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Physics 152 Problem Set 4

Problem 7

Problem 13.1 (Reif)

Since the relaxation time τ is only a function of $|v|$ (or of $\epsilon = \frac{1}{2}mv^2$), show by performing the integrations over angles in (13.4.7) that the expression for the electrical conductivity can be written in the form

$$\sigma_{\text{el}} = -\frac{1}{3}e^2 \int \frac{dg}{d\epsilon}\tau v^2 d^3v$$

or

$$\sigma_{\text{ol}} = -\frac{4\pi}{3}e^2 \int_0^\infty \frac{dg}{d\epsilon}\tau v^4 dv$$

where $g = g(\epsilon) = g\left(\frac{1}{2}mv^2\right)$ is the equilibrium distribution function.

Solution:

The electrical conductivity from Equation (13.4.7) is given by

$$\sigma_{\text{el}} \equiv \frac{j_z}{\delta} = -e^2 \int d^3v \frac{dg}{d\epsilon}\tau v_z^2 \tag{1}$$

Note that the velocity v in cartesian coordinates is given by

$$v^2 = v_x^2 + v_y^2 + v_z^2 \tag{2}$$

Since the velocities in the three axes are equal, this reduces to

$$v^2 = 3v_z^2 \quad \text{or} \quad v_z^2 = \frac{1}{3}v^2 \tag{3}$$

Thus, σ_{el} becomes

$$\sigma_{\text{el}} = -\frac{1}{3}e^2 \int \frac{dg}{d\epsilon}\tau v^2 d^3v \tag{4}$$

We can get the second equation by changing the system from cartesian coordinates, to spherical coordinates using the relation

$$d^3v = dxdydz = v^2 \sin\theta dv d\theta d\phi \tag{5}$$

Substituting this into the expression for σ_{el}, we get

$$\begin{aligned}
\sigma_{\text{el}} &= -\frac{1}{3}e^2 \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} v^2 \sin\theta \frac{dg}{d\theta}\tau v^2 dv d\theta d\phi && (6)\\
&= -\frac{1}{3}e^2 \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \int_0^{\infty} \frac{dg}{d\theta}\tau v^4 dv && (7)
\end{aligned}$$

Therefore, after evaluating the integrals we get

$$\sigma_{\text{ol}} = -\frac{4\pi}{3}e^2 \int_0^{\infty} \frac{dg}{d\epsilon}\tau v^4 dv \tag{8}$$