INTELLIGENT ROBOTS

CHAPTER 11: EXTENDED KALMAN FILTER LOCALIZATION

Outline

- Localization Problem Statement
- Landmark-based Localization
- EKF Localization
- Global EKF Localization

Localization Problem Statement

"Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities." [Cox '91]

- **Given**
  - Map of the environment.
  - Sequence of sensor measurements.
- **Wanted**
  - Estimate of the robot's position.
- **Problem classes**
  - Position tracking
  - Global localization
  - Kidnapped robot problem (recovery)

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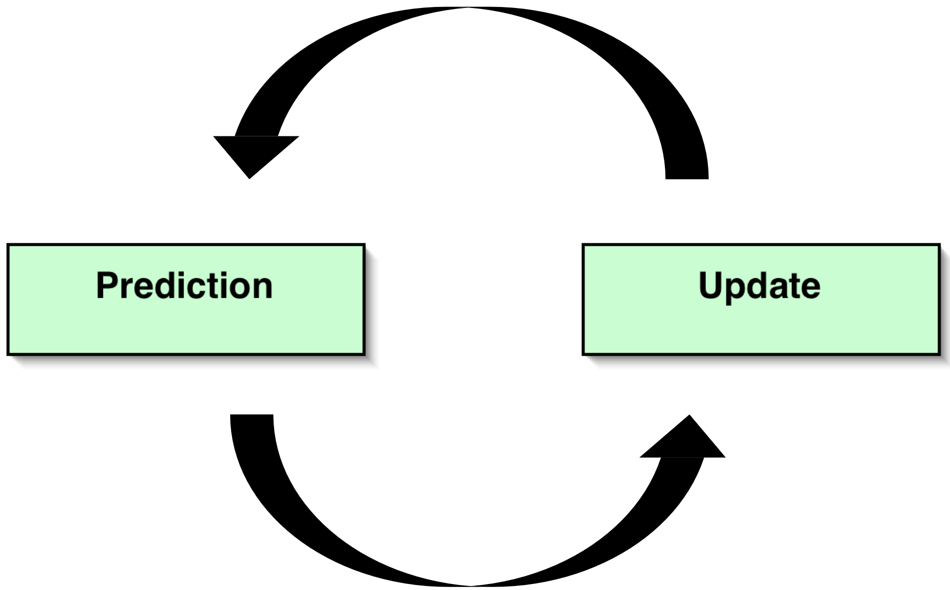
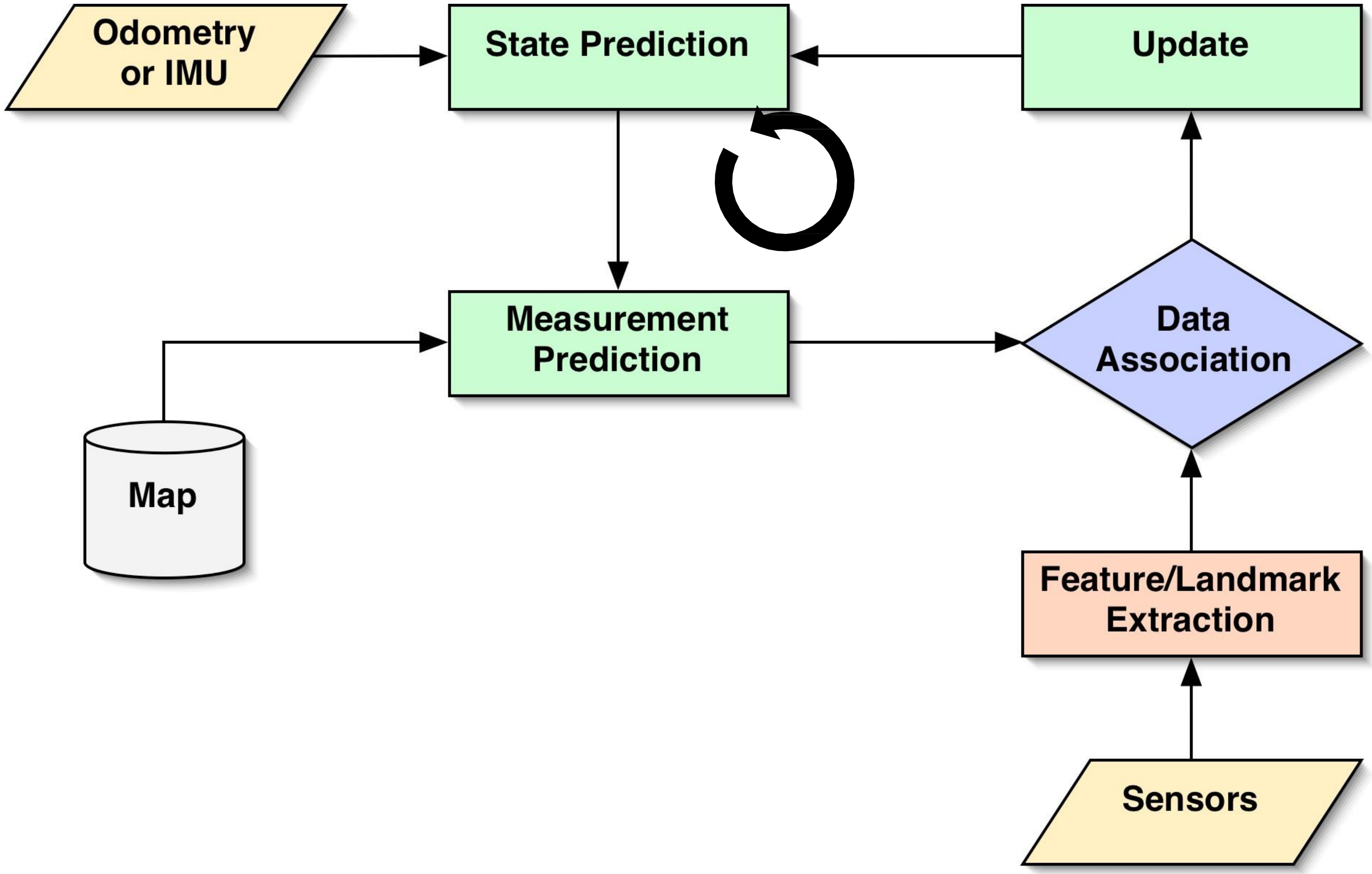
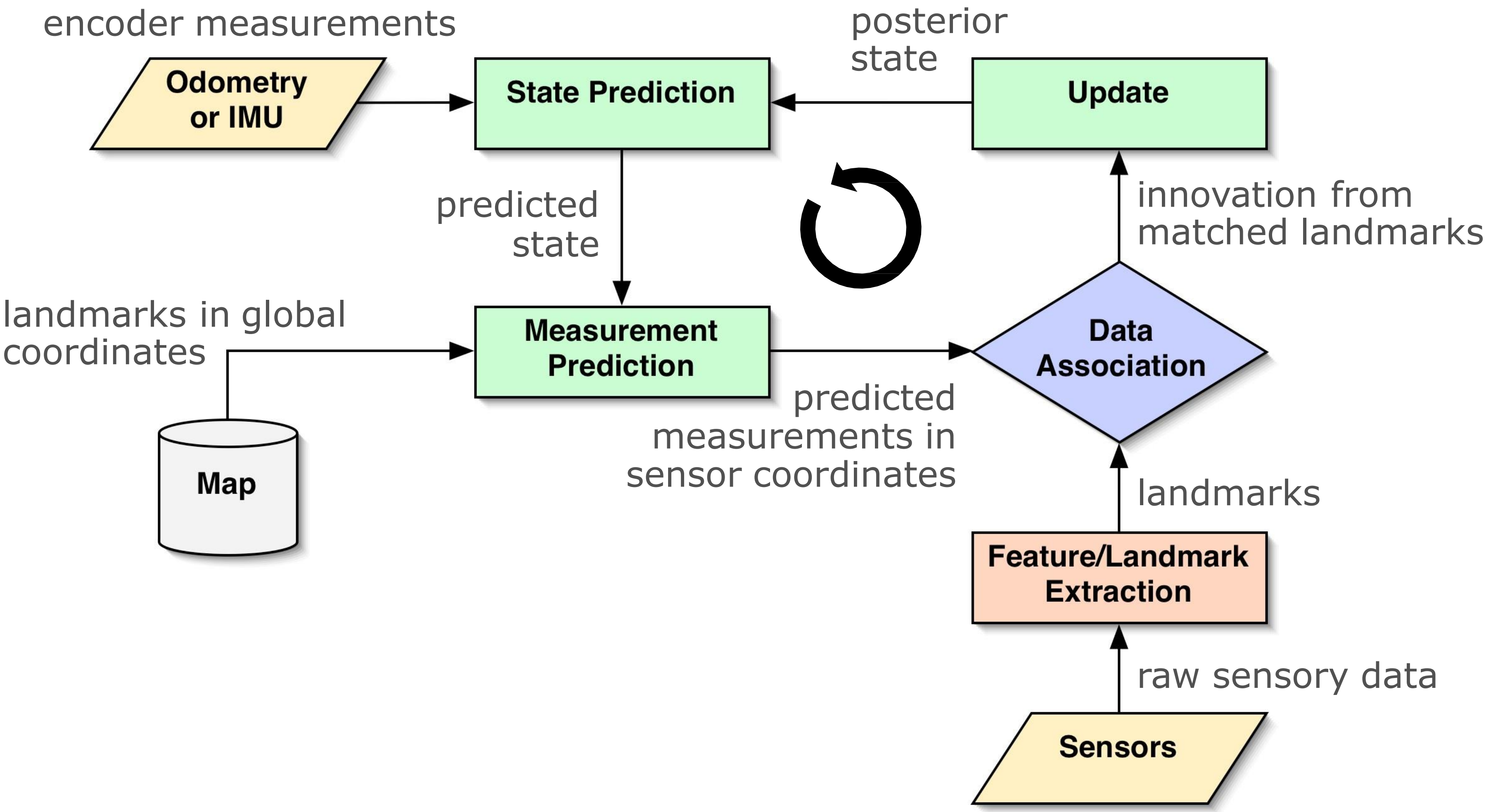
Landmark-based Localization



Landmark-based Localization



Landmark-based Localization

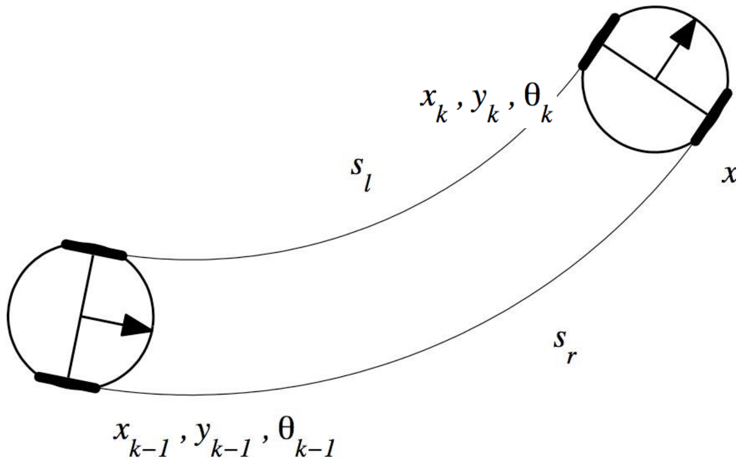


Landmark-based Localization

State Prediction (Odometry)

$$\hat{\mathbf{x}}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k)$$

$$\hat{C}_k = F_x\, C_k\, F_x^T + F_u\, U_k\, F_u^T$$



Control $\mathbf{u}_k$: wheel displacements s_l, s_r

$$\mathbf{u}_k = (s_l \;\; s_r)^T \qquad U_k = \begin{bmatrix} \sigma_l^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}$$

Error model: linear growth

$$\begin{aligned} \sigma_l &= k_l\, |s_l| \\ \sigma_r &= k_r\, |s_r| \end{aligned}$$

Nonlinear process model f:

$$\mathbf{x}_k = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{b}{2}\frac{s_l+s_r}{s_r-s_l}\left(-\sin\theta_{k-1} + \sin(\theta_{k-1} + \frac{s_r-s_l}{b})\right) \\ \frac{b}{2}\frac{s_l+s_r}{s_r-s_l}\left(\cos\theta_{k-1} - \cos(\theta_{k-1} + \frac{s_r-s_l}{b})\right) \\ \frac{s_r-s_l}{b} \end{bmatrix}$$

Landmark-based Localization

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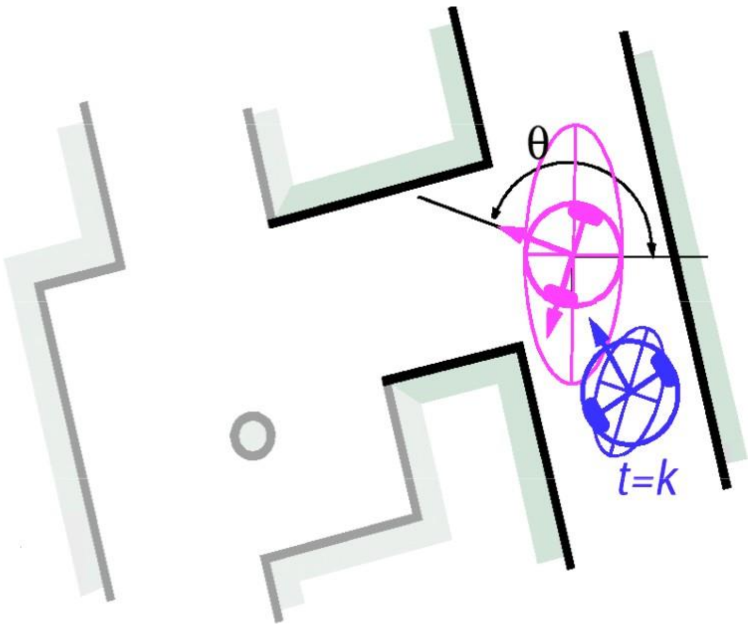

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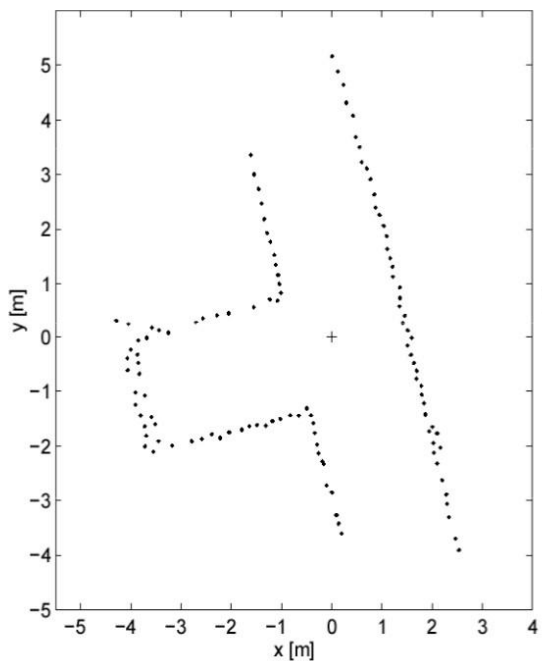
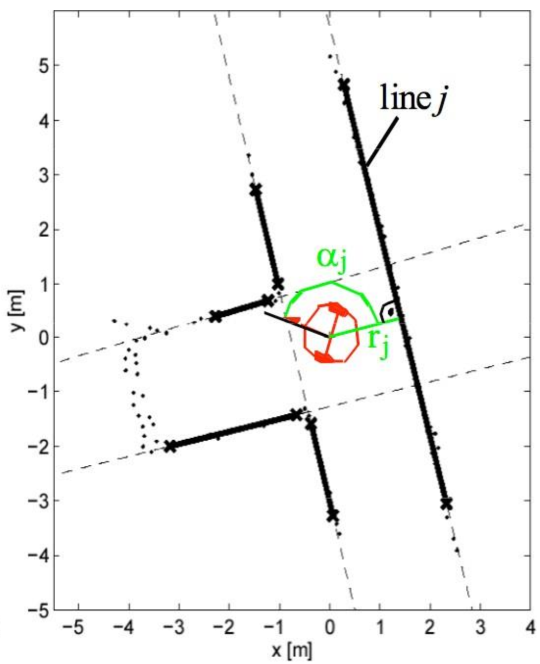
Landmark Extraction

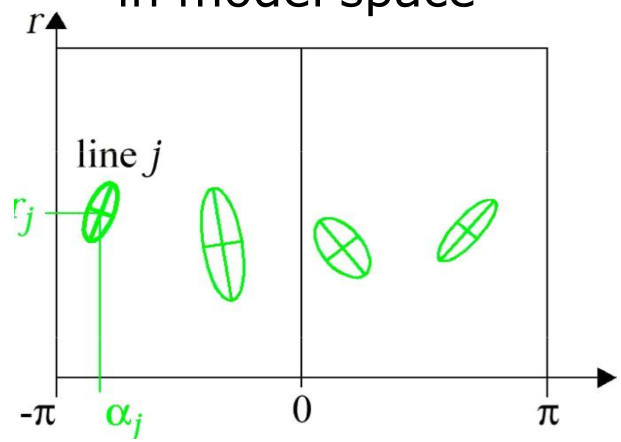
Raw laser range data



Extracted lines



Extracted lines in model space



$$\mathbf{z}_k = \begin{bmatrix} \alpha \\ r \end{bmatrix}$$

$$R_k = \begin{bmatrix} \sigma_\alpha^2 & \sigma_{\alpha r} \\ \sigma_{r\alpha} & \sigma_r^2 \end{bmatrix}$$

Hessian line model

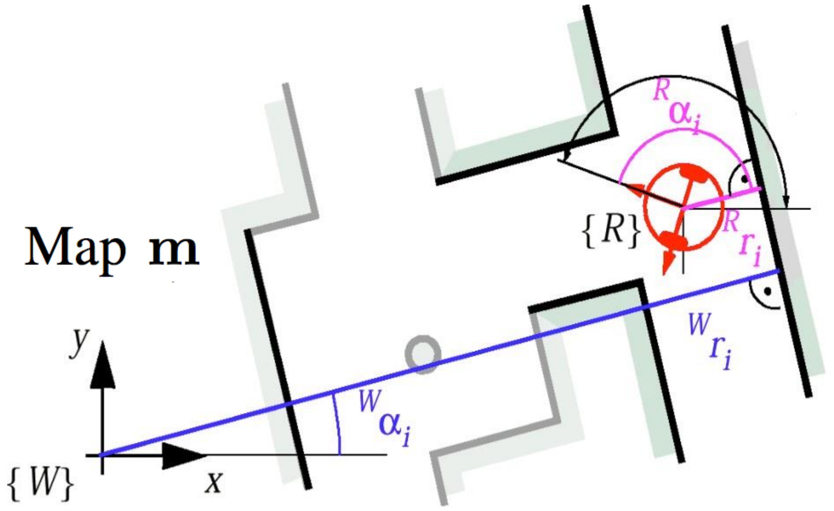
$$x\ cos(\alpha) + y\ sin(\alpha) - r = 0$$

Measurement Prediction

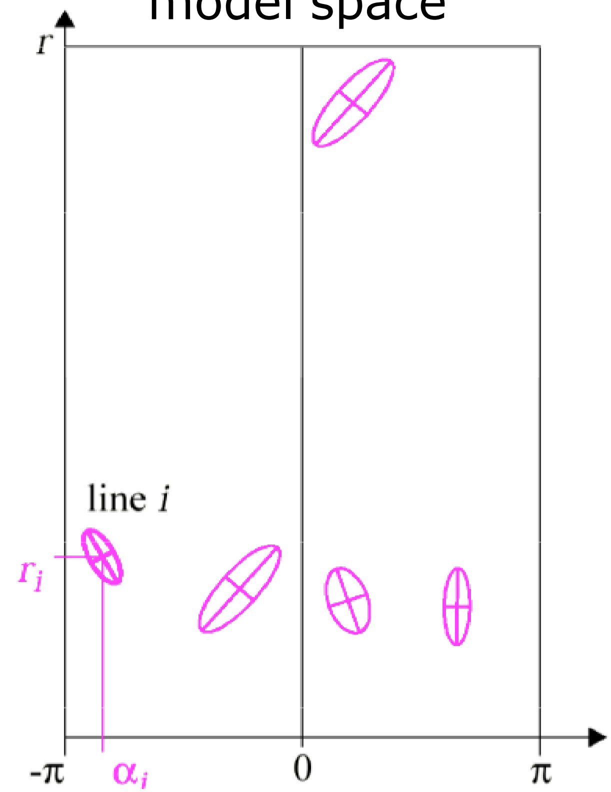
- ...is a coordinate frame transform world-to-sensor
- Given the predicted state (robot pose),

  predicts the location $\hat{\mathbf{z}}_k$ and location uncertainty $H\,\hat{C}_k\,H^T$ of expected observations in sensor coordinates

$$\hat{\mathbf{z}}_k = h(\hat{\mathbf{x}}_k, \mathbf{m})$$



model space



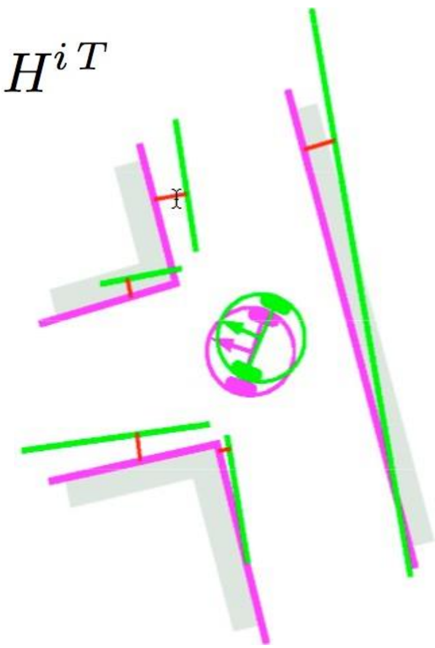
Data Association (Matching)

- Associates predicted measurements $\hat{\mathbf{z}}_k^i$ with observations $\mathbf{z}_k^j$

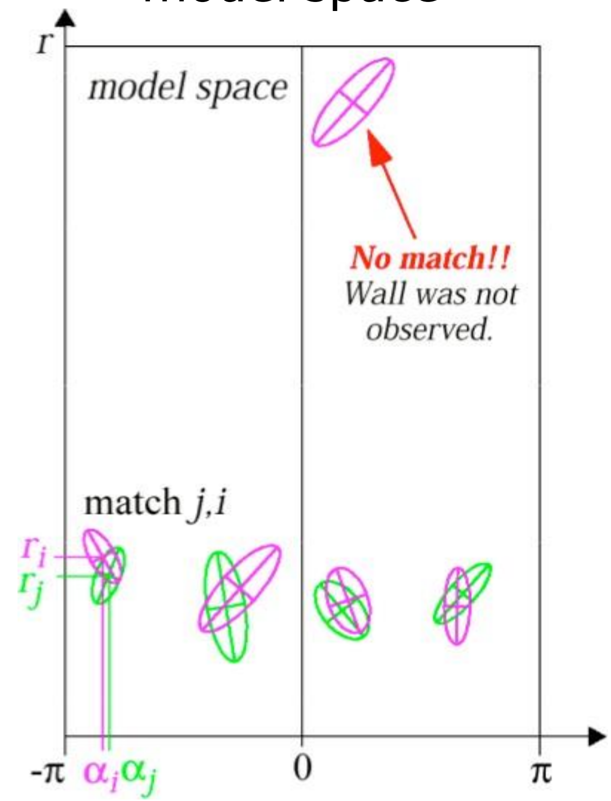
$$
\begin{aligned}
\nu_k^{ij} &= \mathbf{z}_k^j - \hat{\mathbf{z}}_k^i \\
S_k^{ij} &= R_k^j + H^i \, \hat{C}_k \, H^{i\,T}
\end{aligned}
$$

- Innovation ν_k^{ij} and innovation covariance S_k^{ij}

- Matching on significance level alpha



model space



Green: observation
Magenta: measurement prediction

Update

- Kalman gain

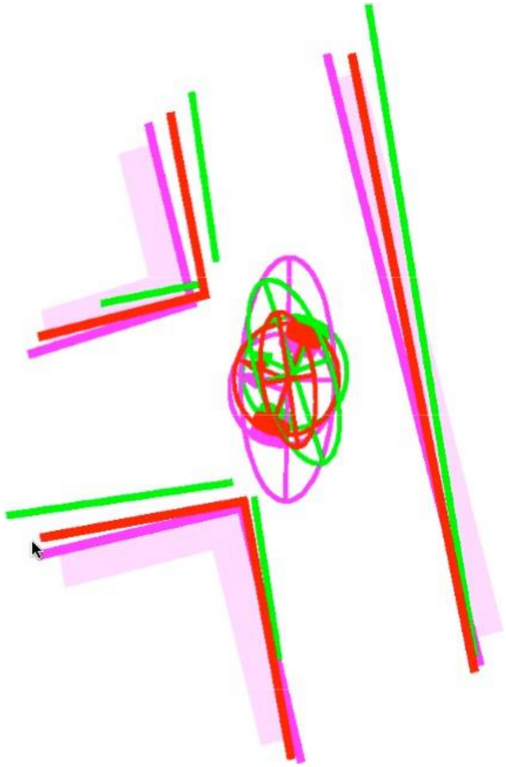
$$K_k = \hat{C}_k \, H^T \, S_k^{-1}$$

- State update (robot pose)

$$\mathbf{x}_k = \hat{\mathbf{x}}_k + K_k \, \nu_k$$

- State covariance update

$$C_k = (I - K_k \, H) \, \hat{C}_k$$



Red: posterior estimate

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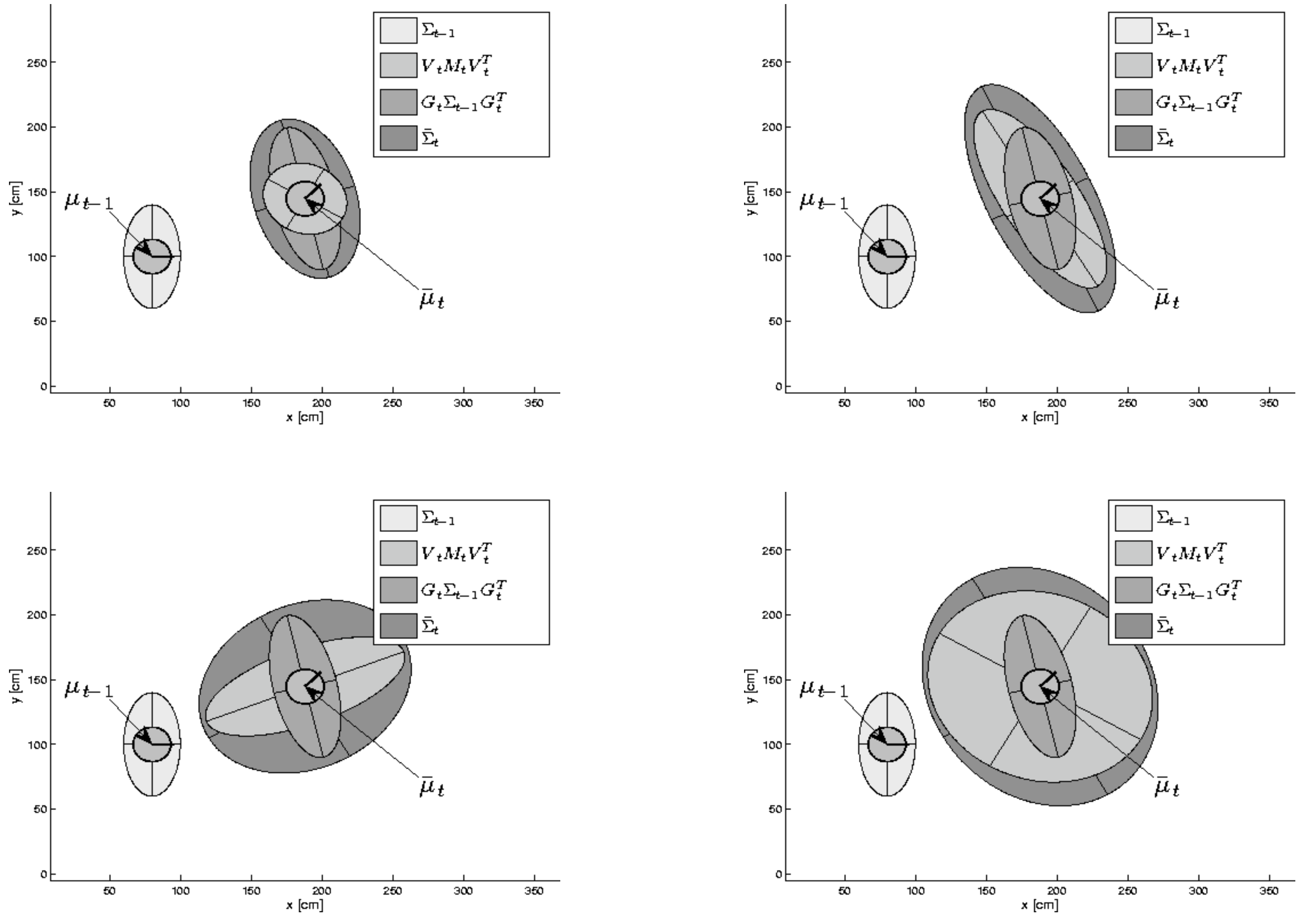
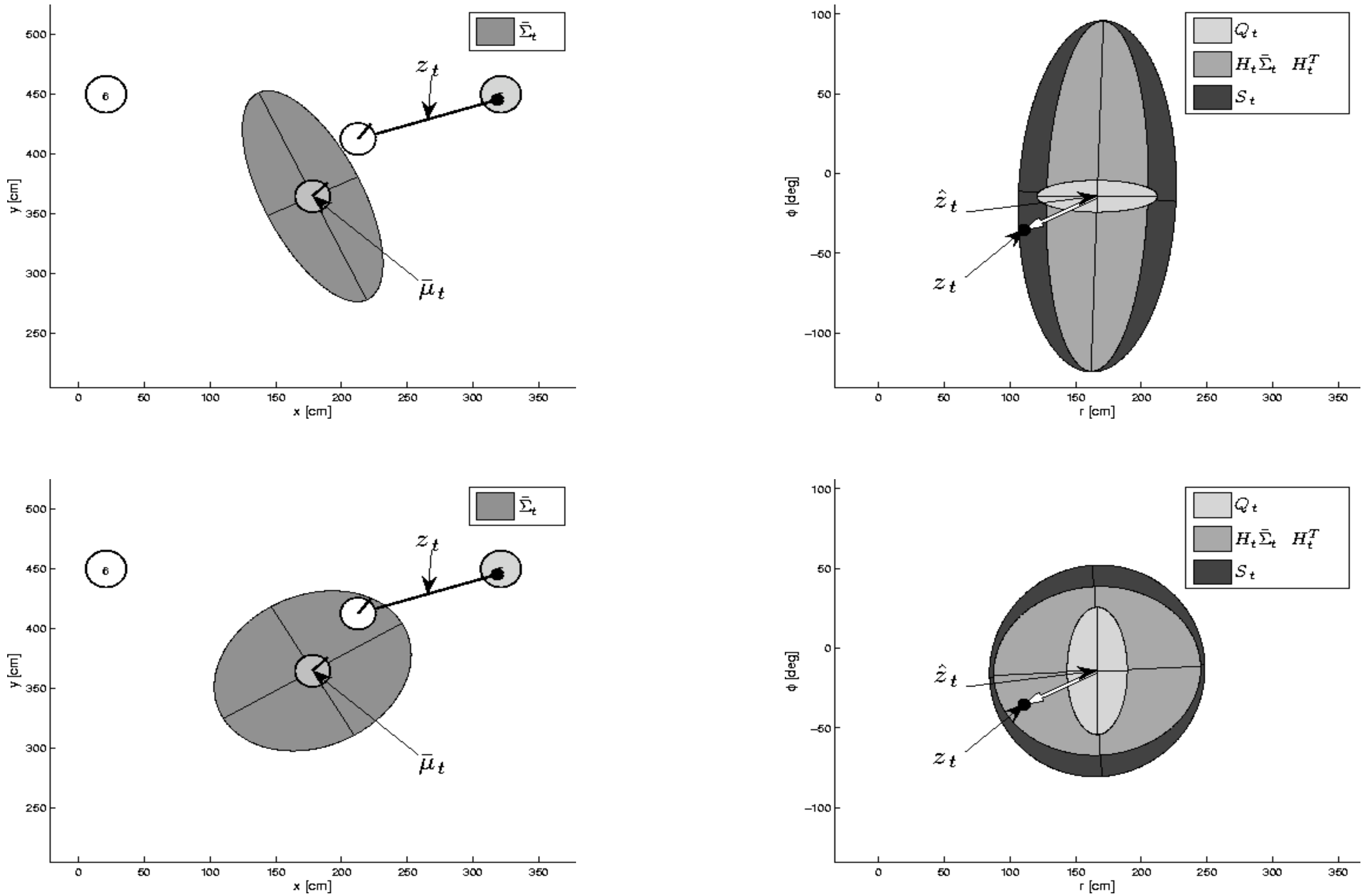
EKF Localization with Point Features

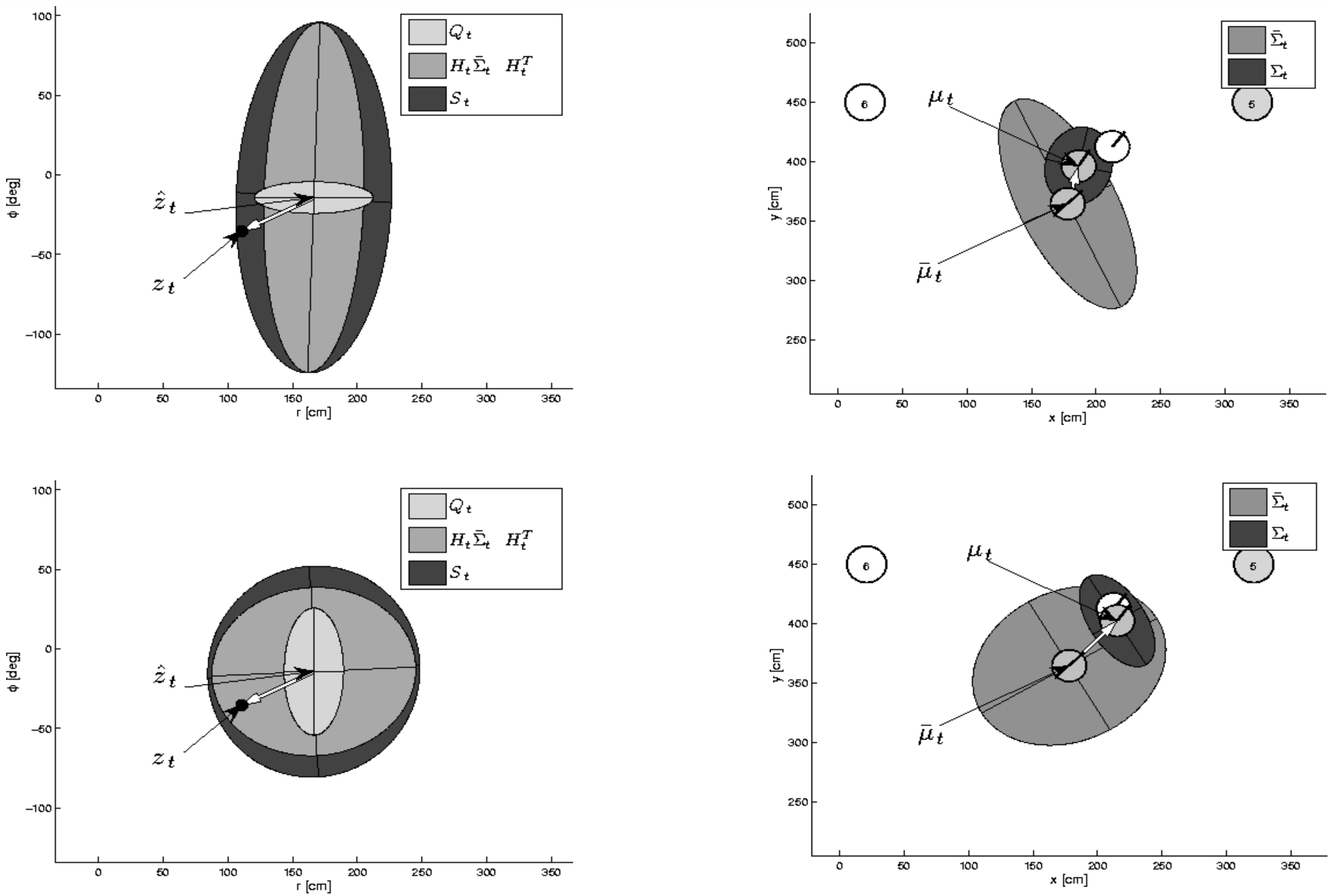
EKF Prediction Step



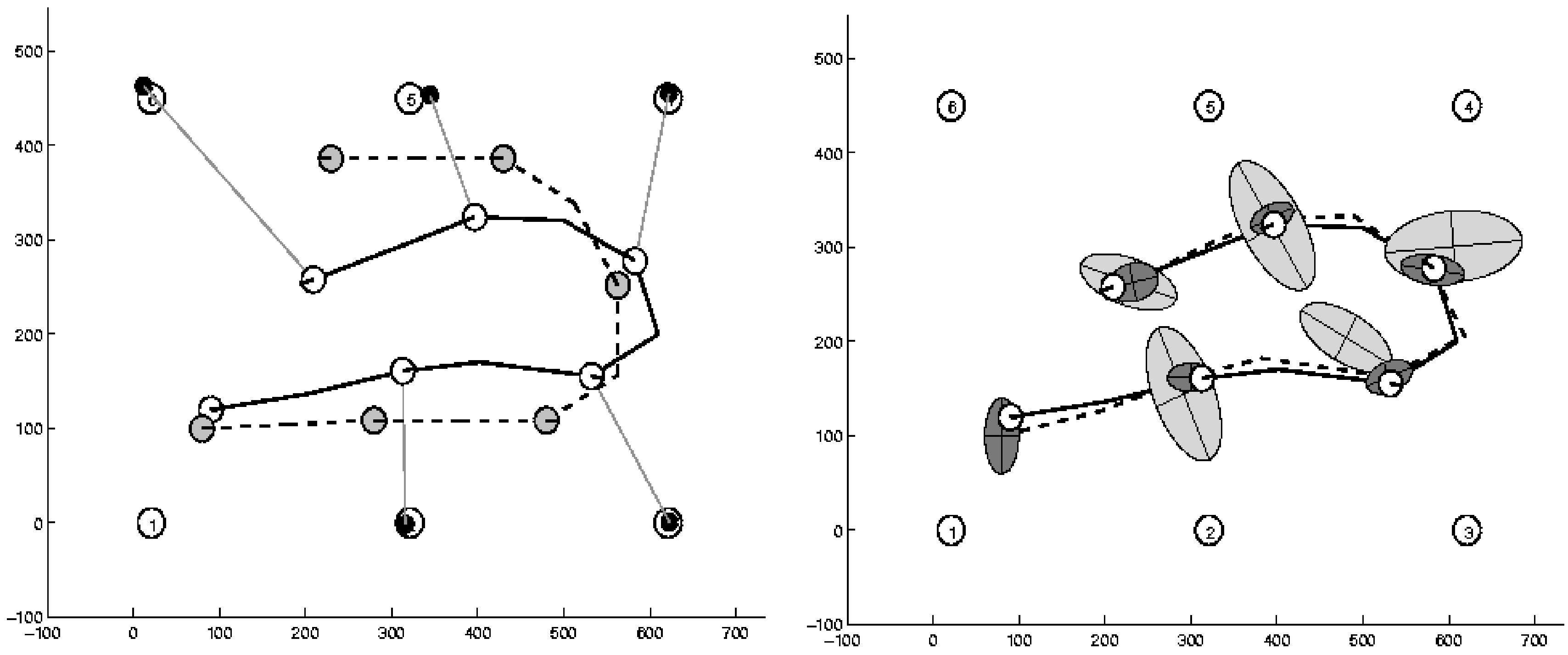
EKF Observation Prediction Step



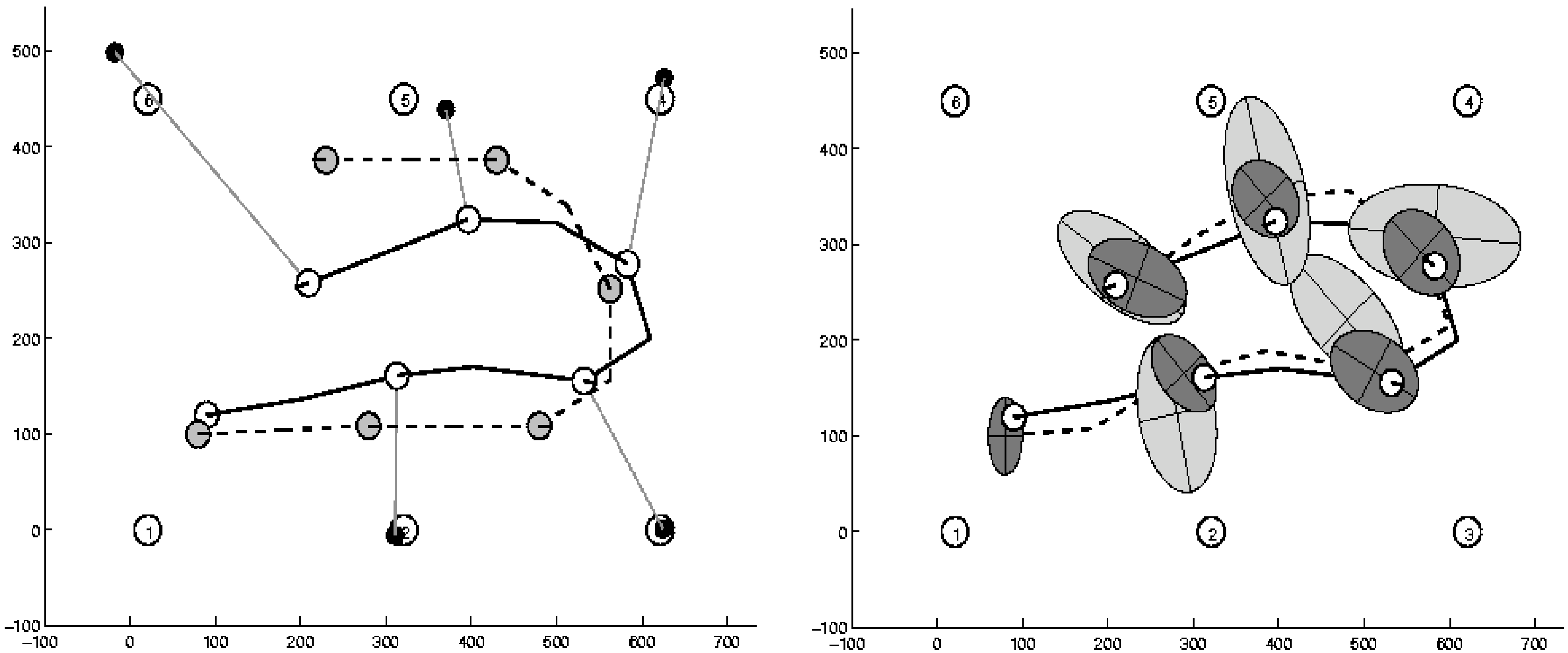
EKF Correction Step



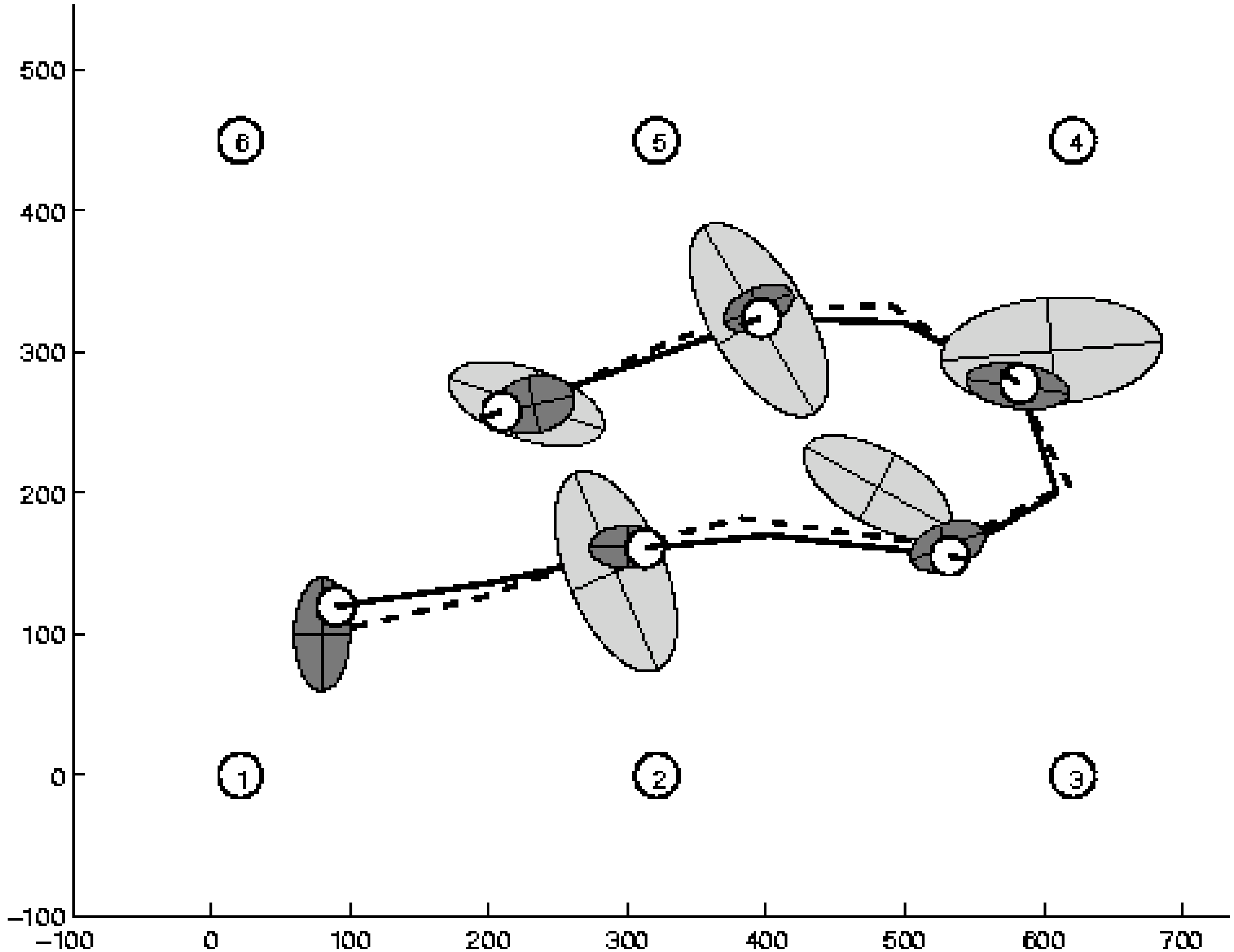
EKF Estimation Sequence



EKF Estimation Sequence



Comparison to Ground Truth



Extended Kalman Filter Summary

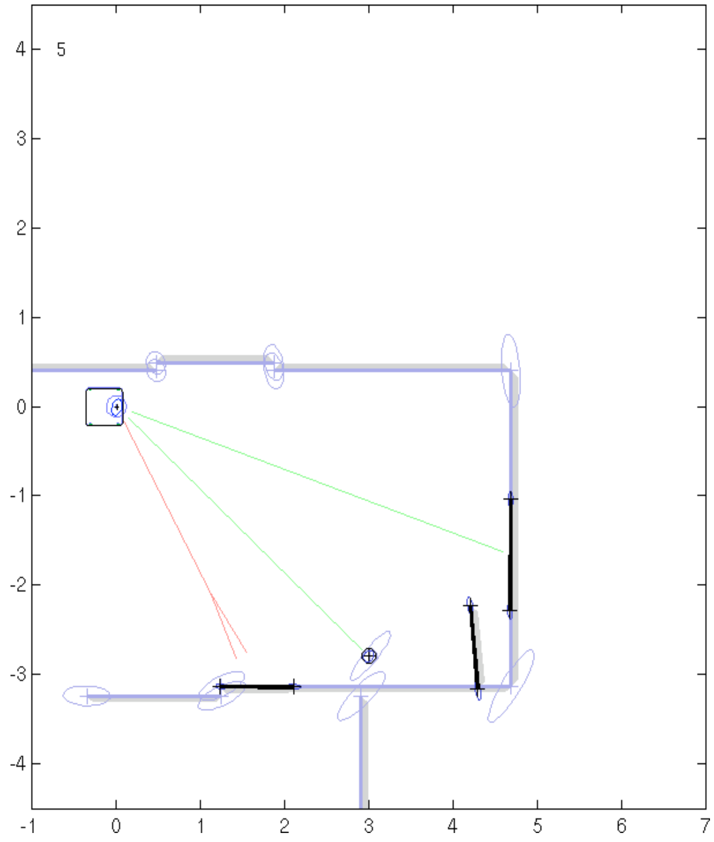
- Highly efficient: Polynomial in measurement dimensionality k and state dimensionality n:

$$O(k^{2.376} + n^2)$$

- Not optimal!
- Can diverge if nonlinearities are large!
- Works surprisingly well even when all assumptions are violated!

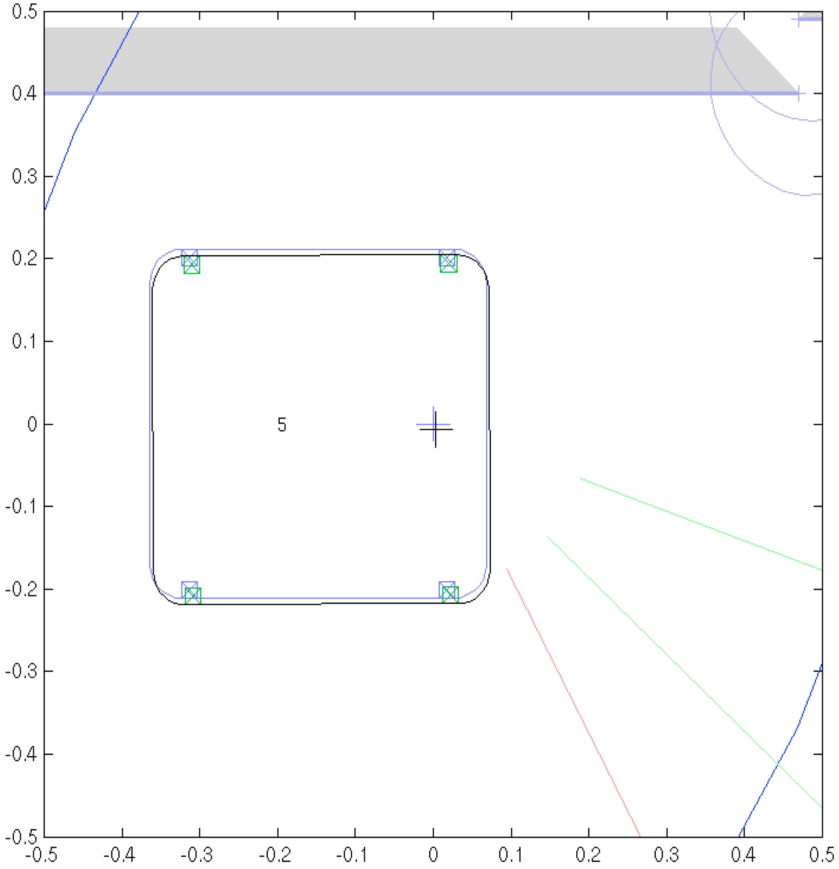
EKF Localization Example

- Line and point landmarks



EKF Localization Example

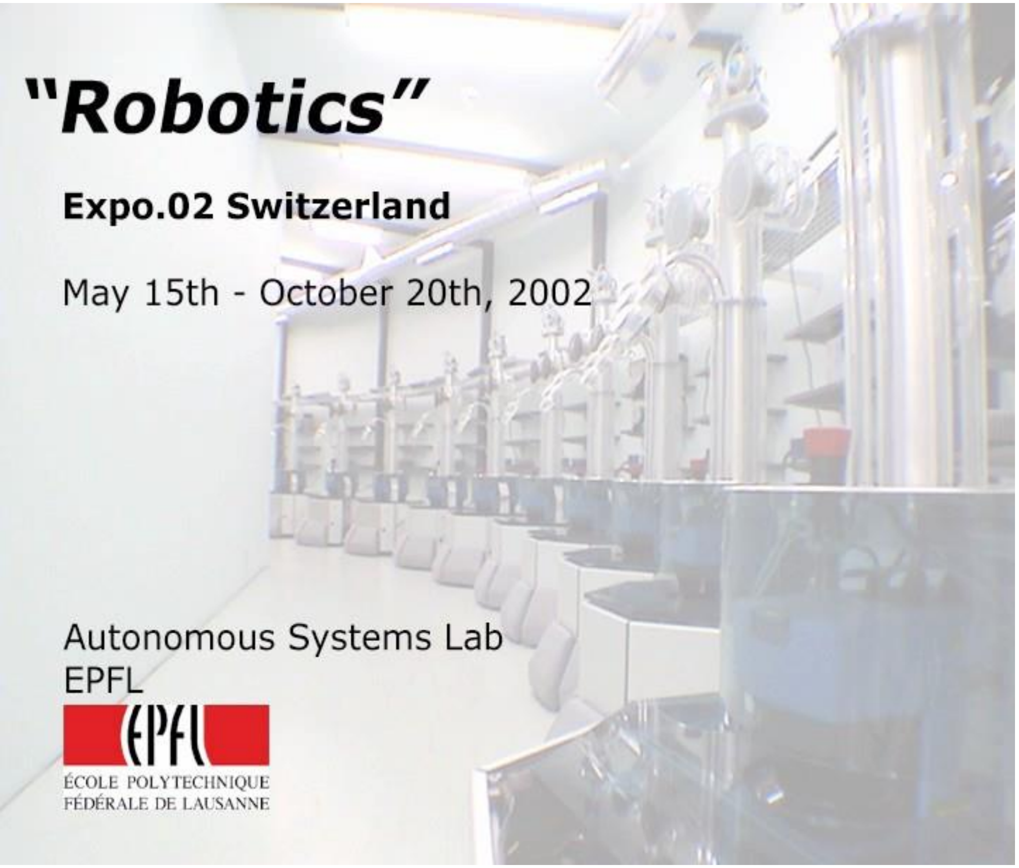
- Line and point landmarks



EKF Localization Example

- Expo.02: Swiss National Exhibition 2002
  - Pavilion "Robotics"
  - 11 fully autonomous robots
  - tour guides, entertainer, photographer
  - 12 hours per day
  - 7 days per week
  - 5 months
  - 3,316 km travel distance
  - almost 700,000 visitors
  - 400 visitors per hour
- Localization method: Line-Based, EKF
- Still the biggest project in mobile robotics of its kind!

EKF Localization Example



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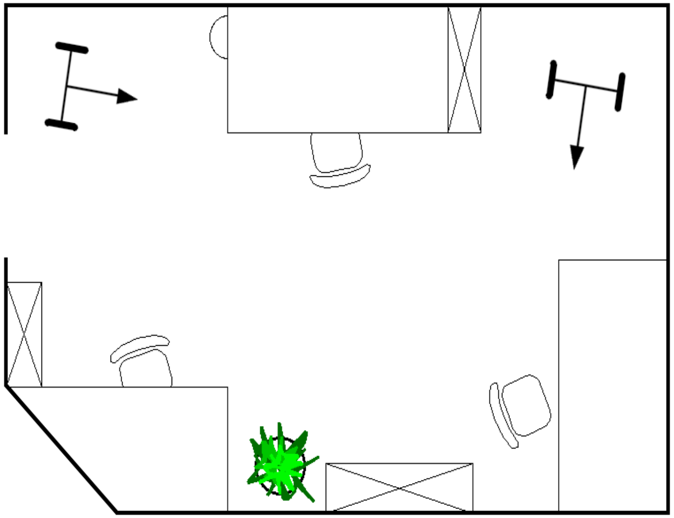
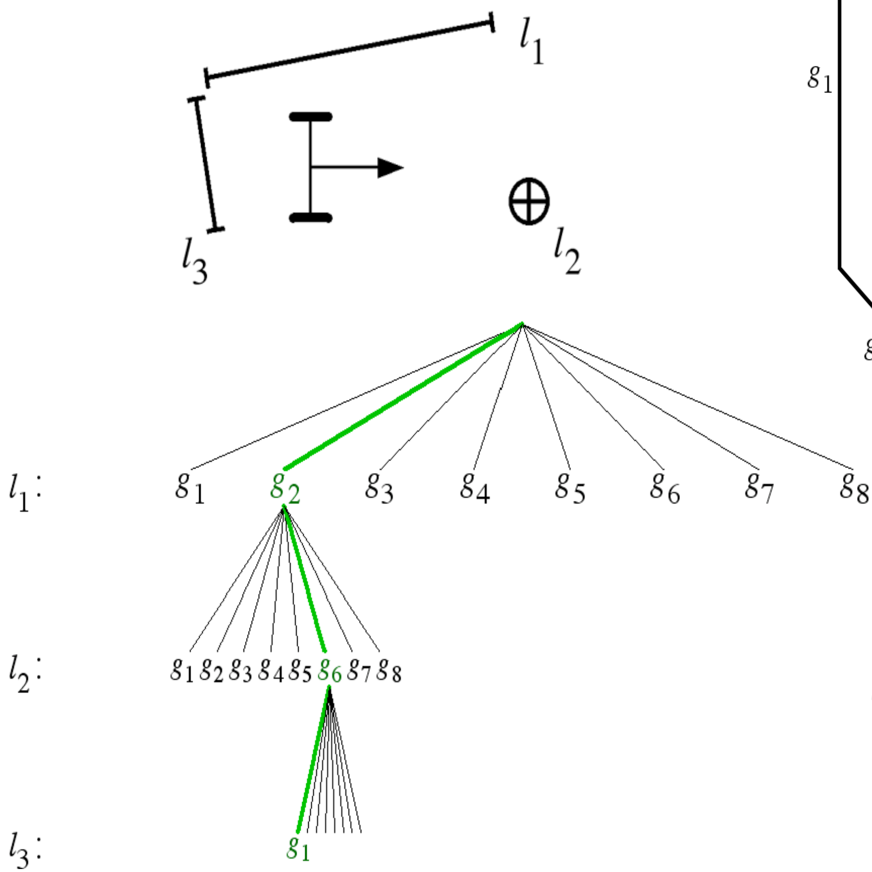
Global EKF Localization

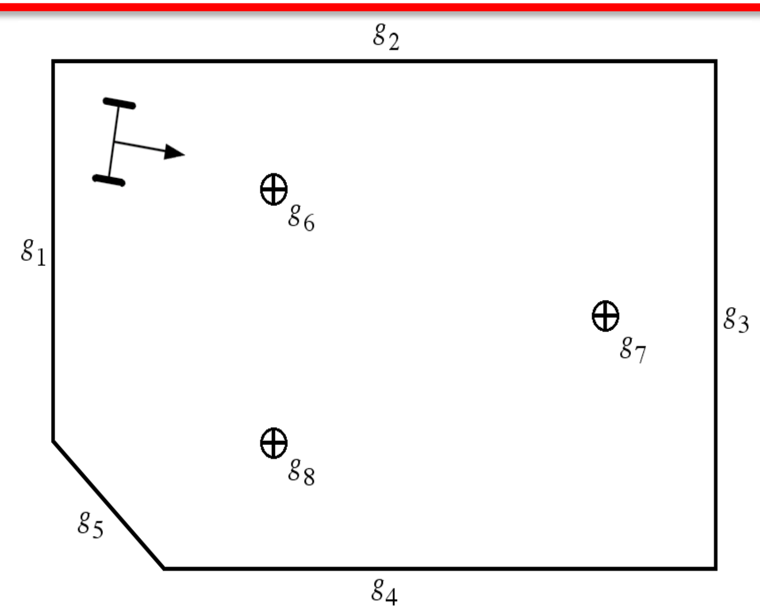
Interpretation tree



Global EKF Localization

Interpretation tree

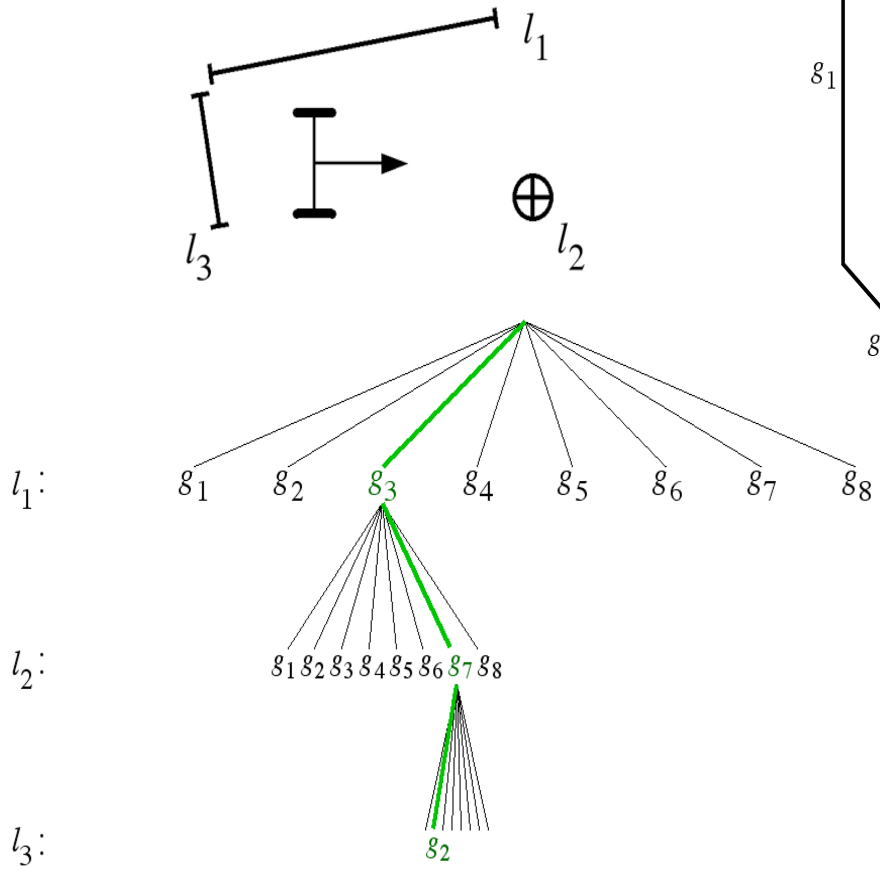


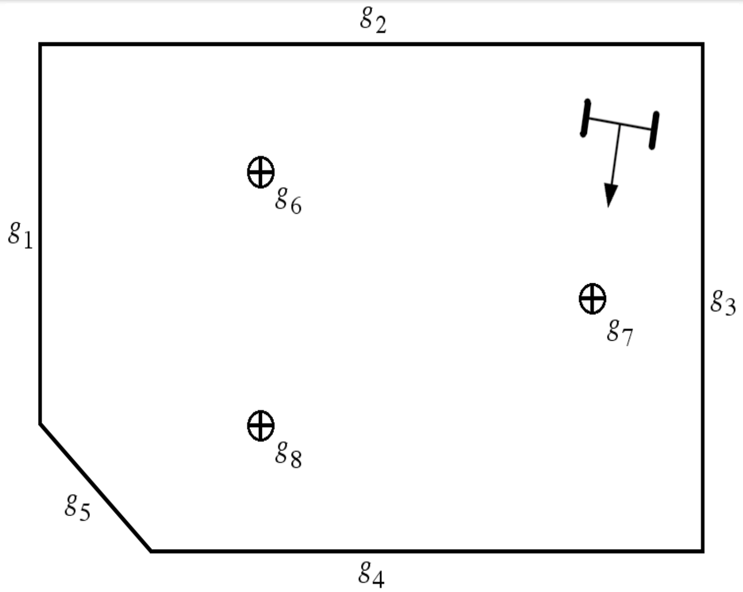


$$S_{h_1} = \{\{l_1, g_2\}, \{l_2, g_6\}, \{l_3, g_1\}\}$$

Global EKF Localization

Interpretation tree

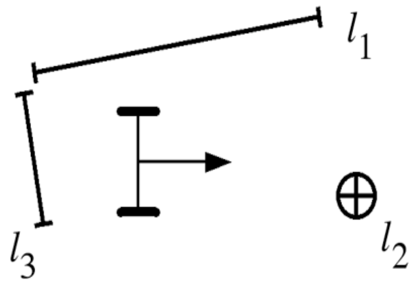


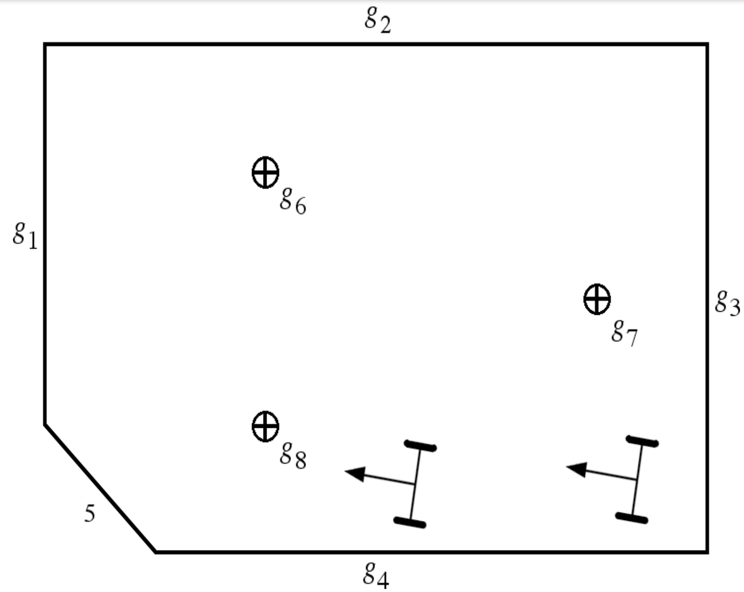


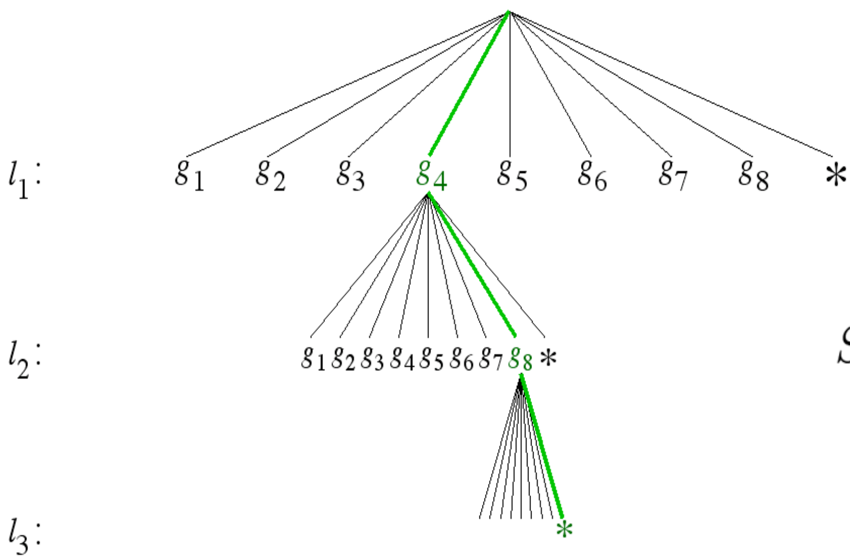
$$S_{h_2} = \{\{l_1, g_3\}, \{l_2, g_7\}, \{l_3, g_2\}\}$$

Global EKF Localization

Env. Dyn.







$$S_h = \{\{l_1, g_4\}, \{l_2, g_8\}, \{l_3, *\}\}$$

Global EKF Localization

Geometric Constraints

Location independent constraints

Unary constraint:
intrinsic property of feature
e.g. type, color, size

Binary constraint:
relative measure between features
e.g. relative position, angle

Location dependent constraints

Rigidity constraint:
"is the feature where I expect it given my position?"

Visibility constraint:
"is the feature visible from my position?"

Extension constraint:
"do the features overlap at my position?"

All decisions on a significance level α

Global EKF Localization

Interpretation Tree

[Grimson 1987], [Drumheller 1987], [Castellanos 1996], [Lim 2000]

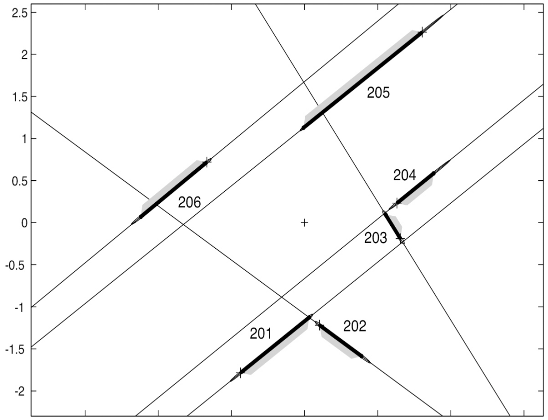
Algorithm

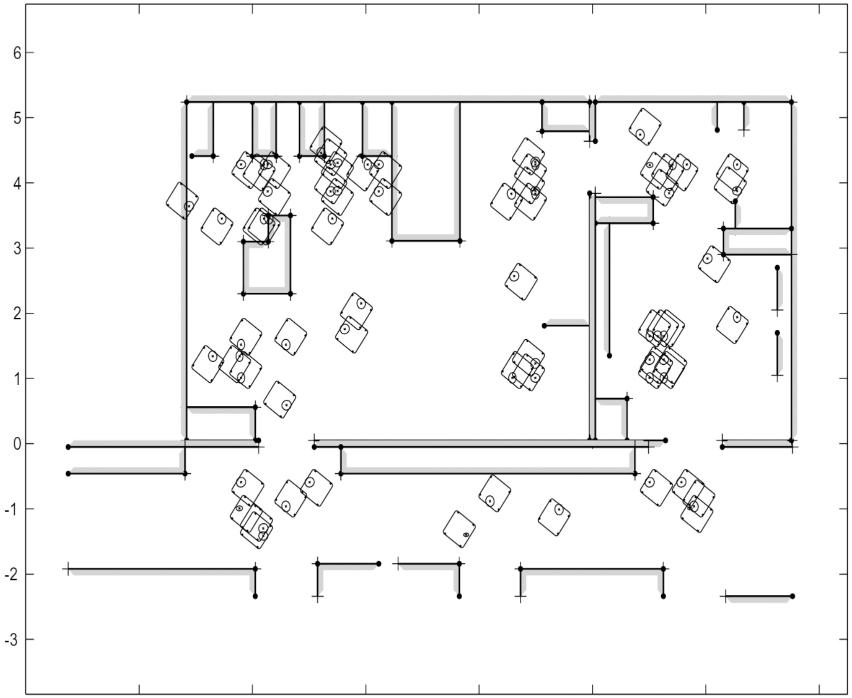
- backtracking
- depth-first
- recursive
- uses geometric constraints
- exponential complexity
- absence of feature: no info.
- presence of feature: info. perhaps

```
function generate_hypotheses( h, L, G )

  H ← {}
  if L = {} then
    H ← H ∪ {h}
  else
    l ← select_observation( L )
    for g ∈ G do
      p ← {l, g}
      if satisfy_unary_constraints( p ) then
        if location_available( h ) then
          accept ← satisfy_location_dependent_cnstr( L_h, p )
          if accept then
A           h' ← h
            S_h' ← S_h ∪ {p}
            L_h' ← estimate_robot_location( S_h' )
          end
        else
          accept ← true
          for p_p ∈ S_h while accept
            accept ← satisfy_binary_constraints( p_p , p )
          end
          if accept then
            h' ← h
            S_h' ← S_h ∪ {p}
B           L_h' ← estimate_robot_location( S_h' )
            if location_available( h' ) then
              for p_p ∈ S_h while accept
                accept ← satisfy_location_dependent_cnstr(L_h' , p)
              end
            end
          end
        end
        if accept then
          generate_hypotheses( h', L \ {l}, G )
        end
      end
    end
    generate_hypotheses( h, L \ {l}, G )
  end

return H
end
```

Global EKF Localization

Pygmalion

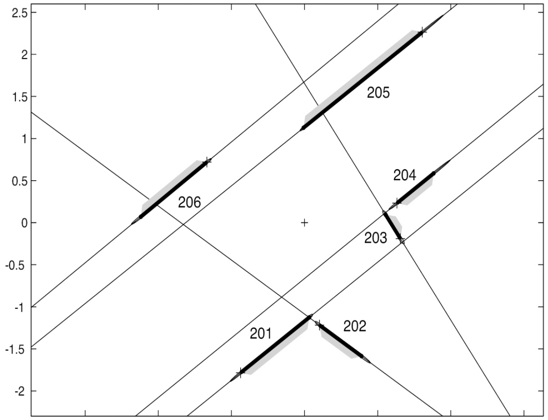


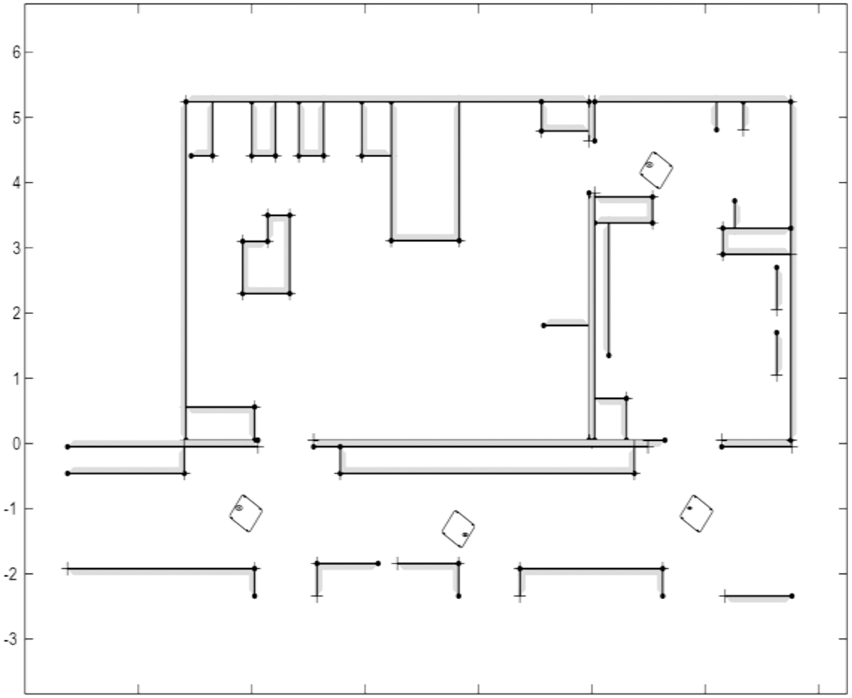


a = 0.95 , p = 2

Global EKF Localization

Pygmalio
n





a = 0.95 , $p = 3$

Global EKF Localization

Pygmalio
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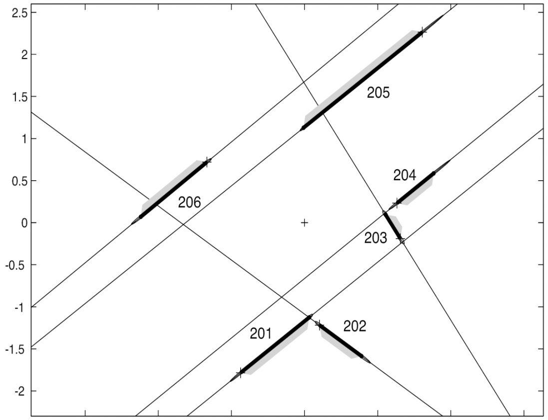
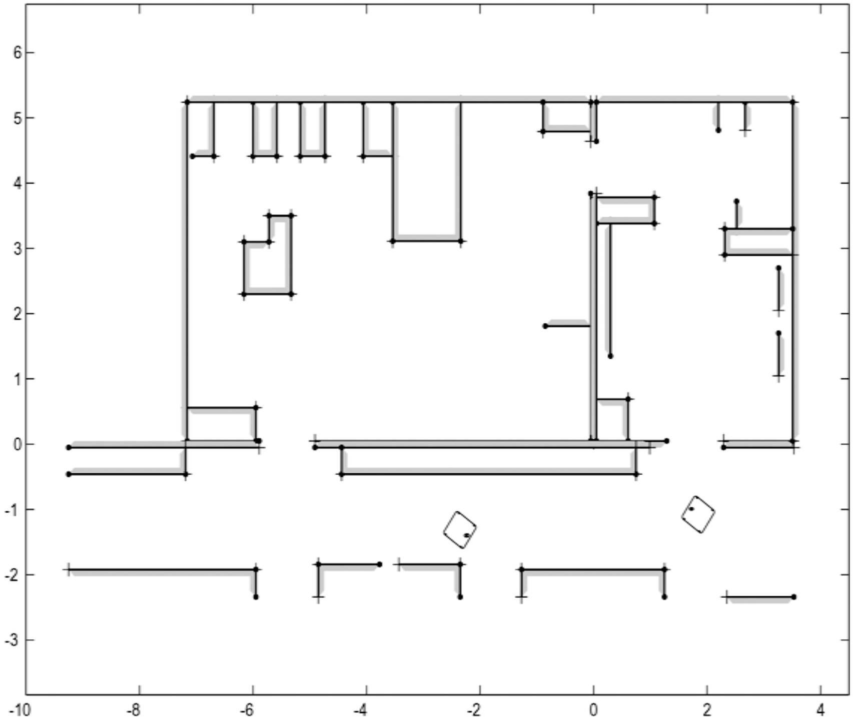
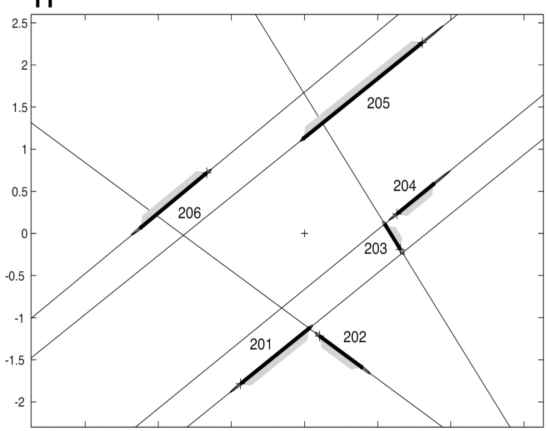


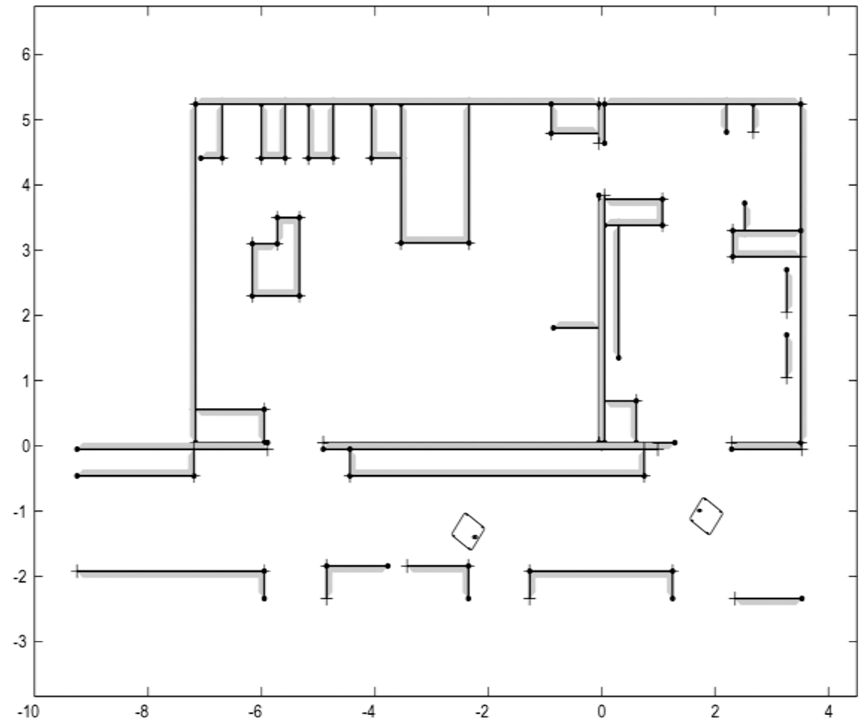


a = 0.95 , $p = 4$

Global EKF Localization

Pygmalio
n





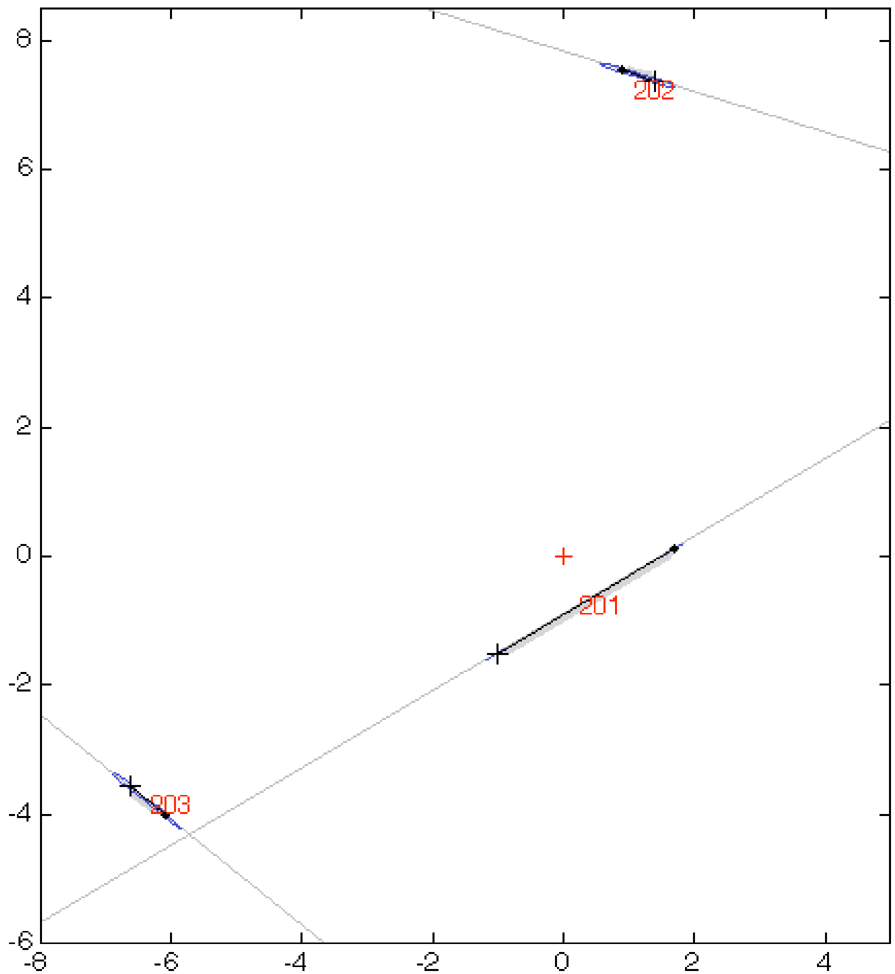
$a = 0.95$, $p = 5$

t_{exe}: **633 ms** (PowerPC at 300 MHz)

Global EKF Localization

At Expo.02

05.07.02, 17.23 h



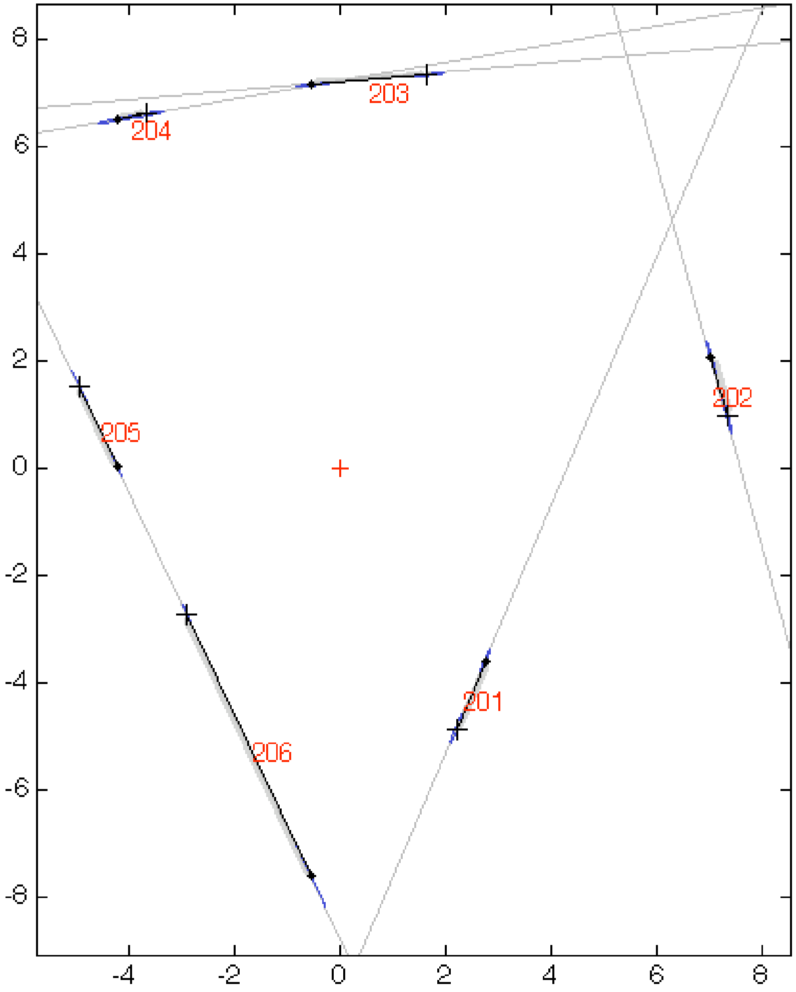
a = 0.999

[Arras et al. 03]

Global EKF Localization

At Expo.02

05.07.02, 17.23 h



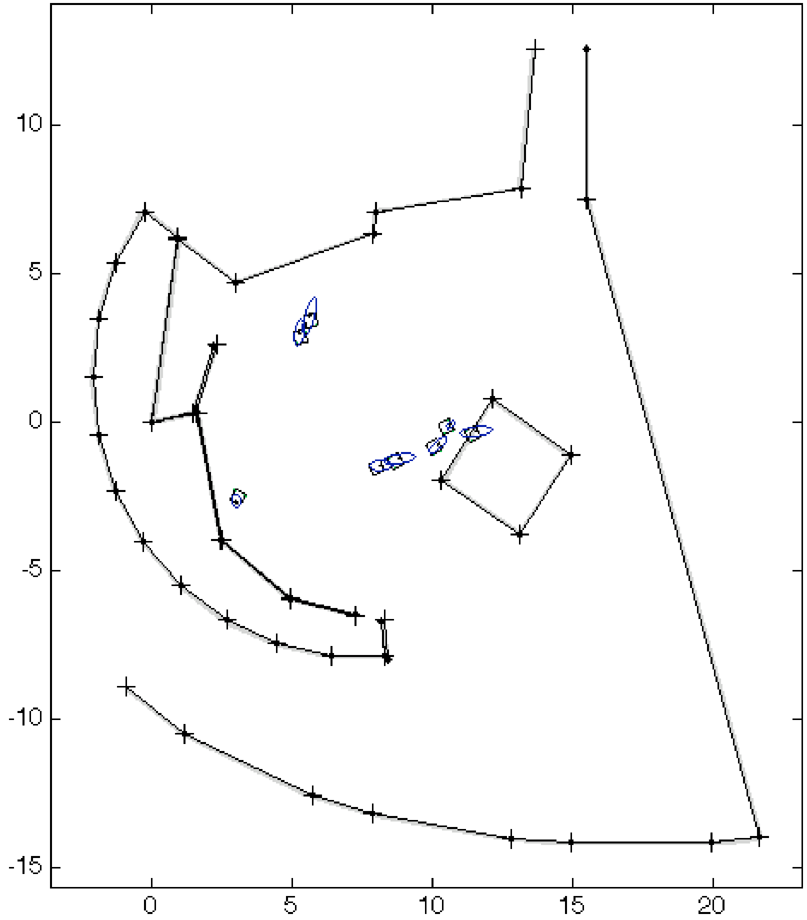
a = 0.999

[Arras et al. 03]

Global EKF Localization

At Expo.02

05.07.02, 17.23 h



a = 0.999

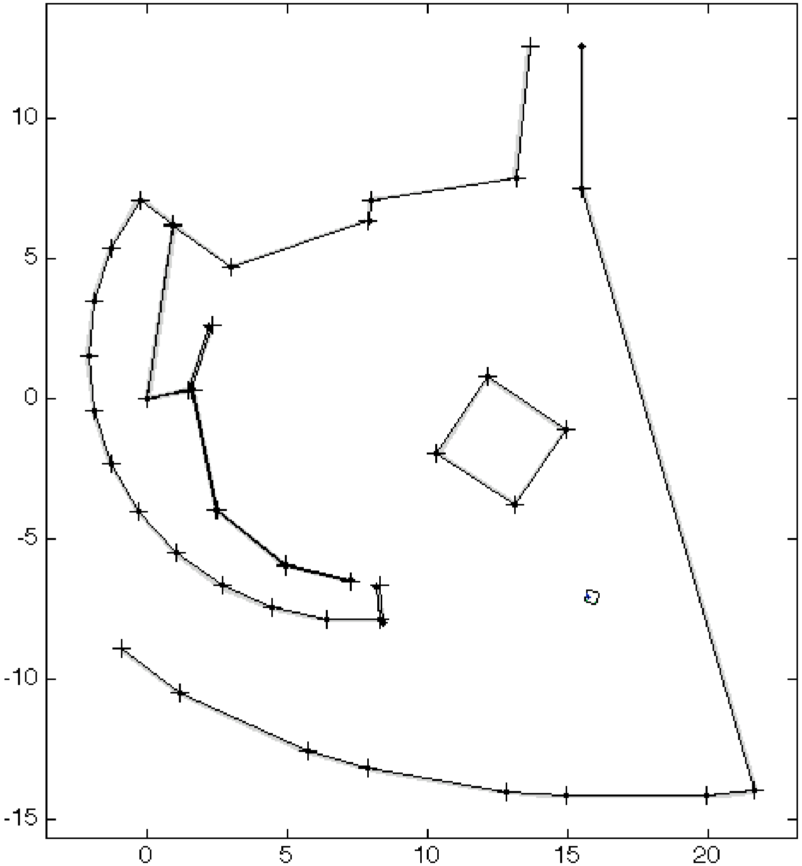
[Arras et al. 03]

t_{exe} = 105 ms

Global EKF Localization

At Expo.02

05.07.02, 17.23 h



a = 0.999

t_{exe} = 446 ms

[Arras et al. 03]