Lizbeth Joy G. Tan February 20, 2019

Physics 151 Problem Set 3

Problem 13

Problem 2.18. Work done in a quasistatic adiabatic process (Gould and Tobochnik)
(a) Use the result that we derived in (2.53) to obtain the alternative form (2.54).

Solution:
The work done of an ideal gas for a quasistatic adiabatic process is

$$W = C_V\left(T_2 - T_1\right) \tag{1}$$

Using the ideal gas law

$$PV = NkT \tag{2}$$

we can make the expression in terms of the pressure P and volume V:

$$W = C_V\left(\frac{P_2V_2}{N_2k_2} - \frac{P_1V_1}{N_1k_1}\right) \tag{3}$$

We can simplify this by noting that N and k are the same all throughout the process so that

$$W = C_V\left(\frac{P_2V_2 - P_1V_1}{Nk}\right) \tag{4}$$

Using the relation between C_P and C_V,

$$C_P = C_V + Nk \tag{5}$$

The work done in a quasistatic adiabatic process becomes

$$W = \left(\frac{C_V}{C_p - C_v}\right)P_2V_2 - P_1V_1 \tag{6}$$

or

$$W = \left(\frac{1}{C_p/C_v - 1}\right)P_2V_2 - P_1V_1 \tag{7}$$

Note that gamma is defined as

$$\gamma = \frac{C_P}{C_V} \tag{8}$$

Therefore, we get an expression for the work done in terms of the pressure P and volume V

$$W = \frac{P_2V_2 - P_1V_1}{\gamma - 1} \tag{9}$$

(b) Show that another way to derive (2.54) is to use the relations (2.14) and (2.46).

Solution:

The relation between the pressure P and volume V for an ideal gas in a quasistatic adiabatic process is given by

$$PV^{\gamma} = C \tag{10}$$

which can also be expressed as

$$P = CV^{-\gamma} \tag{11}$$

The total work done in a quasistatic process from state 1 to 2 is defined as

$$W_{1\to 2} = -\int_{V_1}^{V_2} P(T,V)dV \tag{12}$$

Substituting the relation (11),

$$W = -\int_{V_1}^{V_2} CV^{-\gamma}dV \tag{13}$$

This integral evaluates to

$$W = \left(\frac{CV^{-\gamma+1}}{\gamma-1}\right)\Big|_{V_1}^{V_2} = \frac{C}{\gamma-1}(V_2^{-\gamma+1} - V_1^{-\gamma+1}) \tag{14}$$

We can substitute C from the original relation (10) back into the expression so that we get

$$W = \frac{P_2V_2^{\gamma}V_2^{-\gamma+1} - P_1V_1^{\gamma}V_1^{-\gamma+1}}{\gamma-1} \tag{15}$$

which reduces to

$$W = \frac{P_2V_2 - P_1V_1}{\gamma-1} \tag{16}$$