INTELLIGENT ROBOTS

CHAPTER 9: MAPPING WITH KNOWN POSE

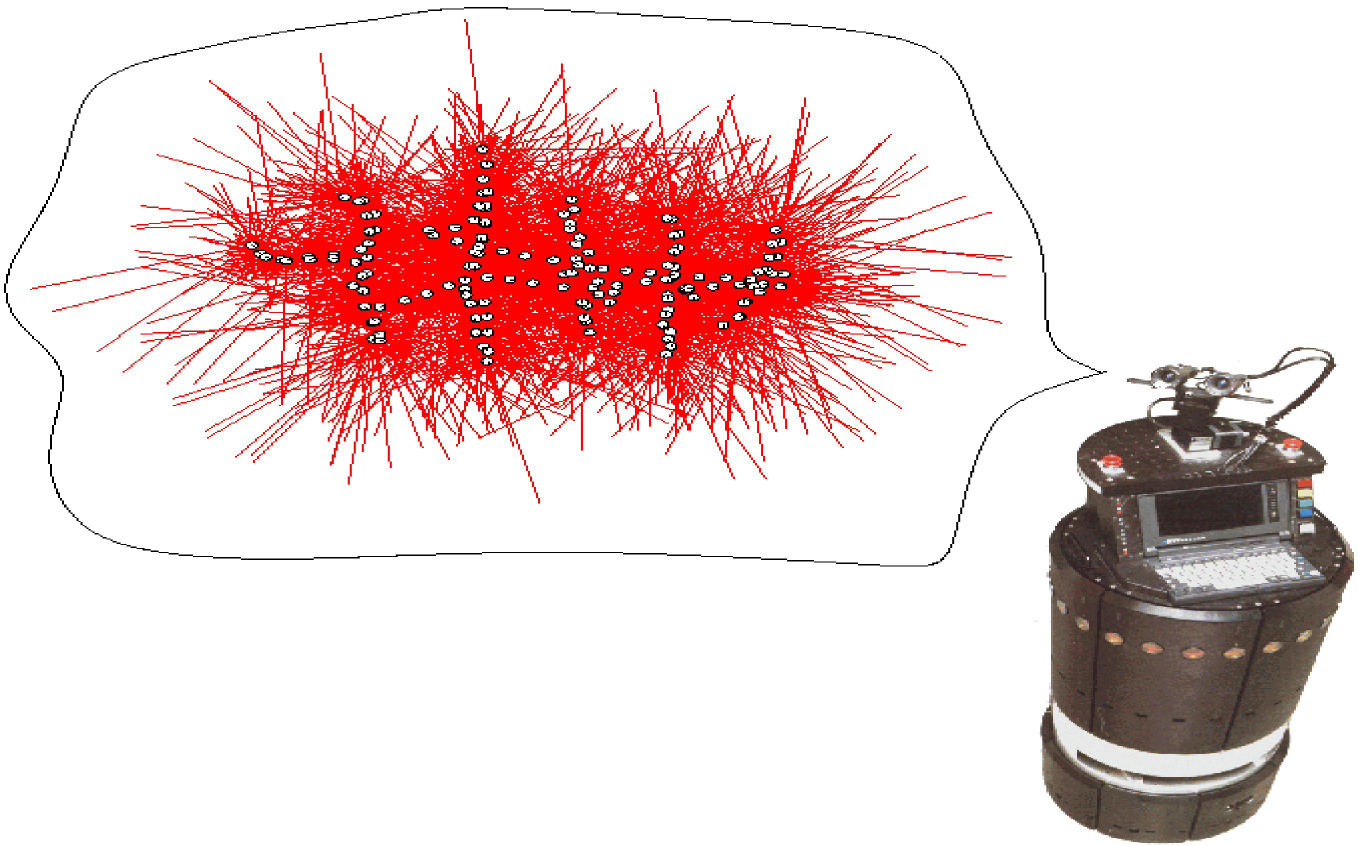
Outline

- Mapping Problems
- Occupancy Grip Maps
- Reflection Maps
- Occupancy v.s. Reflection Maps

Why Mapping?

- Learning maps is one of the fundamental problems in mobile robotics
- Maps allow robots to efficiently carry out their tasks, allow localization
- Successful robot systems rely on maps for localization, path planning, activity planning etc.

General Problem of Mapping



What does the environment look like?

Problem Statement

Formally, mapping involves, given the sensor data,

$$d = \{x_1, z_1, x_2, z_2, \ldots, x_n, z_n\}$$

to calculate the most likely map

$$m^* = \arg\max_m P(m \mid d)$$

Mapping and SLAM

- We have learned how to estimate the pose of the vehicle given the data and the map.

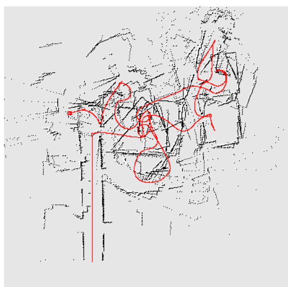
- Mapping, however, involves to simultaneously estimate the pose of the vehicle and the map.

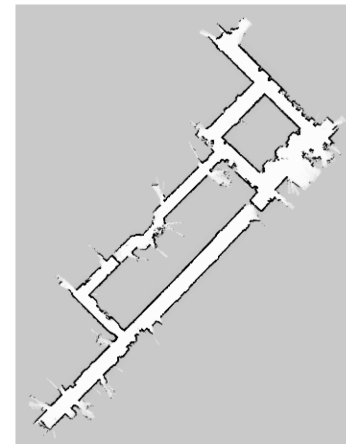
- The general problem is therefore denoted as the simultaneous localization and mapping problem (SLAM).

- In this section we will describe how to calculate a map given we know the pose of the vehicle.

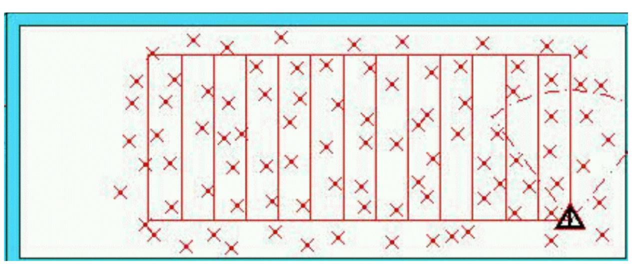
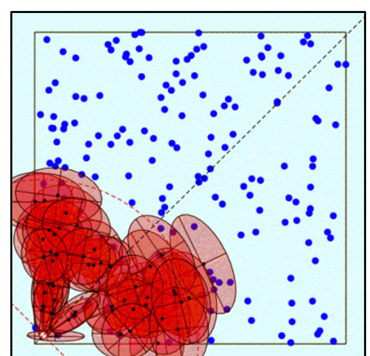
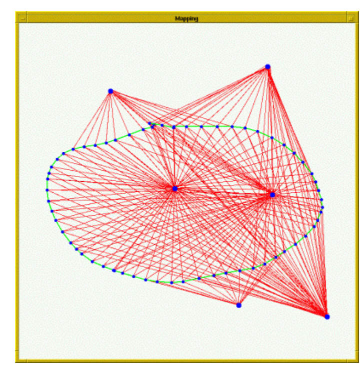
SLAM Problems

- Grid maps or scans





- Landmark-based

Problems in Mapping

- Sensor interpretation
  - How do we extract relevant information from raw sensor data?
  - How do we represent and integrate this information over time?

- Robot locations have to be estimated
  - How can we identify that we are at a previously visited place?
  - This problem is the so-called data association problem.

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Occupancy GridMap

- Introduced by Moravec and Elfes in 1985
- Represent environment by a grid.
- Estimate the probability that a location is occupied by an obstacle.
- Key assumptions
  - Occupancy of individual cells (m[xy]) is independent

$$\boxed{\begin{aligned} Bel(m_t) &= P(m_t \mid x_1, z_2 \ldots, x_{t-1}, z_t) \\ &= \prod_{x,y} Bel(m_t^{[xy]}) \end{aligned}}$$

  - Robot positions *x* are known!

Updating Occupancy Map

- Idea: Update each individual cell using a binary Bayes filter.

$$\boxed{Bel(m_t^{[xy]}) = \eta \; p(z_t \mid m_t^{[xy]}) \int p(m_t^{[xy]} \mid m_{t-1}^{[xy]}, u_{t-1}) Bel(m_{t-1}^{[xy]}) \, dm_{t-1}^{[xy]}}$$

- Additional assumption: Map is static.

$$\boxed{Bel(m_t^{[xy]}) = \eta \; p(z_t \mid m_t^{[xy]}) Bel(m_{t-1}^{[xy]})}$$

Updating Occupancy Map

Update the map cells using the inverse sensor model

$$Bel\left(m_t^{[xy]}\right) = 1 - \left(1 + \frac{P\left(m_t^{[xy]} \mid z_t, u_{t-1}\right)}{1 - P\left(m_t^{[xy]} \mid z_t, u_{t-1}\right)} \cdot \frac{1 - P\left(m_t^{[xy]}\right)}{P\left(m_t^{[xy]}\right)} \cdot \frac{Bel\left(m_{t-1}^{[xy]}\right)}{1 - Bel\left(m_{t-1}^{[xy]}\right)}\right)^{-1}$$

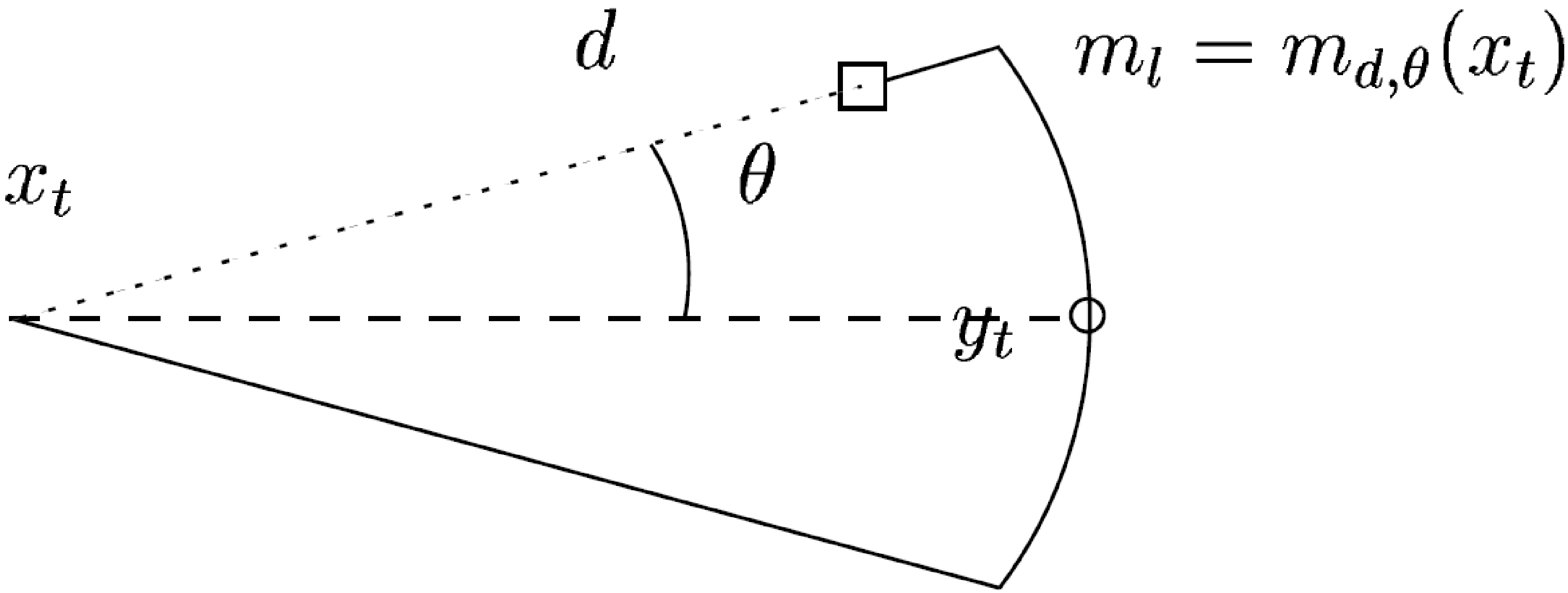
Or use the log-odds representation

$$\begin{aligned} \overline{B}\left(m_t^{[xy]}\right) = {} & \log odds\left(m_t^{[xy]} \mid z_t, u_{t-1}\right) \\ & - \log odds\left(m_t^{[xy]}\right) \\ & + \overline{B}\left(m_{t-1}^{[xy]}\right) \end{aligned}$$

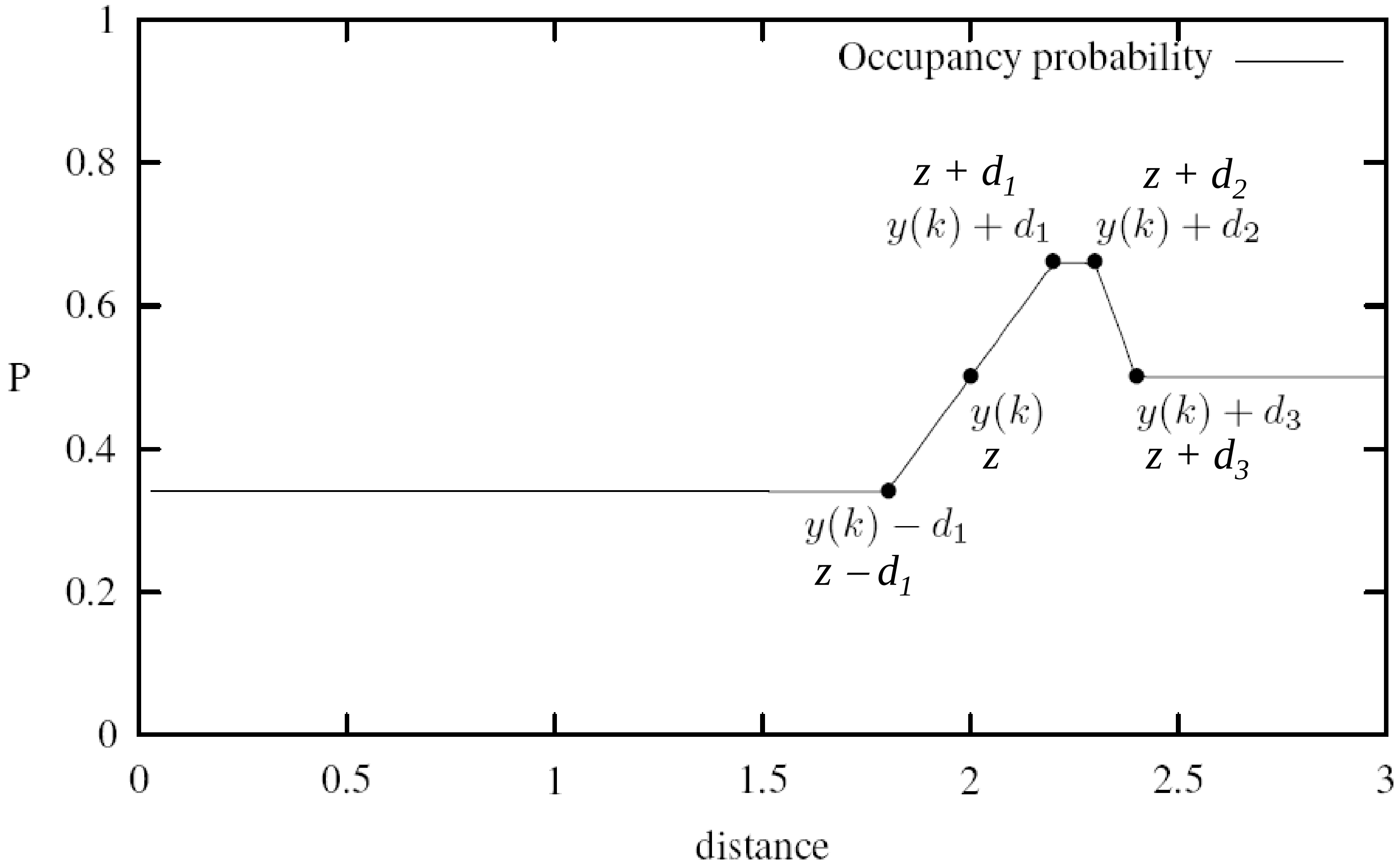
$$\overline{B}\left(m_t^{[xy]}\right) := \log odds(m_t^{[xy]})$$

$$odds(x) := \left(\frac{P(x)}{1 - P(x)}\right)$$

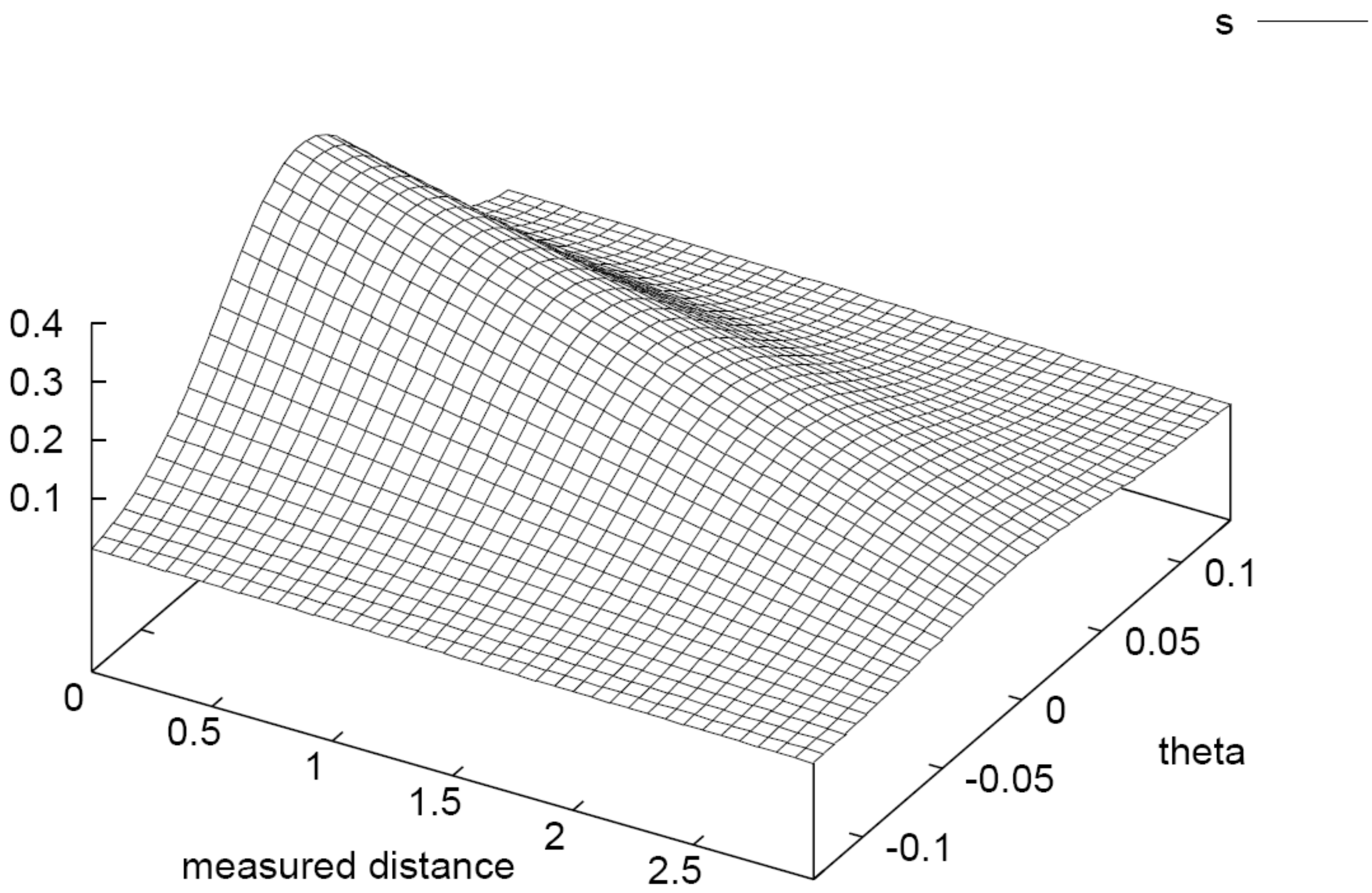
Key Parameters



Occupancy Values



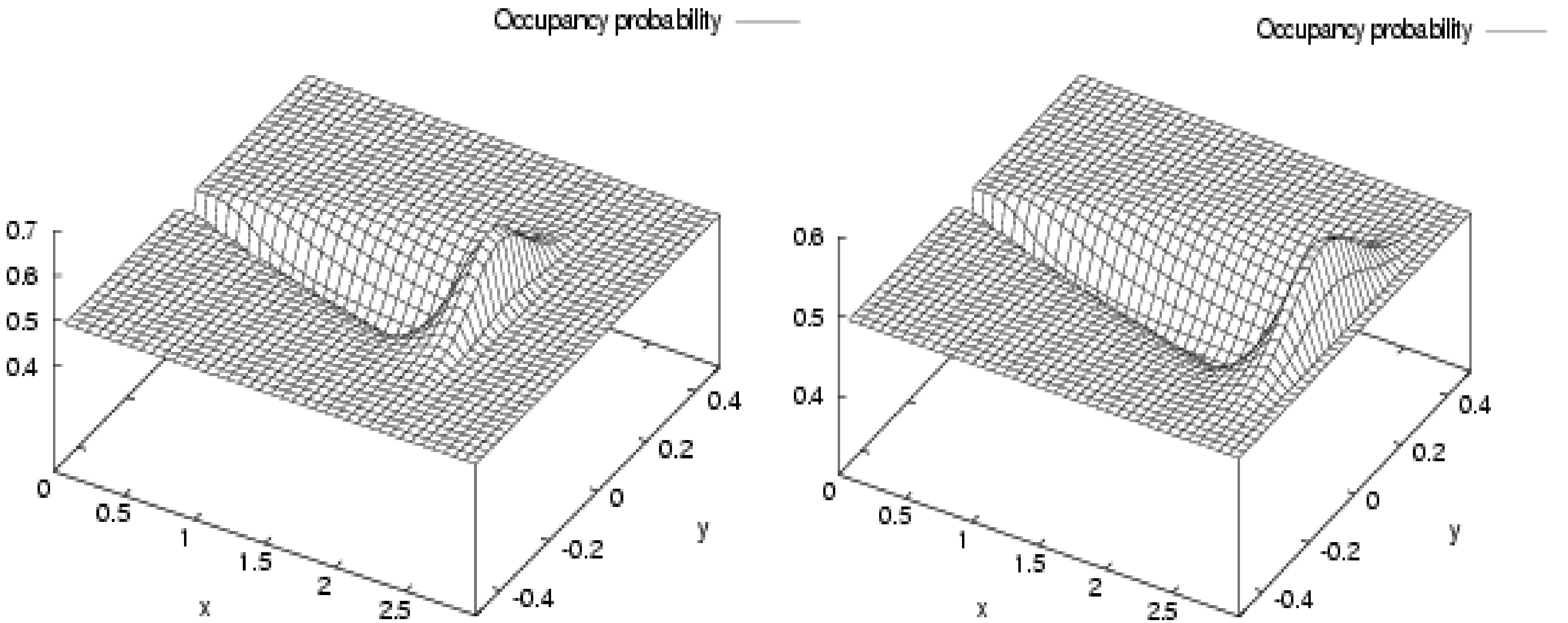
Posterior Belief



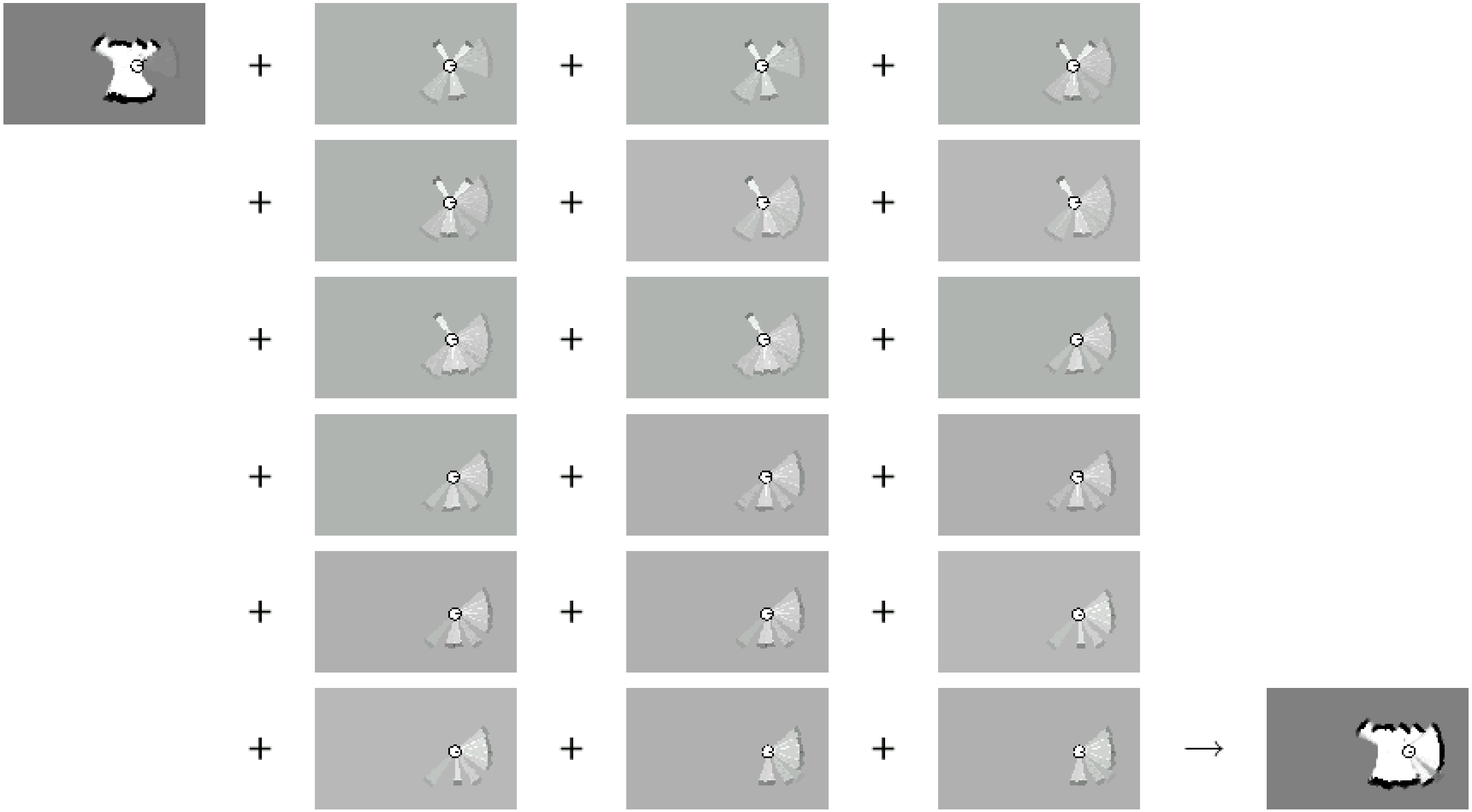
Occupancy Probability

$$P(m_{d,\theta}(x(k)) \mid y(k), x(k)) = P(m_{d,\theta}(x(k)))$$
$$+ \begin{cases} -s(y(k),\theta) & d < y(k) - d_1 \\ -s(y(k),\theta) + \frac{s(y(k),\theta)}{d_1}\ (d - y(k) + d_1) & d < y(k) + d_1 \\ s(y(k),\theta) & d < y(k) + d_2 \\ s(y(k),\theta) - \frac{s(y(k),\theta)}{d_3 - d_2}\ (d - y(k) - d_2) & d < y(k) + d_3 \\ 0 & \text{otherwise.} \end{cases}$$

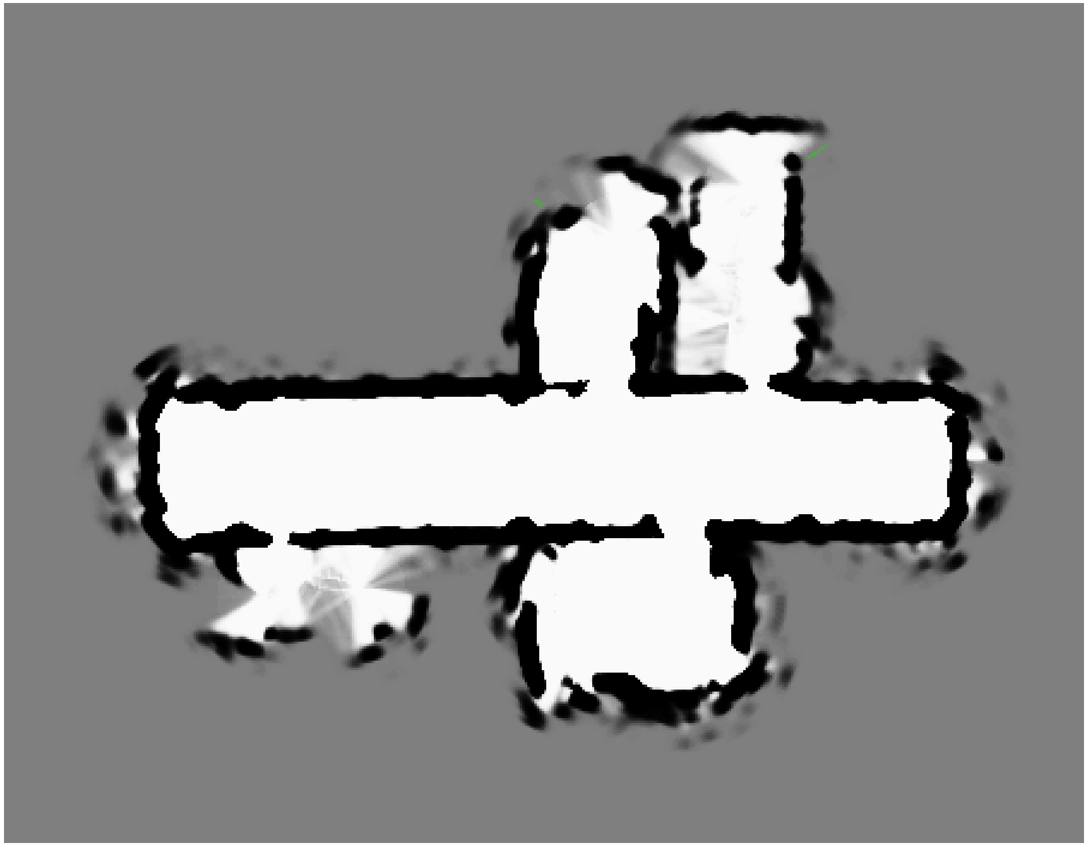
Sensor Model



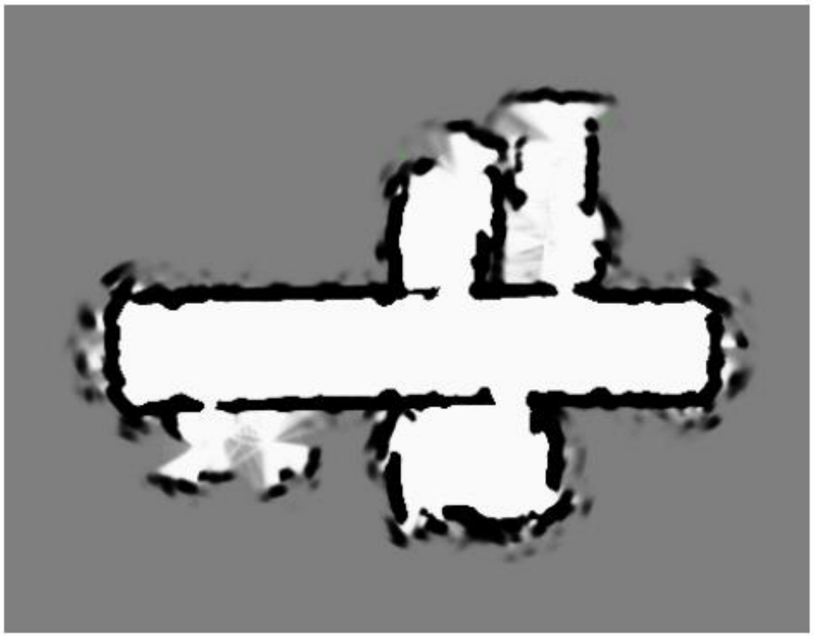
Incremental Updating

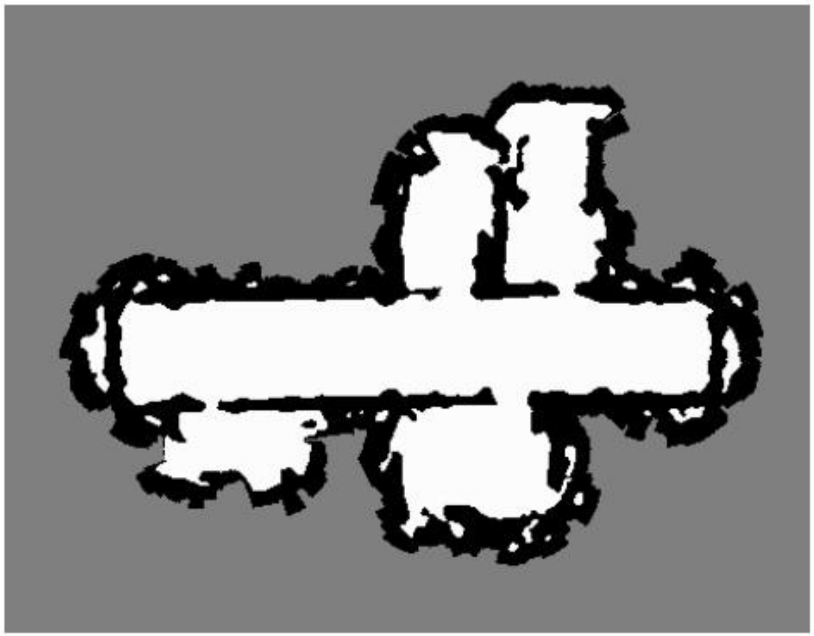


Mapping with Ultrasound Sensors



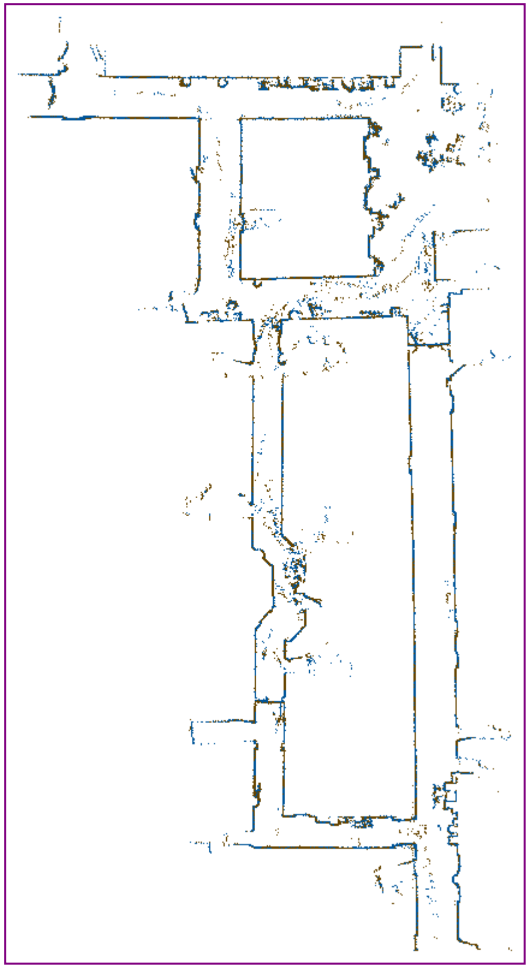
Occupancy Map and ML Map

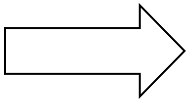


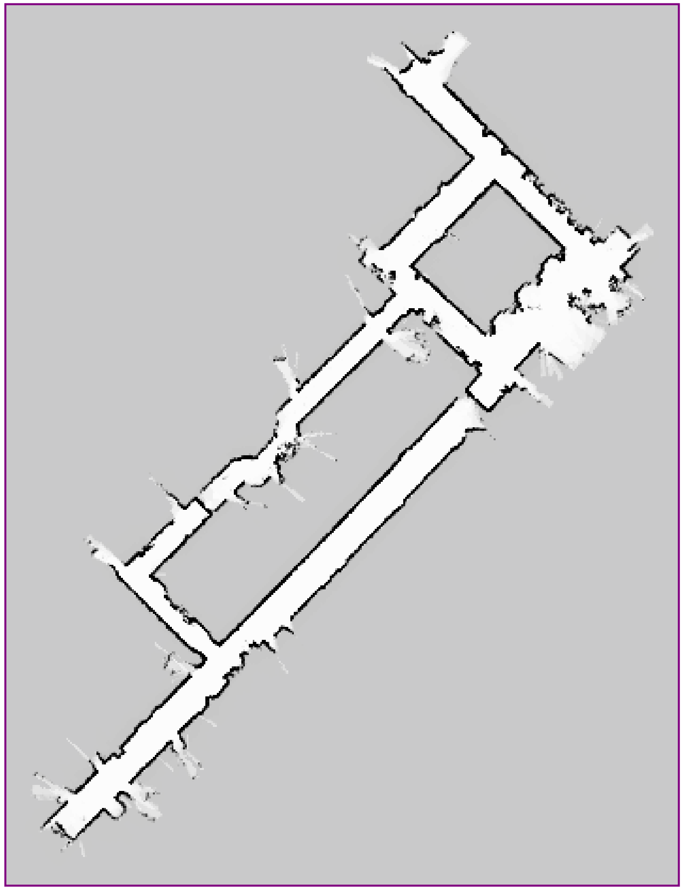


The maximum likelihood map is obtained by clipping the occupancy grid map at a threshold of 0.5

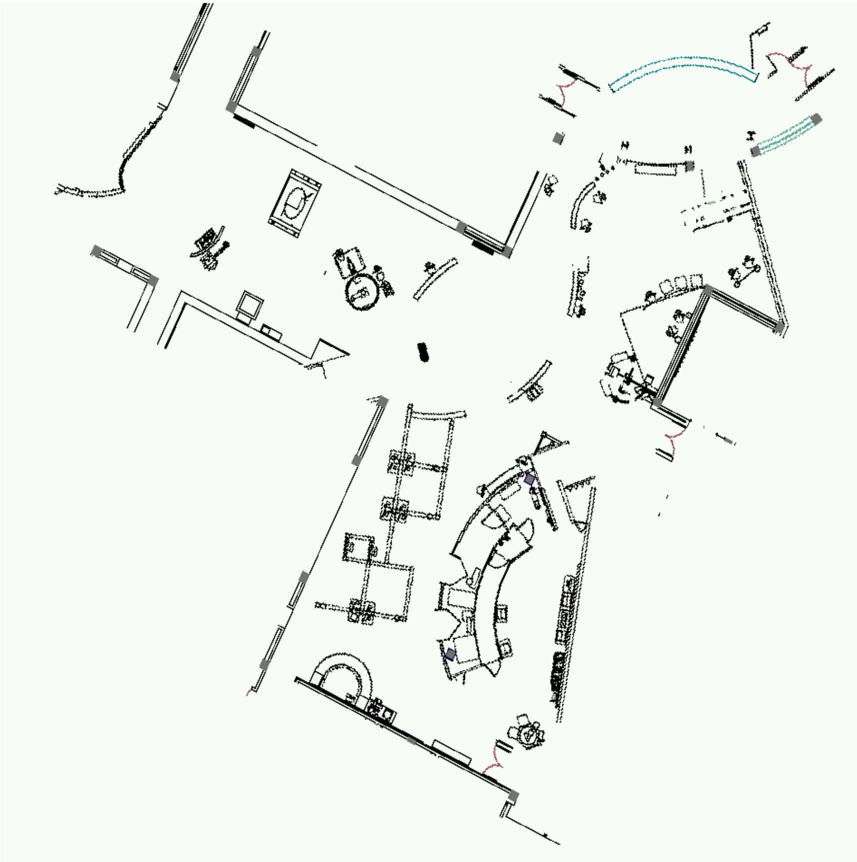
Occupancy Grids: from Scan to Maps







Occupancy Grids: from Scan to Maps



CAD map

occupancy grid map

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Counting

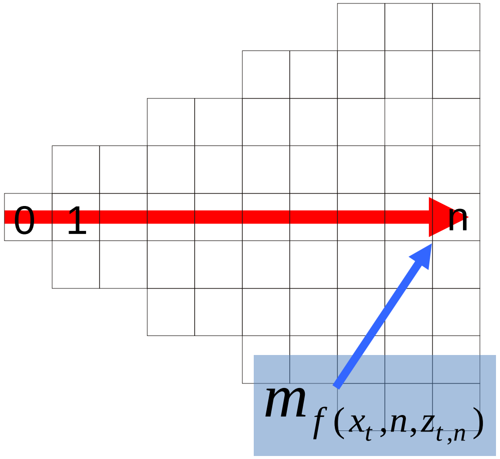
- For every cell count
  - hits(x,y): number of cases where a beam ended at <x,y>
  - misses(x,y): number of cases where a beam passed through <x,y>

$$\boxed{Bel(m^{[xy]}) = \frac{\text{hits}(x,y)}{\text{hits}(x,y) + \text{misses}(x,y)}}$$

- Value of interest: $P(\mathit{reflects}(x,y))$

Measurement Model

1. pose at time *t*: x_t
2. beam *n* of scan *t*: $z_{t,n}$
3. maximum range reading: $\varsigma_{t,n} = 1$
4. beam reflected by an object: $\varsigma_{t,n} = 0$



$$p(z_{t,n} \mid x_t, m) = \begin{cases} \prod_{k=0}^{z_{t,n}-1} (1 - m_{f(x_t, n, k)}) & \text{if } \varsigma_{t,n} = 1 \\ m_{f(x_t, n, z_{t,n})} \prod_{k=0}^{z_{t,n}-1} (1 - m_{f(x_t, n, k)}) & \text{if } \varsigma_{t,n} = 0 \end{cases}$$

Maximum Likelihood Map

- Compute values for m that maximize

$$m^* = \underset{m}{\arg\max}\, P(m \mid z_1, \ldots, z_t, x_1, \ldots, x_t)$$

- Assuming a uniform prior probability for p(m), this is equivalent to maximizing (applic. of Bayes rule)

$$m^* = \underset{m}{\arg\max}\, P(z_1, \ldots, z_t \;\; m, x_1, \ldots, x_t)$$

$$= \underset{m}{\arg\max} \prod P(z_t \mid m, x_t)$$

$$= \underset{m}{\arg\max} \sum \ln P(z_t \mid m, x_t)$$

Maximum Likelihood Map

$$m^* = \arg\max_m \left[\sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{n=1}^{N} \left(I(f(x_t, n, z_{t,n}) = j) \cdot (1 - \varsigma_{t,n}) \cdot \ln m_j \right.\right.$$
$$\left.\left. + \sum_{k=0}^{z_{t,n}-1} I(f(x_t, n, k) = j) \cdot \ln(1 - m_j) \right) \right]$$

Suppose

$$\alpha_j = \sum_{t=1}^{T} \sum_{n=1}^{N} I(f(x_t, n, z_{t,n}) = j) \cdot (1 - \varsigma_{t,n})$$

$$\beta_j = \sum_{t=1}^{T} \sum_{n=1}^{N} \left[\sum_{k=0}^{z_{t,n}-1} I(f(x_t, n, k) = j) \right]$$

Meanings

$$\alpha_j = \sum_{t=1}^{T} \sum_{n=1}^{N} I(f(x_t, n, z_{t,n}) = j) \cdot (1 - \varsigma_{t,n})$$

corresponds to the number of times a beam that is not a maximum range beam ended in cell j (*hits(j)*)

$$\beta_j = \sum_{t=1}^{T} \sum_{n=1}^{N} \left[\sum_{k=0}^{z_{t,n}-1} I(f(x_t, n, k) = j) \right]$$

corresponds to the umber of times a beam intercepted cell j without ending in it (*misses(j)*).

Maximum Likelihood Map

We assume that all cells m_j are independent:

$$m^* = \arg\max_m \left(\sum_{j=1}^{J} \alpha_j \ln m_j + \beta_j \ln(1 - m_j) \right)$$

If we set

$$\frac{\partial m}{\partial m_j} = \frac{\alpha_j}{m_j} - \frac{\beta_j}{1 - m_j} = 0$$

we obtain

$$m_j = \frac{\alpha_j}{\alpha_j + \beta_j}$$

Computing the most likely map amounts to counting how often a cell has reflected a measurement and how often it was intercepted.

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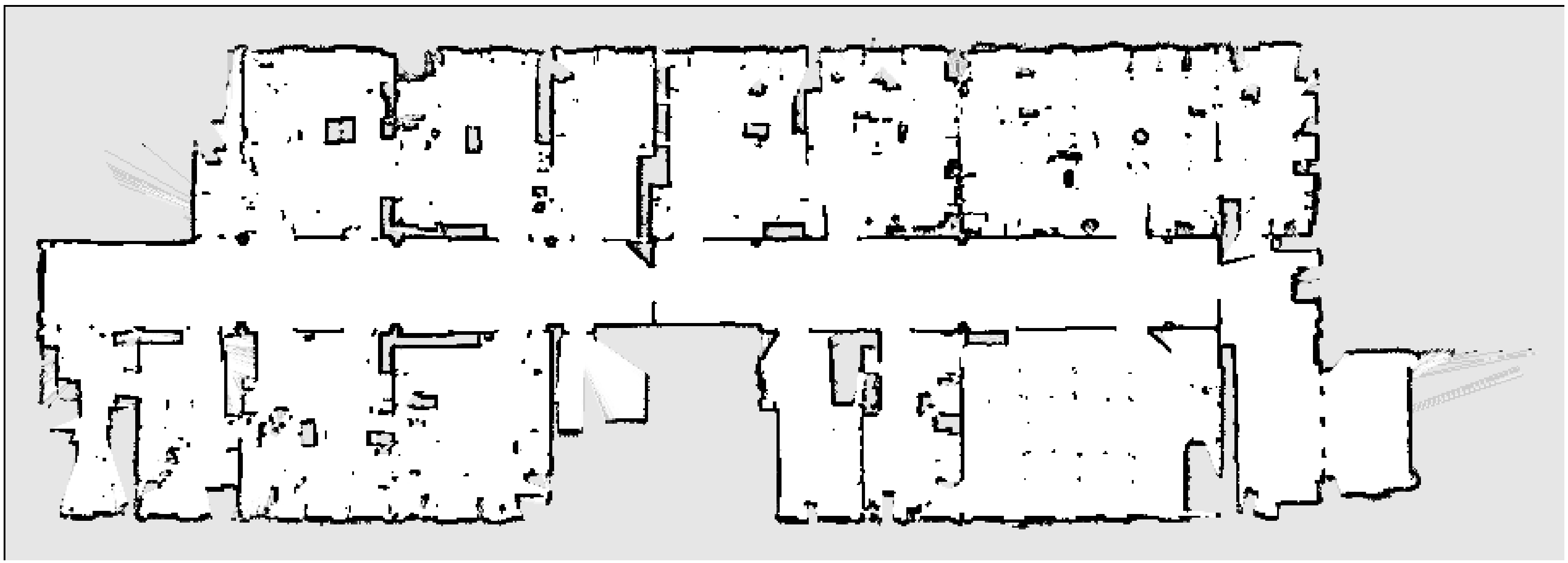
Occupancy v.s. Reflection Maps

- The counting model determines how often a cell reflects a beam.
- The occupancy model represents whether or not a cell is occupied by an object.
- Although a cell might be occupied by an object, the reflection probability of this cell might be very small.

Example: Occupancy Map



Example: Reflection Map

Example

- Out of 1000 beams only 60% are reflected from a cell and 40% intercept it without ending in it.
- Accordingly, the reflection probability will be 0.6.
- Suppose *p(occ | z)* = 0.55 when a beam ends in a cell and *p(occ | z)* = 0.45 when a cell is intercepted by a beam that does not end in it.
- Accordingly, after n measurements we will have

$$\left(\frac{0.55}{0.45}\right)^{n*0.6} * \left(\frac{0.45}{0.55}\right)^{n*0.4} = \left(\frac{11}{9}\right)^{n*0.6} * \left(\frac{11}{9}\right)^{-n*0.4} = \left(\frac{11}{9}\right)^{n*0.2}$$

- Whereas the reflection map yields a value of 0.6, the occupancy grid value converges to 1.

Summary

- Occupancy grid maps are a popular approach to represent the environment of a mobile robot given known poses.
- In this approach each cell is considered independently from all others.
- It stores the posterior probability that the corresponding area in the environment is occupied.
- Occupancy grid maps can be learned efficiently using a probabilistic approach.
- Reflection maps are an alternative representation.
- They store in each cell the probability that a beam is reflected by this cell.
- We provided a sensor model for computing the likelihood of measurements and showed that the counting procedure underlying reflection maps yield the optimal map.

Homework 8

Problem 1: generate occupancy and ML grid maps by using the threshold of 0.5