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Physics 152 Problem Set 4

Problem 8

Problem 13.2 (Reif)

Suppose that the particles in the preceding problem obey Maxwell-Boltzmann statistics. Show that their electrical conductivity can be written in the convenient form

$$\sigma_{\text{el}} = \frac{ne^2}{m}\langle\tau\rangle_\sigma$$

where $\langle\tau\rangle_\sigma$ is a suitably weighted average of $\tau(v)$ over the velocity distribution and is defined by

$$\langle\tau\rangle_\sigma = \frac{8}{3\sqrt{\pi}}\int_0^\infty dse^{-s^2}s^4\tau(\tilde{v}s)$$

Here $\tilde{v} \equiv (2kT/m)^{\frac{1}{2}}$ is the most probable speed of the particle in equilibrium and $s \equiv v/\bar{v}$ is a dimensionless variable expressing molecular speeds in terms of this most probable speed. The average $\langle\tau\rangle_\sigma$ has been defined in such a way that is reduces to τ when this quantity is independent of v.

Solution:

The density of states for a Maxwell-Boltzmann distribution is given by

$$g(\epsilon) = n\left(\frac{m\beta}{2\pi}\right)^{3/2} e^{-\beta\epsilon} \tag{1}$$

The derivative of this wrt ϵ is

$$\frac{dg}{d\epsilon} = -\beta n\left(\frac{m\beta}{2\pi}\right)^{3/2} e^{-\beta\epsilon} \tag{2}$$

Using the expression resulting from the previous problem,

$$\sigma_{\text{ol}} = -\frac{4\pi}{3}e^2\int_0^\infty \frac{dg}{d\epsilon}\tau v^4 dv \tag{3}$$

we can substitute the derivative of g to get

$$\sigma_{\text{ol}} = \frac{4\pi}{3}ne^2\int_0^\infty \beta\left(\frac{m\beta}{2\pi}\right)^{3/2} e^{-\beta\epsilon}\tau v^4 dv \tag{4}$$

Note that $\epsilon = \frac{1}{2}mv^2$ and $\beta = 1/kT$ so

$$\begin{aligned}\sigma_{\text{ol}} &= \frac{4\pi}{3}ne^2\int_0^\infty \beta\left(\frac{m\beta}{2\pi}\right)^{3/2} e^{-\beta(\frac{1}{2}mv^2)}\tau v^4 dv &(5)\\ &= \frac{4\pi}{3}ne^2\int_0^\infty \frac{1}{kT}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\tau v^4 dv &(6)\end{aligned}$$

We then change v into the dimensionless variable s using the relation

$$v = \tilde{v}s \tag{7}$$

where $\tilde{v} \equiv (2kT/m)^{\frac{1}{2}}$ given by the problem.

Thus, the expression for σ_{ol} becomes

$$\sigma_{\text{ol}} = \frac{4\pi}{3} ne^2 \int_0^\infty \frac{1}{kT} \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-s^2} \tau \left(\frac{2kT}{m}\right)^2 s^4 \left(\frac{2kT}{m}\right)^{1/2} ds \tag{8}$$

$$= \frac{4\pi}{3} ne^2 \int_0^\infty \frac{1}{kT} \pi^{-3/2} e^{-s^2} \tau s^4 \left(\frac{m}{2kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^{5/2} ds \tag{9}$$

$$= \frac{4\pi}{3} ne^2 \int_0^\infty \frac{2}{m} \pi^{-3/2} e^{-s^2} \tau s^4 ds \tag{10}$$

$$= \frac{8}{3\sqrt{\pi}} \frac{ne^2}{m} \int_0^\infty ds e^{-s^2} s^4 \tau(\tilde{v}s) \tag{11}$$

The problem defines

$$\langle \tau \rangle_\sigma = \frac{8}{3\sqrt{\pi}} \int_0^\infty ds e^{-s^2} s^4 \tau(\tilde{v}s) \tag{12}$$

Therefore,

$$\sigma_{\text{el}} = \frac{ne^2}{m} \langle \tau \rangle_\sigma \tag{13}$$