Question 1 - CPS

1.1) b.

We will prove that (append$ lst1 lst2 cont) = (cont (append lst1 lst2)) for any lists lst1 and lst2 and a continuation procedure cont by induction on n where n will be the length of lst1.

Base Case: n=0

when n=0 , lst1 is empty then

a-e [(append$ lst1 lst2 cont)] ⇒* a-e [(cont lst2)]

a-e [(cont (append lst1 lst2))] ⇒* a-e [(cont lst2)]

Therefore (append$ lst1 lst2 cont) = (cont lst2) = (cont (append lst1 lst2)).

Assume that the equation holds for n = k. Now we will prove the equation for n = k+1

Inductive Step:

let k∈N, n=k+1 :

n = k+1 so the length of lst1 is k+1

a-e [(append$ lst1 lst2 cont)] ⇒*

a-e [(append$ (cdr lst1) lst2 (lambda (res) (cont (cons (car lst1) res)))]

we will define

From the induction assumption:

a-e [(append$ (cdr lst1) lst2 (lambda (res) (cont (cons (car lst1) res))))] =

a-e [((lambda (res) (cont (cons (car lst1) res))) (append (cdr lst1) lst2))] ⇒*

a-e [(cont (cons (car lst1) (append (cdr lst1) lst2)))]

a-e [(cont (append lst1 lst2))] ⇒*

a-e [(cont (cons (car lst1) (append (cdr lst1) lst2)))]

Therefore (append$ lst1 lst2 cont) = (cont (append lst1 lst2)).

Question 3 - Logic programing

3.1 Unification

1. $unify[t(s(s),\ G,\ s,\ p,\ t(K),\ s),\ t(s(G),\ G,\ s,\ p,\ t(K),\ U)]$

$let\ s\ =\ \{\}\ ,\ A\ =\ t(s(s),\ G,\ s,\ p,\ t(K),\ s)\ ,\ B\ =\ t(s(G),\ G,\ s,\ p,\ t(K),\ U)$

1) $s\ =\ s \circ \{G\ =\ s\}\ =\ \{G\ =\ s\}$

   $A \circ s\ =\ t(s(s),\ s,\ s,\ p,\ t(K),\ s)$

   $B \circ s\ =\ t(s(s),\ s,\ s,\ p,\ t(K),\ U)$

2) $s\ =\ s \circ \{U\ =\ s\}\ =\ \{G\ =\ s\ ,\ U\ =\ s\}$

   $A \circ s\ =\ t(s(s),\ s,\ s,\ p,\ t(K),\ s)$

   $B \circ s\ =\ t(s(s),\ s,\ s,\ p,\ t(K),\ s)$

$result\colon\ s\ =\ \{G\ =\ s\ ,\ U\ =\ s\}$

2. $unify[g(l, M, g, G, U, g, v(M)),\ g(l, v(U), g, v(M), v(G), g, v(M))]$

$let\ s = \{\},\ A\ =\ g(l, M, g, G, U, g, v(M)), B\ =\ g(l, v(U), g, v(M), v(G), g, v(M))$

1) $s\ = s \circ \{M\ =\ v(U)\}\ = \{M\ =\ v(U)\}$
   $A \circ s\ =\ g(l, v(U), g, G, U, g, v(v(U)))$
   $B \circ s\ =\ g(l, v(U), g, v(v(U)), v(G), g, v(v(U)))$
2) $s\ = s \circ \{G\ =\ v(v(U))\}\ = \{M\ =\ v(U),\ G\ =\ v(v(U))\}$
   $A \circ s\ =\ g(l, v(U), g, v(v(U)), U, g, v(v(U)))$
   $B \circ s\ =\ g(l, v(U), g, v(v(U)), v(v(v(U))), g, v(v(U)))$
3) $s\ = s \circ \{U\ =\ v(v(v(U)))\}$
   $U\ appears\ on\ both\ sides\ of\ the\ equation\ so\ the\ algorithm\ will\ fail$
   $when\ running\ the\ occurs\ check$

$result$: $There\ is\ no\ suitable\ substitution.$

3. $unify[m(M, N),\ n(M, N)]$

$let\ s = \{\},\ A\ =\ m(M, N), B\ =\ n(M, N)$

$s\ = s \circ \{m(M, N)\ =\ n(M, N)\}$

$will\ fail\ immediately\ because\ m,\ n\ are\ different\ predicates.$

$result$: $There\ is\ no\ suitable\ substitution.$

4. $unify[p([v \mid [V \mid VV]]),\ p([[v \mid V] \mid VV])]$

$let\ s = \{\},\ A\ =\ p([v \mid [V \mid VV]]), B\ =\ p([[v \mid V] \mid VV])$

$s\ = s \circ \{v\ =\ [v \mid V]\}$

$v\ appears\ on\ both\ sides\ of\ the\ equation\ so\ the\ algorithm\ will\ fail$
$when\ running\ the\ occurs\ check$

$result$: $There\ is\ no\ suitable\ substitution.$

5. $unify[g([T]),\ g(T)]$

$let\ s = \{\},\ A\ =\ g([T]), B\ =\ g(T)$

$s\ = s \circ \{[T]\ =\ T\}$

$T\ appears\ on\ both\ sides\ of\ the\ equation\ so\ the\ algorithm\ will\ fail$
$when\ running\ the\ occurs\ check$

$result$: $There\ is\ no\ suitable\ substitution.$

3.1 Proof Tree

?- le(X,s(zero))),times(X,s(s(zero)),Y))).