Physics 195 Problem Set 5

Problem 2 (Problem 5.2 of Ryden.)

Suppose you are in a flat, matter-only universe that has a Hubble constant $H_0 = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. You observe a galaxy with $z = 1$. How long will you have to keep observing the galaxy to see its redshift change by one part in 10^6? [Hint: use the result from the previous problem.]

Solution:

The observed redshift z changes at a rate

$$\frac{dz}{dt_0} = H_0(1+z) - H_0(1+z)^{3(1+w)/2} \tag{1}$$

For a matter-only universe, $w = 0$ so

$$\frac{dz}{dt_0} = H_0(1+z) - H_0(1+z)^{3/2} \tag{2}$$

With $z = 1$:

$$\frac{dz}{dt_0} = H_0(2 - 2^{3/2}) \tag{3}$$

Thus, the time it takes to observe a change in redshift dz is given by

$$dt_0 = \left| \frac{dz}{H_0(2-2^{3/2})} \right| \tag{4}$$

Given $dz = 10^{-6}$ and $H_0 = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, we get

$$dt_0 = \left| \frac{10^{-6}}{(2-2^{3/2})(68\,\mathrm{km\,s^{-1}\,Mpc^{-1}})} \cdot \frac{3.09\times10^{22}\,\mathrm{m}}{1\,\mathrm{Mpc}} \cdot \frac{1\,\mathrm{km}}{1000\,\mathrm{m}} \right| \tag{5}$$

$$dt_0 = 5.49\times10^{11}\,\mathrm{s} = 1.74\times10^{4}\,\mathrm{yr} \tag{6}$$