Physics 195 Problem Set 2

Problem 1

For an ideal blackbody, plot the ratio of fluxes (or intensities), F_U/F_V, measured with U (centered at $\lambda = 364$ nm) and V filters (centered at $\lambda = 540$ nm), versus its temperature T. $\left(hc/k = 1.44 \times 10^{-2}\ \mathrm{m \cdot K.}\right)$

(a) Is the dependence of F_U/F_V on T monotonic?

Solution:

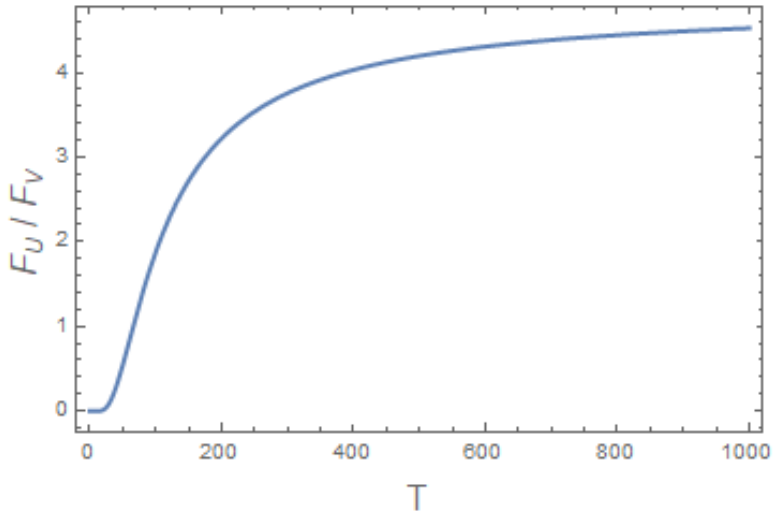

Figure 1: Ratio of fluxes measured with U and V filters

The ratio F_U/F_V is always increasing as shown in the figure above, so it is monotonic in T.

(b) At what temperature (in K) will this flux ratio exceed one?

Solution:

The temperature value T at which the flux ratio is one from the graph is $T = 65.888$ K.

(c) What is the range of possible values for F_U/F_V.

Solution:

From the graph, we can deduce that the plot starts at zero. To get the upper bound of the range, we take the limit of the ratio for $T \to \infty$ so the range of possible values for F_U/F_V is (0, 4.84361).

(d) Estimate the luminosity L of stars whose measured ratios are 1/2 and 2, respectively.

Solution:

The luminosity of a star $L_\star$ is given by

$$L_* = 4\pi R_*^2 \sigma T_{\mathrm{eff}}^4 \tag{1}$$

Note that the temperatures that correspond to the ratio F_U/F_V to be equal to 1/2 and 2 are $T_{1/2} = 48.447$ K and $T_2 = 105.756$ K, respectively. Substituting these into the luminosity we get the value for the luminosities in terms of the radius R of the star: $L_{1/2} = 3.925\, R^2\ \mathrm{W \cdot m^{-2}}$ and $L_2 = 89.132\, R^2\ \mathrm{W \cdot m^{-2}}$.