ECON 308: ECONOMETRICS
ASSIGNMENT 1

Complete each problem to the best of your ability and turn them in on Thursday, September 16. You are encouraged to collaborate with other students, but you should turn in the problem solutions individually. Write legibly, denote your final answers clearly, and show all of your work.

(1) Consider rolling a fair die D with 6 sides (numbered 1 to 6). Each side is equally likely to appear on a given roll.

  (a) What is the probability (on a given roll) that $D = 1$?

  (b) What is $P(D < 3)$? (Notation: $P(E)$ denotes the probability of the event E occurring.)

  (c) What is the expected value of D, i.e., $\mathrm{E}[D]$?

  (d) Compute $\mathrm{E}[D \mid D \leq 3]$.

(2) Suppose we have a set $\{x_1, x_2, \ldots, x_{10}\}$.

  (a) Write an expression for the sum of all the elements using summation notation.

  (b) Write an expression for the mean of all the elements using summation notation.

(3) Consider the random variable X where $\mathrm{E}[X] = 1$ and $\mathrm{Var}(X) = 1$

  (a) Compute $\mathrm{E}\left[\frac{X}{3}\right]$.

  (b) Compute $\mathrm{E}[2X - 1]$.

  (c) Compute $\mathrm{Var}(2X + 2)$.

(4) Consider the random variable Y where

$$Y = \begin{cases} -\$3 & \text{with } p = \frac{1}{3} \\ \$3 & \text{with } p = \frac{2}{3} \end{cases}$$

  (a) Compute $\mathrm{E}[Y]$.

  (b) Compute $\mathrm{E}[Y]^2$.

  (c) Compute $\mathrm{E}[Y^2]$.

  (d) Compute $\mathrm{Var}(Y)$.

(5) Suppose the correlation between two random variables is zero. Does this mean that they are unrelated?

(6) Let X and Y be dependent random variables. Define the distribution of X as

$$X = \begin{cases} 0 & \text{with } p = \frac{3}{4} \\ 1 & \text{with } p = \frac{1}{4} \end{cases}$$

and let the distribution of Y vary with X: When $X = 0$,

$$Y = \begin{cases} -1 & \text{with } p = \frac{1}{4} \\ 1 & \text{with } p = \frac{3}{4} \end{cases}$$

and when $X = 1$,

$$Y = \begin{cases} 5 & \text{with } p = \frac{3}{5} \\ 10 & \text{with } p = \frac{2}{5} \end{cases}$$

(a) Will X and Y have a positive or negative correlation?

(b) Compute $\mathrm{E}[Y \mid X = 0]$.

(c) Compute $\mathrm{E}[Y \mid X = 1]$.

(d) Compute $\mathrm{E}[Y]$.

(7) Suppose we have an i.i.d. random sample $\{x_i \mid i = 1, \ldots, N\}$ of N observations of a random variable x with $\mathrm{E}[x] = \mu$ and $\mathrm{Var}(x) = \sigma^2$. Consider the following estimator for the expected value of x, $\hat{\mu}_n = \bar{x} + \frac{x_n}{n}$, where $\bar{x}$ is the typical sample mean.

(a) Show that this estimator is biased, and quantify the bias.

(b) Derive the sampling variance of this estimator.

(c) Is this a consistent or inconsistent estimator for μ? Explain.

(8) We wish to estimate a population parameter α. Consider the following estimator $\hat{\alpha}_n$ (based on a sample of size n):

$$\hat{\alpha}_n = \begin{cases} \alpha & \text{with probability } = \frac{n-1}{n} \\ \alpha + n & \text{with probability } = \frac{1}{n} \end{cases}$$

(a) Calculate the bias of this estimator.

(b) Calculate the variance of this estimator. What happens to the variance as n grows to infinity?

(c) Is this estimator consistent?