Software Requirements Specification for Time_Freq_Analysis: A program for time-frequency transforms of 1D systems

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Contents

Revision History

Date	Version	Notes
04.11.2020	1.0	Initial Release

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below.

symbol	**unit**	**SI**
s	time	second

Additionally, *frequency*, is derived as the cycles of a repeating signal per second, and its units hertz are: $Hz = 1/\mathrm{s}$ pertaining to 'cycles per second'.

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The symbols are listed in alphabetical order.

symbol	**unit**	**description**
$x(n)$	N/A	discrete signal input
n	N/A	discrete time
N	N/A	the number of samples in a signal, i.e. $x(n)$ for $n[0, \ldots, N]$
f	Hz	frequency
ω	Hz	frequency
i	N/A	the imaginary number s.t. $i^2 = -1$
$w(n)$	N/A	window function
$\phi(t)$	N/A	wavelet function
$X(\omega)$	N/A	Fourier transform of a signal $x(n)$
$X(n, \omega)$	N/A	short time Fourier transform of a signal $x(n)$
$\langle x, y \rangle$	N/A	convolution between x and y
P	s	sampling period of a discrete signal

1.3 Abbreviations and Acronyms

symbol	description
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
STFT	Short Time Fourier Transform
Time_Freq_Analysis	A program for computing a time frequency analysis of a signal
T	Theoretical Model

2 Introduction

Vibrating Screens are large machines used in the mining and aggregate industry to sort aggregate (i.e. gravel) by size. The machines excite the aggregate by vibrating in specific patterns. For the purpose of further research, accelerometers were placed on the machines and the vibrations were recorded. It is hypothesized that there is useful information within the vibrations, for example, the vibrations could hold a unique machine fingerprint or indicate if a part of the machine needs to be replaced. However, the current form of the recording of the vibration, a one dimensional signal, is not ideal for reading that information or for further analysis (such as a statistical analysis or creating a support vector machine). Retrieving the time-frequency content of the recording data (i.e. identify what frequencies occur at what time instance of the sample) is preferred for further analysis.

2.1 Purpose of Document

The purpose of this document is to provide insight on the construction and functionality of Time_Freq_Analysis to the Intended Reader (Section 2.3). This document should clearly communicate all necessary background information and context to the Intended Reader such that they can understand the domain in which the project takes place.

2.2 Scope of Requirements

The domain of this problem is restricted to one dimensional signal inputs. In a practical application, the inputs are intended to be accelerometer data collected by Haver and Beocker Canada. However, restricting the domain to one specific type of input data is unnecessary for this program, specifically as Time_Freq_Analysis is intended to be one tool used within a larger process. Also, only considering the data collected by Haver and Beocker Canada would pose a considerable challenge for verification. Therefore, it has been decided to scope Time_Freq_Analysiss input into any one dimensional signal input (which includes a vibration recording from the accelerometer as mentioned previously, but also be any other one dimensional signal).

Subsequently, since this project is limited to any one dimensional input and not limited to physical data collected by Haver and Beocker Canada, any qualities of the physical data collection (e.xṁissing data points in a recording, flawed recording, noise in recording) are considered out of scope. Essentially, while this program is intended to eventually be used within the context of specific physical data, it is developed and constructed independently and abstractly from that data.

2.3 Characteristics of Intended Reader

The intended reader should have a good understanding of signal processing and Fourier transforms, ideally with a graduate level class in the area.

2.4 Organization of Document

This document includes a general system description where the system is describe in the way that it interacts between the user and the environment, a specific system description which will describe elements of the problem and the tools used to solve the problem in detail, a requirements section, a section on likely changes, and a section on traceability.

3 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

3.1 System Context

The system context is shown in Figure 1 below. The user will input a one dimensional signal to the system. The user will also select a method of time-frequency transform that will be performed. The user must specify the time period within the sample to be analysed. The user could also specify the minimum and maximum frequencies for the analysis to include, although there will also be defaults. If given all valid inputs, the system will then produce a time-frequency representation of the specified portion of the signal.

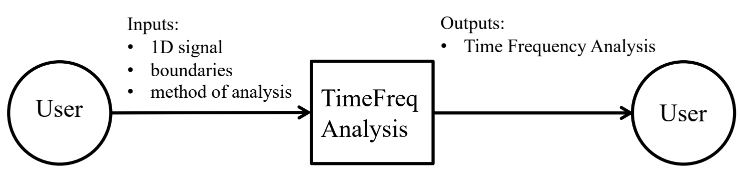

Figure 1: System Context

- User Responsibilities:

- Provide the input data, ensuring it is in correct format.
- Enter other input variables if defaults are not appropriate.

- Time_Freq_Analysis Responsibilities:
  - Detect data type mismatch, such as a string of characters instead of a floating point number.
  - Detect if input data sample is too short for analysis.
  - Ensure analysis boundaries are an appropriate size for the sample.
  - Calculate the output.

3.2 User Characteristics

The end user of Time_Freq_Analysis should have a good understanding of signal processing and Fourier transforms, ideally with a graduate level class in the area.

3.3 System Constraints

There are no known constraints for this system at the present time.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

4.1 Problem Description

Time_Freq_Analysis is a computer program that is intended to transform any one dimensional signal into a two dimensional time-frequency representation of that signal, time localizing the frequencies of the signal.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- Time-frequency analysis/transform - Representing and understanding a 1D signal by its time localized frequency content.
- Short Time Fourier Transform - A method of time-frequency transform using discrete Fourier transforms.

- Wavelet - A function that represents a portion of a ‘wave’ or periodic function, it is mainly 0 over its domain expect for around 0, the maximum amplitude should be 1.

- Wavelet Transform - A method of time-frequency transform using wavelets.

4.1.2 Goal Statements

Given a one dimensional input signal, a minimum frequency, a maximum frequency, and a time period within the recording the goal is:

GS1: Compute the STFT representation of the data.

4.2 Solution Characteristics Specification

The instance models that govern Time_Freq_Analysis are presented in Subsection 4.2.4. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

A1: The input signal is longer than the time period over which the time-frequency analysis is computed (Ref by DD2, IM1, LC3).

A2: The minimum frequency of the analysis is significantly larger than the sampling frequency of the input signal (Ref by DD2,DD3, ,IM1, LC3).

A3: The maximum frequency of the analysis is significantly smaller than the $1/P$ where P is the time period of the analysis (Ref by DD2, IM1, LC3).

A4: The recording of the signal is an accurate representation of the signal it represents, it is not missing data points and does not contain anomalies.(Ref by DD2, IM1).

A5: A morelet wavelet will be a sufficient wavelet to compute the time-frequency analysis of a signal $x(n)$. Currently, it is the only wavelet being considered for the wavelet transform, but there are other options available (Ref by IM1, LC1, LC2).

A6: The discrete sampling period of the signal $x(n)$ is known (Ref by DD2, IM1).

4.2.2 Theoretical Models

This section focuses on the general equations and laws that Time_Freq_Analysis is based on.

Number	T1
Label	**Discrete-time Fourier transform**
SI Units	N/A
Equation	$X(\omega) = \sum_{n=0}^{N-1} x(n)e^{-i\omega n}$
Description	The discrete-time Fourier transform transforms a discrete signal $x(n), n[0, \ldots, N]$ into a function of frequency $X(\omega)$.
Source	https://cpb-us-w2.wpmucdn.com/sites.gatech.edu/dist/5/462/files/2016/08/DFT-of-Noise.pdf
Ref. By	T2

Number	T2
Label	**Short Time Fourier Transform (Discrete)**
Equation	$X(n, \omega) = \sum_{m=-\infty}^{\infty} x(m)w(m-n)e^{-i\omega m}$
Description	The above equation of represents the transformation from a signal $x(n)$ to a function $X(n, \omega)$ which gives the frequency content of frequency ω at discrete time n. $w(n)$ is the *analysis window* which is normalized such that $w(0) = 1$ and $w(n \to \infty) = w(n \to -\infty) = 0$
Source	**?**
Ref. By	DD2

Number	T3
Label	**Wavelets and their properties**
Equation	Let $\psi(t)$ represent the mother wavelet. The child wavelets are $\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a})$ where a is positive and defines the scale and b is a real number that defines the shift.
Description	A wavelet is a brief oscillation constructed from any periodic function. Child wavelets are versions of the mother wavelet that have been scaled and shifted in time.
Source	https://cpb-us-w2.wpmucdn.com/sites.gatech.edu/dist/5/462/files/2016/08/DFT-of-Noise.pdf
Ref. By	DD3

4.2.3 Data Definitions

This section collects and defines all the data needed to build the instance models.

Number	DD1
Label	**Morlet Wavelets**
Symbol	$\psi_{Mor}(t)$
Equation	$\psi_{Mor}(t) = c_\sigma \pi^{-\frac{1}{4}e^{-\frac{1}{2}t}}(e^{i\sigma t} - \kappa_\sigma)$ where $\kappa_\sigma = e^{1\frac{1}{2}\sigma^2}$ and $c_\sigma = (1 + e^{-\sigma^2} - 2e^{-\frac{3}{4}\sigma^2})^{\frac{1}{2}}$
Description	A Morelet wavelet is one option of wavelet to be used to analyse the signal with the wavelet transform in **??**. Here σ is a tuning parameter that allows for a trade off between time-frequency resolutions, conventionally $\sigma < 5$.
Source	https://cpb-us-w2.wpmucdn.com/sites.gatech.edu/dist/5/462/files/2016/08/DFT-of-Noise.pdf
Ref. By	IM3

Number	DD2
Label	**Matrix representation of STFT**
Equation	for a signal $x(n)$ with a corresponding STFT $X(n, \omega)$ the matrix representation is $$M(x(n)) = \begin{bmatrix} X(n_0, \omega_0) & X(n_1, \omega_0) & \dots & Tn_N, \omega_0) \\ X(n_0, \omega_1) & X(n_1, \omega_1) & \dots & TX(n_N, \omega_1) \\ \vdots & \vdots & \ddots & \vdots \\ X(n_0, \omega_F) & X(n_1, \omega_F) & \dots & X(n_N, \omega_F) \end{bmatrix}$$
Description	For a signal $x(n)$ the matrix at $M_{a,b}(x(n)) = X(n_a, \omega_b)$. The frequency ω_0 will be the minimum frequency that should be analysed and ω_F will be the maximum frequency that should be analysed, with the remaining frequencies evenly spaced intervals.
Source	N/A
Ref. By	DD3

Number	DD3
Label	**Matrix representation of Wavlet Transform**
Equation	The wavelet coefficients are $WT_\psi\{f(n)\}(a, b) = \langle f, \psi_{a,b} \rangle$. Thus, the matrix representation will be $$M(x(n)) = \begin{bmatrix} \langle f, \psi_{n_0,\omega_0} \rangle & \langle f, \psi_{n_1,\omega_0} \rangle & \dots & \langle f, \psi_{n_N,\omega_0} \rangle \\ \langle f, \psi_{n_0,\omega_1} \rangle & \langle f, \psi_{n_1,\omega_1} \rangle & \dots & \langle f, \psi_{n_N,\omega_1} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle f, \psi_{n_0,\omega_F} \rangle & \langle f, \psi_{n_1,\omega_F} \rangle & \dots & \langle f, \psi_{n_N,\omega_F} \rangle \end{bmatrix}$$
Description	The wavelet coefficients can be assembled into a matrix for a time-frequency representation of the signal. The frequency ω_0 will be the minimum frequency that should be analysed and ω_F will be the maximum frequency that should be analysed, with the remaining frequencies evenly spaced intervals.
Source	`https://en.wikipedia.org/wiki/Wavelet`
Ref. By	DD1

4.2.4 Instance Models

This section transforms the problem defined in Section 4.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 4.2.3 to replace the abstract symbols in the models identified in Section 4.2.2.

The goal statement 4.1.2 is solved by IM1.

Number	IM1
Label	**Time-frequency representation of a signal** $x(n)$
Input	$x(n)$, T, n_i, Δ_n, ω_{min}, ω_{max} and a method to compute the time frequency representation T where $T \in S, W$, S for a STFT and W for a wavelet transform.
Output	$M(x(n))$ from $n[ni, \ldots, ni + \Delta_n]$ and from $\omega[\omega_{min}, \ldots, \omega_{max}]$
Description	$x(n)$: discrete input signal of length N, $n[0, \ldots, N]$ P: sampling period of $x(n)$ (s) n_i: discrete time within signal to begin time-frequency transform Δ_n: the length of the time period over which the transform is performed. ω_{min}: the minimum frequency analysed for the transform (Hz). ω_{max}: the maximum frequency analysed for the transform (Hz). $M((x(n))$: the matrix representing of the time-frequency transform of $x(n)$.
Sources	N/A
Ref. By	

4.2.5 Input Data Constraints

Table 1 shows the data constraints on the input output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The software constraints will be helpful in the design stage for picking suitable algorithms. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise.

The specification parameters in Table 1 are listed in Table **??**.

Table 1: Input Variables

Var	Physical Constraints	Software Constraints	Typical Value	Uncertainty
n_i	$0 \leq n_i \leq (N - \Delta_n)$	N/A	26	N/A*
Δ_n	$0 < \Delta_n$, $t_i + \Delta_n \leq N$	N/A	2000	N/A *
ω_{min}	$0 < \omega_{min} << \omega_{max}$	$1/\Delta_n << \omega_{min} << 1/P$	2 Hz	N/A *
ω_{max}	$\omega_{min} << \omega_{max}$	$1/\Delta_n << \omega_{max} << 1/P$	200 Hz	N/A *
$x(n)$	N/A	$x_{min} \leq x(n) \leq x_{max}$ for all n	0.03627	10%
P	$P > 0$	N/A	0.25 s	10%

(*) The input discrete time indexes and frequencies do not have an uncertainty in the context of this project. They are merely the boundaries for the time-frequency transformed to be performed within, it would not effect the accuracy of the transform, just the portion of the signal it is performed upon.

(**) P is determined by the method of recording/sampling. It is not chosen by the user but the user must specify it.

4.2.6 Properties of a Correct Solution

The output of this Time_Freq_Analysis is a matrix representation of a 1D signal, for this reason, it is not easy to describe the properties of a correct solution. A correct solution should look like a heat map when graphed, essentially the matrix values should not be 'random' or sporadically scattered, and there should be 'clusters' at the prominent frequencies at the time they occur.The output should obviously be consistent for the same input signal and similar for similar input signals. Expressed in table 2 any one value in a transform cannot be greater than the value of the signal itself, at the same time n.

Table 2: Output Variables

Var	Physical Constraints
$M_{a,b}$	$M_{n,b} \leq x(n)$

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

R1: Program shall require the signal to be analysed as input. All other inputs will have defaults, but program will accept user inputs for those as well (IM 1).

R5: The time-frequency representations of simple input signals (such as sinusoids of a constant frequency or an impulse) should be comparable to existing time-frequency transforms of that signal (IM 1).

5.2 Nonfunctional Requirements

- Program shall plot time-frequency representation as a heat map.
- The time complexity for this program should be $O(n)$,
- Program will not have a graphical user interface but should still be easy to use, the input parameters besides the signal shall all have default values, there should be at most 6 optional inputs.
- The program code should be clear and readable.
- The program should easily integrate with other software programs.

6 Likely Changes

LC1: Currently there is only one option for type of wavelet: a Morlet wavelet. Depending on the resources available, the type of wavelet could be specified, as many different wavelets exist that all do slightly different things. This will have to be added as an input IF the user chooses wavelet transform for the input T as in table 1. Essentially, this would be a conditional input, only if the user selects wavelet transform. Per assumption A5.

LC2: σ a parameter used by the Morlet wavelet transform 1 can not be set by the user, as the end goal of this program is to analyse a large among of similar signals and then compare them, and σ should remain constant for all of those signals. However, it may be beneficial for the user to be able to input this parameter to find the best time-frequency resolution for their problem.

LC3: Currently, the user specifies the following four input parameters: n_i, Δ_n, ω_{min} and ω_{max}. However, this could lead to the user inputting inappropriate values for the sample. A time-frequency at too small an interval will not yield useful results, but this depends on the content of the sample itself, so it cannot be a parameter set by the system. However, it might be fit to restrict the range of these input values more then they already are.

7 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Table 3 shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. Table 4 shows the dependencies of instance models, requirements, and data constraints on each other. Table 5 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

	T1	T3	T2	DD1	DD2	DD??	IM1
T1			X		X		
T3						X	
T2	X				X		
DD1					X		
DD2	X		X				X
DD3		X					X
IM1				X	X	X	

Table 3: Traceability Matrix Showing the Connections Between Items of Different Sections

	IM1
R1	X
R2	X
R3	X
R4	X
R5	X

Table 4: Traceability Matrix Showing the Connections Between Requirements and Instance Models

	A1	A2	A3	A??	A5	A6
T1						
T3		X	X			
T2		X	X	X		X
DD1						
DD2	X					
IM3						
IM1	X	X	X	X	X	X
LC1					X	
LC2						
LC3	X	X	X			

Table 5: Traceability Matrix Showing the Connections Between Assumptions and Other Items

References

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