$\boldsymbol{Kernel\ SVM}$

$Our\ goal\ is$

$$min\ \frac{1}{2}\left||w|\right| + C\sum_{i=1}^{m}[1 - y_i(w \cdot x_i + b)]_+$$

$By\ Lagrange\ multiply\ method,$

$$w = \sum_{i=1}^{m}\alpha_i y_i x_i$$

$Take\ w\ back\ to\ our\ goal$

$$min\ \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_i\,\alpha_j\,y_i y_j x_i \cdot x_j + C\sum_{i=1}^{m}[1 - y_i(\sum_{j=1}^{m}\alpha_j y_j x_j \cdot x_i + b)]_+$$

$In\ the\ following, use\ K\ to\ denote\ kerel\ function\ of\ two\ vector$

$$K(x_i, x_j) = \emptyset(x_i, x_j)$$

$For\ sing\ example\ x_i$:

$\mathbf{1.}\,\boldsymbol{if}\ \ \left[\mathbf{1} - \boldsymbol{y_i}\left(\sum_{\boldsymbol{j=1}}^{\boldsymbol{m}}\boldsymbol{\alpha_j y_j x_j \cdot x_i + b}\right)\right] > \mathbf{0}$:

$$\frac{\partial L}{\partial \alpha_i} = \frac{1}{2}\alpha_i K(x_i, x_i) + \frac{1}{2}y_i\sum_{j=1}^{m}[\alpha_j y_j K(x_i, x_j)]\ +\ C(-y_j)y_i K(x_i, x_j)$$

$$\frac{\partial L}{\partial b} = C(-y_i)$$

$\mathbf{2.}\,\boldsymbol{else}$:

$$\frac{\partial L}{\partial \alpha_i} = \frac{1}{2}\alpha_i K(x_i, x_i) + \frac{1}{2}y_i\sum_{j=1}^{m}[\alpha_j y_j K(x_i, x_j)]$$

$$\frac{\partial L}{\partial b} = 0$$