Statement of Problem

Determine the trade-offs and balance for *Pieris brassicae* through the usage of anti-aphrodisiacs (shorthand for "*a.a.*"*)*

Model Design

We simulate an idealized ecosystem with only two species involved, the white butterfly *Pieris brassicae* and the egg parasitoid wasps from the genus *Trichogramma*. Male butterflies may encounter other males during reproduction. To either mask or dissuade other males, male butterfly transfers a chemical signal, a.a., to the mated female. Thus, such interactions between female and male butterflies increase the probability for females to fertilize eggs. On the other hand, the *a.a.* that mated females carry attracts parasitic wasps, making the newly laid eggs more likely to be eaten by the wasps. This model analyzes the dynamic trade-offs affecting the survival of eggs through the usage of *a.a.*

Assumptions

1. This is an idealized ecosystem focusing solely on wasps and butterflies. The mortality rate for butterflies and the birth rate for wasps are of this model's interest. We assume that the birth rate for butterflies and the mortality rate for wasps are consistent with natural conditions.
2. Even though *T. brassicae* needs a successful ride to develop *T. evanescens'* innate ability, other aspects of predatory strategies are practically similar for the two wasp species (Huigens et al., 2010). Thus, we combine them.
3. The ratio of female *P. brassicae* to male *P. brassicae* is 1:1, and the probability of giving birth to a male progeny is 50%.

Variables

t: time measured in days

B, W: the total population of butterflies *P. brassicae* and wasps *T. brassicae,* respectively

g_B: the growth rate of butterflies with the presence of *a.a.*

f_1: butterflies' mortality factor caused by supplying eggs to wasp with the presence of *a.a.*

f_2: wasps' growth factor caused by consuming host butterfly eggs with the presence of *a.a.*

α: the amount of *a.a.* secreted by one male butterfly in μg

Constants

K_B , K_W: the carrying capacity of butterflies and wasps in the idealized ecosystem, respectively

d_W: the natural mortality rate of wasps.

g_{B0}: the natural growth rate of butterflies with the absence of *a.a.*

f_{10}: butterflies' natural mortality factor with the absence of *a.a.*

f_{20}: wasps' natural growth factor with the absence of *a.a.*

m*:* the effectiveness of *a.a.* helping female butterflies to better reproduce eggs

h: the effectiveness of *a.a.* in helping wasps eat the eggs

Models

$$\frac{dB}{dt} = g_B B\left(1 - \frac{B}{K_B}\right) - f_1 BW \qquad (1)$$

$$\frac{dW}{dt} = -d_W W\left(1 + \frac{W}{K_W}\right) + f_2 BW \qquad (2)$$

g_B, f_1, and f_2 have following system of equations

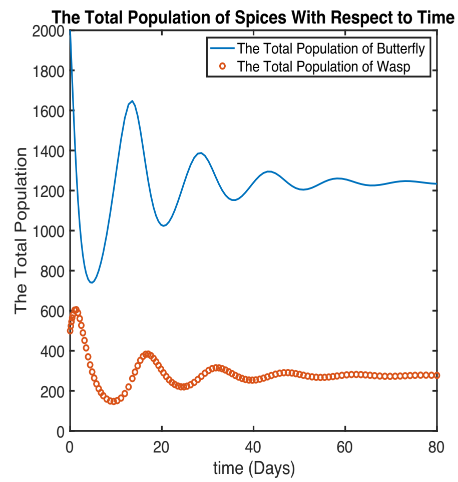
$$g_B = g_{B0}(1 + m\alpha) \qquad (3)$$

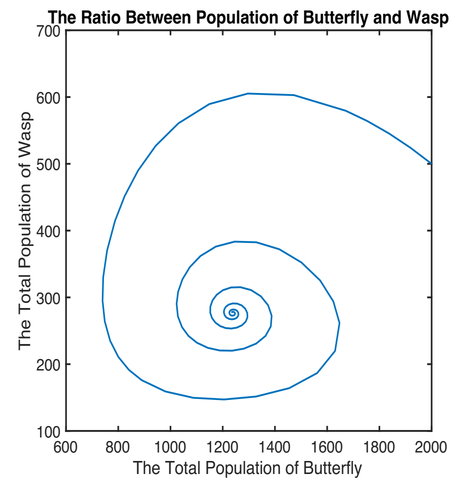
$$f_1 = f_{10}(1 + h\alpha) \qquad (4)$$

$$f_2 = f_{20}(1 + h\alpha) \qquad (5)$$

Models Explanation

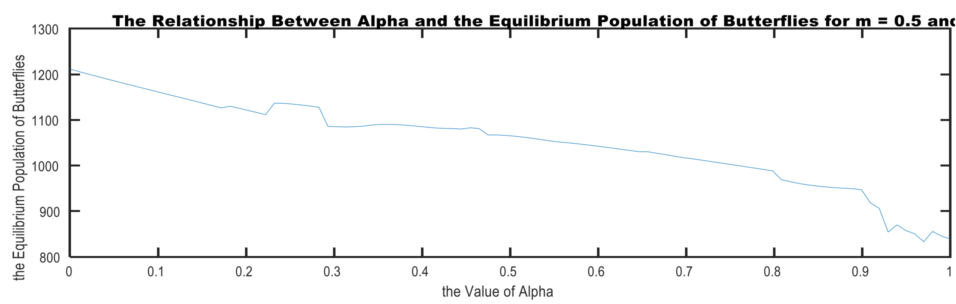
The established Lotka-Volterra Model fits well into our simulated situation of the competition between butterflies and wasps. Instead of the classic LV models, we introduced the influence of a.a. on the butterflies' mortality factor due to interactions with wasps, f_1, wasps' growth factor due to interactions with butterflies, f_2, and the natural growth rate of butterflies, g_B. We suppose that the level of secretion of a.a. has a linear relationship with them. Thus, we set up a linear relationship of f_1 and f_2 with new variables: h and α. Also, we set similar equation for g_B with new variables: m and α. Our goal is to find the best level of α without changing other constants in order to have the largest population of butterflies in the long run. Our initial conditions are that there are 2000 butterflies and 500 wasps at the first place. m = 0.5, h = 0.5 (Fatouros et al., 2005), g_{B0}= 0.348, d_W = 0.57(Southon et al., 2015), K_B = 6000, K_W = 3000, f_{10} = 0.01, and f_{20} = 0.0005.

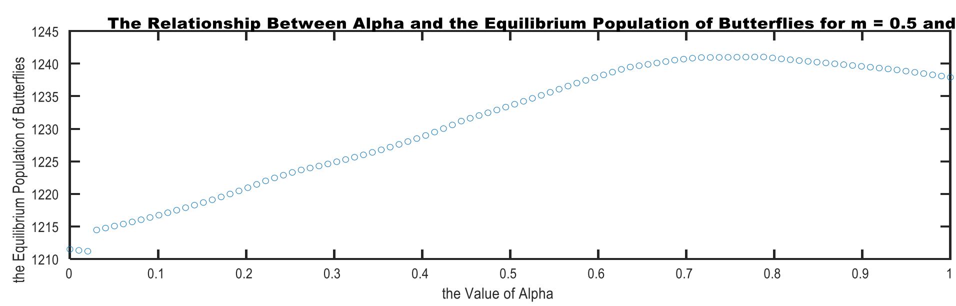




Simulations and Analysis

Our simulation has several characteristics of LV models. Initially, with many wasps endangering the butterflies, the butterfly population reaches its minimum on around 5 days. Then, as the wasp population continues to increase, butterfly population starts to increase. With increasing number of butterflies, the wasps can stage a comeback. This cyclical process repeats. As time increases, these curves will have low amplitude sinusoidal shape, and both butterfly population and wasp population will vary slightly with time. We call it equilibrium population or fixed point.





Under our assumed conditions m = 0.5 and h = 0.05, we found that when α = 0, the equilibrium population of butterfly has the maximum value of 1212. We find that the side effect of releasing a.a. is too large compared with its benefits. However, when we decrease h, for example, to 0.05, we find: as the α increase from 0 to 0.78, the equilibrium population increase. In this situation, a.a. helps female butterflies lay their eggs in better place significantly, which surpass the side effect that wasps will more likely eat these eggs. When α is larger than 0.78, the equilibrium population decrease significantly. During this process, the disadvantage caused by releasing a.a. outweigh the benefits of releasing it. Studies show that the male butterflies are under selective pressure and tend to minimize the use of a.a., which is consistent with our simulations.