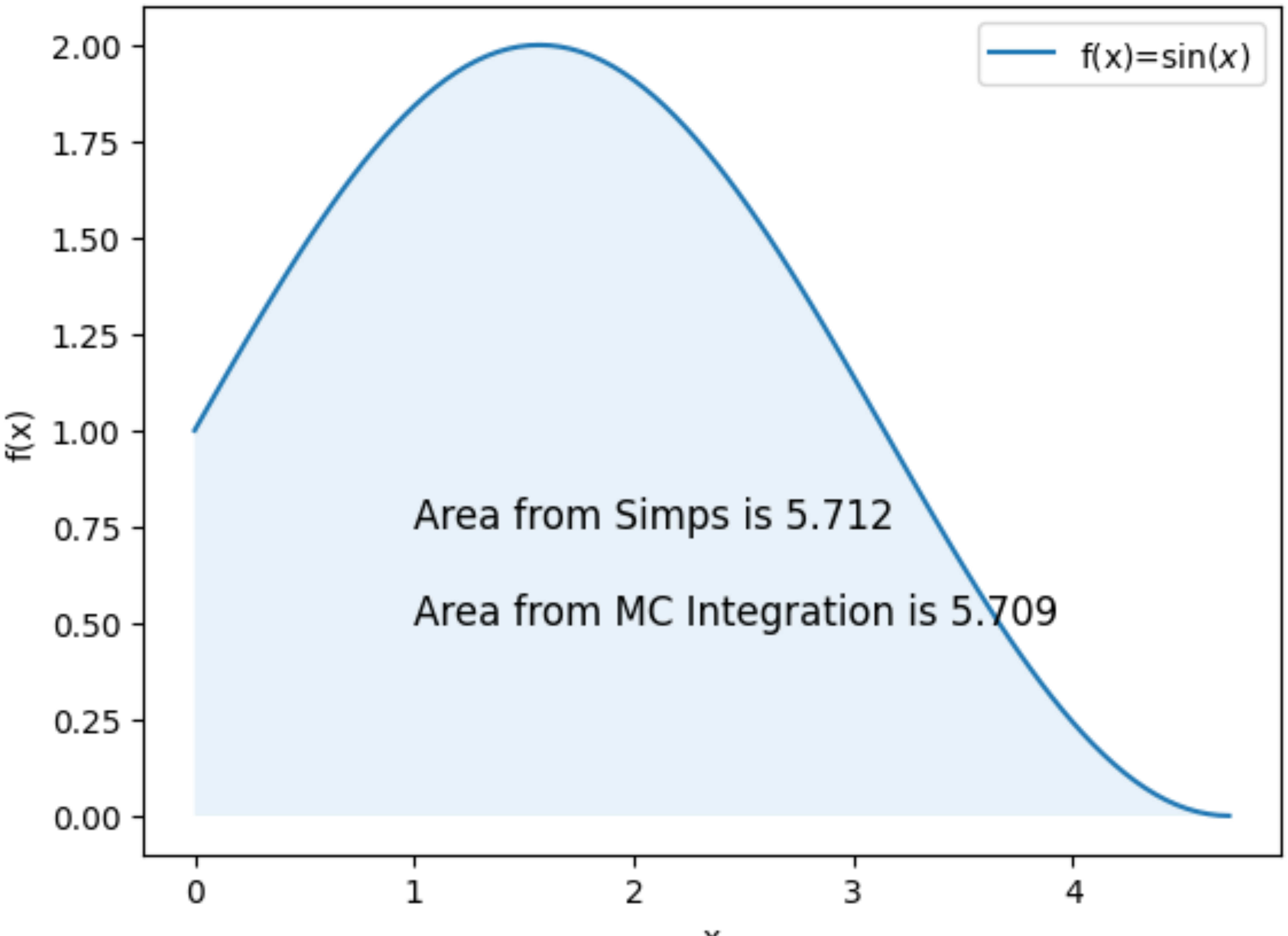
How about Monte Carlo method?

- Given $$I = \int_0^1 f(x)dx$$
- Divide $[0,1]$ into N subintervals with endpoints, $x_0 = 0, x_N = 1$.
- Then approximate I as:

$$I \approx \frac{1}{N}\sum_{n=1}^{N} f(x_n)$$

using sampling from $x_1, x_2, \ldots x_N$ in $[0,1]$ with equal weight (i.e. 1) at each point for large N.

Monte Carlo with Python



Application in astrophysics

- Blackbody Radiation

$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\exp \frac{hc}{\lambda k_B T} - 1}$$

- Let's integrate with wavelength, λ

$$B = \int_0^\infty B_\lambda d\lambda$$

Application in astrophysics

- Our integral now become:

$$B = 2hc^2 \int_0^\infty \frac{1}{\lambda^5} \frac{1}{exp\frac{hc}{\lambda k_B T - 1}} d\lambda$$

- Simplifying the wavelength in term of x

$$B = \frac{2(k_B T)^4}{h^3 c^2} \int_0^\infty \frac{x^3}{\exp(x) - 1} dx$$

Blackbody radiation with Monte Carlo

- We are going to test the integration to infinity using Monte Carlo method.
- Since integral is approaching infinity , we put the highest limit that computer could handle and check our answer with the analytical value