Cross Phase Calculation Memo

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FHD calibrates to 'XX' and 'YY' (or, in equivalent notation, '*pp*' and '*qq*') visibilites only. The 'crosspol' visibilities are excluded from calibration. This choice limits the information used in calibration but we expect it to have little effect on the quality of our power spectrum results.

However, excluding the crosspol visibilities from calibration means we have an extra calibration degree of freedom that must be calculated before we can achieve sky-polarized images. This degree-of-freedom corresponds to the overall phase offset between the East-West and North-South dipoles, called ϕ for the purposes of this memo. ϕ roughly corresponds to a mixing between the Stokes U and V modes (although a full polarization analysis involves the instrument Jones matrix and mixes all instrumental and Stokes polarization modes).

In the most general case, calibration solutions are calculated by minimizing the quantity

$$\chi^2 = \sum_f \sum_{ij} \left[\frac{1}{\sigma^2_{pp,ij}} \left| v_{pp,ij} + g_{p,i} g^*_{p,j} m_{pp,ij} \right|^2 + \frac{1}{\sigma^2_{qq,ij}} \left| v_{qq,ij} + g_{q,i} g^*_{q,j} m_{qq,ij} \right|^2 \right.$$
$$\left. + \frac{2}{\sigma^2_{pq,ij}} \left| v_{pq,ij} + g_{p,i} g^*_{q,j} m_{pq,ij} \right|^2 \right], \tag{1}$$

where p denotes the East-West dipole direction, q denotes the North-South dipole direction, and i and j index antennas. For example, $v_{pq,ij}$ is the correlated visibility between the East-West polarization of antenna i and the North-South polarization of antenna j. All variables are implicitly per-frequency.

In general, FHD only calibrates using the first two the terms in this χ-squared, omitting the cross-visibilities in calibration. However, this introduces a calibration degeneracy corresponding to the average phase between p and q. Therefore, we need another calibration step to break that phase degeneracy.

Let $g_{p,i} = h_{p,i} e^{-i\phi/2}$ and $g_{q,i} = h_{q,i} e^{i\phi/2}$ where ϕ is the overall phase between p

and q. Assuming that $\sigma^2_{pq,ij} = \sigma^2_{qp,ij} = \sigma^2_{ij}$, we get that

$$\begin{aligned}
&\frac{\partial \chi^2}{\partial \phi} \\
&= \frac{\partial}{\partial \phi} \sum_f \sum_{ij} \frac{2}{\sigma^2_{ij}} \left[\left| v_{pq,ij} + h_{p,i} h^*_{q,j} e^{-i\phi} m_{pq,ij} \right|^2 \right] \\
&= \frac{\partial}{\partial \phi} \sum_f \sum_{ij} \frac{2}{\sigma^2_{ij}} \left[\mathrm{Re}(v_{pq,ij} h^*_{p,i} h_{q,j} m^*_{pq,ij}) \cos\phi - \mathrm{Im}(v_{pq,ij} h^*_{p,i} h_{q,j} m^*_{pq,ij}) \sin\phi \right] \\
&= \frac{\partial}{\partial \phi} \sum_f \sum_{ij} \frac{2}{\sigma^2_{ij}} \left[\mathrm{Re}(v_{pq,ij} h^*_{p,i} h_{q,j} m^*_{pq,ij}) \cos\phi - \mathrm{Im}(v_{pq,ij} h^*_{p,i} h_{q,j} m^*_{pq,ij}) \sin\phi \right] \\
&= -\sin\phi \sum_f \sum_{ij} \frac{2}{\sigma^2_{ij}} \mathrm{Re}(v_{pq,ij} h^*_{p,i} h_{q,j} m^*_{pq,ij}) - \cos\phi \sum_f \sum_{ij} \frac{2}{\sigma^2_{ij}} \mathrm{Im}(v_{pq,ij} h^*_{p,i} h_{q,j} m^*_{pq,ij}).
\end{aligned} \tag{2}$$

Setting $\frac{\partial \chi^2}{\partial \phi} = 0$ gives

$$\tan\phi = \frac{\sum_f \sum_{ij} \frac{1}{\sigma^2_{ij}} \mathrm{Im}(v^*_{pq,ij} h_{p,i} h^*_{q,j} m_{pq,ij})}{\sum_f \sum_{ij} \frac{1}{\sigma^2_{ij}} \mathrm{Re}(v^*_{pq,ij} h_{p,i} h^*_{q,j} m_{pq,ij})}. \tag{3}$$

In other words,

$$\phi = \mathrm{Arg}\left(\sum_f \sum_{ij} \frac{1}{\sigma^2_{ij}} v^*_{pq,ij} h_{p,i} h^*_{q,j} m_{pq,ij} \right) \tag{4}$$

where Arg denotes the complex argument.

This calculation is implemented in FHD with the function `vis_calibrate_crosspol_phase`.