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Physics 152 Problem Set 3

Problem 6

Problem 12.12 (Reif)

A long cylindrical wire of radius a and electrical resistance R per unit length is stretched along the axis of a long cylindrical container of radius b. This container is maintained at a fixed temperature T_0 and is filled with a gas having thermal conductivity κ. Calculate the temperature difference ΔT between the wire and the container walls when a small constant electrical current I is passed through the wire, and show that a measurement of ΔT provides a means for determining the thermal conductivity of the gas. Assume that a steady-state condition has been reached so that the temperature T at any point has become independent of time. (Suggestion: Consider the condition which must be satisfied by any cylindrical shell of the gas contained between radius r and radius $r + dr$.)

Solution:

Joule heating is the process by which a conductor produces heat when an electric current passes through it. The power dissipated through heat is given by Joule's first law:

$$P \propto I^2 R \tag{1}$$

For the cylindrical shell, the heat generation per unit length will be

$$Q = -\kappa \left(\frac{A}{L}\right) \frac{dT}{dr} \tag{2}$$

where the area A is

$$A = 2\pi r L \tag{3}$$

Assuming that the cylindrical shell will conduct all of the heat from the wire, we equate (1) and (2) to get the temperature difference between the wire and the container walls:

$$I^2 R = -\kappa \left(\frac{A}{L}\right) \frac{dT}{dr} = -2\pi r \kappa \frac{dT}{dr} \tag{4}$$

We can rearrange this equation so we can integrate both sides:

$$-\int_T^{T_0} dT = \frac{I^2 R}{2\pi\kappa} \int_a^b \frac{dr}{r} \tag{5}$$

Evaluating the integral,

$$-\left(T_0 - T\right) = \frac{I^2 R}{2\pi\kappa} \ln\left(\frac{b}{a}\right) \tag{6}$$

Therefore, the temperature difference is

$$T - T_0 = \Delta T = \frac{I^2 R}{2\pi\kappa} \ln\left(\frac{b}{a}\right) \tag{7}$$