**General Form of Distributions**

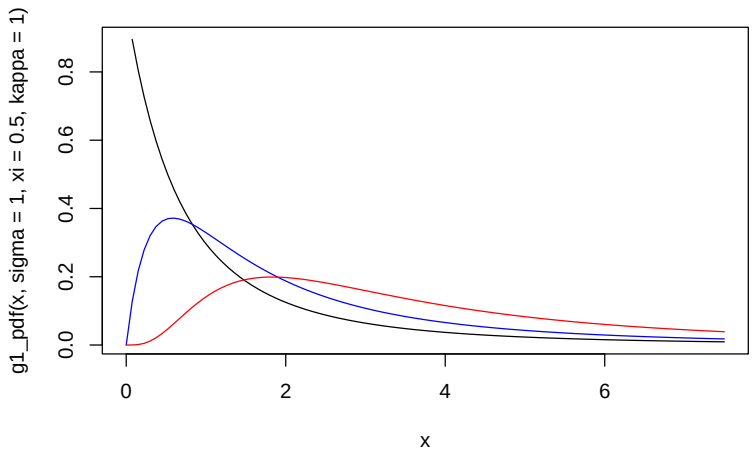
$$F(x) = G\{H_\xi(\frac{x}{\sigma})\}$$

**Parametric Families**

1. $G(v) = v^\kappa,\ \kappa > 0$

$$F_1(x) = \{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^\kappa$$

$$f_1(x) = \frac{\kappa}{\sigma}[1 + \xi(\frac{x}{\sigma})]^{-(1/\xi+1)}\{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa-1}$$

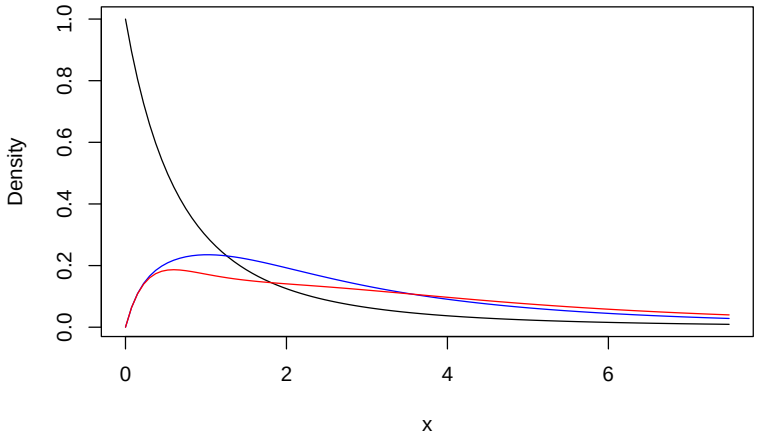



$$F_1^{-1}(U) = \frac{\sigma}{\xi}[(1 - U^{1/\kappa})^{-\xi} - 1]$$

2. $G(v) = pv^{\kappa_1} + (1-p)v^{\kappa_2},\ \kappa_1, \kappa_2 > 0$

$$F_2(x) = p\{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa_1} + (1-p)\{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa_2}$$

$$f_2(x) = \frac{1}{\sigma}[1 + \xi(\frac{x}{\sigma})]^{-(1/\xi+1)}\left(\kappa_1 p\{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa_1-1} + \kappa_2(1-p)\{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa_2-1}\right)$$



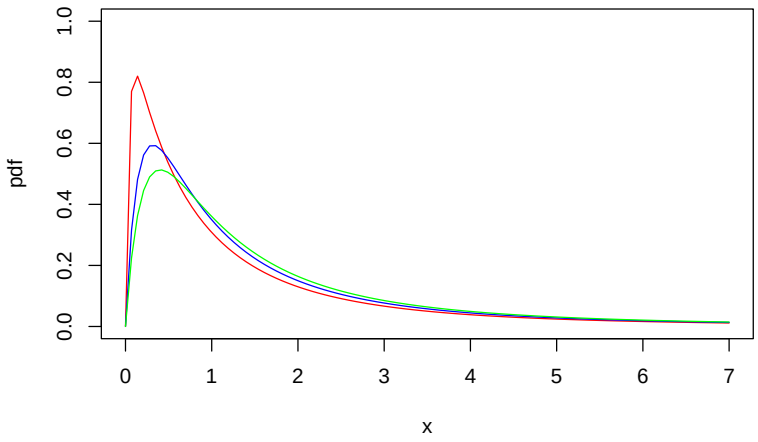
3. $G(v) = 1 - Q_\delta\{(1-v)^\delta\},\ \delta > 0, \quad Q_\delta \sim \text{Beta}(1/\delta, 2)$

$$F_3(x) = 1 - Q_\delta\{[1+\xi(\frac{x}{\sigma})]^{-\delta/\xi}\}, \quad Q_\delta \overset{d}{=} \text{Beta}(1/\delta, 2)$$

$$f_3(x) = \frac{1+\delta}{\delta\sigma}[1+\xi(\frac{x}{\sigma})]^{-(1/\xi+1)}\left(1-[1+\xi(\frac{x}{\sigma})]^{-\delta/\xi}\right)$$

```
g3_pdf <- function(x, sigma = sigma, xi = xi, delta = delta) {
    lpdf <- log(1 + delta) - log(delta * sigma) - (1/xi + 1) *
        log(1 + xi * (x/sigma)) + log(1 - (1 + xi * (x/sigma))^(-delta/xi))
    return(exp(lpdf))
}
```

```
curve(g3_pdf(x, sigma = 1, xi = 0.5, delta = 25), xlim = c(0,
    7), ylim = c(0, 1), ylab = "pdf", col = "red")
curve(g3_pdf(x, sigma = 1, xi = 0.5, delta = 5), xlim = c(0,
    7), add = TRUE, col = "blue")
curve(g3_pdf(x, sigma = 1, xi = 0.5, delta = 3), xlim = c(0,
    7), add = TRUE, col = "green")
```

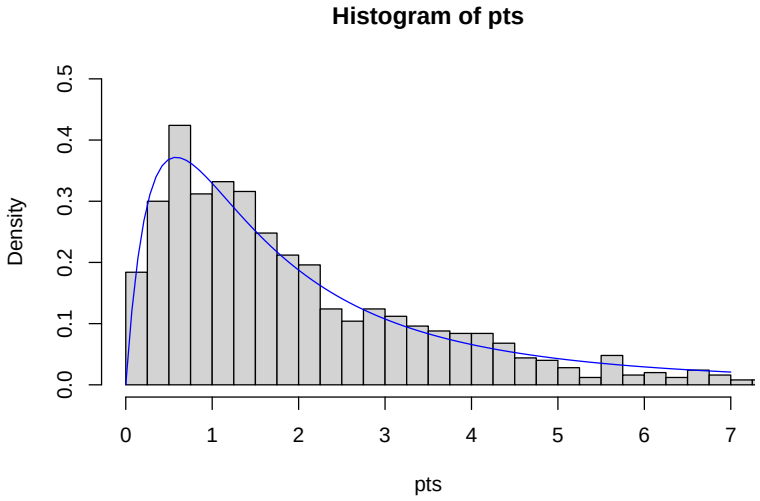



$$F_3^{-1}(U) = \frac{\sigma}{\xi}\left([Q_\delta^{-1}\{1-U\}]^{-\xi/\delta} - 1\right)$$

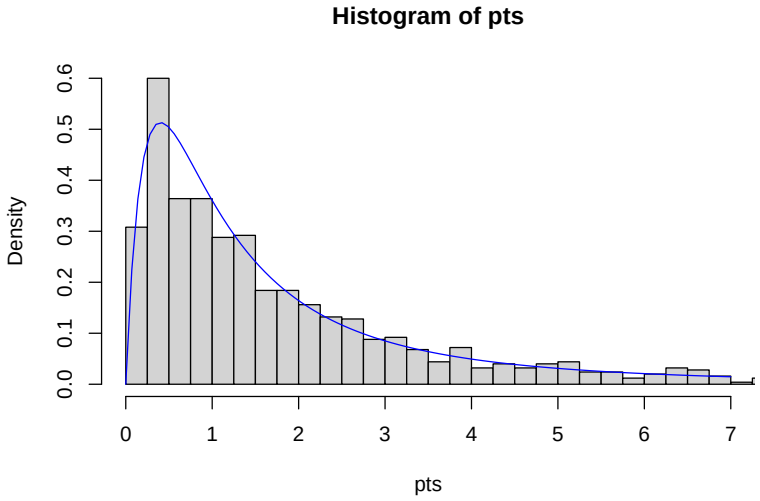
```
g3_cdf_inv <- function(u, sigma = sigma, xi = xi, delta = delta) {
    (sigma/xi) * ((qbeta((1 - u), (1/delta), 2)^(-xi/delta)) -
        1)
}

g3_rng <- function(n, sigma = sigma, xi = xi, delta = delta) {
    g3_cdf_inv(runif(n), sigma, xi, delta)
}
```

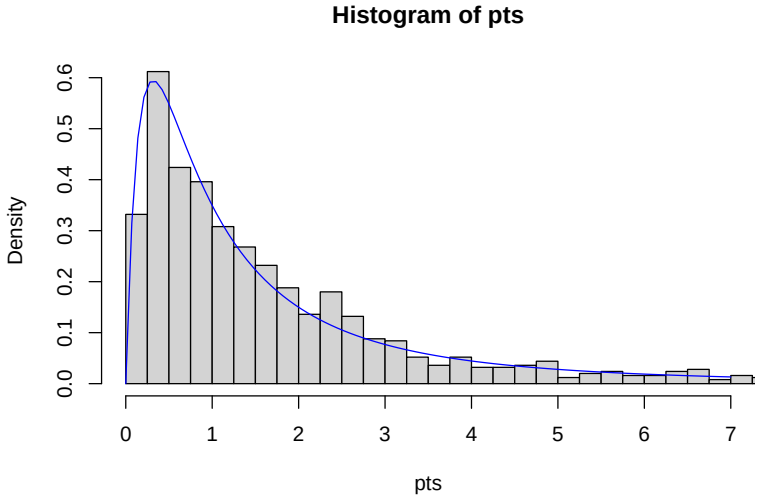
$\delta = 1$




$\delta = 3$




$\delta = 5$



4. $G(v) = [1 - Q_\delta\{(1-v)^\delta\}]^{\kappa/2},\ \kappa, \delta > 0$

$$F_4(x) = [1 - Q_\delta\{[1 + \xi(\frac{x}{\sigma})]^{-\delta/\xi}\}]^{\kappa/2}, \quad Q_\delta \stackrel{d}{=} \text{Beta}(1/\delta, 2)$$