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Physics 151 Problem Set 4

Problem 17

Problem 2.24. Applications of (2.133) (Gould and Tobochnik)

(a) Use (2.133) to derive the relation (2.44) between T and V for a quasistatic adiabatic process.

Solution:

The relation between T and V from Equation (2.44) in Gould and Tobochnik is given by

$$TV^{\gamma-1} = constant \tag{1}$$

The entropy of an ideal gas is given by

$$\Delta S = \frac{3}{2}Nk\ln\frac{T_2}{T_1} + Nk\ln\frac{V_2}{V_1} \tag{2}$$

For a quasistatic adiabatic process, $\Delta S = 0$ so

$$\frac{3}{2}Nk\ln\frac{T_2}{T_1} + Nk\ln\frac{V_2}{V_1} = 0 \tag{3}$$

Note that for an ideal monatomic gas, we have the relations

$$C_v = \frac{3}{2}Nk \quad \text{and} \quad C_p = \frac{5}{2}Nk \tag{4}$$

We can use substitute these into Equation (3), to get

$$C_v\ln\frac{T_2}{T_1} + (C_p - C_v)\ln\frac{V_2}{V_1} = 0 \tag{5}$$

or

$$\left(\frac{C_v}{C_p - C_v}\right)\ln\frac{T_2}{T_1} = -\ln\frac{V_2}{V_1} \tag{6}$$

Using the definition of γ

$$\gamma = \frac{C_p}{C_v}, \tag{7}$$

the equation becomes

$$\ln\frac{T_2}{T_1} = -(\gamma - 1)\ln\frac{V_2}{V_1} \tag{8}$$

This can be rewritten as

$$\ln\frac{T_2}{T_1} = \ln\left(\frac{V_1}{V_2}\right)^{\gamma-1} \tag{9}$$

Taking the exponential on both sides,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \tag{10}$$

Rearranging the terms, we get the relation from Equation (1)

$$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1} \tag{11}$$

(b) An ideal gas of N particles is confined to a box of chamber V_1 at temperature T. The gas is then allowed to expand freely into a vacuum to fill the entire container of volume V_2. The container is thermally insulated. What is the change in entropy of the gas?

Solution:
The change in entropy of an ideal gas is given by the relation

$$\Delta S = \frac{3}{2} Nk \ln \frac{T_2}{T_1} + Nk \ln \frac{V_2}{V_1} \tag{12}$$

Since the container is thermally insulated, there is no change in temperature. Thus,

$$T_1 = T_2 = T \tag{13}$$

which corresponds to

$$\ln \frac{T_2}{T_1} = 0 \tag{14}$$

Therefore, the change in entropy reduces to

$$\Delta S = Nk \ln \frac{V_2}{V_1} \tag{15}$$

(c) Find $\Delta S(T, P)$ for an ideal classical gas.

Solution:
The equation of state of an ideal gas is given by

$$PV = NkT \tag{16}$$

We can rearrange this to get

$$V = \frac{NkT}{P} \tag{17}$$

which we can use to transform the expression for the change of entropy given by

$$\Delta S(T, V) = \frac{3}{2} Nk \ln \frac{T_2}{T_1} + Nk \ln \frac{V_2}{V_1} \tag{18}$$

to an expression in terms of T and P:

$$\Delta S(T, P) = \frac{3}{2} Nk \ln \frac{T_2}{T_1} + Nk \ln \left(\frac{NkT_2}{P_2} \right) \left(\frac{P_1}{NkT_1} \right) \tag{19}$$

This simplifies to

$$\Delta S(T, P) = \frac{3}{2} Nk \ln \frac{T_2}{T_1} + Nk \ln \frac{T_2 P_1}{P_2 T_1} \tag{20}$$

Using a property of logarithms, we can rewrite this into

$$\Delta S(T,P) = \frac{3}{2}Nk\ln\frac{T_2}{T_1} + Nk\ln\frac{T_2}{T_1} + Nk\ln\frac{P_1}{P_2} \tag{21}$$

Therefore, the expression for the change in entropy ΔS in terms of T and P is

$$\Delta S(T,P) = \frac{5}{2}Nk\ln\frac{T_2}{T_1} + Nk\ln\frac{P_1}{P_2} \tag{22}$$