Implementing Bayesian Methods to Rank College Football Teams during the Playoff Era

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1 Introduction

The challenge of ranking college football teams (specifically Division 1A Football Bowl Subdivision teams) has been controversial for decades. A national championship team has been crowned since 1869 [4], though the NCAA does not specifically hold a championship event for FBS teams, unlike in other sports. Since 1936, various polls of relevant experts have been conducted to rank the top college football teams in the country, the most widely accepted of which are the Associated Press poll (which polls leading sportswriters) and the Coaches' Poll (which polls members of the American Football Coaches Association). Because there were multiple polls that were widely accepted, in multiple years, two or even three teams were declared "national champions". [8] From 1998 to 2013, the Bowl Championship Series was the system that selected the two teams that would play in the national championship game; rankings were based on the results of two polls, the Coaches Poll and the Harris Poll; a team's "strength of schedule", a calculation based on the team's opponents' win/loss record and their opponents' opponents' win/loss record, and a variety of computer ranking systems. The formula for the BCS rankings was tweaked multiple times in response to controversy over the years, particularly when the teams chosen as the top 2 differed from the teams ranked at the top by the various human polls. Some statisticians objected to the role that so-called computer ranking systems played in the BCS rankings, with Stern [7] calling for a quantitative boycott of the BCS, because statistics were being used and manipulated to bolster the results of the human polls.

Since the 2014 football season, the college football national champion has been the winner of the College Football Playoff National Championship Game. In contrast to the BCS era, the College Football Playoff rankings are decided by a committee of 13 members, comprised of people with experience as coaches, college administrators, student-athletes, journalists, and sitting athletic directors. Each member serves a three year term, and terms on the committee are staggered. College Football Playoff rankings of the top 25 teams in the country are released starting in the 9th or 10th week of the season, and are released 6 to 7 times throughout the season, with the final rankings being released after the regular season has been completed and all conference championship games have been played. The top 4 teams in the final rankings are selected to play in the College Football Playoff games, taking place on New Year's Eve or New Year's Day, with the number 1 team playing the number 4 team, and the number 2 team playing the number 3 team. The winners of these games play in the College Football Playoff Championship game approximately one week later, and the winner of this game is crowed the national champion for the season.

Unlike in the BCS era, the Playoff Selection Committee does not use any specific metric to rank teams. In contrast, the Committee is required to "honor regular season success while at the same time providing enough flexibility and discretion to select a non-champion or independent under

circumstances where that particular non-champion or independent is unequivocally one of the four best teams in the country" [5]. When breaking ties between teams, the Committee is instructed to consider conference championships won, strength of schedule, head-to-head competition where applicable, and comparative outcomes among common opponents (without considering margin of victory). Only once in the history of the CFP has a non-conference champion been ranked in the top 4 and selected to participate in the College Football Playoff (Alabama in 2017). Further, the Playoff Selection Committee has on four occasions ranked an undefeated team (2014 Florida State, 2016 Western Michigan, 2017 UCF, 2018 UCF) behind teams with one, two, or even three losses. [5]

Since the Playoff Selection Committee rankings are by nature subjective, and controversy frequently arises over its treatment of teams outside the so-called "Power 5" conferences (Southeastern Conference, Atlantic Coast Conference, Big Ten, Big 12, Pac-12), it is worthwhile to consider a statistical ranking system. Many statistical ranking systems already exist, and many researchers have discussed methods of ranking college football teams. We will implement a statistical ranking system and compare the outcome of our system for each season to the final rankings of the Playoff Selection Committee.

2   Data

Data for this project was obtained using the R package `cfbscrapR` [3], which scrapes data from the API of the website collegefootballdata.com. We obtained the game data from the 2014-2019 seasons, as well as the teams' overall records for those seasons. For the purposes of this project, we omit games played between teams in the FBS (Division 1A) and teams outside the FBS. We consider only games played during the regular season for model fitting, up to and including conference championship games. We do not include bowl games, national championship semifinal games, and the national championship game, since the last set of Playoff Selection Committee rankings are released prior to those games and determine which teams play in the National Championship Semifinals. The number of games played each season and the number of FBS teams are listed in Table 1.

Table 1: Data for FBS Games and Teams, 2014-2019

Year	Number of Teams	Number of Games
2014	128	721
2015	128	724
2016	128	719
2017	130	736
2018	130	733
2019	130	734

Each team plays approximately 12 games in a regular season. Some teams play one or more games against FCS (Division 1AA) opponents, which are excluded from our analysis. Teams that win the most games in their conference play in their conference championship game, where applicable. Not all FBS conferences hold conference championship games, and several teams are not members of an FBS conference. Ties are always broken using overtime periods, and thus every game has a winner and a loser.

3 Bradley-Terry Model

Since every football game involves two teams competing with one winner and one loser, the Bradley-Terry model [1] is the most suitable model for this data.

The Bradley-Terry model requires each team to have an ability parameter, denoted $\alpha_K \in \mathbb{R}$. The probability that team i beats team j is then given by

$$\Pr[i \text{ beats } j] = \text{logit}^{-1}(\alpha_i - \alpha_j),$$

where logit, the log odds function, is defined by

$$\text{logit}(u) = \log\left(\frac{u}{1-u}\right)$$

and its inverse is given by

$$\text{logit}^{-1}(v) = \frac{e^v}{1+e^v}.$$

The parameters α_K can be estimated using a maximum likelihood approach. We use the statistical modeling software Stan and the R package `RStan` [6] to implement the algorithm, as well as to use Bayesian methods to find the posterior distributions of rankings. Code was adapted from an example model in the StanDev GitHub repository [2].

We assume that the parameters α_K are distributed according to a standard normal distribution, and we have no prior knowledge about these ability parameters. (This is for ease of calculation; in reality, preseason rankings, recruiting information, prior season results, number of elite players returning from year to year, and other information offer a glimpse of a team's ability parameter.)

The generated parameters α_K and the MLE parameters $\alpha_K^\star$ are plotted for each year of data in Figure 1. $\alpha_K^\star$ represents the parameter values that maximize the probability of the game results that were observed each season. With the exception of a few outliers each season, the relationship between the parameters is linear.

Once the estimated values for α_K have been generated, we rank the teams according to these estimates. The rankings for each season are presented in tables 2 through 7, along with the rankings of each team by the College Football Playoff committee, where applicable. All teams ranked by the CFP are included in the table, so the number of teams in each table varies.
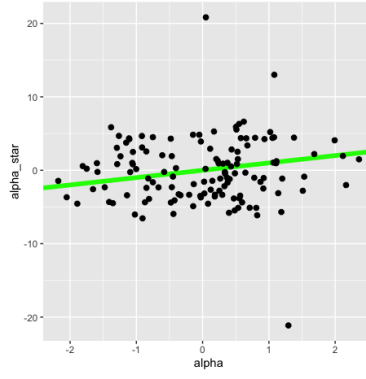
4 Bayesian Bradley-Terry Model

We now wish to implement Bayesian methods to fit a Bradley-Terry model to the data. As in the sample code [2], we use a standard normal distribution as the prior for the parameter α_K. We fit the model and use Markov chain Monte Carlo methods to sample from the posterior distribution, performing 4 chains with 4000 iterations each. Each chain has a burn-in of 1000 iterations, so we have 12000 iterations with which to calculate our means for each parameter. For brevity, the output of this process, including the mean, standard error, standard deviation, and 5th, 50th, and 95th percentiles for each α are listed in the attached Excel spreadsheets. The Rhat value for each α approaches 1 but does not reach exactly 1.
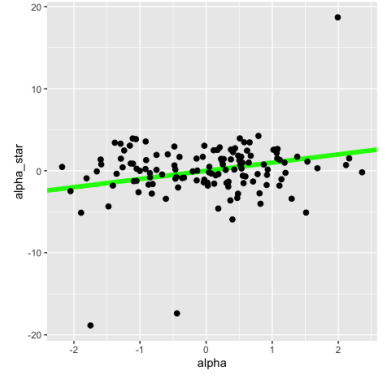
Since we are ultimately interested in the rankings, we again ranked the teams after performing the sampling. Because the posterior parameters are calculated with some level of uncertainty, the rankings are returned with uncertainty as well. The rankings sorted according to the posterior means, with higher rankings representing better teams, are displayed in Tables 8-13. We include the CFP rankings as well.
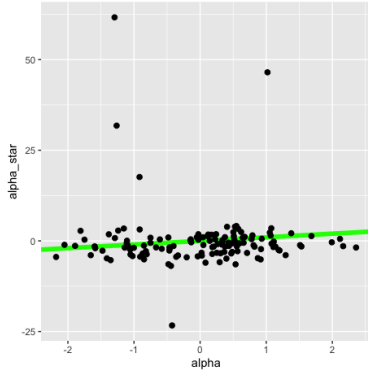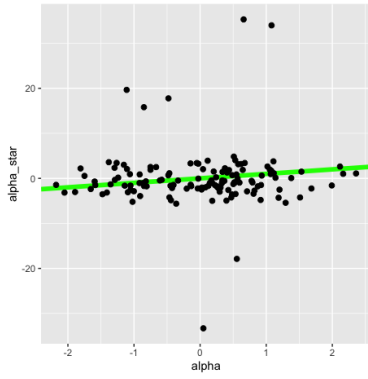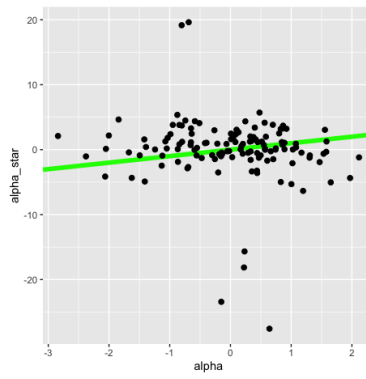
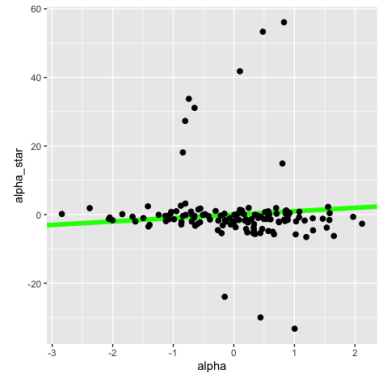(a) 2014 season   (b) 2015 season   (c) 2016 season

(d) 2017 season   (e) 2018 season   (f) 2019 season

Figure 1: Plots of true α_K vs MLE for Bradley-Terry Model

Table 2: 2014 Bradley-Terry Model Rankings and CFP Rankings

Bradley-Terry Rank	Team	CFP Rank
1	Florida State	3
2	Alabama	1
3	TCU	6
4	Baylor	5
5	Oregon	2
6	Mississippi State	7
7	Ole Miss	9
8	Auburn	19
9	Kansas State	11
10	Ohio State	4
11	LSU	23
12	Arizona	10
13	UCLA	14
14	Missouri	16
15	Georgia	13
16	Michigan State	8
17	Texas A&M	unranked
18	Oklahoma	unranked
19	West Virginia	unranked
20	Arkansas	unranked
21	Arizona State	15
22	Clemson	17
23	Georgia Tech	12
24	Utah	22
25	Wisconsin	18
26	Florida	unranked
27	USC	24
28	Boise State	20
29	Oklahoma State	unranked
30	Texas	unranked
31	Louisville	21
32	Nebraska	unranked
33	Stanford	unranked
34	Tennessee	unranked
35	Minnesota	25

Table 3: 2015 Bradley-Terry Model Rankings and CFP Rankings

Bradley-Terry Rank	team	CFP Rank
1	Clemson	1
2	Michigan State	3
3	Iowa	5
4	Oklahoma	4
5	Alabama	2
6	Ohio State	7
7	Notre Dame	8
8	Stanford	6
9	Northwestern	13
10	TCU	11
11	Oklahoma State	16
12	Houston	18
13	Oregon	15
14	Michigan	14
15	Ole Miss	12
16	Baylor	17
17	Navy	21
18	Florida	19
19	LSU	20
20	Florida State	9
21	Utah	22
22	USC	25
23	Memphis	unranked
24	Temple	24
25	Washington State	unranked
26	Wisconsin	unranked
27	Texas A&M	unranked
28	Tennessee	23
29	Texas Tech	unranked
30	Arkansas	unranked
31	Mississippi State	unranked
32	UCLA	unranked
33	California	unranked
34	Georgia	unranked
35	West Virginia	unranked
36	North Carolina	10

Table 4: 2016 Bradley-Terry Model Rankings and CFP Rankings

Bradley-Terry Rank	team	CFP Rank
1	Western Michigan	15
2	Alabama	1
3	Ohio State	3
4	Michigan	6
5	Penn State	5
6	Wisconsin	8
7	Washington	4
8	Iowa	unranked
9	Nebraska	unranked
10	Colorado	10
11	Clemson	2
12	USC	9
13	Stanford	18
14	Minnesota	unranked
15	Northwestern	unranked
16	Oklahoma	7
17	Washington State	unranked
18	Florida State	11
19	Liberty	non FBS
20	Coastal Carolina	non FBS
21	West Virginia	16
22	LSU	unranked
23	Auburn	14
24	Florida	17
25	Utah	19
26	Pittsburgh	23
27	South Florida	unranked
28	Tennessee	unranked
29	Boise State	unranked
30	Louisville	13
31	Texas A&M	unranked
32	UAB	unranked
33	Oklahoma State	12
34	Virginia Tech	unranked
35	Georgia Tech	unranked
36	Houston	unranked
37	Temple	24
38	North Carolina	unranked
39	Miami	unranked
40	Tulsa	unranked
41	Georgia	unranked
42	Arkansas	unranked
43	Kansas State	unranked
44	Kentucky	unranked
45	Navy	25

Table 5: 2017 Bradley-Terry Model Rankings and CFP Rankings

Bradley-Terry Rank	Team	CFP Rank
1	UCF	12
2	Memphis	20
3	Georgia	3
4	Clemson	1
5	Oklahoma	2
6	Wisconsin	6
7	Ohio State	5
8	Alabama	4
9	Auburn	7
10	Notre Dame	14
11	USC	8
12	Penn State	9
13	Miami	10
14	Michigan State	16
15	Stanford	13
16	Washington	11
17	TCU	15
18	Northwestern	21
19	Washington State	18
20	LSU	17
21	Virginia Tech	22
22	Michigan	unranked
23	Mississippi State	23
24	Oklahoma State	19
25	NC State	24
26	Iowa	unranked
27	San Diego State	unranked
28	Boston College	unranked
29	South Carolina	unranked
30	Louisville	unranked
31	Arizona State	unranked
32	UCLA	unranked
33	Wake Forest	unranked
34	Texas A&M	unranked
35	Florida State	unranked
36	West Virginia	unranked
37	Boise State	25

Table 6: 2018 Bradley-Terry Model Rankings and CFP Rankings

Bradley-Terry Rank	Team	CFP Rank
1	Alabama	1
2	Clemson	2
3	Notre Dame	3
4	UCF	8
5	Georgia	5
6	LSU	11
7	Florida	10
8	Kentucky	14
9	Texas A&M	19
10	Missouri	23
11	Mississippi State	18
12	South Carolina	unranked
13	Ohio State	6
14	Michigan	7
15	Auburn	unranked
16	Oklahoma	4
17	Vanderbilt	unranked
18	Tennessee	unranked
19	Syracuse	20
20	Penn State	12
21	West Virginia	16
22	Washington	9
23	Washington State	13
24	Texas	15
25	NC State	unranked
26	Pittsburgh	unranked
27	Northwestern	22
28	Cincinnati	unranked
29	Stanford	unranked
30	Iowa State	24
31	Utah	17
32	Army	unranked
33	Ole Miss	unranked
34	Michigan State	unranked
35	Temple	unranked
36	Iowa	unranked
37	Duke	unranked
38	Boston College	unranked
39	Fresno State	21
40	Appalachian State	unranked
41	Wake Forest	unranked
42	Boise State	25

Table 7: 2019 Bradley-Terry Model Rankings and CFP Rankings

Bradley-Terry Rank	Team	CFP Rank
1	LSU	1
2	Ohio State	2
3	Clemson	3
4	Georgia	5
5	Florida	9
6	Penn State	10
7	Wisconsin	8
8	Auburn	12
9	Michigan	14
10	Memphis	17
11	Oklahoma	4
12	Notre Dame	15
13	Alabama	13
14	Minnesota	18
15	Baylor	7
16	Iowa	16
17	Navy	23
18	Oregon	6
19	Cincinnati	21
20	SMU	unranked
21	Boise State	19
22	Utah	11
23	Texas A&M	unranked
24	Air Force	unranked
25	Appalachian State	20
26	USC	22
27	Kansas State	unranked
28	Oklahoma State	25
29	Texas	unranked
30	Iowa State	unranked
31	Virginia	24

Table 8: 2014 Posterior Rankings and CFP Rankings

team	mean	se_mean	5%	50%	95%	Rhat	Post. Rank	CFP Rank
Florida State	125.702667	0.06819587	111	128	131	1.0000076	1	3
Alabama	123.372667	0.07923157	105	127	131	0.9999059	2	1
Oregon	121.969167	0.09339491	101	125	131	0.9999562	3	2
Ohio State	121.895083	0.0838311	101	125	131	0.9999774	4	4
TCU	118.346583	0.11201231	92	123	131	0.9998842	5	6
Baylor	116.824583	0.11490947	88	121	131	0.9999077	6	5
Boise State	113.989917	0.10824227	86	118	130	0.9999821	7	20
Mississippi State	113.371417	0.12290361	82	118	130	0.9997493	8	7
Marshall	111.90625	0.1259009	79	116	130	1.0000051	9	unranked
Ole Miss	111.235333	0.12240698	79	115	129	1.0003282	10	9
Arizona	111.183583	0.12547994	79	115	129	0.9997842	11	10
Michigan State	111.067417	0.12991751	78	115	130	1.0000179	12	8
UCLA	110.646917	0.12683263	79	115	129	0.9998801	13	14
Georgia Tech	108.265917	0.1324042	74	112	128	0.9997813	14	12
Missouri	106.87675	0.12768616	73	111	128	0.9998415	15	16
Wisconsin	105.77375	0.13322453	71	110	128	0.9998678	16	18
Kansas State	104.804417	0.13650369	68	109	128	0.9997495	17	11
Georgia	103.638667	0.14205619	67	108	127	0.9997296	18	13
Auburn	103.398	0.14080936	68	107	127	0.9998286	19	19
Colorado State	103.360833	0.14029254	65	107	128	0.9999191	20	unranked
Clemson	103.140917	0.14343843	65	107	128	0.9999303	21	17
Arizona State	102.71625	0.13824431	66	106.5	127	0.9998874	22	15
Louisville	100.151417	0.14159571	61	104	127	0.9998524	23	21
Nebraska	99.811583	0.14337002	61	103	126	0.9998391	24	unranked
Northern Illinois	99.188167	0.14633823	59	103	127	0.9999605	25	unranked
LSU	97.593417	0.15097452	57	101	125	0.9999035	26	23
USC	96.059	0.15087799	57	99	124	1.0000999	27	24
Oklahoma	95.295583	0.1483991	55	98	124	0.999807	28	unranked
Utah	94.43925	0.15312135	53	98	124	0.9998923	29	22
Cincinnati	93.909167	0.14424576	54	97	124	0.9997934	30	unranked
Air Force	92.56275	0.15958799	51	96	124	0.9997157	31	unranked
Minnesota	92.073833	0.14691255	50	95	123	0.9998488	32	25

Table 9: 2015 Posterior Rankings and CFP Rankings

team	mean	se_mean	5%	50%	95%	Rhat	Post. Rank	CFP Rank
Clemson	124.66058	0.07212804	108	128	131	0.9997832	1	1
Alabama	122.85808	0.07507448	105	126	131	0.9999581	2	2
Michigan State	122.41883	0.08112237	102	126	131	0.9998168	3	3
Oklahoma	119.48375	0.09124207	96	123	131	1.0001548	4	4
Iowa	118.46692	0.09587521	93	122	131	0.99984	5	5
Ohio State	118.29167	0.09567495	94	122	131	0.9998633	6	7
Stanford	117.21692	0.09036555	93	121	130	0.9998488	7	6
Houston	115.40267	0.11518355	87	119	131	0.9997771	8	18
Notre Dame	114.00233	0.10799348	85	118	130	0.9998267	9	8
Northwestern	112.24683	0.1087981	82	116	130	1.0000315	10	13
Oklahoma State	108.89283	0.12109286	76	113	129	0.9999827	11	16
TCU	108.81275	0.12056877	75.95	113	129	0.9998858	12	11
Navy	107.5535	0.12124668	74	111	129	0.9997965	13	21
Florida	107.3735	0.11593215	76	111	128	0.9997506	14	19
Michigan	107.032	0.11823542	75	111	128	0.9998531	15	14
Florida State	106.87008	0.12160647	72	111	129	0.9999385	16	9
North Carolina	106.28817	0.12541878	72	110	128	0.999838	17	10
LSU	105.67975	0.12281464	72	109	128	0.9999853	18	20
Oregon	105.41308	0.12759365	71	109	128	0.9997713	19	15
Ole Miss	105.22983	0.12343598	71	109	128	0.999714	20	12
Western Kentucky	104.308	0.1161484	70	107	127	0.9997908	21	unranked
Memphis	102.10283	0.13091007	66	105	127	0.9998482	22	unranked
Utah	102.08358	0.12840353	68	105	127	0.9997166	23	22
Temple	101.23367	0.12649478	66	104	126	0.9997557	24	24
Toledo	100.8135	0.12813528	64	104	127	0.9998308	25	unranked
Bowling Green	98.58433	0.12622797	63	101	125	0.9998645	26	unranked
Appalachian State	98.26592	0.13055031	60	101	126	0.9997887	27	unranked
Baylor	98.03108	0.13618214	60	101	126	0.9998912	28	17
Wisconsin	98.02825	0.1302716	61	101	126	0.9997461	29	unranked
Washington State	96.72492	0.12685674	59	100	125	0.99985	30	unranked
USC	96.54783	0.12842585	62	99	123	0.9998352	31	25
Georgia	95.89875	0.13120813	58.95	98	124	0.9998699	32	unranked
BYU	93.82883	0.12767657	57	96	123	0.9998401	33	unranked
Mississippi State	93.63317	0.12945858	57	96	123	0.9999004	34	unranked
Tennessee	93.51517	0.13541051	56	96	123	0.9998809	35	23

Table 10: 2016 Posterior Rankings and CFP Rankings

team	mean	se_mean	5%	50%	95%	Rhat	Post. Rank	CFP Rank
Alabama	126.6215	0.05879126	114	129	131	0.9998725	1	1
Ohio State	123.716417	0.08283311	106	127	131	1.0000491	2	3
Clemson	122.08025	0.08735793	101	125	131	0.999767	3	2
Western Michigan	120.97242	0.1005865	97	125	131	0.9997972	4	15
Washington	120.713417	0.09123514	98	124	131	0.9998033	5	4
Penn State	118.685417	0.09853855	94	122	130	0.9998808	6	5
Michigan	116.77075	0.10826463	89	121	130	0.9999813	7	6
Oklahoma	112.97725	0.12672518	82	117	130	0.9998144	8	7
Wisconsin	112.60575	0.1162897	83	117	129	0.9999792	9	8
Colorado	109.464333	0.12312926	77	113	129	0.9999029	10	10
Boise State	108.608417	0.12576215	75	113	129	1.0000285	11	unranked
USC	107.273333	0.13194147	73	111	128	0.9998073	12	9
West Virginia	107.07683	0.133066	71	111	129	0.9998115	13	16
Florida State	106.49025	0.13203195	72	110	128	0.9999887	14	11
South Florida	106.27275	0.14324422	70	110	128	0.9998782	15	unranked
Nebraska	105.434167	0.13414923	70	109	128	0.9998212	16	unranked
Stanford	104.575	0.13947306	69	109	127	0.9999799	17	18
Iowa	104.217833	0.14234393	67	108	128	0.9999438	18	unranked
Washington State	103.5635	0.13661724	66	108	127	0.9999574	19	unranked
Louisville	100.251	0.14237829	62	104	126	0.999808	20	13
Temple	98.26875	0.15108054	59	102	126	0.9997925	21	24
Oklahoma State	98.2465	0.14867497	59	102	126	0.9998904	22	12
Tennessee	97.588917	0.15475083	59	101	125	0.9998331	23	21
Florida	97.263917	0.14253767	58	101	125	0.9999282	24	17
Houston	97.168667	0.15686332	57	101	126	0.9998984	25	unranked
Auburn	96.811417	0.14871658	57	100	125	0.9998167	26	14
Western Kentucky	95.97775	0.1567301	56	99	125	0.9998482	27	unranked
Appalachian State	95.861583	0.14926535	57	99	125	0.9998439	28	unranked
Tulsa	94.055333	0.16132423	53	97	125	0.9998076	29	unranked
Toledo	93.503917	0.15588971	52	97	124	0.9999702	30	unranked
Pittsburgh	93.280333	0.15016179	53	96	124	0.9998465	31	23
Virginia Tech	92.441667	0.15516139	52	95	123	0.9999263	32	22
LSU	92.1175	0.15522872	50	95	123	1.0000787	33	20
Air Force	91.695417	0.16523195	50	94	123	0.9997698	34	unranked
Texas A&M	91.630417	0.15432723	51	94	123	0.9997688	35	unranked
Navy	91.53075	0.15318638	51	94	122	0.9997794	36	25
Minnesota	91.369917	0.15255595	50	94	123	0.9998165	37	unranked
Troy	90.930333	0.16762394	49	94	123	0.9997918	38	unranked
Georgia Tech	90.7585	0.16595842	50	93	122	0.9997832	39	unranked
San Diego State	89.55975	0.15981188	48	92	122	0.9998338	40	unranked
Utah	87.699	0.16764885	47	90	121	0.9998452	41	19

Table 11: 2017 Posterior Rankings and CFP Rankings

team	mean	se_mean	5%	50%	95%	Rhat	Post. Rank	CFP Rank
UCF	123.8335	0.08219554	105	127	131	0.9998396	1	12
Clemson	122.459333	0.08284794	103	125	131	1.0001239	2	1
Georgia	122.192167	0.08227549	103	125	131	1.0001797	3	3
Wisconsin	121.02383	0.08828528	100	124	131	1.0000146	4	6
Oklahoma	120.73925	0.09146882	99	124	131	0.9998509	5	2
Ohio State	119.2715	0.09192868	96	122	131	0.9998358	6	5
Alabama	119.02275	0.0922969	95	123	131	0.9997698	7	4
USC	115.99025	0.10558157	90	119	130	1.0000526	8	8
Penn State	114.79725	0.10933644	87	118	130	0.999799	9	9
Miami	113.30875	0.11263324	83	117	130	0.999842	10	10
Notre Dame	113.236667	0.11002039	85	117	129	1.0000515	11	14
Auburn	113.086333	0.11176959	84	117	129	0.9999274	12	7
Memphis	110.458667	0.13004549	77	115	129	0.9999224	13	20
Michigan State	108.794833	0.12409946	76	112	129	0.9997306	14	16
Washington	108.767333	0.14239393	75	113	129	1.0005303	15	11
Toledo	105.548333	0.13724962	69	109	128	1.0000189	16	unranked
Stanford	105.155583	0.13034064	71	109	127	0.999921	17	13
TCU	105.154667	0.13919391	70	109	128	0.9997873	18	15
San Diego State	105.052583	0.15249809	69	109	128	0.9998111	19	unranked
Boise State	104.375583	0.12931842	69	108	127	0.9999623	20	25
Northwestern	103.862917	0.14285066	68	107	127	0.9997904	21	21
Washington State	102.29225	0.14393225	65	106	127	0.9998062	22	18
LSU	101.923083	0.14110953	64	105	127	1.0000655	23	17
Florida Atlantic	100.0605	0.1446541	63	104	126	0.9998448	24	unranked
Troy	99.631833	0.15522992	60	103	126	1.0000391	25	unranked
Virginia Tech	99.5595	0.13948065	61	103	126	0.9999107	26	22
Oklahoma State	98.277083	0.15420241	58	102	126	1.0002136	27	19
Michigan	97.71725	0.14666821	60	101	124	0.9997927	28	unranked
South Florida	95.934667	0.16709517	52	100	126	0.9997612	29	unranked
NC State	94.669083	0.15950993	55	98	123	0.9998159	30	24
Mississippi State	94.211083	0.15471988	53	98	123	0.9998628	31	23

Table 12: 2018 Posterior Rankings and CFP Rankings

team	mean	se_mean	5%	50%	95%	Rhat	Post. Rank	CFP Rank
Alabama	125.222	0.05788354	113	127	130	1.0000564	1	1
Clemson	124.96	0.0637054	112	127	130	1.0001163	2	2
Notre Dame	124.300583	0.06728201	109	127	130	1.0000032	3	3
UCF	120.988167	0.09557704	99	125	130	0.9998672	4	8
Oklahoma	119.763583	0.08625827	99	123	130	0.9998797	5	4
Ohio State	119.483417	0.08748482	97	123	130	0.9998106	6	6
Georgia	118.472	0.0933502	96	122	129	0.9998243	7	5
Michigan	115.103083	0.10692646	88	119	129	0.9998564	8	7
Fresno State	106.848417	0.13997504	71	111	127	0.9998426	9	21
Washington State	105.88475	0.14988938	69	111	127	0.9999369	10	13
LSU	105.742333	0.14021772	69	110	127	0.9998718	11	11
Penn State	105.1065	0.13380833	69	110	126	0.9998079	12	12
Cincinnati	104.73525	0.14756686	68	109	127	0.999896	13	unranked
Kentucky	104.454583	0.14160189	68	109	126	0.9998492	14	14
Washington	104.42525	0.14391676	68	109	126	0.9998536	15	9
Appalachian State	103.570083	0.1473336	65	108	127	0.9997555	16	unranked
Florida	103.14025	0.1509281	64	108	126	0.9998069	17	10
Army	102.96725	0.14958659	64	108	127	0.9997498	18	unranked
Boise State	102.920833	0.13753258	66	107	126	0.9997357	19	25
Syracuse	101.675583	0.14937302	62	106	126	0.9997389	20	20
Texas A&M	101.086833	0.15735805	62	105	125	0.9999271	21	19
Utah State	98.018417	0.17521254	55	103	126	0.9997361	22	unranked
Missouri	97.039917	0.16646617	56	101	124	0.9998835	23	23
Texas	96.905167	0.15351	58	101	123	1.0001501	24	15
Utah	96.20425	0.16050249	57	100	123	0.9999104	25	17
Temple	96.07075	0.16114376	54	100	124	0.9998095	26	unranked
Mississippi State	95.719333	0.16153132	55	100	124	0.999925	27	18
NC State	95.569417	0.16373291	53	99	124	0.9998252	28	unranked
West Virginia	95.34175	0.1610799	53	99	124	0.9997877	29	16
Stanford	93.89875	0.16217374	53	97	123	0.9998992	30	unranked
UAB	92.332667	0.16678685	51	96	123	0.9999418	31	unranked
Buffalo	91.698	0.16118802	49	95	123	0.9998009	32	unranked
Georgia Southern	91.501667	0.16430614	48	95	123	0.9997282	33	unranked
Northwestern	90.351083	0.15419231	51	93	121	0.9997134	34	22
Troy	89.9985	0.1669571	47	93	122	1.0000033	35	unranked
Iowa	87.77875	0.1614587	45	91	120	0.9998741	36	unranked
Iowa State	87.679667	0.16863421	44	91	121	0.9999923	37	24

Table 13: 2019 Posterior Rankings and CFP Rankings

team	mean	se_mean	5%	50%	95%	Rhat	Post. Rank	CFP Rank
Ohio State	125.97025	0.05416804	115	128	130	0.9998898	1	2
LSU	124.956333	0.05850373	112	127	130	1.0000421	2	1
Clemson	121.513	0.08281368	103	125	130	0.9997314	3	3
Memphis	118.591	0.09091761	96	122	130	0.9998359	4	17
Oklahoma	117.732083	0.09943185	95	121	129.05	1.0000359	5	4
Boise State	115.8255	0.10341614	91	119	129	0.9999034	6	19
Georgia	115.6125	0.10454621	92	119	129	1.0000154	7	5
Appalachian State	113.393417	0.11787199	86	117	129	0.9998986	8	20
Notre Dame	113.342833	0.11534021	86	117	128	0.9998626	9	15
Penn State	113.248167	0.11578145	86	117	129	1.0001478	10	10
Wisconsin	111.89508	0.1192769	84	115	128	0.9998164	11	8
Oregon	111.50625	0.11934576	83	115	128	0.9998906	12	6
Baylor	110.680417	0.13202502	81	114	128	0.9998866	13	7
Florida	110.125583	0.1265286	80	114	128	0.9997235	14	9
Cincinnati	109.4085	0.1280077	79	113	127	1.0000523	15	21
Utah	109.351583	0.12757517	79	113	127	0.9999404	16	11
Navy	108.898167	0.12949897	77	113	128	0.9998071	17	23
Michigan	108.689	0.13253042	78	112	127	1.0000395	18	14
Auburn	108.073417	0.12712574	77	111	127	0.9999365	19	12
Alabama	107.618833	0.13079123	76	111	127	1.00004	20	13
SMU	107.6175	0.12639145	75	111	127	0.9999617	21	unranked
Minnesota	106.610333	0.13558169	73	110	127	0.9998995	22	18
Air Force	105.275917	0.14390795	71	109	127	0.9998649	23	unranked
Iowa	104.138083	0.15481461	71	108	126	1.0002471	24	16
Florida Atlantic	100.217667	0.15750284	64	104	125	0.9998653	25	unranked
Louisiana	97.237917	0.15263477	60	101	124	0.9998274	26	unranked
USC	94.802667	0.17030658	57	98	122	0.9999839	27	22
UCF	93.620667	0.17355801	54	97	122	1.0000012	28	unranked
Virginia	92.308833	0.16413202	53	95	121	1.0000794	29	24
Western Kentucky	90.22325	0.1660046	49	93	121	0.9999623	30	unranked
Oklahoma State	88.146	0.17610979	47	91	119	1.0001418	31	25

5 Discussion

The Bradley-Terry model is based solely on wins and losses. The goal is to maximize the likelihood that a team with a higher ability level α beats a team with a lower ability level. As a result, it does not take into account strength of schedule or conference championships when ranking one team above another. Strength of schedule is one of the more controversial measures in college football, and there are many different metrics that measure strength of schedule. Future models could incorporate the strength of schedule metrics that already exist, or devise a new strength of schedule metric.

Despite using 12000 post-burn in iterations, the MCMC algorithm did not converge to an Rhat of 1 in our experiment. It is possible that additional iterations would result in better convergence, and this is also worth exploring further.

The normal distribution used for the prior is a non-informative prior. Past research in this field has used the uniform distribution or the Dirichlet distribution as a prior. These are still non-informative priors, and as mentioned earlier, there is always information known about the ability of each team prior to the beginning of the college football season. Future studies could implement other informative or non-informative prior distributions to help improve the model. Prior season results should not be used to inform the prior distribution, both because the College Football Playoff committee is supposed to only consider the current season when devising its rankings, and because the nature of college football is such that teams' ability can be vastly different from one season to the next. UCF, for example, went from a 0 win season in 2015 to undefeated seasons in 2017 and 2018.

As expected, the Bradley-Terry model ranked undefeated teams above all other non-undefeated teams. An unintended oversight resulted in teams that were not part of the FBS during particular seasons (Liberty and Coastal Carolina in 2016) being ranked in the top 25. These teams were not listed in any game data for these seasons, so this is clearly an issue that needs to be remedied in future models. In addition, in the 2017 rankings, Memphis was ranked number 2, behind only UCF (who beat them twice, during the regular season and in the conference championship game). Georgia, meanwhile, who lost one game during the regular season and won the SEC Championship, is ranked behind Memphis, with two losses and no conference championship.

Comparing the posterior rankings with the non-Bayesian Bradley-Terry model for each year of interest, these unexpected events seem to no longer be a factor in the rankings. The most relevant ranks are ranks 1 through 4, as those 4 teams are invited to compete in the College Football Playoff and play for the National Championship. In 2014 and 2015, the posterior rankings selected the same top 4 teams as the College Football Playoff committee, though the order was different in 2014. In 2016, the posterior rankings would have resulted in undefeated Western Michigan participating in the playoff instead of one-loss Pac-12 champion Washington. In 2017, both Oklahoma (Big 12 champion) and Alabama (one loss team that did not participate in the conference championship game) would have been left out of the playoff, in favor of undefeated UCF and Big Ten runner-up Wisconsin. (It is interesting that the posterior mean rankings rank Wisconsin ahead of Ohio State, who beat Wisconsin to win the 2017 Big Ten championship.) In 2018, undefeated UCF would have been selected for the playoff in lieu of Big 12 champion Oklahoma, and in 2019, Oklahoma (Big 12 champion, one loss) would have again been left out in favor of AAC champion and one-loss team Memphis.

A perpetual complaint about the Playoff Selection Committee is that teams from Power 5 conferences are ranked above teams with better records from non-Power 5 conferences. This model, in general, disregards conference membership. The argument in favor of Power 5 teams being ranked higher is usually due to strength of schedule–Power 5 teams are perceived to play more challenging

17

schedules than non-Power 5 teams. It would be worthwhile to analyze the differences in rankings between P5 and non P5 teams–the teams that would have been selected for the playoff using our model are, with one exception, non-P5 teams. In addition, examining the results of the model in conjunction with time effects (comparing teams whose one loss came at the beginning of the season instead of the end) would be worthwhile. The Playoff Committee does not begin ranking teams until they have played at least 7 to 9 games, and thus they may be less likely to be swayed by a single loss in week 1 of the season.

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