

Physics 151 Problem Set 3

Problem 11

Problem 2.15. (Gould and Tobochnik)

Use (2.44) and the ideal gas pressure equation of state in (2.8) to show that in a quasistatic adiabatic processes P and V are related as

$$PV^\gamma = \text{constant}.$$

Also show that T and P are related as

$$TP^{(1-\gamma)/\gamma} = \text{constant}.$$

Solution:

For a quasistatic adiabatic process, the relation between the temperature T and the volume V is given by

$$TV^{\gamma-1} = C \tag{1}$$

where C is a constant.

The ideal gas law

$$PV = NkT \tag{2}$$

can also be written as

$$T = \frac{PV}{Nk} \tag{3}$$

Substituting this into the relation, we get

$$\left(\frac{PV}{Nk}\right)V^{\gamma-1} = C \tag{4}$$

Note that we can absorb N and k into the constant C (since they are both constants for this process) such that we get a relation between P and V given by

$$PV^\gamma = C \tag{5}$$

Similarly, we can also get a relation between T and P for a quasistatic adiabatic process by using the expression

$$V = \frac{NkT}{P} \tag{6}$$

from the ideal gas law to get

$$T \left(\frac{NkT}{P}\right)^{\gamma-1} = C \tag{7}$$

Again, we can absorb N and k into the constant and the expression simplifies to

$$T^\gamma P^{1-\gamma} = C \tag{8}$$

or

$$TP^{(1-\gamma)/\gamma} = C \tag{9}$$