INTELLIGENT ROBOTS

CHAPTER 2: LOCOMOTION

Outlines

- Turtlebot Hardware Specifications

- Wheeled Mobile Robot Kinematics

- Mobile Robot Motion Control

Turtlebot Specifications

- Base
- Motor
- Motor Encoder
- Gyroscope
- Accelerometer
- Magnetometer

Base Specifications

SIZE AND WEIGHT

EXTERNAL DIMENSIONS (L x W x H)	354 x 354 x 420 mm (14.0 x 14.0 x 16.5 in)
WEIGHT	6.3 kg (13.9 lb)
WHEELS (Diameter)	76 mm (3 in)
GROUND CLEARANCE	15 mm (0.6 in)

SPEED AND PERFORMANCE

MAX. PAYLOAD	5 kg (11 lb)
MAX. SPEED	0.65 m/s (2.1 ft/s)
MAX. ROTATIONAL SPEED	180°/S

BATTERY AND POWER SYSTEM

STANDARD BATTERY	2200 mAh Li-Ion
EXTENDED BATTERY	4400 mAh Li-Ion
USER POWER	5 V and 19V (1A), 12 V (1.5A), 12V (5A)

Kobuki Motor Specifications

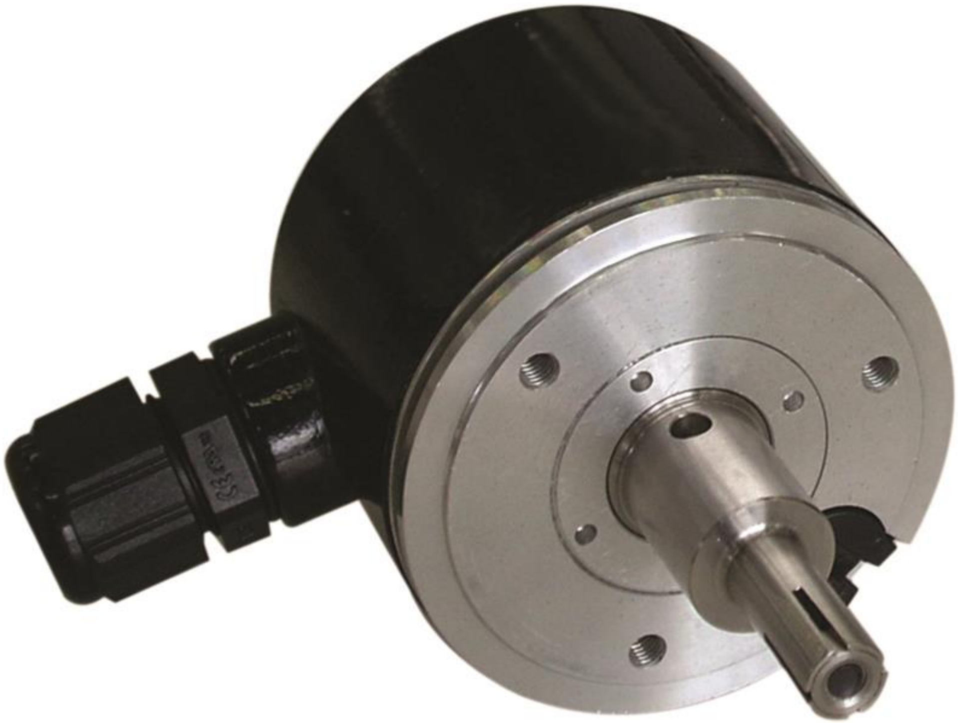
Brushed DC Motor

- Manufacturer: Standard Motor
- Rated Voltage: 12 V
- Rated Load: 5 mN·m
- Load Current: 210 mA
- Load Speed: 9960 rpm ± 15%
- Rated Load Current: 750 mA
- Rated Load Speed: 8800 rpm ± 15%
- Armature Resistance: 1.5506 Ω at 25° C
- Armature Inductance: 1.51 mH
- Torque Constant(Kt): 10.913 mN·m/A
- Velocity Constant(Kv): 830 rpm/V
- Stall Current: 6.1 A
- Stall Torque: 33 mN·m

Rotary Encoder Based Odometry

- Odometry is the use of data from motion sensors to estimate change in position over time.

Rotary Encoder： A rotary encoder, also called a shaft encoder, is an electro-mechanical device that converts the angular position or motion of a shaft or axle to an analog or digital signal



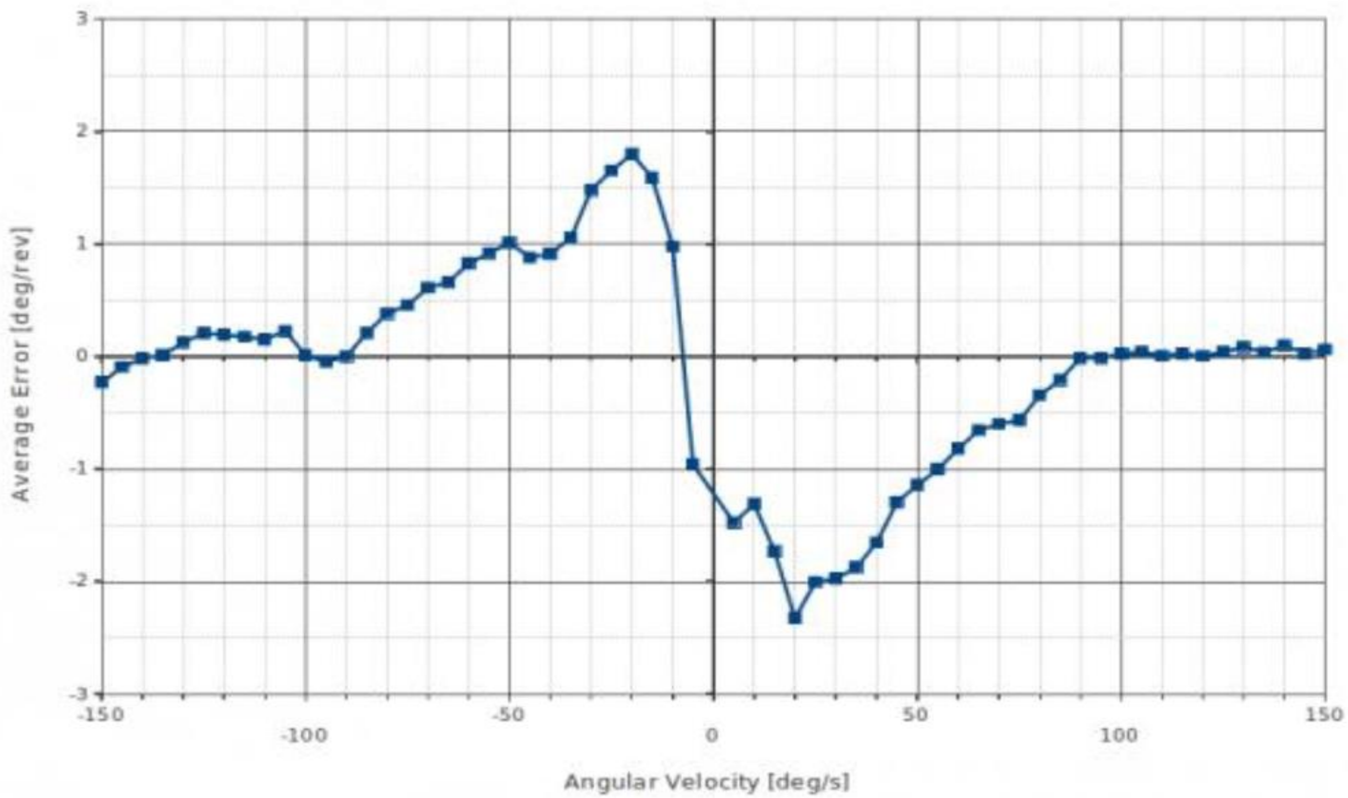
Gyro Specifications

3-Axis Digital Gyroscope

- Manufacturer : STMicroelectronics
- Part Name : L3G4200D
- Measurement Range: ±250 deg/s
- Yaw axis is factory calibrated within the range of ±20 deg/s to ±100 deg/s

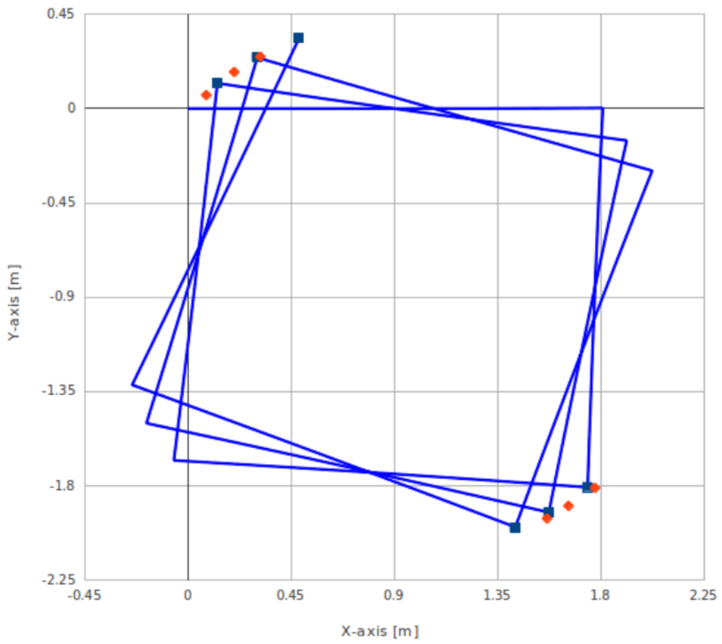
Gyro Signals

- This graph shows the average heading error per revolution of gyro, when robot rotates with a given velocity :



Gyro Based Odometry

- This graph shows the position error of fused odometry with gyro, when robot moves along a square path. Robot moved with 0.1 m/s on the line segment and rotated with 30 deg/s on the corner.

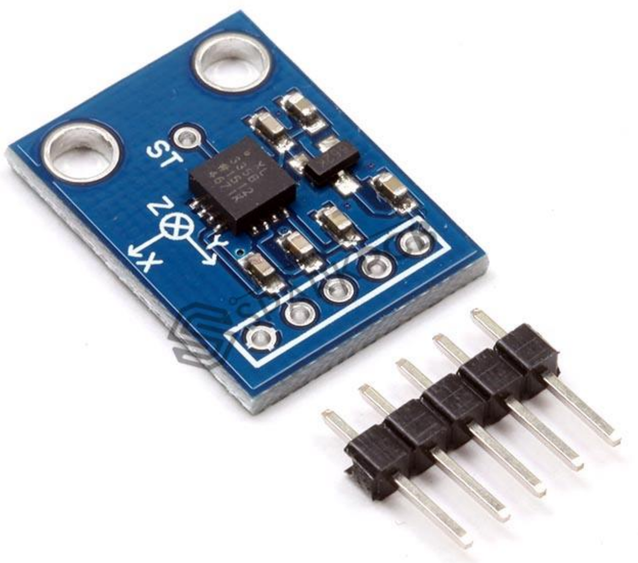


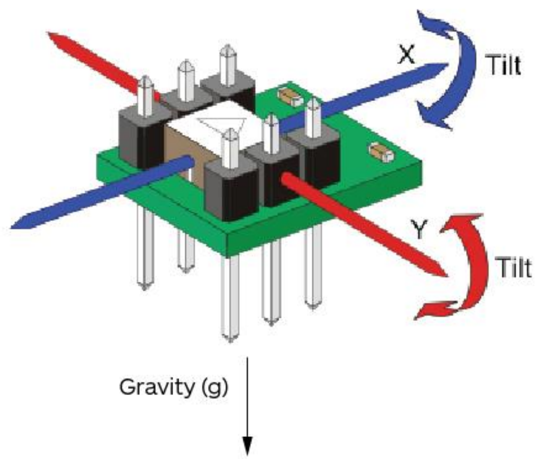
- This table shows the calculated angular error, when robot arrived at the diagonally opposite corner from the starting point(0.0, 0.0) .

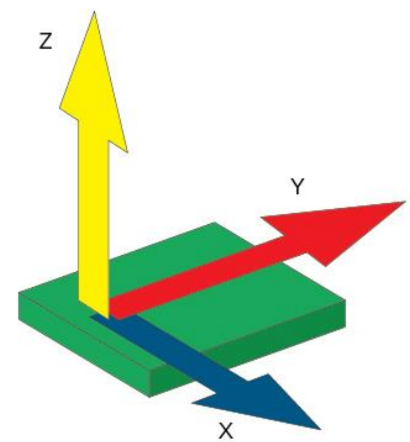
Number of turns of square path	Angular Error [deg]
0.5	0.47
1.5	1.99
2.5	3.18

Accelerometer

An accelerometer is a device that measures proper acceleration.





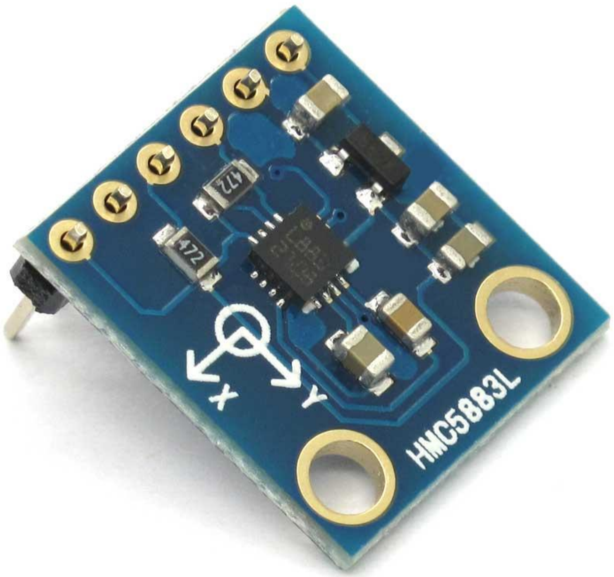


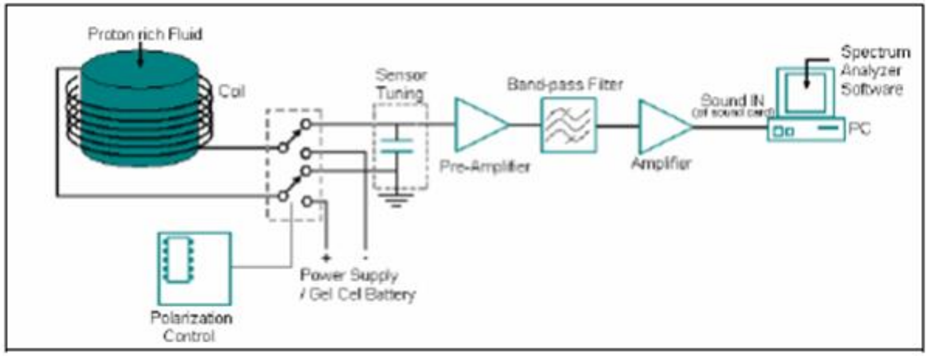
Diagrams illustrating the axes of 2-axis (left) and 3-axis accelerometers. This particular 2-axis sensor is also capable of tilt measurement.

Image credit: Parallax | Kerry Wong

Magnetometer

A magnetometer is an instrument that measures the direction, strength, or relative change of a magnetic field at a particular location.





Outlines

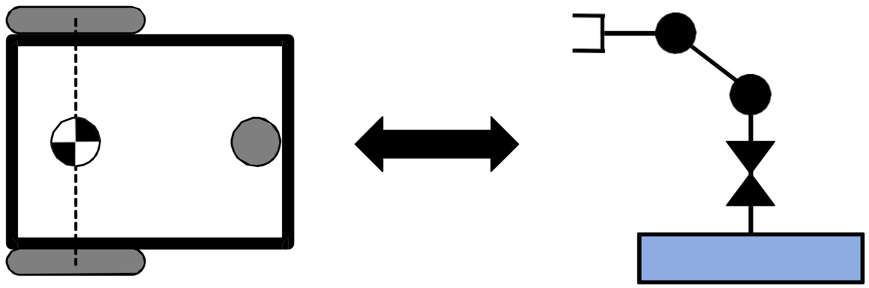
- Turtlebot 2 hardware Specifications
- Wheeled Mobile Robot Kinematics
- Mobile Robot Motion Control

Mobile Robot Kinematics

- Overview
- Locomotion of Wheeled Robot
- Kinematic Constraints
- Odometry of Mobile Robot

Overview

- Manipulator- vs. Mobile Robot Kinematics
  - Both are concerned with forward and inverse kinematics
  - However, for mobile robots, encoder values don‘t map to unique robot poses
  - However, mobile robots can move unbound with respect to their environment
    - There is no direct (=instantaneous) way to measure the robot’s position
    - Position must be integrated over time, depends on path taken
    - Leads to inaccuracies of the position (motion) estimate
  - Understanding mobile robot motion starts with understanding wheel constraints placed on the robot’s mobility

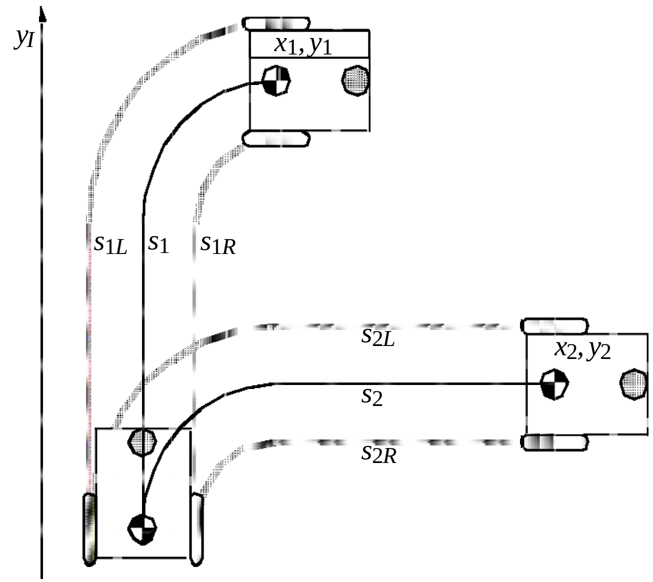


Non-Holonomic Systems

- Non-holonomic systems
  - differential equations are not integrable to the final position.
  - the measure of the traveled distance of each wheel is not sufficient to calculate the final position of the robot. One has also to know how this movement was executed as a function of time.
  - This is in stark contrast to actuator arms

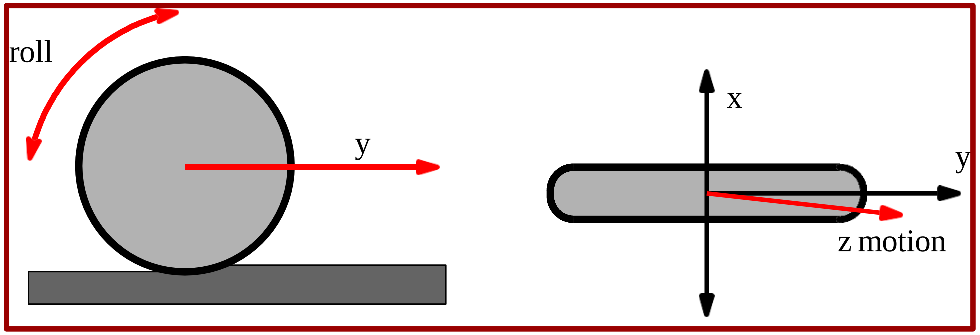
$$s_1 = s_2, s_{1R} = s_{2R}, s_{1L} = s_{2L}$$

$$x_1 \neq x_2, y_1 \neq y_2$$



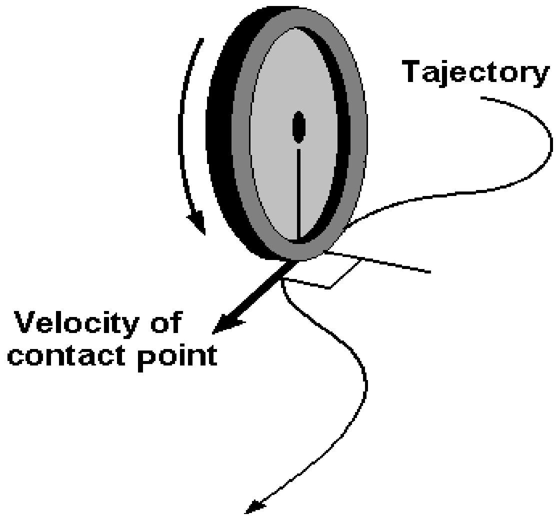
Locomotion of Wheeled Robots

Locomotion : Power of motion from place to place



- Differential drive (AmigoBot, Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive (B21)
- Mecanum wheels, XR4000

Idealized Rolling Wheel



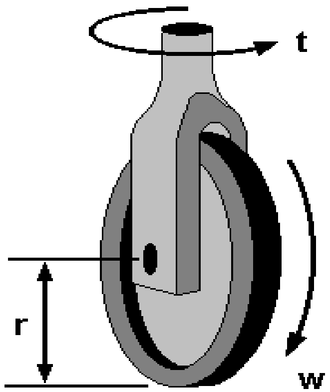
Non-slipping and pure rolling

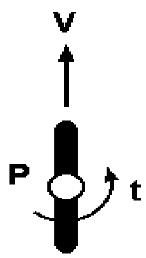
Assumptions

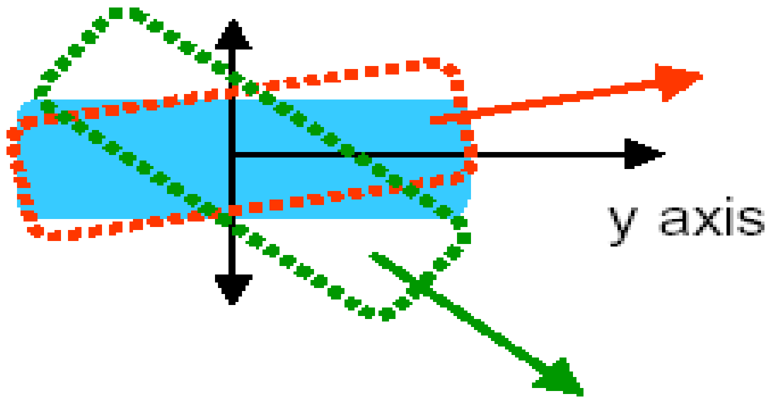
1. The robot is built from rigid mechanisms.
2. No slip occurs in the orthogonal direction of rolling (non-slipping).
3. No translational slip occurs between the wheel and the floor (pure rolling).
4. The robot contains at most one steering link per wheel.
5. All steering axes are perpendicular to the floor.

Steered Wheel

The orientation of the rotation axis can be controlled

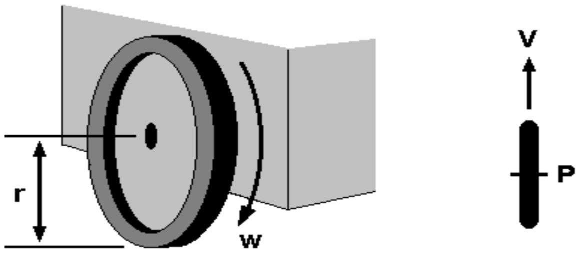




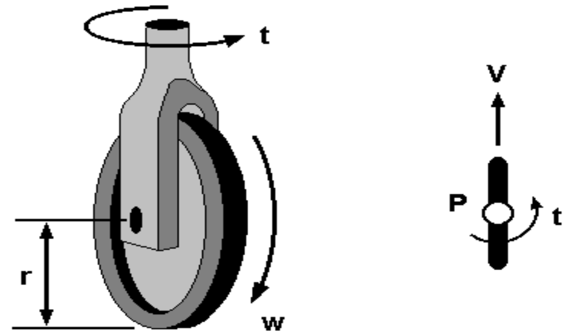


Wheel Types

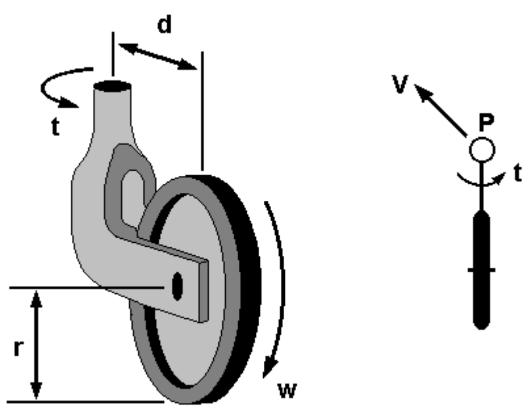
Fixed wheel



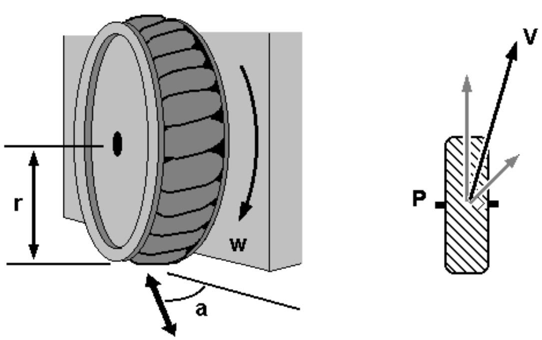
Centered orientable wheel



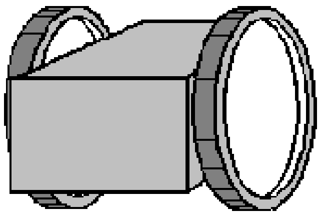
Off-centered orientable wheel
(Castor wheel)



Swedish wheel: omnidirectional property

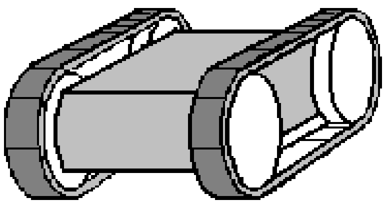


Examples of Wheeled Mobile Robots



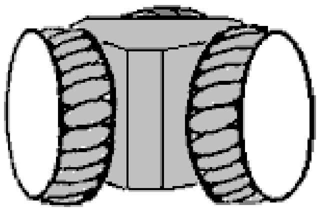
Bi-wheel type robot

- Smooth motion
- Risk of slipping
- Some times use roller-ball to make balance



Caterpillar type robot

- Exact straight motion
- Robust to slipping
- Inexact modeling of turning



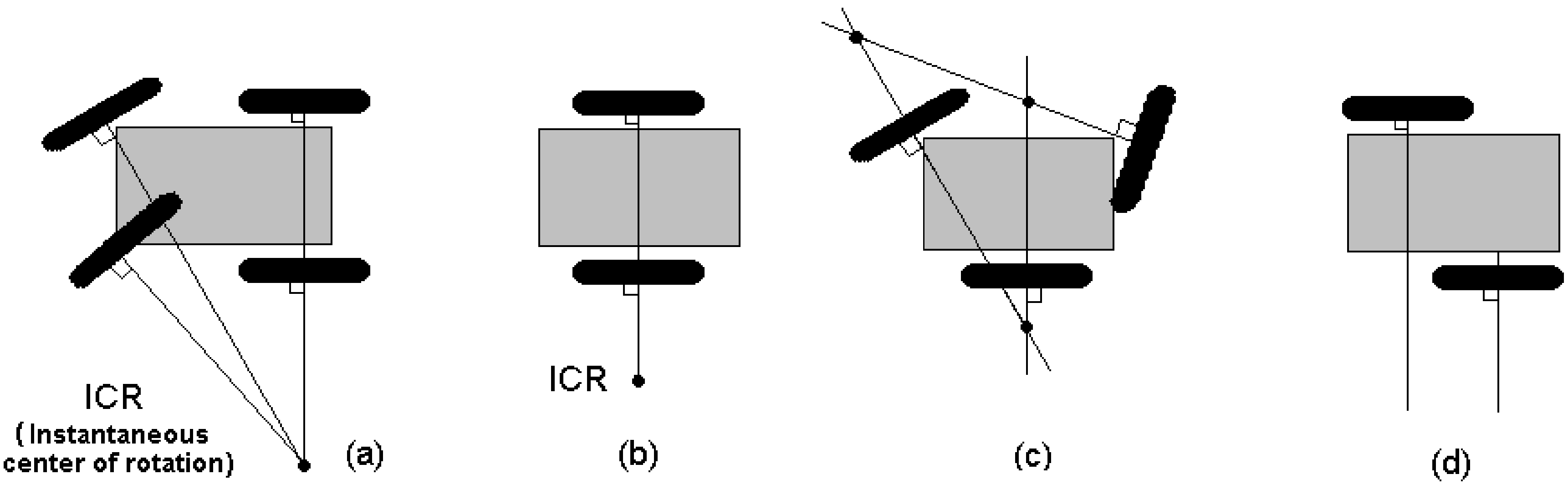
Omnidirectional robot

- Free motion
- Complex structure
- Weakness of the frame

Mobile Robot Locomotion

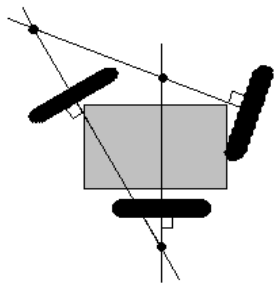
Instantaneous center of rotation (ICR) or Instantaneous center of curvature (ICC)

A cross point of all axes of the wheels



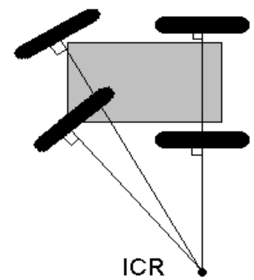
Degree of Mobility

The degree of freedom of the robot motion



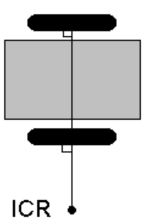
Cannot move
anywhere (No ICR)

- Degree of mobility : 0



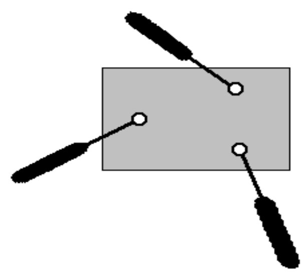
Fixed arc motion
(Only one ICR)

- Degree of mobility : 1



Variable arc motion
(line of ICRs)

- Degree of mobility : 2



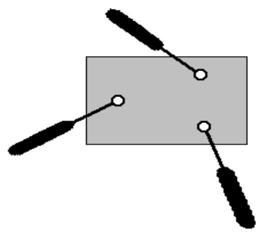
Fully free motion

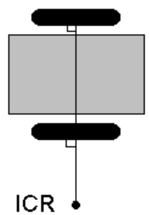
(ICR can be located
at any position)

- Degree of mobility : 3

Degree of Steerability

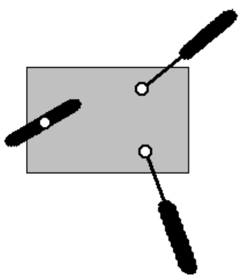
The number of centered orientable wheels that can be steered independently in order to steer the robot



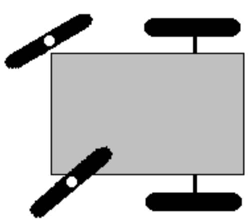


No centered orientable wheels

- Degree of steerability : 0

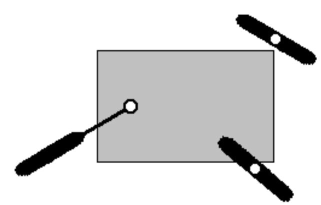


One centered orientable wheel



Two mutually dependent centered orientable wheels

- Degree of steerability : 1

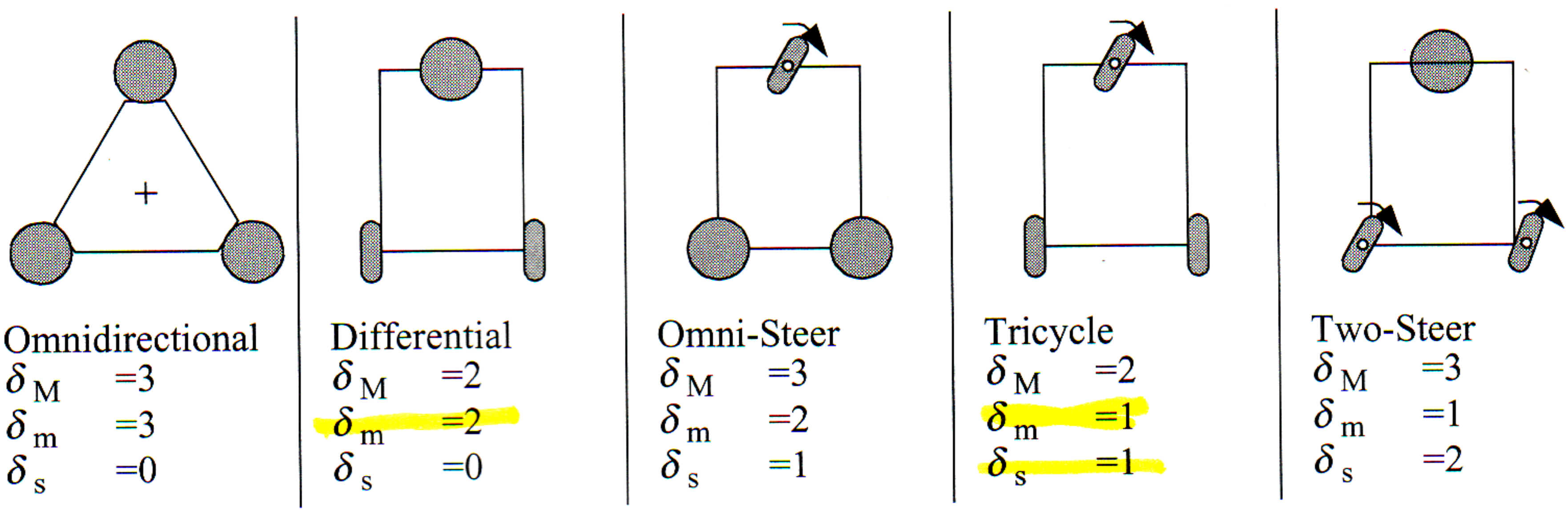


Two mutually independent centered orientable wheels

- Degree of steerability : 2

Degree of Maneuverability

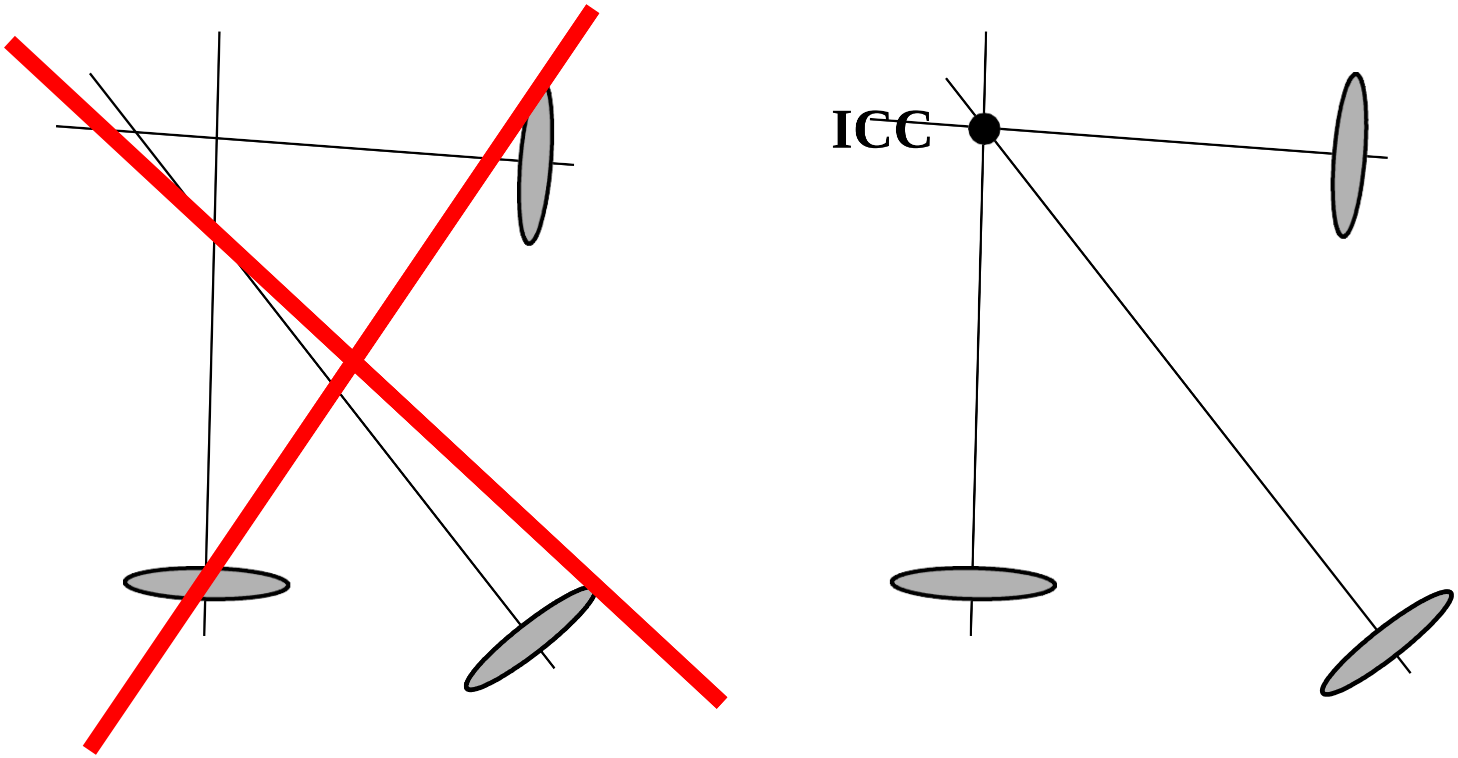
$$\delta_M = \delta_m + \delta_s$$



Non-Holonomic Constraints

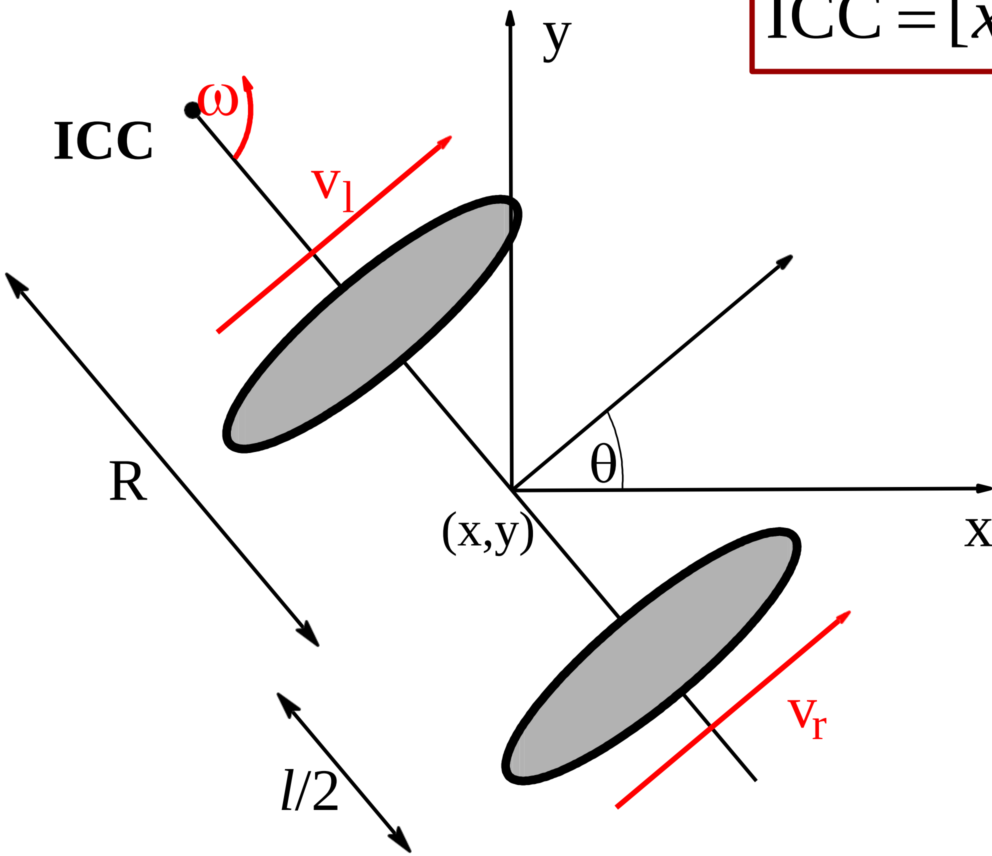
- Non-holonomic constraints limit the possible incremental movements within the configuration space of the robot.
- Robots with differential drive or Ackerman drive synchronous drive move on a circular trajectory and cannot move sideways.
- XR-4000 or Mecanum-wheeled robots can move sideways (they have no non- holonomic constraints).

Instantaneous Centre of Curvature



- For rolling motion to occur, each wheel has to move along its y-axis

Differential Drive



$$ICC = [x - R\sin\theta,\ y + R\cos\theta]$$

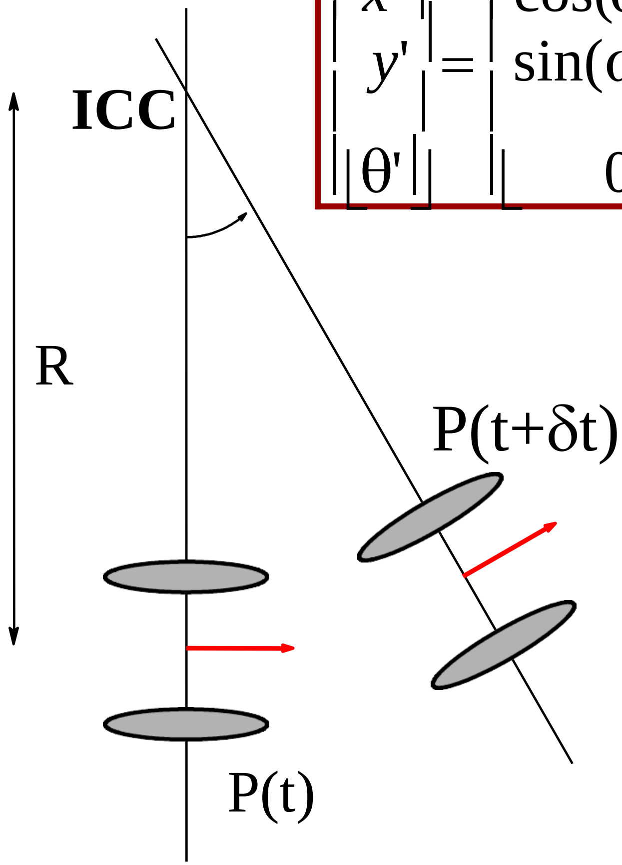
$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

Differential Drive: Forward Kinematics



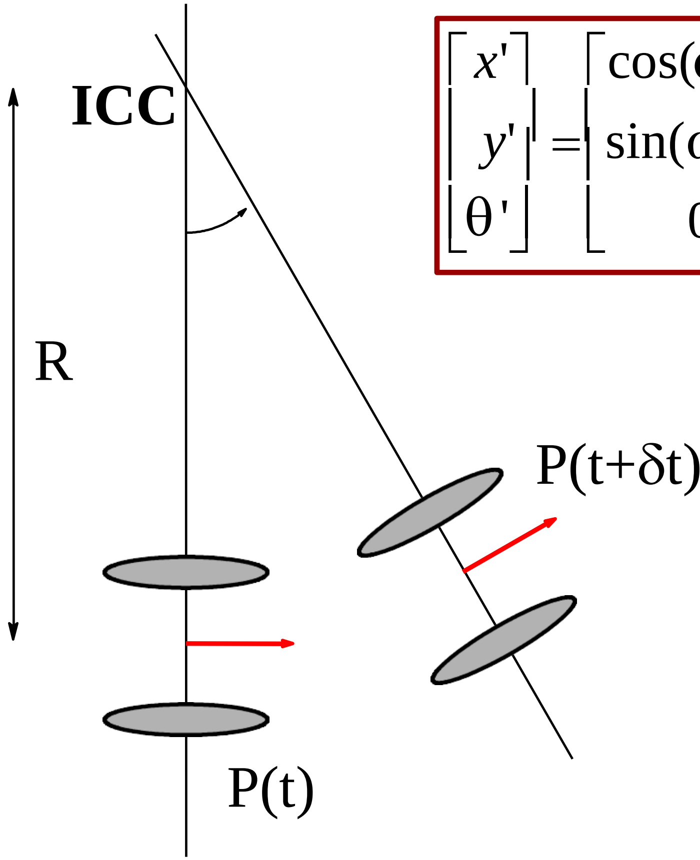
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t v(t') \cos[\theta(t')]dt'$$

$$y(t) = \int_0^t v(t') \sin[\theta(t')]dt'$$

$$\theta(t) = \int_0^t \omega(t')dt'$$

Differential Drive: Forward Kinematics



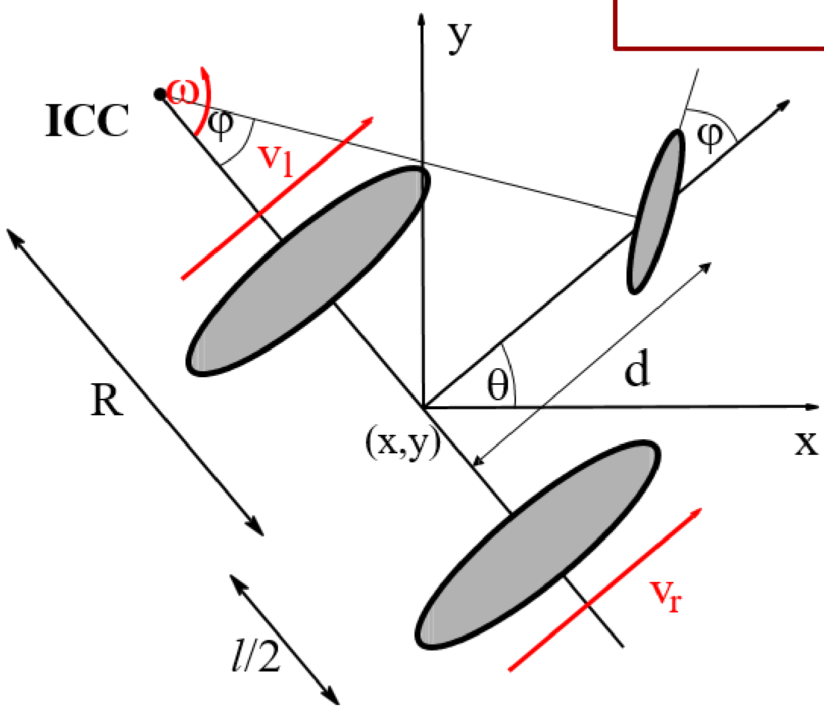
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \frac{1}{2}\int_0^t [v_r(t') + v_l(t')] \cos[\theta(t')]dt'$$

$$y(t) = \frac{1}{2}\int_0^t [v_r(t') + v_l(t')] \sin[\theta(t')]dt'$$

$$\theta(t) = \frac{1}{l}\int_0^t [v_r(t') - v_l(t')]\, dt'$$

Ackerman Drive



$$ICC = [x - R\sin\theta,\ y + R\cos\theta]$$

$$R = \frac{d}{\tan\varphi}$$

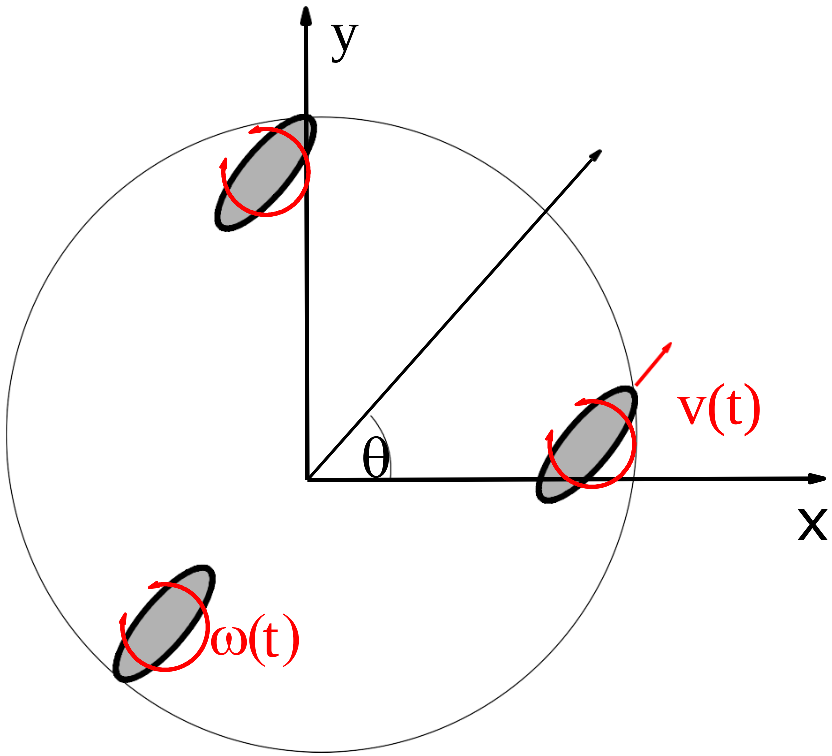
$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

Synchonous Drive

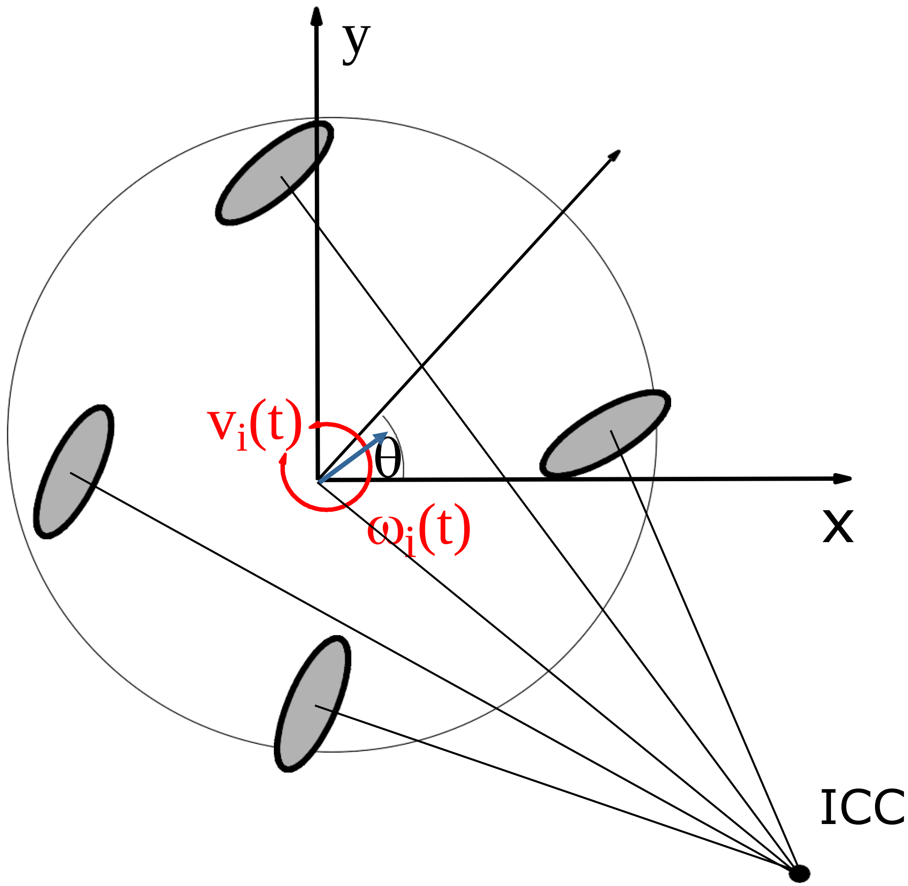


$$x(t) = \int_0^t v(t') \cos[\theta(t')] dt'$$

$$y(t) = \int_0^t v(t') \sin[\theta(t')] dt'$$

$$\theta(t) = \int_0^t \omega(t') dt'$$

XR4000 Drive



$$x(t) = \int_0^t v(t') \cos[\theta(t')]dt'$$

$$y(t) = \int_0^t v(t') \sin[\theta(t')]dt'$$

$$\theta(t) = \int_0^t \omega(t')\,dt'$$

Mecanum Drive

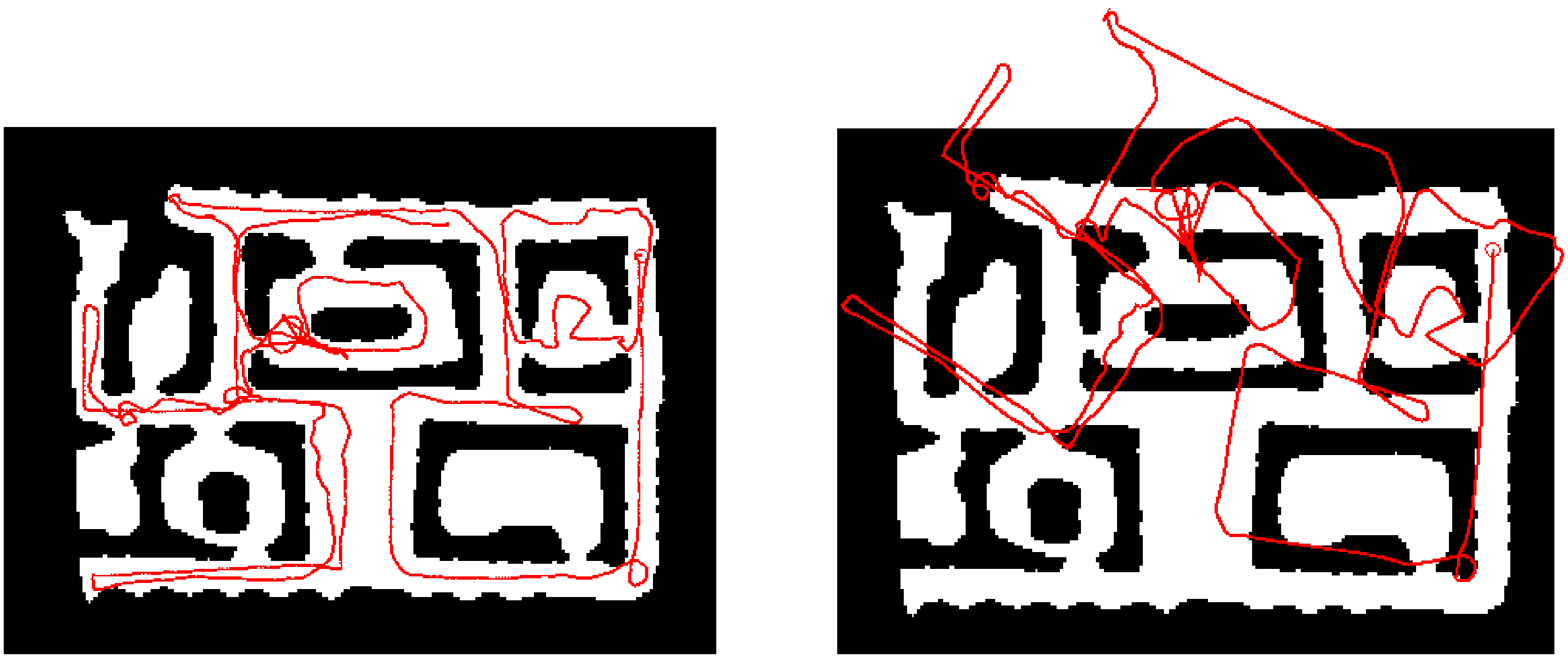
$$v_y = (v_0 + v_1 + v_2 + v_3)/4$$

$$v_x = (v_0 - v_1 + v_2 - v_3)/4$$

$$v_\theta = (v_0 + v_1 - v_2 - v_3)/4$$

$$v_{error} = (v_0 - v_1 - v_2 + v_3)/4$$

Mobile Robot Odometry



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- Wheeled Mobile Robot Kinematics
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Wheeled Mobile Robot Motion Control

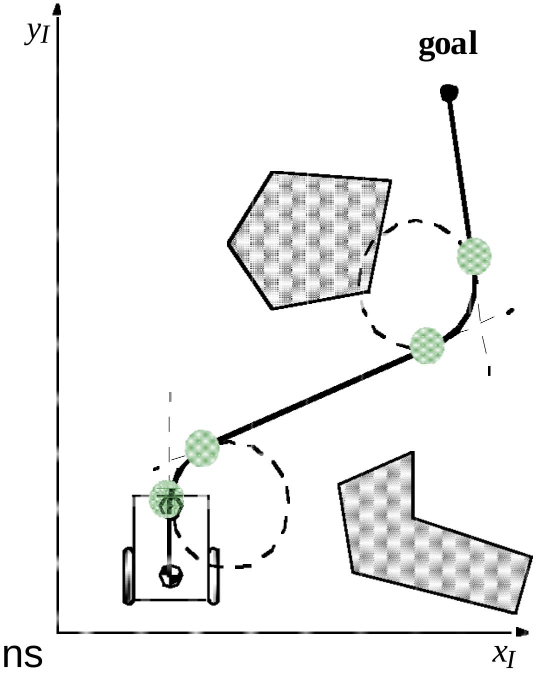
- Overview
- Open Loop Control
- Feedback Control
- Kinematic Position Control
- Coordinates Transformation
- Control Law
- Resulting Path

Overview

- The objective of a kinematic controller is to follow a trajectory described by its position and/or velocity profiles as function of time.
- Motion control is not straight forward because mobile robots are typically non-holonomic and MIMO systems.
- Most controllers (including the one presented here) are not considering the dynamics of the system

Open Loop Control

- Trajectory (path) divided in motion segments of clearly
- Defined shape:
  - straight lines and segments of a circle
  - Dubins car, and Reeds-Shepp car
- Control problem:
  - pre-compute a smooth trajectory
    based on line, circle (and clothoid) segments
- Disadvantages:
  - It is not at all an easy task to pre-compute a feasible trajectory
  - limitations and constraints of the robots velocities and accelerations
  - does not adapt or correct the trajectory if dynamical changes
    of the environment occur.
  - The resulting trajectories are usually not smooth (in acceleration, jerk, etc.)



Feedback Control

- Find a control matrix K, if exists

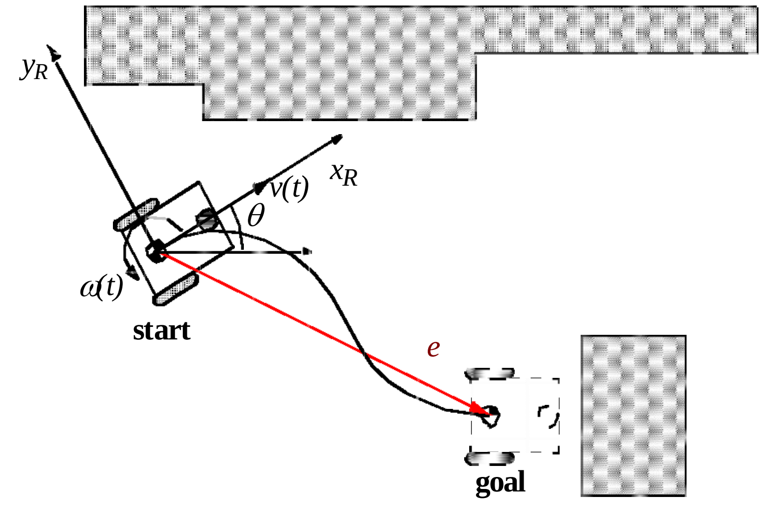
$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} \quad \text{with } k_{ij}=k(t,e)$$

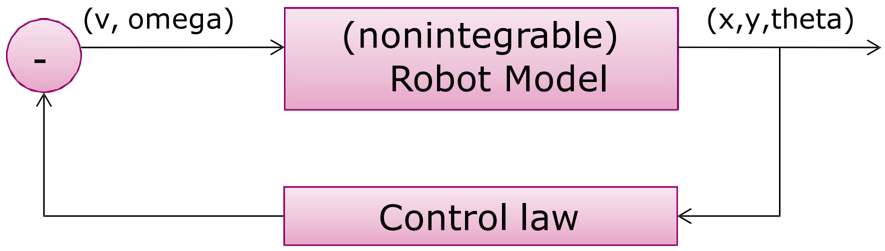
- such that the control of $v(t)$ and $\omega(t)$

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \cdot {}^{R}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- drives the error e to zero $\lim_{t\to\infty} e(t) = 0$

- MIMO state feedback control

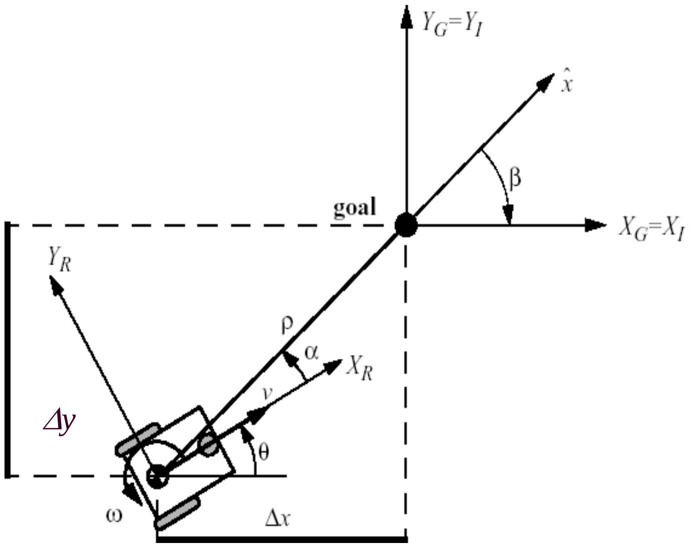




Kinematic Position Control

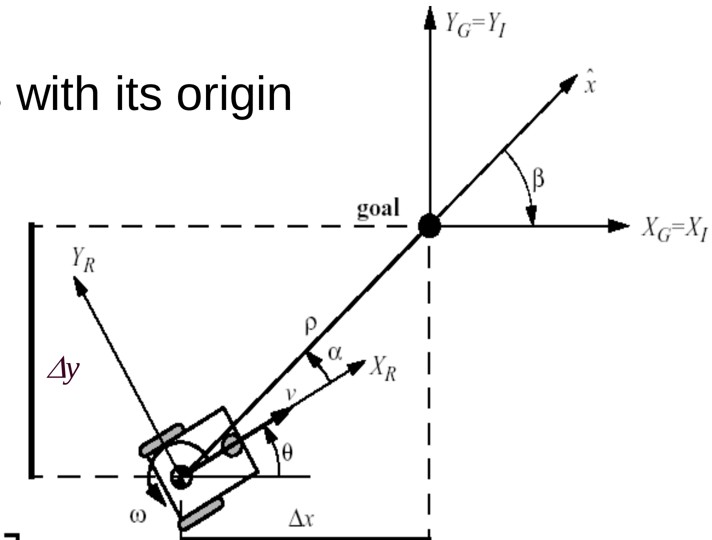
- The kinematics of a differential drive mobile robot described in the inertial frame $\{x_I, y_I, \theta\}$ is given by,

$$
{}^{I}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}
$$



- where $\dot{x}$ and $\dot{y}$ are the linear velocities in the direction of the x_I and y_I of the inertial frame.
- Let alpha denote the angle between the x_R axis of the robots reference frame and the vector connecting the center of the axle of the wheels with the final position.

Coordinates Transformation



- Coordinates transformation into polar coordinates with its origin at goal position:

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$

- System description, in the new polar coordinates

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \frac{\sin\alpha}{\rho} & -1 \\ -\frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad \textit{for } I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ -\frac{\sin\alpha}{\rho} & 1 \\ \frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad \textit{for } I_2 = (-\pi, -\pi/2] \cup (\pi/2, \pi]$$

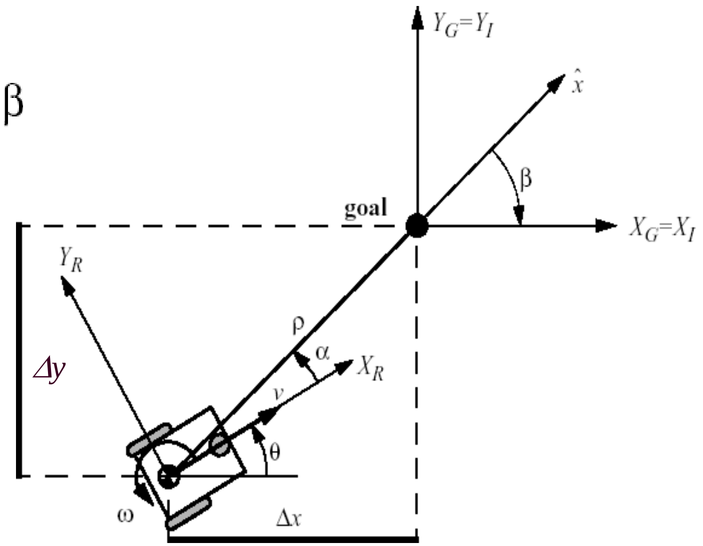
Control Law

- It can be shown, that with $v = k_\rho \rho \qquad \omega = k_\alpha \alpha + k_\beta \beta$

  the feedback controlled system

  $$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

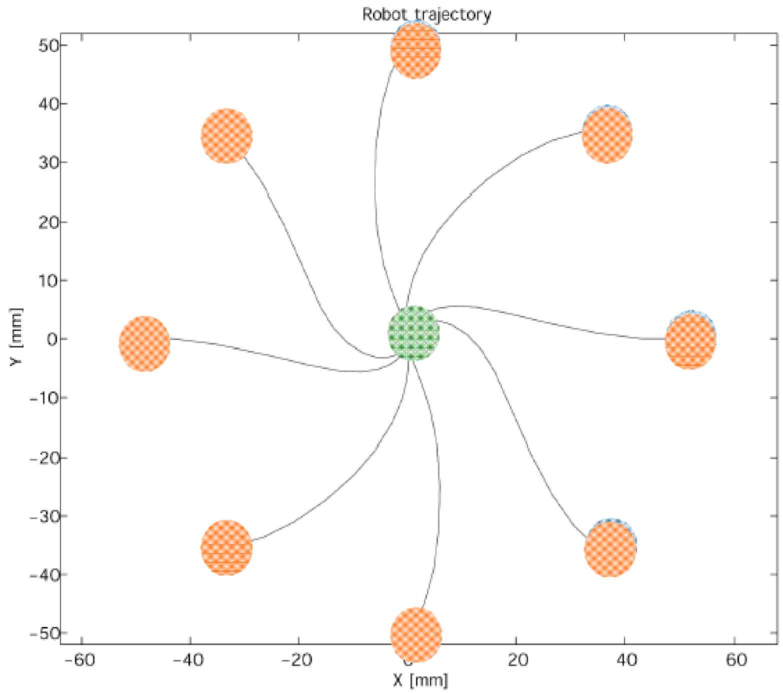
  will drive the robot to $(\rho, \alpha, \beta) = (0, 0, 0)$

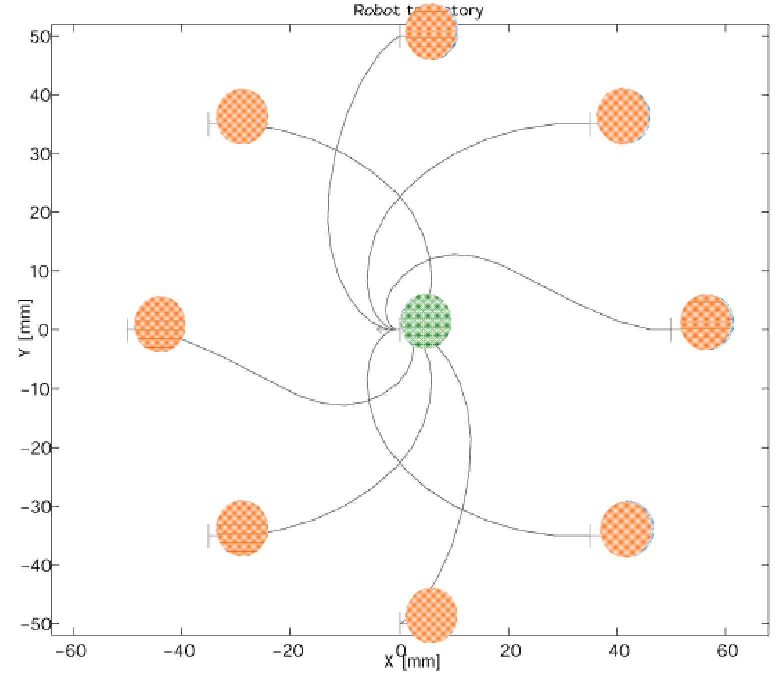


- The control signal v has always constant sign,
  - the direction of movement is kept positive or negative during movement
  - parking maneuver is performed always in the most natural way and without ever inverting its motion.

Resulting Path

- The goal is in the center and the initial position on the circle.





$$k = (k_\rho, k_\alpha, k_\beta) = (3, 8, -1.5)$$

Homework I

(1) Discuss the advantages and limits of different wheel drives

(2) Simulate the kinematics of a robot with a differential drive

(3) Simulate the motion control of a robot with a differential drive and show the resulting paths w.r.t. different control laws

Due March 20 8AM