

General Form of Distributions

$$F(x) = G\{H_\xi(\frac{x}{\sigma})\}$$

Parametric Families

1. $G(v) = v^\kappa, \kappa > 0$

$$F_1(x) = \{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^\kappa$$

$$f_1(x) = \frac{\kappa}{\sigma} [1 + \xi(\frac{x}{\sigma})]^{-(1/\xi+1)} \{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa-1}$$

$$F_1^{-1}(U) = \frac{\sigma}{\xi} [(1 - U^{1/\kappa})^{-\xi} - 1]$$

2. $G(v) = pv^{\kappa_1} + (1-p)v^{\kappa_2}, \kappa_1, \kappa_2 > 0$

$$F_2(x) = p\{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa_1} + (1-p)\{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa_2}$$

$$f_2(x) = \frac{1}{\sigma} [1 + \xi(\frac{x}{\sigma})]^{-(1/\xi+1)} \left(\kappa_1 p \{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa_1-1} + \kappa_2 (1-p) \{1 - [1 + \xi(\frac{x}{\sigma})]^{-1/\xi}\}^{\kappa_2-1} \right)$$

3. $G(v) = 1 - Q_\delta\{(1-v)^\delta\}, \delta > 0, Q_\delta \sim \text{Beta}(1/\delta, 2)$

$$F_3(x) = 1 - Q_\delta\{[1 + \xi(\frac{x}{\sigma})]^{-\delta/\xi}\}, Q_\delta \stackrel{d}{=} \text{Beta}(1/\delta, 2)$$

$$f_3(x) = \frac{1+\delta}{\delta\sigma} [1 + \xi(\frac{x}{\sigma})]^{-(1/\xi+1)} \left(1 - [1 + \xi(\frac{x}{\sigma})]^{-\delta/\xi} \right)$$

$$F_3^{-1}(U) = \frac{\sigma}{\xi} \left([Q_\delta^{-1}\{1-U\}]^{-\xi/\delta} - 1 \right)$$

4. $G(v) = [1 - Q_\delta\{(1-v)^\delta\}]^{\kappa/2}, \kappa, \delta > 0$

$$F_4(x) = [1 - Q_\delta\{[1 + \xi(\frac{x}{\sigma})]^{-\delta/\xi}\}]^{\kappa/2}, Q_\delta \stackrel{d}{=} \text{Beta}(1/\delta, 2)$$