Physics 195 Problem Set 4

Problem 3 (Problem 4.2 of Ryden.)

Consider Einstein's static universe, in which the attractive force of the matter density ρ is exactly balanced by the repulsive force of the cosmological constant, $\Lambda = 4\pi G\rho$. Suppose that some of the matter is converted into radiation (by stars, for instance). Will the universe start to expand or contract? Explain your answer.

Solution:

The acceleration equation with the cosmological constant is given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P) + \frac{\Lambda}{3} \tag{1}$$

with the equation of state

$$P = \omega\epsilon \tag{2}$$

Note that $\omega = 0$ for baryonic matter and $\omega = 1/3$ for radiation.

When there is only matter, the acceleration equation reduces to

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\epsilon_m + \frac{\Lambda}{3} \tag{3}$$

If some of the matter gets converted into radiation, the equation becomes

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\left\{\epsilon_m + \left[\epsilon_r + 3\left(\frac{1}{3}\epsilon_r\right)\right]\right\} + \frac{\Lambda}{3} \tag{4}$$

This reduces to

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\left(\epsilon_m + 2\epsilon_r\right) + \frac{\Lambda}{3} \tag{5}$$

We can see from this equation that the acceleration becomes slower because of the added radiation term. Therefore, the universe will start to contract when some of the matter gets converted into radiation.