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Physics 151 Problem Set 8

Problem 37

Problem 4.10. Rapid increase in the number of states (Gould and Tobochnik)

(a) Consider an Einstein solid of $N = 20$ distinguishable particles. What is the total number of accessible microstates $\Omega(E)$ for $E = 10, 10^2, 10^3$,...? Is $\Omega(E)$ a rapidly increasing function of E for fixed N?

Solution:

The number of microstates of an Einstein solid composed of N particles with total energy E is given by

$$\Omega = \frac{(E + N - 1)!}{E!(N - 1)!} \tag{1}$$

Using this expression, we get different values of Ω for varying E and with $N = 20$. This is shown in Table 1. The log plot in Figure 1 shows that Ω is a rapidly increasing function of E.

Table 1: Number of microstates for fixed N and varying E

Energy	Number of microstates
10	2.00×10^7
10^2	4.91×10^{21}
10^3	9.93×10^{39}
10^4	8.38×10^{58}
10^5	8.24×10^{77}

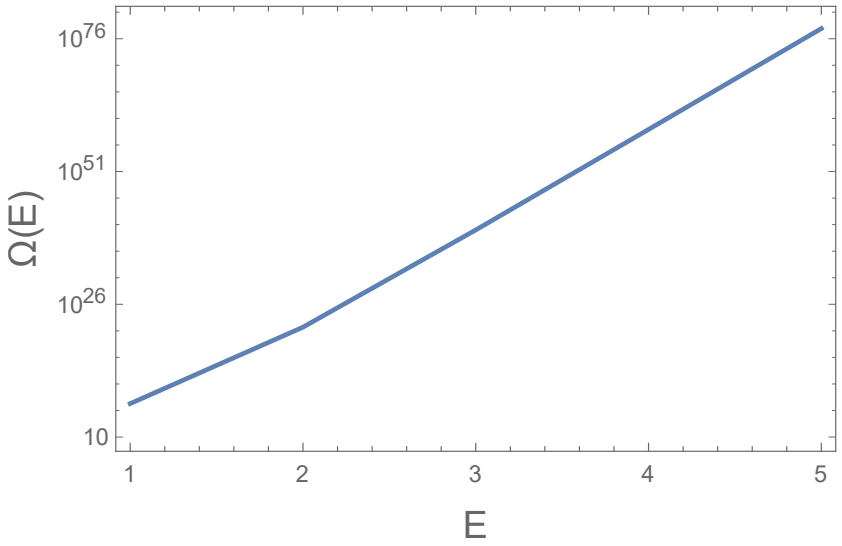

Figure 1: Log plot of the number of microstates for different energies

(b) Is Ω a rapidly increasing function of N for fixed E?

Solution:

A plot of the number of microstates Ω for different values of N and a fixed $E = 10$ is shown in Figure 2. From this plot, we can see that Ω is also a rapidly increasing function of N since the curve is exponential-like.

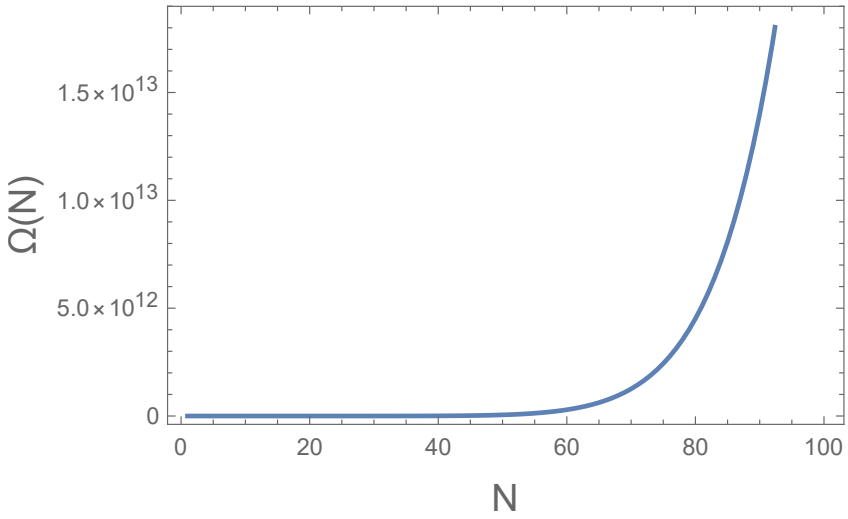

Figure 2: Log plot of the number of microstates for different number of particles