Team Number:	apmcm24201203
Problem Chosen:	C

2024 APMCM summary sheet

In our study, we focus on the rapid growth of the pet industry in China and globally, analyzing key factors driving this trend and forecasting future dynamics. Using data analysis and modeling techniques such as gray relational analysis, time series analysis (ARIMA), and machine learning (logistic regression and neural networks), we achieved the following objectives:

China's Pet Industry Analysis: We identified key drivers of industry growth, including per capita disposable income and urbanization rate, segmented by pet types. Our predictions indicate sustained growth in market size over the next three years.

Global Pet Industry Analysis: We analyzed trends in pet numbers and demand in Europe, the U.S., and emerging markets, using statistical models to forecast changes in pet food demand.

Forecasting China's Pet Food Industry: By employing time series models and the Facebook Prophet algorithm, we predicted future production and export trends for China's pet food industry. We also proposed sustainable development strategies for the sector.

Policy Impact Analysis: Government policies significantly influence China's pet industry. Domestic factors like urbanization and rising incomes boost pet ownership, while regulations on pet food safety drive innovation. Using game theory, we analyze the strategic interactions between the government, producers, and consumers. For instance, stricter regulations may lead to higher production costs, affecting consumer demand. On the global stage, international trade policies, such as tariffs and agreements, create competitive dynamics, where China navigates trade barriers and explores emerging markets.

Our findings suggest that China's pet market, particularly in the cat sector, will continue expanding over the next three years, while global demand for pet food will grow steadily. The report concludes with recommendations for enhancing model applicability to address the complexity of real-world challenges.

Keywords: ARIMA Neural Network Prophet algorithm Game Theory

Contents

I. Introduction

1.1 Background

The global pet industry has experienced significant growth in recent decades, driven by evolving consumer attitudes toward pets and increasing economic prosperity. As pets are increasingly seen as companions and family members, spending on their well-being has surged. The development of pet-related sectors, such as pet food, healthcare, supplies, and services, reflects the growing demand for higher-quality products and specialized care.

In China, the pet industry has shown rapid expansion since the 1990s. The establishment of the China Small Animal Protection Association in 1992 and the entry of international pet brands like Royal Canin and Mars in 1993 marked key milestones in the industry's evolution. Rising incomes, urbanization, and the shift toward nuclear families have further fueled the industry's growth, as more people seek the emotional benefits of pet companionship. In addition, the growing middle class has shown a willingness to invest in premium pet products and services, creating opportunities for both domestic and international brands. The current trends in China's pet industry reveal strong growth

Figure 1 China Pet Industry Brand

in pet food and nutrition, veterinary services, grooming, and pet tech solutions. However, challenges remain, including regulatory barriers, fragmented supply chains, and the need for localized innovations. With the growing of China's economic environment and shifting demographics, strategic recommendations are essential to sustain growth and meet evolving consumer expectations in this promising market.

1.2 Problem Restatement

Considering the background information and the specific constraints outlined in the problem statement, we need to address the following questions:

- **Problem 1**
  Using the data to analyze the development of China's pet industry over the past years by pet type. Identify key factors influencing this growth to create a mathematical model predicting the industry's development over the next three years.
- **Problem 2**
  Analyze the global pet industry's development by pet type, noting the rapid growth in overseas markets like Europe and America. Create a mathematical model to forecast global pet food demand for the next three years.
- **Problem 3**
  Based on China's pet food production and export values, analyze the development of the pet food industry and forecast its production and exports over the next three years, considering global demand trends and China's growth.
- **Problem 4**
  To quantitatively analyze impact from new foreign economic policies, create a mathematical model using the data in the attachments. Propose feasible strategies for the sustainable development of China's pet food industry.

II. What's the trend: Analysis for Chinese Pet Industry

2.1 Data Description

To analyze the key factors influencing the development of pet industry in China, it is essential to gather relevant data sets. Due to different statistical departments being responsible for various elements, Table 1 below presents the sources of the data sets used in this chapter, along with explanations of the variables.

Since much of the data collected in the initial phase isn't in a directly usable format, it requires some preliminary processing.

To mitigate the impact of varying dimensions across the data sets and to facilitate the construction of a multivariate regression model, each variable's data needs to be normalized. Given the relatively stable nature of the data, which lacks extreme outliers in its maximum and minimum values, this paper employs min-max normalization for

data processing. This can be expressed using the following formula:

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

x represents the value of a specific variable, $x^{'} \in [0, 1]$, $max(x)$ and $min(x)$ respectively denote the maximum and minimum values of the given data. The variables mentioned below are all normalized in the way of the given formula.

Data	Sources
China's GDP	National Bureau of Statistics
Capital disposable income	National Bureau of Statistics
Urbanization rate	China Economic Development Report
Engel's coefficient	China Economic Development Report
Terms-of-trade	China Shippers' Association
Pet food market scale	China Pet Industry Association

Table 1 Data sources

2.2 Analysis for the development of China's pet industry

2.2.1 *Development of China's pet industry by pet type*

In recent years we can see more and more people walking on the street with a pet around them. According to the data released by the Asian Pet Research Institute, the scale of China's pet market has grown rapidly in the past 10 years. To get a more convincing conclusion, we did a numerical analysis based on the additional data collected by our team and given data in attachment 1.

After finding the data by all means, we found that the current domestic pet category is mainly dogs and cats, and the two are generally accepted by the public, so we can focus on pet types on cats and dogs.

To better analyze the data, we choose to visualize the number of cats and dogs and the development situation of pet industry from 2019 to 2023 in figure 2.

We can see that the number of dogs fluctuates little between 5100 to 5500, and cats remain stable increase year by year. The pet industry scale and pet food industry scale

are also continuously increasing. A conclusion can be drawn that the future prospects for the pet industry are promising, with potential for continued growth.

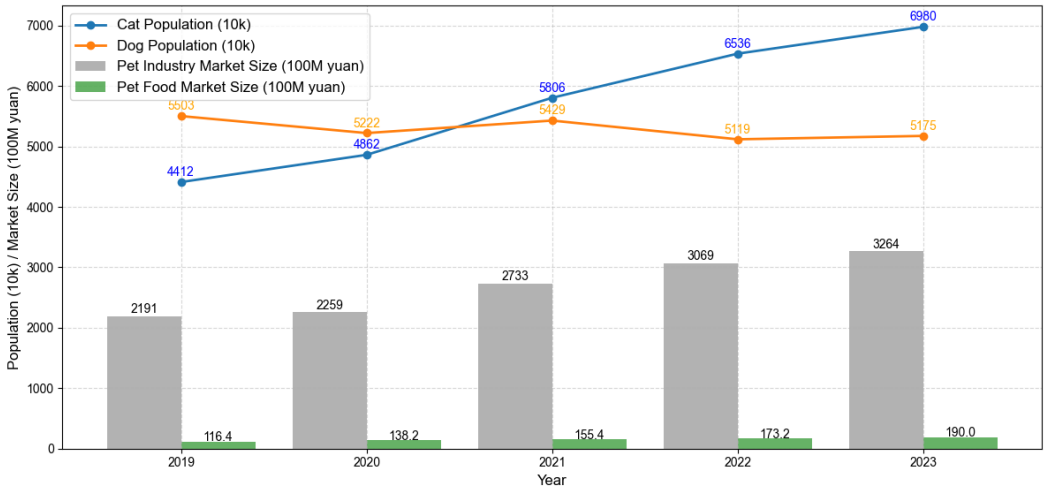

Figure 2 Trends in China's Pet Industry and Pet Food Market(2019-2023)

2.2.2 *Analysis for factors in the development of China's pet industry*

The development of pet industry can be influenced by many factors. We conducted a PEST analysis (Policy, Economics, Society, Technology) on the driving factors of China's pet industry development, selecting key indicators to collect data. By querying different reliable databases mentioned in 4.1, we obtained the following data:

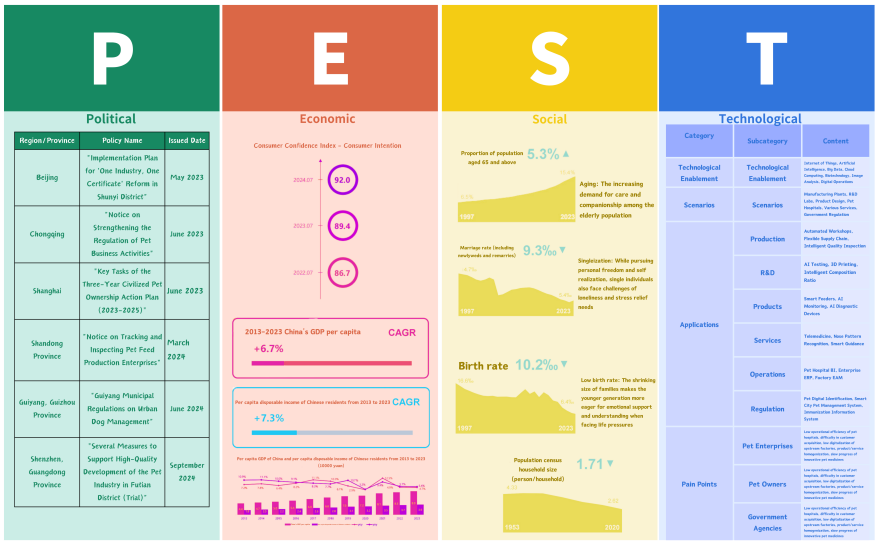

Figure 3 PEST Model Structure

Policy: Exchange rate, total customs duties, import and export commodity price index. Economics: Per capita disposable income and Engel coefficient for residents. Society: Total population, population aged 65 and above, birth rate, marriage registration, urbanization rate. Technology: Pet hospital number.

Then we use gray relation analysis to get the key factors that affect the development of pet industry. The calculation process can be represented as:

$$\gamma(x_0, x_i) = \frac{1}{n} \sum_{k=1}^{n} y(x_0(k), x_i(k))$$

where γ is the relation, n is the number of factors, y is the dependent variable and x_0 x_i respectively represent the selected variable and other variable.

The result is shown in Figure 4, from which we can see that the cat number, dog number and market size are influenced by key factors, which are GDP, per capita disposable income, total population, population aged 65 and above, urbanization rate and pet hospital number.

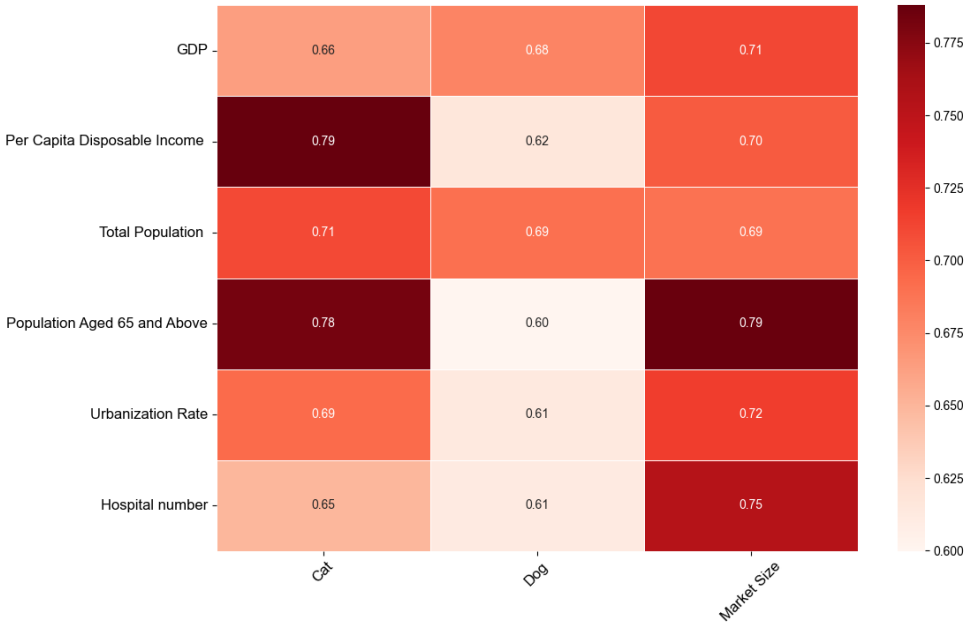

Figure 4 Gray Correlation Analysis

2.2.3 *Prediction for development of China's pet industry*

For the development pattern of China's pet industry in the next three years, a time series analysis was conducted using the ARIMA model.The basic idea of the ARIMA model

is to use the historical information of the data itself to predict the future, and it consists of three main parts, which are the autoregressive mode (AR), the difference process (I) and the moving average model (MA).

To ensure the smoothness of the series, d-difference processing is used to achieve this. For the three parameters in the $ARIMA(p, d, q)$ model, where:

p denotes the autoregressive model part, describing the lagged values of the observations used in the model, i.e., the observation is a linear combination of the p preceding observations; q denotes the moving average model component, describing the lagged values of the error term, i.e., the observation is a linear combination of the q preceding white noise terms; d denotes the differencing order, which transforms the time series data into a stationary series before fitting the ARIMA model.

The complete mathematical expression is:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q} + \epsilon_t$$

where Y_t is the differenced series, ϕ_i are the autoregressive coefficients, θ_i are the moving average coefficients, and ϵ_t is the white noise error term.

Using the outer product of gradients (OPG) method for model evaluation, the following model evaluation metrics were obtained:

Metrics	Values
Log Likelihood	-20.844
AIC	45.68722312236989
BIC	43.884447699706115
HQIC	42.06341443283669

Plotting the results obtained from the ARIMA model gives the following Figure4:

From the chart, it can be seen that China's pet industry will maintain steady growth over the next three years, with the market size expected to increase from 351.179 billion yuan in 2024 to 397.524 billion yuan in 2026. Among them, the cat market continues to show strong growth, projected to reach 840.109 million in 2026, while the dog-related market is expected to remain stable or slightly decline.

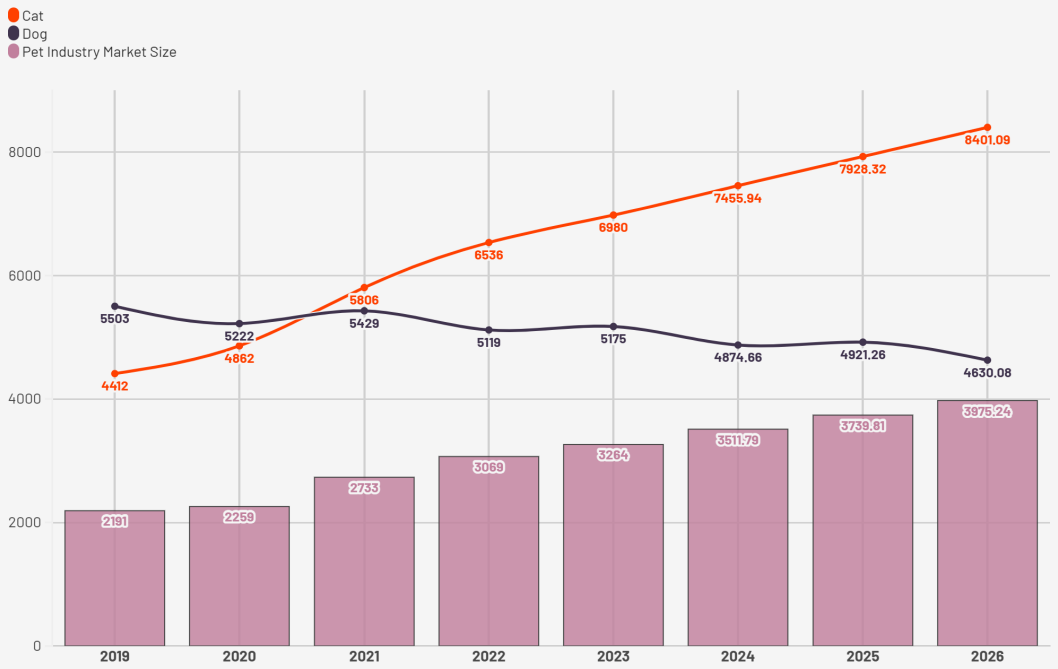

Figure 5 Forecast Trends for the Next Three Years

III. What's the trend: Analysis for Global Pet Industry

3.1 Development of Global pet industry by pet type

Not only in China, but also in Worldwide the pet need is growing. Pet pictures are posted to media platform and have received millions of likes. People see pets as good friends and heal to soul. We also find data to support this view and analyze the global trend. Use statistical analysis to investigate and analyze the economic development of the global pet industry using statistical theory, and the heat map for cats and dogs are drawn as follows:

As can be seen from the Figure 6, some hot markets such as the US market are cooling down, but there are also some emerging markets such as France and Germany market is heating up. By substituting the data of cats and dogs into the ARIMA model of the first question, the number of cats and dogs in each country in the next three years can be obtained as follows:

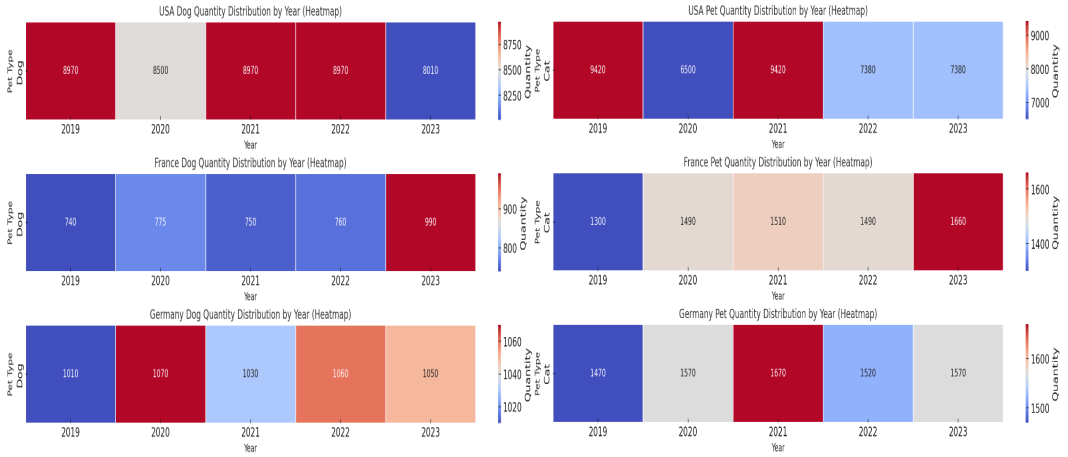

Figure 6 Visualization for the Past Five Years

Years	Cat (America)	Dog (America)	Cat (France)	Dog (France)	Cat (Germany)	Dog (Germany)
2024	7060	8249	1706	948	1605	1065
2025	6740	8104	1778	997	1620	1072
2026	6420	7959	1850	1046	1635	1079

3.2 Forecast for Global Pet Food Need

When it comes to the food need globally, the previous data shows that it is also increasing, the future for pet food industry is promising. PLOS ONE study's survey shows that the average food need for cat is 29.3kg per year, and the need for dog is 182.5kg per year. We can use the following formula to calculate the specific food need, where t is number of a type of animal and n is the food need per year.

$$Food = t * n$$

The annual consumption of pet food in the United States, France and Germany is shown in the table below, the units are 10,000 kilograms:

Years	Pet food in America	Pet food in France	Pet food in Germany
2024	1712300.5	222995.8	241389
2025	1676462	234047.9	243106
2026	1640623.5	245100	244823

To conduct the analysis, we also collected global data. We build a logic regression model and neural network model to predict.

The origin of logistic regression method can be explained as follows: Logistic regression was initially developed to model binary outcomes, where the dependent variable is categorical, typically taking values of 0 or 1. For example, it might predict the probability of a certain event occurring (such as success or failure). The core of logistic regression is the logistic function (or sigmoid function), which transforms any real-valued input into a value between 0 and 1, making it ideal for probability estimation.

First, we start by assuming that population growth follows an S-curve, where growth is initially exponential but slows down as it approaches a maximum carrying capacity due to limiting factors. This behavior can be mathematically modeled using the logistic function:

$$p(t) = \frac{1}{1 + e^{-r(t-t_0)}}$$

where $p(t)$ represents the probability of an event occurring at time t, r is the growth rate, and t_0 is the time at which the probability is 50

Over time, logistic regression was extended to model more complex phenomena beyond binary classification, such as population growth. By adapting the logistic function, it has become a powerful tool for modeling population dynamics, where growth starts exponentially but slows down as resources become limited, leading to a saturation point.

A neural network is a computational model inspired by the way biological neural networks in the human brain process information. It consists of layers of interconnected nodes (neurons), where each connection has a weight that adjusts during training. Neural networks are powerful tools for modeling complex relationships between inputs and outputs, especially when the underlying data has nonlinear patterns.

The basic structure of a neural network consists of: Input Layer: The layer that receives input data. Hidden Layers: Layers that process the input data using weighted connections, activation functions, and nonlinear transformations. Output Layer: The layer that produces the final prediction.

During the training process, the network learns by adjusting the weights of connections based on the difference between its predicted output and the actual output. This adjustment is done through a process called backpropagation, which uses the gradient descent algorithm to minimize the loss function.

For predicting pet population growth, a neural network can model more complex trends in the data that might not be captured by traditional methods like logistic regression. For example, nonlinear relationships between various factors influencing pet population growth, such as economic conditions, urbanization, or cultural shifts, can be

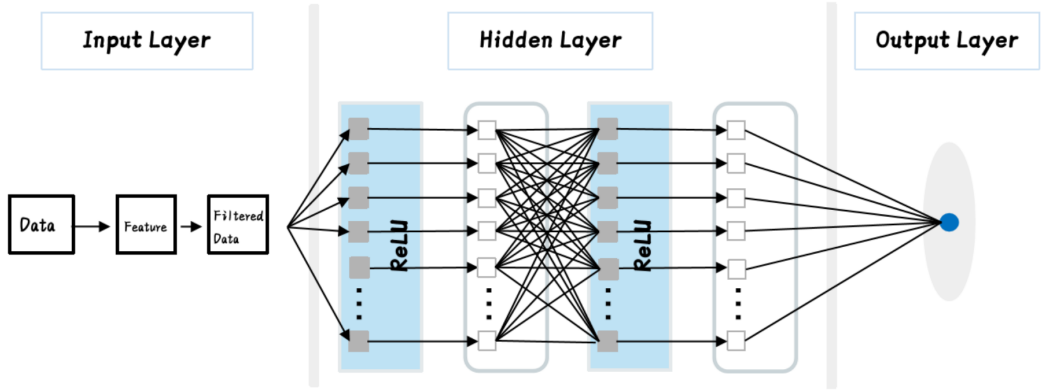

Figure 7 The structure of neural network

learned by the neural network. This allows it to make accurate predictions about future pet food needs based on more nuanced patterns.

To get a more accurate result, we combine these two results together. The food need for pets in the following 3 years are shown in figure 8, they are 153, 163 and 171 billion dollars respectively.

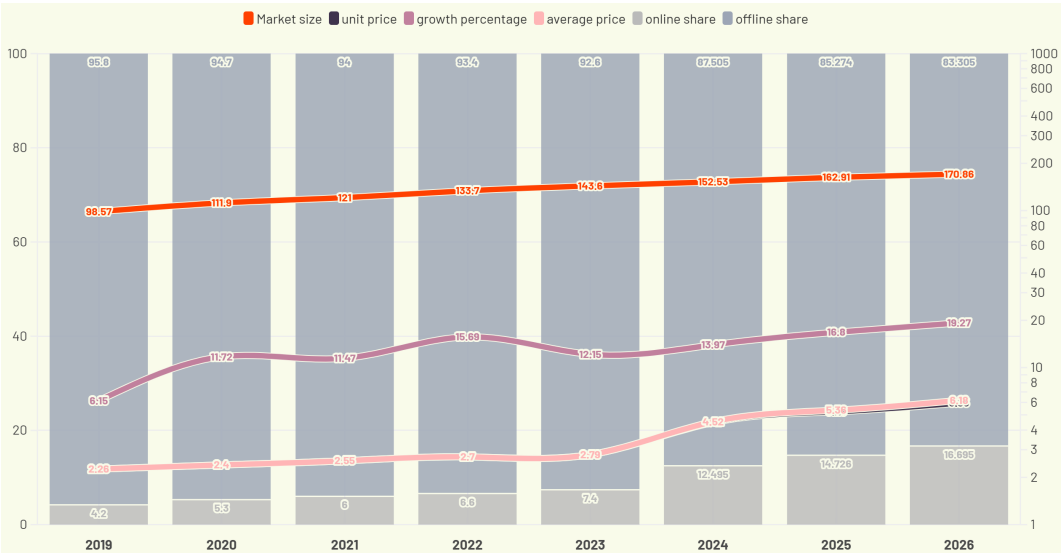

Figure 8 Forecast for Global Pet Food Industry

IV. What's next: Prediction for Pet Food Industry

4.1 China's pet food industry analysis

With the rapid development in pet industry, the food demand is also surging fast. We conclude data from attachment 3 to make figure 9. This figure shows that the total pet food production is continue to grow, which indicate that the future is prosperous. When

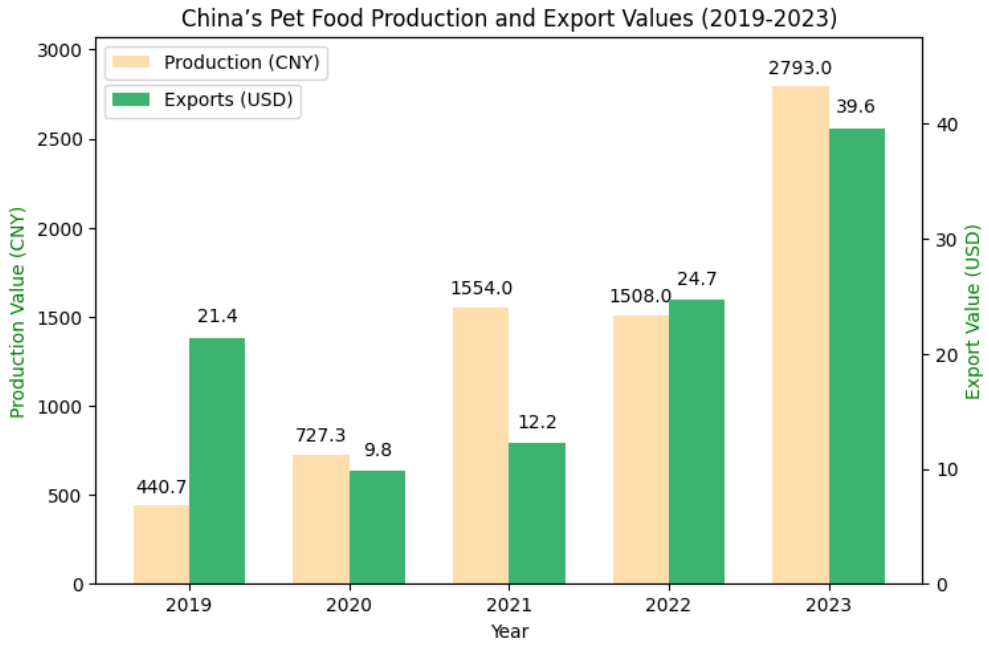

Figure 9 China's Pet Food Production and Expert Values(2019-2023)

it comes to the total value of China's pet food exports, we can see a decrease in 2020, this can be because of the pandemic. The restore of total export value in the next years proves this.

4.2 Prediction for the next 3 years

The future trend is necessary for investors to decide whether to put more money on it. We urgently need to know the pet food development in China and pet food export development.

To solve this problem, we implement Prophet method to predict the values for the next 5 years. FBprophet is a time-series forecasting algorithm open-sourced by Facebook. It is applicable to business data with clear underlying patterns. We can get result by using the following formula:

$$y(t) = g(t) + s(t) + h(t) + \epsilon_{(t)}$$

Among them:

g(t) represents the trend component, which reflects the non-periodic changes in the time series; s(t) represents the big small year component, generally measured in years; h(t) represents the accident event component, accounting for the impact of potential non-fixed periodic holidays on forecast values; And the error term $\epsilon_{(t)}$, also known as the residual term, represents the unanticipated fluctuations in the model, following a Gaussian distribution.

The trend term has two important functions, one based on a logistic regression function (nonlinear growth) and the other based on a piecewise linear function (linear growth)

In the real environment, holidays or some major events will have a great impact on the time series, and these time points often do not have periodicity, and the analysis of these points is extremely necessary.

$$s(t) = (k + \boldsymbol{a}(t)^T\boldsymbol{\delta}) \cdot t + (m + \boldsymbol{a}(t)^T\boldsymbol{\gamma})$$

Due to the possibility of various seasonal trends in time series, such as daily, weekly, monthly, or yearly cycles, the Fourier series can be used to approximate these periodic characteristics.

$$g(t) = \frac{C(t)}{1 + \exp\left(-\left(k + \boldsymbol{a}(t)^T\boldsymbol{\delta}\right) \cdot \left(t - \left(m + \boldsymbol{a}(t)^T\boldsymbol{\gamma}\right)\right)\right)}$$

$$a(t) = (a_1(t), \cdots, a_S(t))^T, \boldsymbol{\delta} = (\delta_1, \cdots, \delta_S)^T, \boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_S)^T$$

c(t) represents carrying capacity, k denotes the growth rate, and m represents the offset.

The result is shown in Figure 10, from which we can see that in the following years, scale of pet food industry will be 3081.06, 3611.18, 4165.85, and export value will be 383.61, 438.75, 514.66.

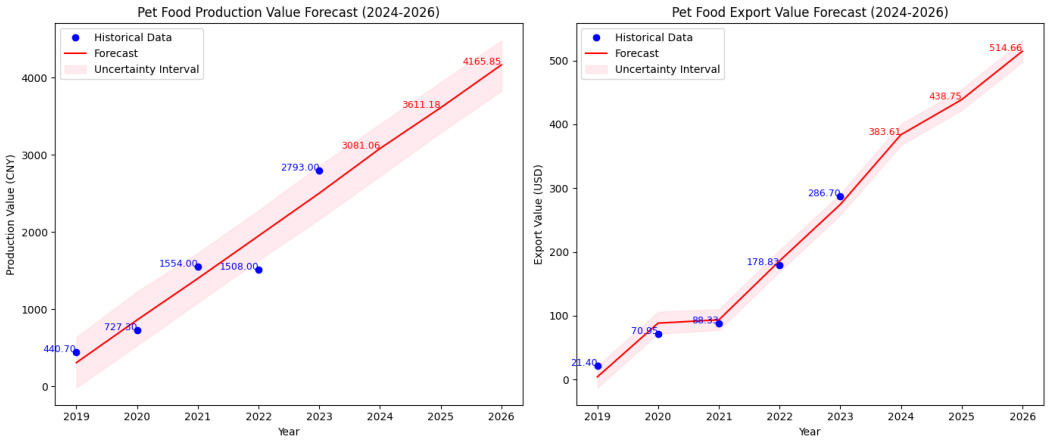

Figure 10 Pet Food Production (Expert) Value Forecast

V. The impact of foreign trade policy on pet food industry in China

With the advancement of global economic integration, the foreign trade economic policies of Europe and the United States, especially the tariff policies, are having a profound impact on the import and export trade of countries around the world. As an important producer and exporter in the global pet food market, China is bound to be affected by these policy changes. We apply game theory to analyze the impact of foreign trade economic policies in Europe and the United States on China's pet food industry, and provide strategic suggestions for the sustainable development of China's pet food industry.

5.1 Research hypothesis:

Players: Key players in the model include European countries, the United States, China, and producers and consumers in the pet food industry.

Strategic space: the foreign trade policy strategies of Europe and the United States (such as raising tariffs, implementing import quotas, etc.); China's response strategy (such as price adjustment, technological innovation, market diversification, etc.).

Payment function: The revenue function for each country and participant is constructed based on factors such as market share, export volume, tariff cost, etc.

5.2 Game theory framework:

Static games assume that all decision makers are making decisions at the same time, and the information is complete, and the players can adjust their strategies according to the policies of other countries.

Non-cooperative games: Each party makes independent decisions to maximize its own interests.

Nash Equilibrium: The ultimate goal of the model is to solve the Nash equilibrium in the game, that is, under the premise that each party knows the strategy of the other party, no party can obtain a higher return by unilaterally changing its strategy.

5.3 Model construction

The impact of tariff policy on pet food industry in China

Us and European tariff strategies: Suppose the US and Europe impose different tariffs on Chinese pet food. According to the attached data, the influence of different tariff levels on the export volume of China's pet food industry is set and expressed by mathematical functions (such as linear or non-linear functions).

China's response strategy: China's response strategy can include reducing costs, optimizing the supply chain, and improving the added value of products.

5.4 The game between producers and consumers

Manufacturer's decision: Manufacturers adjust product prices and production according to the tariff policy to maximize their own profits.

Consumer decisions: Consumers make purchasing decisions based on price and product quality.

Game modeling: The interaction between the producer and the consumer is regarded as a game, the consumer chooses the strategy of buying or not buying, and the producer chooses the production and pricing strategy.

5.5 Mathematical Formulation of the Model

Let P_C be the price of Chinese pet food, and P_U and P_E be the market prices in the U.S. and Europe, respectively. Let T_U and T_E represent the tariffs imposed by the U.S. and Europe.

The producer's profit function is:

$$\Pi_C = (P_C - C) \times Q_C$$

where C is the cost, Q_C is the quantity sold, and P_C is affected by tariff policies.

The consumer's utility function is:

$$U_C = f(P_C, Q_C)$$

Consumers make purchasing decisions based on price and product quality, which in turn affect the producer's sales and profits.

5.6 Strategic Recommendations

5.6.1 *Recommendations for the Chinese Government*

- **Market Diversification:** Open up emerging markets to reduce dependence on the U.S. and European markets, mitigating the risk of policy changes from these regions.
- **Technological Innovation and Brand Enhancement:** Increase the technological content and brand value of pet food to reduce the risks associated with price competition.
- **Diplomatic Negotiations:** Engage in diplomatic and international trade negotiations to reduce tariff barriers and ensure the stability of pet food exports.

5.6.2 *Recommendations for China's Pet Food Industry*

- **Cost Control and Supply Chain Optimization:** Optimize supply chains and reduce production costs to maintain competitiveness in the face of rising tariffs.
- **Market Segmentation and Differentiation:** Adjust product positioning according to different market needs, adopting a differentiated competitive strategy to reduce price competition pressures.

VI. Conclusion

Through game theory analysis, this paper reveals the impact of European and U.S. trade policies on China's pet food industry and offers strategic recommendations based on Nash equilibrium results. Future research could consider dynamic games and long-term policy effects, providing further insights into the sustainable development of China's pet food industry.

VII. References

[1] John Nash. *Non-Cooperative Games*. Annals of Mathematics, 1951.

[2] James Friedman. *Game Theory with Applications to Economics*. Oxford University Press, 2002.

[3] M. Osborne. *An Introduction to Game Theory*. Oxford University Press, 2004.