Resilience Evaluation of the London Underground and Simulation of Flows under Different Scenarios[1]

Word Count:3276

1 PART 1: Resilience Evaluation

1.1 Measures

1.1.1 Centrality measures

This report selects 3 centrality measures to evaluate the importance of different stations: **Degree Centrality, Betweenness Centrality,** and **Closeness Centrality**, as they apply to traffic networks better.

Degree centrality (D.C., $C_d(i)$)

D.C. is the simplest centrality. It represents the number of nodes directly connected to the node and reflects the local value of a node. It's defined as: $C_d(i) = \sum_i a_{ij}$, where $a_{ij} = 1$ if there is a direct link between node i and node j, otherwise $a_{ij} = 0$.

Closeness centrality (C.C., $C_c(i)$)

C.C. is a more global characteristic representing the general accessibility of a node. It is the inverse of the mean geodesic distance between that node and all other nodes and describes how close the node is to all the other nodes on average. A node that is very close to most nodes has high in C.C. It can be defined as: $C_c(i) = \frac{n}{\sum_j d_{ij}}$, Where d_{ij} is the shortest distance from $nodes\ i$ to j, and n is the number of nodes in a network.

Betweenness centrality (B.C., $C_b(i)$)

Like C.C.(Brandes, Borgatti and Freeman, 2016), B.C. better illustrates the global value of a node, reflected via the number of shortest paths passing through the node compared to all shortest paths. It can be defined as: $C_d(i) = \sum_{st} \frac{g_{st}^i}{g_{st}}$, Where g_{st}^i is the number of the shortest path between $nodes\ s$ and t passing i, and g_{st} is the total number of the shortest path between $nodes\ s$ and t.

1.1.2 Evaluation measures

We choose **Global Efficiency** and the **Normalised Size of the Giant Component** to evaluate the impact of node removals and identify the importance of the removed node through the change in the measures' values. Both measures evaluate the performance of a network on a global scale.

Global efficiency (G.E., $E(g)$)

G.E. is the average of all the local efficiency for each pair of nodes and is based on computing all possible shortest paths between each pair. It evaluates the efficiency of the network on a global level and has been applied to optimizations of transportation systems and brain connectivity (Ek, VerSchneider and Narayan, 2015). It's defined as: $E(g) = \frac{1}{n(n-1)} \sum_i \sum_{j \neq i} \frac{1}{d_{ij}}$

Where d_{ij} is the shortest distance between $nodes\ i$ and j, and $\frac{1}{d_{ij}}$ is the local efficiency for $nodes\ i$ and j, n the number of nodes in a network.

Normalized size of the giant component (S.G., $S(g)$)

After node removals, connected networks can be divided into connected components, among which the component with the most nodes is called the giant component. S.G. is the relative size of the giant component compared to the whole network, which is defined as: $S(g) = \frac{n_{max}}{n}$

Where n is the number of nodes in a network, and n_{max} is the number of nodes in the largest component.

In Part 1, we simplify London underground into an undirected unweighted network and ignore directions nor weights of links.

1.2 Analysis

1.2.1 Initial evaluation

The original S for the London underground is **1.0**, as it's a fully connected network with **306** stations. The original E is calculated to be **0.102125**, suggesting that the London underground is a large network with large shortest paths on average, but only a few interchanges.

Baker Street and **King's Cross St. Pancras** both have the largest D.C. of 7, meaning they are directly connected to 7 stations. **Green Park** has the largest B.C. and C.C. The highest C.C. suggests that it is located in the topological center of the underground network and closest to all the other stations on average. The highest B.C. indicates that many pairs of nodes have it on their shortest paths. If removed, the lengths of shortest paths for many pairs will significantly increase.

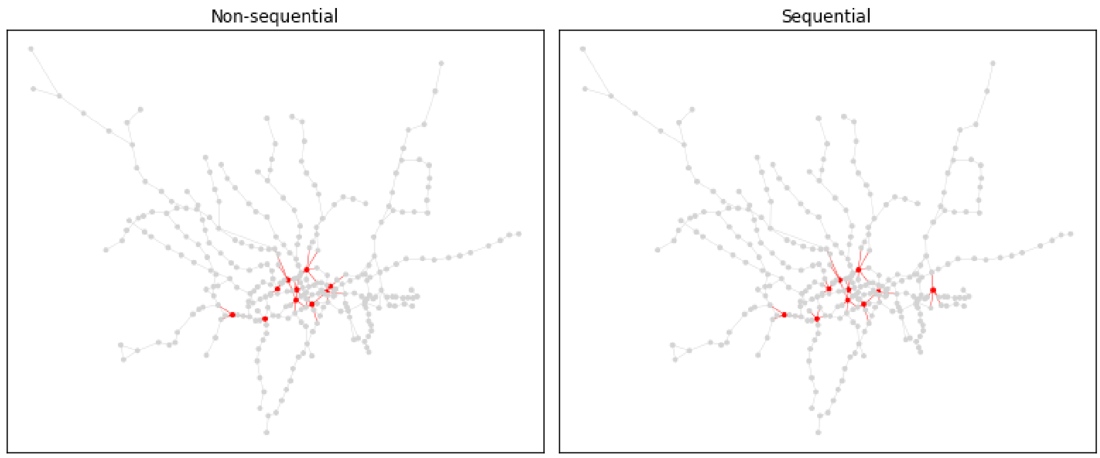
1.2.2 Removal strategies

we remove 10 nodes with the highest values of 3 different centrality measures separately, following two strategies, **'non-sequential'** and **'sequential'**. In 'non-sequential' scenarios, we use centrality values calculated from the original network for the whole removal process, while being 'sequential', we recompute the centrality values of the remaining nodes after each removal. In both strategies, $\boldsymbol{E(g)}$ and $\boldsymbol{S(g)}$ of the system are recomputed for evaluation after each removal. Thus we have **6 scenarios in total**.

1.2.3 Result Presentation

Removal based on D.C.

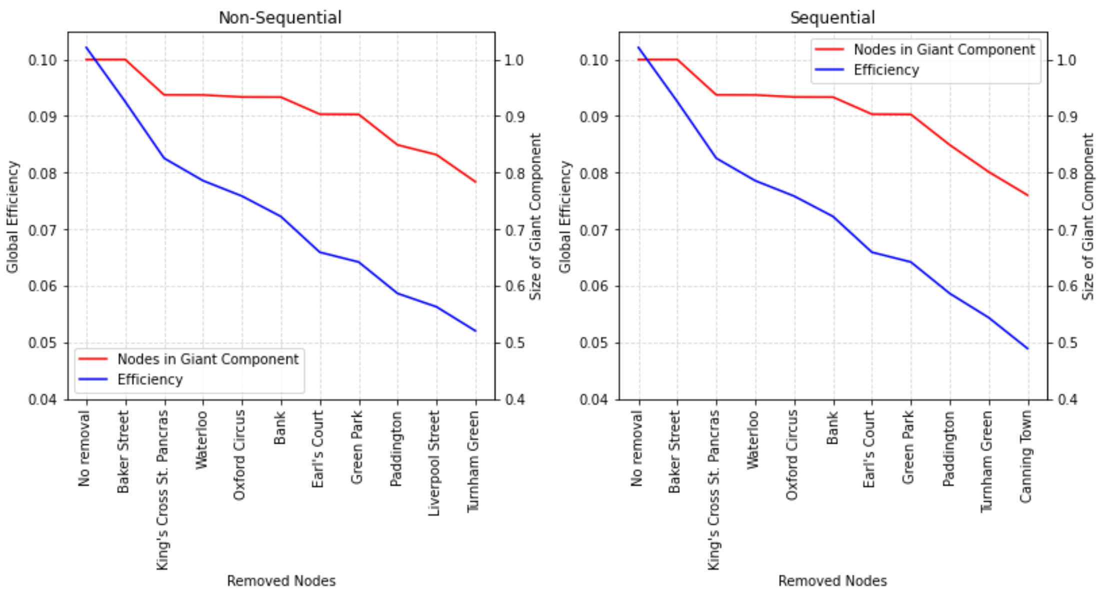
Figure 1: Nodes removed based on D.C.



In D.C. scenarios, with non-sequential strategy, E dropped from 0.102125 to 0.052010, and S dropped from 1.0 to 0.783784; while with sequential strategy, E dropped to 0.048881, and S dropped to 0.760135 (Figure 2). Sequential strategy achieves slightly greater declines in both measures, and 2 removal strategies share **9** common removed nodes (Figure 1).

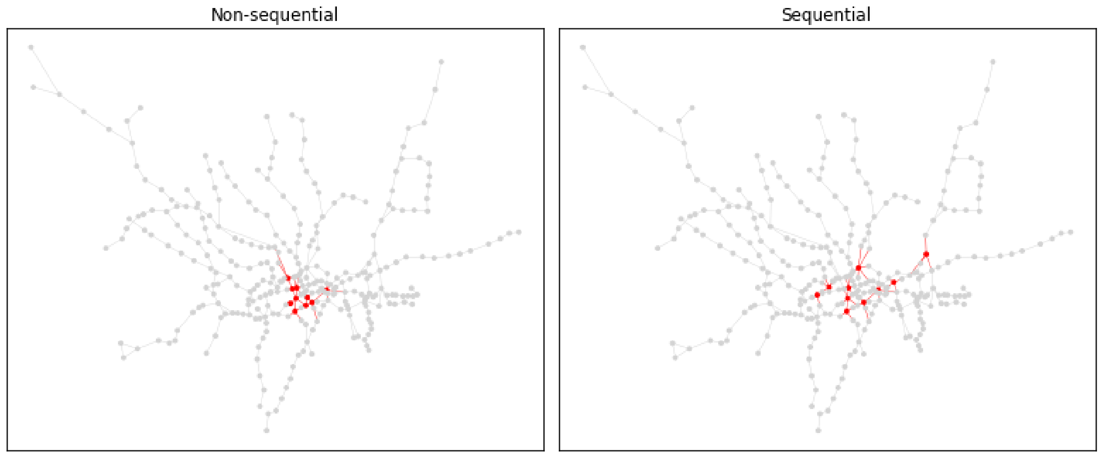
Non-sequential and sequential outcomes are similar because the degree is a local static characteristic that seldom changes after node removals. Besides, the London underground only has limited degrees from **1 to 7**, meaning stations are very likely to have equal degrees. Among top 10 stations with highest degrees in the original network, 2 stations have degrees of 7, and 5 stations degrees of 6. D.C. can't distinguish the importance of different nodes effectively and is more suitable for static non-sequential strategy.

Figure 2: Change in E and S after node removals based on D.C.



2 Removals based on C.C.

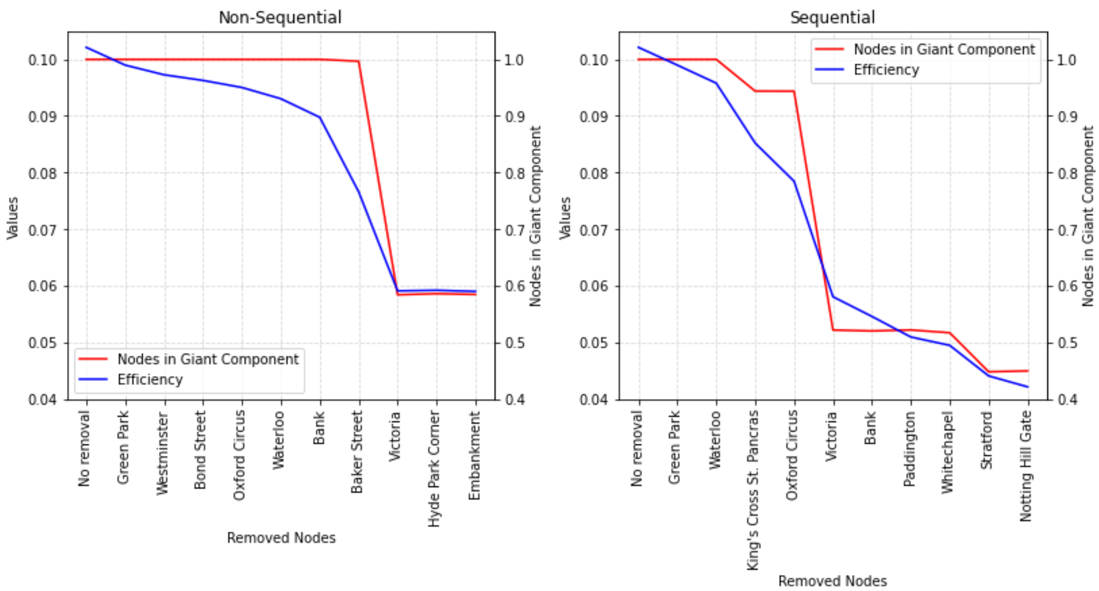
Figure 3: Nodes removed based on C.C.



In C.C. scenarios, with non-sequential strategy, E dropped to 0.584459, and S dropped to 0.058990; while with sequential strategy, E dropped to 0.042123, and S dropped to 0.449324 (Figure 4). Sequential strategy achieved greater declines in both measures, and the 2 strategies have **5** removed nodes in common (Figure 3). Compared to D.C., C.C. produces greater declines with sequential strategy but smaller ones in non-sequential strategies.

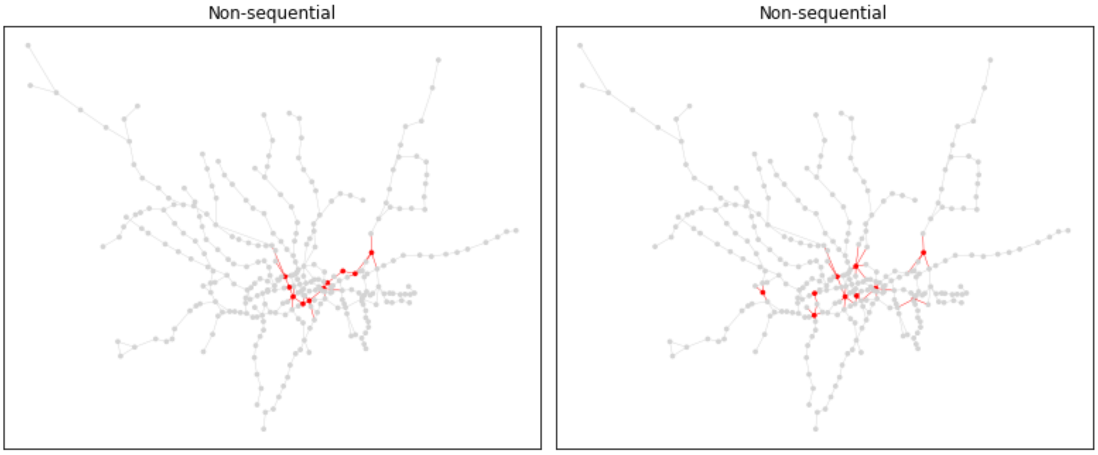
Unlike D.C., C.C. is a global characteristic and changes after each node removal, which can be more precise on targeting key stations using sequential strategies, but not as effective in non-sequential scenarios as non-sequential removals are more static and not sensitive to changes in centrality values. In the London underground, there are **292** different C.C. values, which can better distinguish the importance of different stations with fewer repetitive values.

Figure 4: Change in E and S after node removals based on C.C.



3 Removals based on B.C.

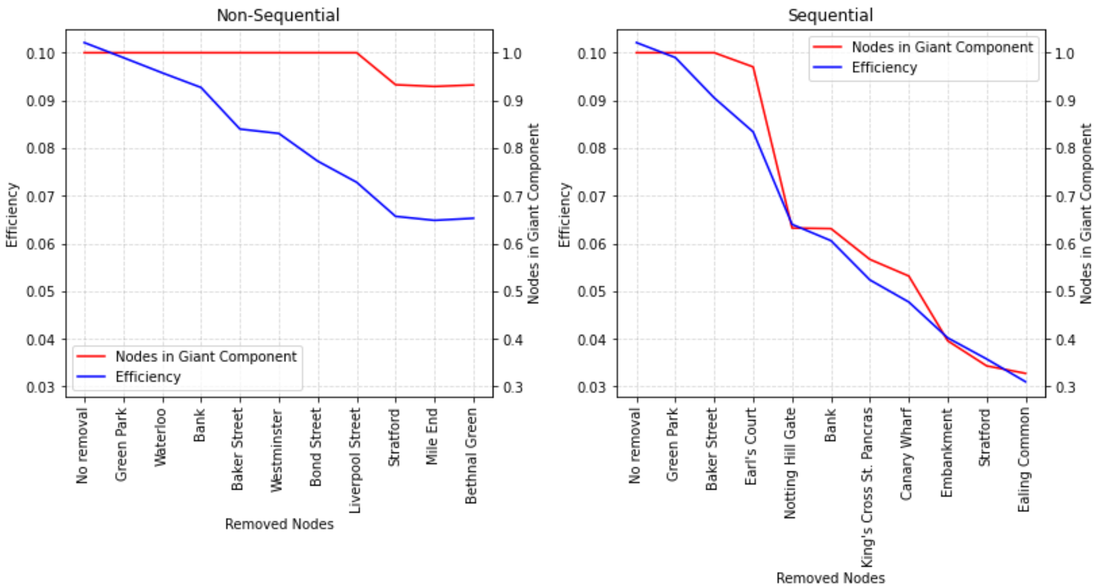
Figure 5: Nodes removed based on B.C.



In B.C. scenarios, with non-sequential strategy, E dropped to 0.065291, and S dropped to 0.932432; while with sequential strategy, E dropped to 0.031017, and S dropped to 0.327702 (Figure 6). The 2 strategies have only **4** removed nodes in common (Figure 5). With sequential strategy, B.C. achieves the greatest declines in both evaluation measures than degree and closeness, but with non-sequential strategy, it performs worst among 3 centrality measures.

The hugely different outcomes from different strategies can be explained that B.C. is the most global characteristics, closely relevant to even very remote nodes and edges, and changes after each node removal. Thus it suits sequential scenarios and doesn't work well with non-sequential strategy. Besides, the **167** different B.C. values in the London underground also outnumber the 7 values in degree, which specifically adds to precision for sequential scenarios.

Figure 6: Change in E and S after node removals based on B.C.



1.3 Discussion

1.3.1 Comparing evaluation measures

With sequential strategy, B.C. performs best among 3 measures in targeting key stations and causing declines in evaluation measures, but in non-sequential scenarios, with the least declines, it is the least suitable measure. **With non-sequential strategies, D.C. and C.C. have won and lost against each other, but both perform better than B.C.**

The greatest decline in both evaluation measures is produced with the combination of B.C. and sequential removals, which coincides with former researches (Xing *et al.*, 2017)(De-Los-Santos *et al.*, 2012).

It's also interesting that D.C. has only 7 values, while B.C. and C.C. have 167 and 292 values, respectively, suggesting that **C.C. is most precise for selecting specific nodes, especially in sequential scenarios. Meanwhile, D.C. is very vague and could lead to high randomness and terrible reproductivity of the removal progress.**

As a result, B.C. is highly recommended for best performance. If non-sequential strategies are given as premises, C.C. is recommended for better reproductivity and precision.

All centrality measures have wide applications apart from simulating traffic networks, especially in neural network and social network analyses (Ek, VerSchneider and Narayan, 2015).

1.3.2 Choosing removal strategies

Based on the above results, the London underground is less resilient to sequential removals, as sequential removal produces greater declines with all 3 centrality measures in both evaluation measures. **It is more effective in targeting key stations important to systematic resilience.**

For explanation, B.C. and C.C. values of most stations in the London underground would change significantly after node removals, and the station with the highest centrality value,i.e., the next one to be removed, would also change. This is also why B.C. and C.C. suit sequential strategy naturally. Only the D.C. values are less likely to change after node removals, which consequently favors non-sequential strategies. But we wouldn't recommend D.C. and have explained above. **Thus sequential strategy would be more precise and recommended, especially when using global and dynamic centrality measures.**

1.3.3 Refecting evaluation measures

Generally, greater declines are witnessed in G.E. for most scenarios. But the limitation of G.E. is also obvious. As G.E. only remains effective When the network is still connected. When the network is no longer connected, distances between nodes in different components will become infinite, which means with more and more components emerging and the network less connected, G.E. will become less effective.**S.G. works as a good supplement for G.E.** When the network is connected, S.G. always remains 1.0, but it would begin to decline when components emerge. This is also why S.G. only plunges after certain numbers of removals in C.C. and B.C. scenarios.

We argue that for evaluation in attack simulations, only one measure is not enough.Using G.E. and S.G. as a combination is preferable and helps the evaluation maintain sensitivity.

1.3.4 Limitation of simulations

This analysis has 3 limitations. Firstly, as an unweighted topological simulation, the analysis has not taken the number of commuters, the physical distance, or the directions of links into consideration. Secondly, as stated above, G.E. becomes dull when the network becomes unconnected, and S.G. is the other way around, which means we depend on different values at different stages of node removal. Thirdly, with repeating values in centrality measures, the removal process has certain randomness and might not be perfectly replicated.

2 PART 2: Simulation of Flows

In Part 2, we take the flows and directions into consideration and regard the underground system as a directed weighted network.

2.1 Resilience evaluation with flows

2.1.1 Evaluating flows affected[2]

Based on the removed stations in Part 1, we re-evaluate the impact of node removals by the cumulative number of trips affected (Figure 7 - 9). **In general, the outcome of the flow-based evaluation is similar to the results of G.E. and S.G. evaluation in Part 1, but not exactly the same.** Our major findings are as follows:

1 Unlike Part 1 where London's underground is most vulnerable to the betweenness-based sequential attacks and least sensitive to betweenness-based non-sequential attacks, in terms of affected flows, the largest affected flow is witnessed in closeness-based sequential attacks, and the corresponding cumulative affected inflow and outflow are 614,542 and 554,331 separately. The mildest decline is produced in betweenness-based sequential removals, and the corresponding cumulative affected inflow and outflow are 442,194 and 395,450 separately.

2 When **using the non-sequential strategy**, **D.C**. produces the greatest declines in affecte d flows, which is consistent with Part 1. The corresponding cumulative affected inflow an d outflow are 562,067 and 482,689 separately. When **using the sequential strategy**, the g reatest declines in affected flows come with **C.C.**, which is different from Part 1.

3 The difference between sequential and non-sequential still exists in flow-based evaluation. The most significant difference exists in closeness-based scenarios, and the least obvious difference is in betweenness-based scenarios. Whereas in Part 1, the difference between the 2 strategies is most significant in betweenness-based scenarios and least obvious in degree-based scenarios.

4 In all scenarios, **affected inflows are greater than the outflows**. As inflows indicate jobs and creation and outflows represent residence. This phenomenon can be explained by the fact that key stations are destinations for jobs and recreations in the middle of the network, and their residential functions are less important.

Figure 7: Cumulative affected flow after node removals based on D.C.

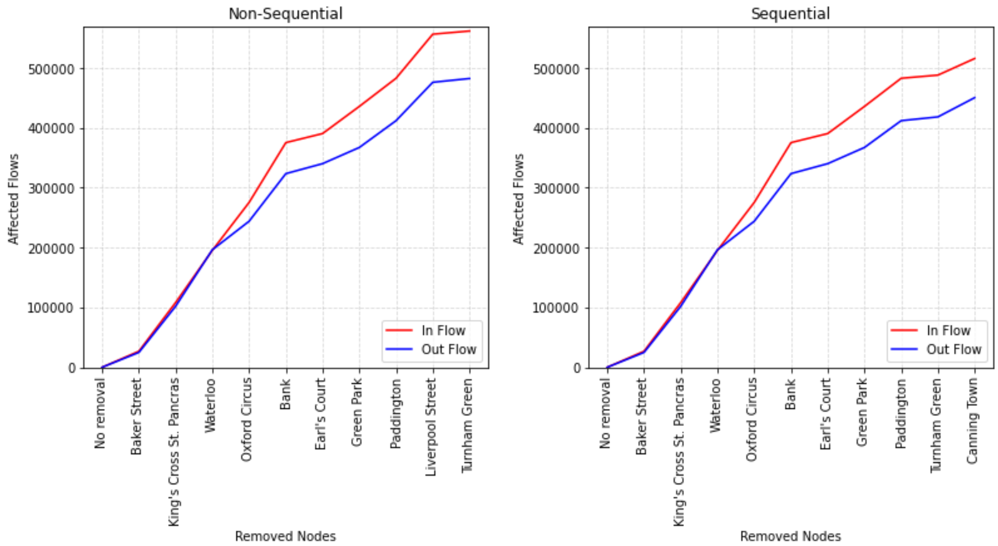


Figure 8: Cumulative affected flow after node removals based on C.C.

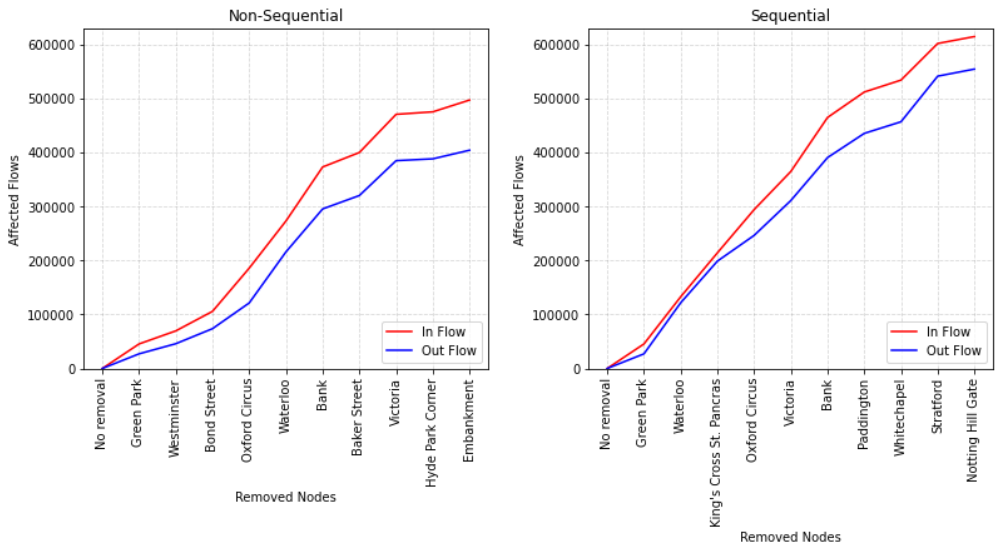
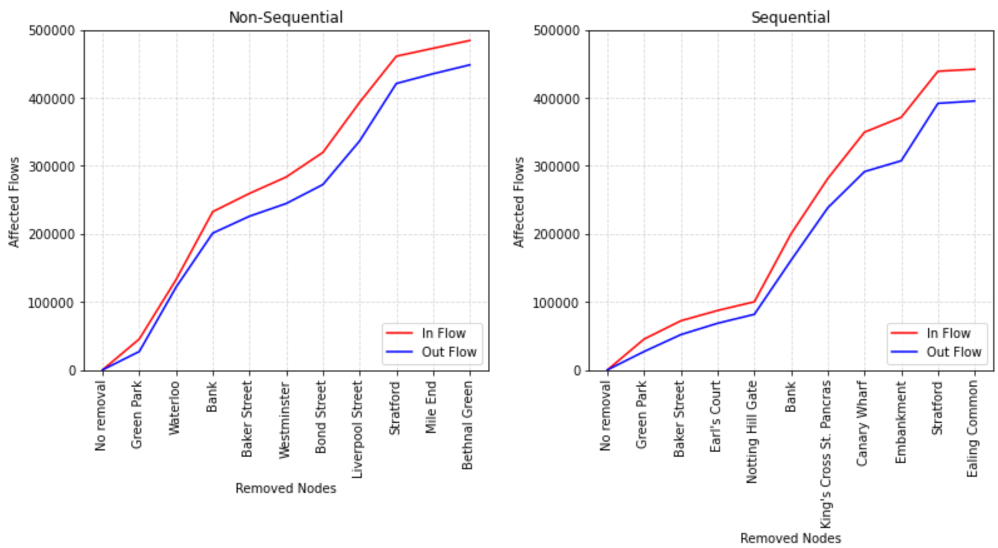


Figure 9: Cumulative affected flow after node removals based on B.C.

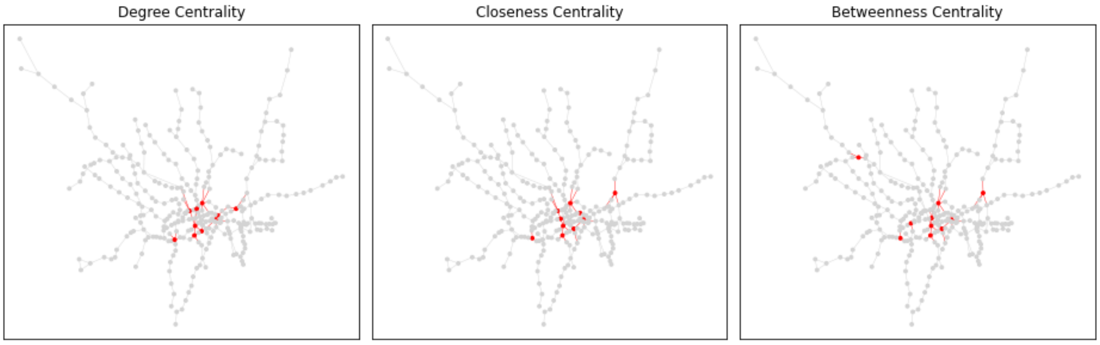


2.1.2 Evaluating flow-weighted networks

We use G.E.[3] and sequential remuvals to evaluate the weighted network, as G.E. is more effective than S.G. when only a few nodes are removed, and generally sequential removal performs better than non-sequantial removals.

1 Station of highest importance[4]

Figure 10: Nodes removed sequentially with 3 centrality measures



For D.C.-based scenarios, unweighted and weighted evaluations share 5 stations in common, among which none has equal ranks; for C.C.-based scenarios, different evaluations share 6 stations in common, among which 2 stations have equal ranks; for B.C.-based scenarios, different evaluations share 6 stations in common, among which 2 stations have equal ranks (Figure 10 and Table 1).

Stations of top importance are similar but not the same with or without considering flows, regardless of centrality measures being used. Besides, different centrality measures also have common important stations. Specifically, **Green Park, Bank, King's Cross St. Pancras** are the 3 stations that appear in the top 10 important station lists for all 6 scenarios (unweighted or weighted, D.C., C.C., or B.C. based).

Table 1: Top 10 important stations using G.E. sequentially (unweighted and flows-weighted)

Rank	Degree Centrality		Closeness Centrality		Betweenness Centrality	
	Unweighted	Weighted	Unweighted	Weighted	Unweighted	Weighted
1	**Baker Street**	**Bank**	**Green Park**	**Green Park**	**Green Park**	**Green Park**
2	**King's Cross St. Pancras**	**Green Park**	**Waterloo**	Barbican	Baker Street	**Bank**
3	Waterloo	**King's Cross St. Pancras**	**King's Cross St. Pancras**	**Bank**	Earl's Court	**King's Cross St. Pancras**
4	Oxford Circus	**Baker Street**	Oxford Circus	**King's Cross St. Pancras**	**Notting Hill Gate**	Waterloo
5	**Bank**	Mile End	**Victoria**	**Waterloo**	**Bank**	Victoria
6	**Earl's Court**	Westminster	**Bank**	**Victoria**	**King's Cross**	Oxford Circus

					St. Pancras	
7	**Green Park**	Victoria	Paddington	Bond Street	Canary Wharf	**Stratford**
8	Paddington	Warren Street	Whitechapel	Baker Street	Embankment	**Notting Hill Gate**
9	Turnham Green	Liverpool Street	**<u>Stratford</u>**	**<u>Stratford</u>**	**Stratford**	Hammersmith
10	Canning Town	**Earl's Court**	Notting Hill Gate	Hammersmith	Ealing Common	Harrow-on-the-Hill

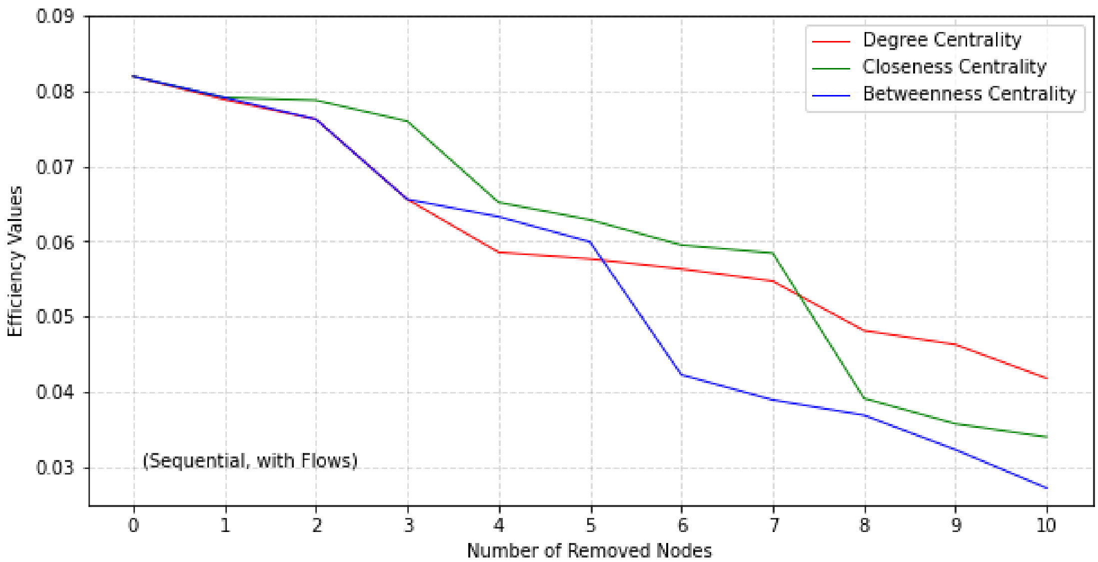
2 Selection of centrality measures

In Part 1, B.C. yields the greatest decline, and D.C. produces the least decline in G.E, which are 0.071108 and 0.053243 separately. Considering flows and directions, D.C., C.C., and B.C.-based scenarios make G.E. drop by 0.060313, 0.068112, and 0.074899 separately (Figure 11).

So the most suitable centrality measure for sequential removal with or without flow is the same, i.e., B.C., and the ranks of different centrality measures also remain unchanged after considering directed flows.

Besides, considering flows also boosts the drop in G.E.

Figure 11: Change in G.E. with sequential node removals using 3 centrality measures



2.2 Spatial interaction models

2.2.1 Explaining models

According to Wilson (Wilson, 1971), there are 4 SIMs based on different constraints, namely the Unconstrained Model, Origin-constrained Model, Destination-constrained Model, and Doubly constrained Model. These models originate from Newton's law of gravitation and assume flows are proportional to the product of the mass of the origin and destination and inversely proportional to the distance between them. In the unconstrained model, each flow needs to be estimated; the Origin-constrained and destination-constrained models assume that the outflows or the inflows from nodes remain fixed; while the doubly constrained model, assuming that all stations have fixed inflows and outflows, focuses on flows' distribution and calibrating parameters (Williams, 1976). The models are defined as:

Unconstrained Model (U.M.):

$$T_{ij} = kO_i^{\alpha} D_j^{\gamma} f(c_{ij}), \text{ w.r.t. } T = \sum_i \sum_j T_{ij};$$

Origin-constrained Model (O.M.):

$$T_{ij} = A_i O_i D_j^{\gamma} f(d_{ij}), \text{ w.r.t. } O_i = \sum_j T_{ij};$$

Destination-constrained Model (D.M.):

$$T_{ij} = B_j O_i^{\alpha} D_j f(d_{ij}), \text{ w.r.t. } D_j = \sum_i T_{ij};$$

Doubly Constrained Model (D.C.M.):

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}), \text{w.r.t. } O_i = \sum_j T_{ij} \text{ and } D_j = \sum_i T_{ij}.$$

Where T_{ij} is the flow from origin O_i to destination D_j;

k is the constant of proportionality, all estimated flows in the U.M. will be summed up to calibrate it;

O_i is the mass or population at the origin; similarly D_j is the mass or attraction (jobs) at the destination;

A_i is the balancing factor for each origin, and the flow estimates from each origin sum to the known totals O_i; similarly B_j is the balancing factor for each destination, and the flow estimates to each destination sum to D_j;

α, γ are the parameters of how the origin or destination motivate movements;

d_{ij} is the distance between O_i and D_j, and $f(d_{ij})$ is the cost function reflecting the distance's influence on T_{ij}.

Many people researched cost functions (Wilson, 1971)(Taylor, 1971), and there are generally 2 common functions, namely Inverse Power-law and Negative Exponential, which are defined as:

Inverse Power (I.P.): $f(d_{ij}) = d_{ij}^{-\beta}$; or

Negative Exponential (N.E.): $f(d_{ij}) = exp(-\beta d_{ij})$.

Where β is a key parameter that defines the rate at which the distance influences T_{ij}.

2.2.2 Calibration of β

Flows from and to each station are defined as the population and jobs of the station separately[5], and the topological distance between stations is used as distance[6]. Given that we can calculate the population and jobs for every station, **D.C.M. is the best choice to calibrating β, as it has the most constraints and renders better simulation of the system.**

We still run all 4 models to test how good each model fit with 2 cost functions (Figure 12 and Table 2) for the purpose of comparing different cost functions. As I.P. produces higher R^2 and lower RMSE than N.E. in all 4 models, **we choose the I.P. cost function**.

The final equation is $\lambda_{ij} = \exp(\alpha_i + \gamma_j - \beta \ln d_{ij})$, Where λ_{ij} is the Poisson distributed mean of T_{ij}, and α_i γ_j are the dummy variables for the origins and destinations. The final calibrated β is **0.69593**8, with an R2 at **45.72%,** indicating that our D.C.M. accounts for about 45.72% of the variation of flows.

Figure 12: Goodness-of-fit coefficients for different spatial interaction models

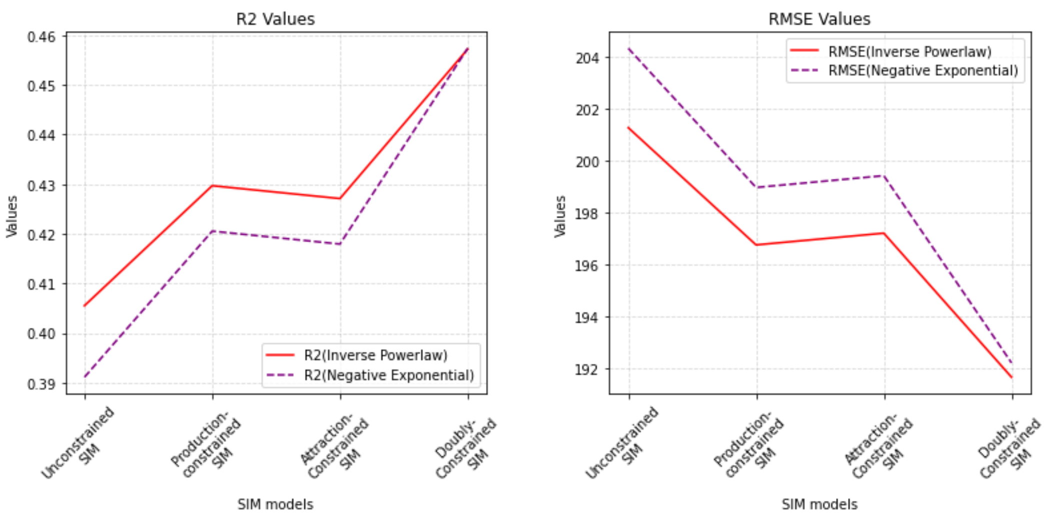


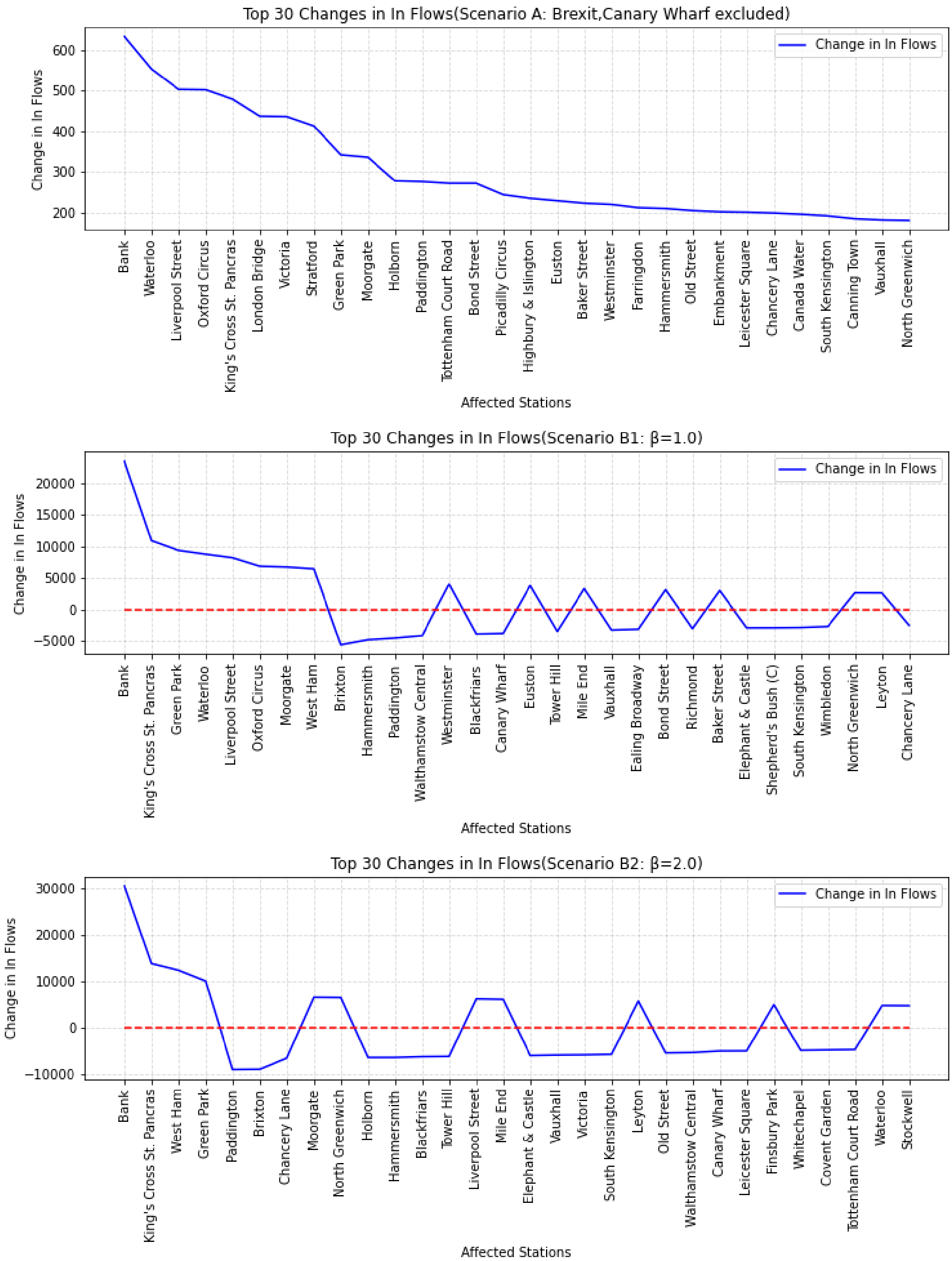
Table 2: Calibrated βs and goodness-of-fit coefficients for different SIMs and cost functions

Model	Unconstrained Model		Origin-Constrained Model		Destination-Constrained Model		Doubly-Constrained Model	
Cost-Function	I.P.	N.E.	I.P.	N.E.	I.P.	N.E.	I.P.	N.E.
β	0.518647	0.082229	**0.632881**	0.114760	0.592808	0.104857	**0.695938**	0.132374
R^2	40.55%	39.11%	**42.97%**	42.05%	42.71%	41.79%	**45.72%**	45.73%
RMSE	201.25	204.31	**196.75**	198.96	197.20	199.41	**191.66**	192.21

2.3 Simulating certain scenarios

As both scenario A and B have fixed populations at origins and only variate in jobs at destinations, we suggest using **O.M. and I.P.**, with an original $\boldsymbol{\beta}$ calibrated at **0.632881** and γ at 0.697999.

Figure 13: Top 30 change in station inflows under different scenarios

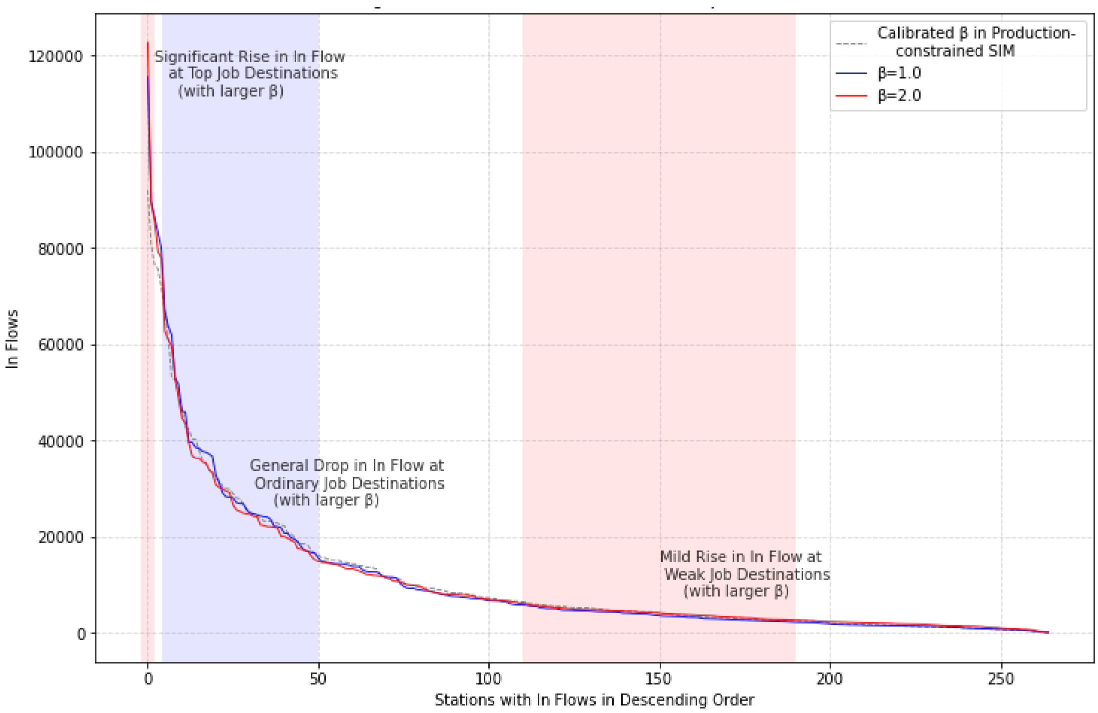


In scenario A, Canary Wharf has a 50% decrease in jobs after Brexit. Based on our simulation, the inflow at Canary Wharf declines by **18775, but the inflows at other stations witness mild and similar increases. The highest increase is 633 at Bank** (Figure 13).

In scenario B, apart from the original $\boldsymbol{\beta}$ **0.632881**, we select **1.0** and **2.0** to simulate a significant increase in the cost of transport. With increases in β, inflows are variating rapidly. When $\boldsymbol{\beta = 1.0}$, the maximum increase is **23536**. When $\boldsymbol{\beta = 2.0}$, the maximum increase is **30669** (Figure 13). **Both inflow changes in scenario B are several orders of magnitude higher than the inflow changes in scenario A.** Largest changes in inflows in both scenario are recorded at stations like **Baker Street**, **King's Cross St. Pancras**, **Green Park,** which are all key stations identified in Part 1 and Part 2.1.

To conclude, scenario B has more impact on the redistribution of flow. For explanation, we plot the top 30 inflows with 3 different βs in scenario B, finding that with larger βs, significant rises in inflow at major job destinations, general decreases in inflow at ordinary job destinations, and mild rise in inflow at weak job destinations can be witnessed (Figure 14).

Figure 14: Top 30 inflows with 3 different βs (scenario B)



2.4 Discussion

This report finds that **sequential attacks based on B.C.** are most harmful to the resilience of the London underground in both topological and weighted simulation. Key stations are also identified, among which the most important 3 are **Green Park, Bank, King's Cross St. Pancras**. Using S.I.M.s, we also calibrate $\boldsymbol{\beta}$s for the underground system and simulate possible scenarios to find out that the change in overall travel cost has greater impacts on flow distribution than changes in job supplies at certain stations. There are certain limitations as the **interchanges** between different lines are not considered in this report which requires further explorations.

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APPENDIX

1 **Github repository** for codes, data and paper:
https://github.com/lizhiyuan913/Urban-Simulation-Assessment-2021
All the values and graphs can be automatically reproduced after clicking **'Restart & Run All'**.

2 I had to rewrite my codes due to the **"create_using" mistake** when calculating "g_flows" in the practical. After comparing different options, I decided to use 'create_using= nx.DiGraph';

3 The directed network with inflows and out flows was **converted into an undirected one before calculating G.E.** Because unlike R, the Python 'networkx' library can only calculate the G.E. of an undirected graph with 'nx.global_efficiency()';

4 I **inverted the flows** when calculating of C.C. B.C. values;

5 I calculated the **'population'**, **'job'** for each station and the **'distance'** for each pair of stations on my own, despite the fact that Signe has provided her results in Slack. **Related codes is provided** in 'Codes for US assessment.ipynb' and the process from the original dataset "ODtube2017.csv" and 'underground.shp' to final dataframe can be reproduced automatically;

6 I used the '**topological distance', i.e. the number of nodes in the shortest path** to represent the distance between a pair of nodes, not because it's difficult to calculate, but **for 2 reasons**. **Firstly,** underground rails are **winding** rather than straight. Euclidean distances between adjacent stations can't represent the actual length covered. **Secondly,** as the underground stops at each station, considering **the stay** at each station stopped along the path, the actual commuting time is less affected by physical rail length, but instead better reflected by the topological distance.