Lizbeth Joy G. Tan March 27, 2019

Physics 151 Problem Set 8

Problem 40

Problem 4.16. Density of states of an ideal gas (Gould and Tobochnik)
Use (4.51) to calculate the density of states $g(E, V, N)$ and verify that $\Gamma(E, V, N)$ and $g(E, V, N)$ are rapidly increasing functions of E, V, and N.

Solution:
We can transform this expression

$$\ln \Gamma(E, V, N) = N \ln \frac{V}{N} + \frac{3}{2} N \ln \frac{mE}{3N\pi\hbar^2} + \frac{5}{2} N \tag{1}$$

into $\Gamma(E, V, N)$ by taking the logarithm of both sides such that

$$\Gamma(E, V, N) = \left(\frac{V}{N}\right)^N \left(\frac{mE}{3N\pi\hbar^2}\right)^{3N/2} e^{5N/2} \tag{2}$$

From this, we can see that $\Gamma(E, V, N)$ is a rapidly increasing function of E, V, and N.
For an E dependence only, $g(E)$ has the form

$$g(E)\Delta E = \Gamma(E + \Delta E) - \Gamma(E) \approx \frac{d\Gamma(E)}{dE} \Delta E \tag{3}$$

When g becomes a function of E, V, and N, it takes the form

$$g(E, V, N) = \frac{\partial \Gamma}{\partial E} + \frac{\partial \Gamma}{\partial V} + \frac{\partial \Gamma}{\partial N} \tag{4}$$

where the derivatives are given by

$$\frac{\partial \Gamma}{\partial E} = \left(\frac{V}{N}\right)^N \left(\frac{m}{3N\pi\hbar^2}\right)^{3N/2} e^{5N/2} \frac{3N}{2} E^{(3N-2)/2} \tag{5}$$

$$\frac{\partial \Gamma}{\partial V} = \left(\frac{V}{N}\right)^{N-1} \left(\frac{mE}{3N\pi\hbar^2}\right)^{3N/2} e^{5N/2} \tag{6}$$

$$\begin{split}\frac{\partial \Gamma}{\partial N} = \left(\frac{V}{N}\right)^N \left(\frac{mE}{3N\pi\hbar^2}\right)^{3N/2} \frac{5}{2} e^{5N/2} - \frac{5N}{2} \frac{5}{2} V^N \left(\frac{mE}{3N\pi\hbar^2}\right)^{3N/2} N^{(5N-2)/2} e^{5N/2} \\ + \left(\frac{V}{N}\right)^N \frac{3}{2} \left(\frac{mE}{3N\pi\hbar^2}\right)^{(3N-2)/2} e^{5N/2}\end{split} \tag{7}$$