Physics 195 Problem Set 5

Problem 6 (Problem 5.9 of Ryden.)

A universe is spatially flat, and contains both matter and a cosmological constant. For what value of $\Omega_{m,0}$ is t_0 exactly equal to H_0^{-1}?

Solution:

From Equation (5.104) of Ryden, we have t_0 in terms of $\Omega_{m,0}$ and H_0^{-1}:

$$t_0 = \frac{2H_0^{-1}}{3\sqrt{1-\Omega_{m,0}}} \ln\left[\frac{\sqrt{1-\Omega_{m,0}}+1}{\sqrt{\Omega_{m,0}}}\right] \tag{1}$$

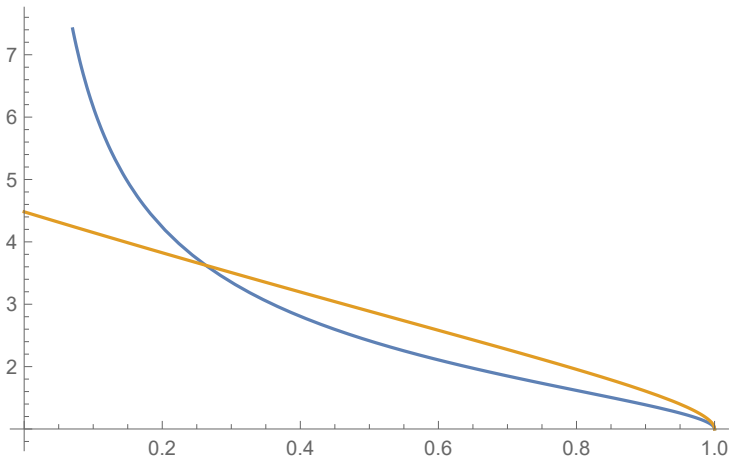
If $t_0 = H_0^{-1}$, then

$$\frac{2}{3\sqrt{1-\Omega_{m,0}}} \ln\left[\frac{\sqrt{1-\Omega_{m,0}}+1}{\sqrt{\Omega_{m,0}}}\right] = 1 \tag{2}$$

We can rewrite this as

$$\frac{\sqrt{1-\Omega_{m,0}}+1}{\sqrt{\Omega_{m,0}}} = \exp\left(\frac{3\sqrt{1-\Omega_{m,0}}}{2}\right) \tag{3}$$

Plotting the left and right sides of the equation:



From this plot, the intersection of the two curves is the solution of the equation above. Therefore, we get $\Omega_{m,0} = 0.263$.