# A Semiparametric Life Cycle Labor Supply Model with Non-Additive Fixed Effects *

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**Abstract**

This paper estimates the Frisch elasticity of labor supply, which represents the intertemporal elasticity of substitution. Estimation of this elasticity has previously required assuming that utility is either separable between consumption and leisure or quasi-homothetic with respect to leisure. These restrictions are required to generate hours equations in which individual effects, representing the marginal utility of wealth, enter additively and can be differenced out. Using PSID data, I relax this assumption for a sample of prime-age men. I estimate a semiparametric labor supply equation, using a control function to control for the individual effects. This strategy allows fixed effects to be both non-additive and correlated with the regressors. The average structural function and average partial effects of wages on hours are identified and estimated. In addition, variation in the elasticity over different points in the data is explored.

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# 1 Introduction

The intertemporal substitution elasticity is used in many macroeconomic models and is essential to the analysis of tax and benefit policies. This elasticity is often estimated with household data using marginal utility of wealth-constant labor supply functions. The wage elasticity in these equations is known as the Frisch elasticity, and measures the change in hours of labor supplied over time in response to receiving different wages at different periods of the life-cycle. It is interpreted as the response of labor supply to *anticipated* changes in the wage.

Frisch labor supply equations are typically derived from utility functions that are separable in consumption and leisure, which generate labor supply equations that are additive in marginal utility of wealth. At a minimum, estimation of Frisch, or "lambda-constant," labor supply equations has previously required utility to be quasi-homothetic, in which case marginal utility can be treated as additive even if preferences are not separable. These assumptions are made so that marginal utility, which is unobserved, can be modeled as a fixed effect. The assumption that utility is separable between consumption and leisure is widely recognized in the literature as unlikely to be true. Quasi-homotheticity is also a strong and potentially misleading assumption. If these assumptions are false, estimates of the Frisch elasticity used for policy analysis could be severely biased.

I examine the severity of this bias by estimating a life-cycle labor supply equation for married men that allows for a more general form of utility, and thus does not require additive fixed effects. Specifically, I allow the fixed effect to enter the hours equation in an unspecified, nonlinear, way. The hours equation is estimated using the double-index semiparametric least squares estimator of Ichimura and Lee (1988). A control function is employed to account for the fixed effect, which contains the marginal utility of wealth. This strategy allows the individual effects to be both non-additive and correlated with other variables. An additional benefit of this approach is that the Frisch elasticity is allowed to vary across the wealth distribution, which has implications for the distributional effects of policies such as tax reforms.

The remainder of this paper is organized as follows. Section 2 reviews the literature on the Frisch elasticity. Section 3 provides the identification strategy and estimation technique. Section 4 discusses the empirical results, and Section 5 concludes.

# 2 Literature

Modern life-cycle labor supply estimation began with Heckman and MaCurdy (1980) and MaCurdy (1981 and 1983). These papers formulate the labor supply decision in a given period as a function of current state variables, including wages and household characteristics. An individual solves the following problem.

$$V(a_{it}, t) = \max\left[U\left(c_{it}, h_{it}\right) + \beta E_t V\left(a_{i,t+1}, t+1\right)\right] \tag{1}$$

$$\text{s.t. } a_{i,t+1} = \left(1 + r_{t+1}\right)\left(a_{it} + w_{it}h_{it} - c_{it}\right) \tag{2}$$

The first order conditions, assuming an interior solution for consumption, are:

$$U_c(c_{it}, h_{it}) = \lambda_{it} \tag{3}$$

$$U_h(c_{it}, h_{it}) \geq \lambda_{it} w_{it} \tag{4}$$

where $\lambda_{it}$ is the marginal utility of wealth, $\frac{\delta V}{\delta a_{it}}$. Demands for quantities of goods and leisure can then be written as functions of current prices and the marginal utility of wealth. These demands are known as Frisch demands. A Frisch labor supply equation takes the following form.

$$h_{it} = f_{it}(w_{it}, \lambda_{it}) \tag{5}$$

Here, $f_{it}()$ may be a function of preference-shifting variables such as household or individual characteristics. Using this equation to estimate labor supply elasticities has two benefits. First, equation (5) does not include consumption, and so can be estimated without consumption data. Second, past and future realizations of wages and any preference variables enter the hours decision only through their effect on current marginal utility of wealth. Marginal utility is unobserved, but the solution to the agent's optimization problem keeps expected marginal utility constant over the life cycle. Assuming rational expectations and perfect capital markets, marginal utility evolves according to the following Euler equation.

$$\lambda_{it} = E[\beta(1 + r_{t+1})\lambda_{i,t+1}] \tag{6}$$

Estimation typically employs a log-approximation of the Euler equation, which breaks $\lambda_{it}$ into distinct components.

$$\ln \lambda_{it} = \mu_t + \ln \lambda_{i0} + \varepsilon_{it} \tag{7}$$

Individuals determine their marginal utility of wealth at the beginning of the life cycle, setting $\lambda_{i0}$. Marginal utility in each subsequent period differs from this initial level by a time effect $\mu_t$, which is a function of the common discount rate and interest rate, and an idiosyncratic forecast error $\varepsilon_{it}$. The Frisch labor supply equation can now be estimated, given proper treatment of $\lambda_{i0}$.

In order to handle the unobserved marginal utility of wealth, past studies impose a labor supply function of the following form (Browning 1986).

$$g(h_{it}) = \psi_{it}(w_t) + \phi_i(\lambda_t) \tag{8}$$

The estimate of $\frac{\partial h_{it}}{\partial w_{it}}$ gives an estimate of the Frisch elasticity, the effect of a change in wages holding $\lambda$ constant. If the function $\phi_i()$ is the natural log, equation (7) can be substituted in for the last term, and the time-invariant individual effect, $\ln \lambda_{i0}$, can be treated as a fixed effect.

The obvious benefit of this framework is that accounting for the fixed effect controls for the influence of all past and future time periods on the current hours choice. The cost, however, is that generating a labor supply equation in which the marginal utility term is additive or log-additive requires restrictions on preferences. A common strategy follows Heckman and MaCurdy (1980) and Macurdy (1981), and is summarized by Blundell and MaCurdy (1999). A log specification is generated by a utility function that is separable in consumption and labor.

$$U_{it} = g(c_{it}, Z_{it}) + \exp(-Z_{it}\rho - v_{it})(h_{it})^{\sigma} \tag{9}$$

Which gives the first order condition:

$$\ln h_{it} = \frac{1}{1-\sigma}\left(\ln w_{it} + \rho Z_{it} + \ln \lambda_{it} - \ln \sigma + v_{it}\right) \tag{10}$$

$$\ln h_{it} = \delta \ln w_{it} + \alpha_{i0} + \mu_t + \beta Z_{it} + e_{it} \tag{11}$$

where $\alpha_{i,t} = \delta(\ln \lambda_{i,t} - \ln \sigma)$ and $\alpha_{i0}$, the individual effect, comes from substituting in the updating process for $\lambda_{i,t}$.[1] The individual effect contains time zero marginal utility of wealth and is thus theoretically correlated with $\mathrm{w}_{it}$ and $\mathrm{Z}_{it}$. Since wages in time t affect wealth, and the preference variables contained in Z affect utility, the wages and preference variables in all time periods can be expected to be correlated with the marginal utility of wealth. The marginal utility term is therefore treated as a fixed effect, and the hours equation is estimated in first differences.

$$\Delta \ln h_{it} = \delta \Delta \ln w_{it} + \beta \Delta Z_{it} + \theta_t + \Delta e_t \tag{12}$$

The estimate of $\delta$ is an estimate of the Frisch elasticity.

MaCurdy (1981) estimates an equation of this form and finds the Frisch elasticity to be 0.23, which is reduced to 0.10 when time dummies are included to control for the interest rate effects $\mu_t$. Since then, estimates have tended to fall within or close to this range, include those of Altonji (1986) and Ham (1986), despite different specifications for preferences and different instruments used to control for remaining time-varying endogeneity of the wage.

There is ample evidence, however, rejecting intratemporal separability between consumption and leisure, which is assumed by both MaCurdy and Altonji. Altonji estimates an equation similar to (12), but then tests the separability assumption by adding terms for cross substitution between consumption and hours. He concludes that the assumption of separability is unlikely to be true. Browning and Meghir (1991) devise a methodology for testing for weak separability. Using a system of conditional demand functions for household commodities in the UK Family Expenditure Survey, they test whether these demands depend on labor supply. The authors find that variation in labor supply variables is

[1] Here, $\delta = \frac{1}{\sigma - 1}$, $\beta = \delta\rho$, and $e_{it} = \delta v_{it}$.

important in explaining variation in budget shares. They conclude that separability of demand for goods from both hours of work and labor force participation is rejected, for both males and females. Blundell, Browning and Meghir (1993) use this dataset to extend the idea of conditioning on labor supply to the marginal utility of wealth-constant framework. They specify a set of preferences that allow them to exploit results on two-stage budgeting (Gorman 1959), first estimating within-period preferences, and then aggregating cohorts and estimating intertemporal preferences. They find that labor market variables have significant effects on consumption growth, which is a further rejection of separability.

Blundell, Fry and Meghir (1990) show that relaxing additive separability in a log-hours labor supply equation can only be done by imposing homothetic preferences. An alternative specification is found in Browning, Deaton and Irish (1985). The authors relax additive separability by defining an individual's profit function and deriving the corresponding duel problem to utility maximization. Here, the transformation for g() in equation (5) is linear. The authors show, however, that treatment of $\lambda$ or $\ln(\lambda)$ as additive in the hours equations implies intra-period quasi-homotheticity. (This point is also discussed in Browning (1986) and Nickell (1988)). Preferences of this type restrict hours of work and expenditures to be linearly related to within-period full income. Browning, Deaton and Irish estimate this model using a pseudo panel created with cohort means. While the elasticity is not parameterized as it is in the MaCurdy-type specification, they find the intertemporal elasticity at the mean of hours to be around 0.4 when allowing nonseparability.

Blundell, Fry and Meghir discuss the limitations of quasi-homothetic preferences. As expenditures increase, preferences become linear Leontief, implying that the rich have zero within-period substitution effects. In addition, the intertemporal elasticity of substitution tends to zero as expenditures rise. The authors conclude that these may be strong restrictions to impose, and a high price to pay for relaxing separability between consumption and leisure.

An alternative approach to estimating the intertemporal elasticity of substitution is to parameterize utility in such a way that preference parameters can be estimated in two stages. First, within-period preferences can be estimated using the first order conditions for consumption and labor. Next, a suitably parameterized intertemporal Euler equation can be estimated to recover intertemporal parameters. This approach requires specification of intertemporal preferences, but has the advantage of removing the restrictions on utility discussed above, as it does not require marginal utility to be additive in the first order condition for hours. It does require data on consumption, however, which can be difficult to obtain at the individual level, since purchases are often made at the household level. The two-stage approach also requires dealing with the endogeneity of consumption in the hours decision.

I relax the restrictions on utility that are needed to estimate an additive fixed effects Frisch labor supply equation, but do not require data on consumption. I estimate a log-hours equation in which the marginal utility term is allowed to enter the equation in an unspecified manner, allowing for nonseparability and nonhomotheticity. I identify and estimate the average structural function, which gives the expected value of the hours function at a given X, averaged over the marginal utility of wealth.

This methodology also has the advantage of allowing the Frisch elasticity to vary at different points in the data. Models that imply a constant Frisch elasticity are typically rejected, and the literature has found significant differences in elasticity among wealth quantiles. For example, using a two-stage approach similar to that discussed above, Ziliak and Kniesner (1999) find that the Frisch elasticity rises with wealth, so that the hours response to a wage change is about 40% higher for the wealthiest quartile of men than for the poorest quartile. The authors conclude that examining only average elasticities obscures the distributional effects of tax policy. I therefore also estimate the distribution of the wage elasticity, both averaged over the individual effect and unconditionally.

# 3 Estimation and identification

I relax the assumptions that estimating equation (8) imposes on utility by allowing for an arbitrary relationship between the marginal utility of wealth and the observed variables that determine labor supply. This is achieved by estimating an hours equation that allows the individual effect to enter in an unspecified way. Denote the observed variables $X_{it} = [\ln w_{it}, Z_{it,}\mu_t]$. Let $\eta_{it} = [\lambda_{i0}, \varepsilon_{it}]$. The equation of interest is

$$\ln h_{it} = h^*(X_{it}, \eta_{it}) + e_{it} \tag{13}$$

Here, $e_{it}$ is a zero-mean error term, assumed to be uncorrelated with X and $\eta$.[2] The function $h^*()$ is unspecified, and allows for interactions between its two arguments. Life-cycle theory tells us that the marginal utility of wealth is a function of an individual's wages and other characteristics in every period of the life cycle. The unobserved $\lambda_{i0}$ is therefore correlated with the variables in $X_{it}$, and correlation between the two arguments of $h^*()$ must be taken into account. The objective is to estimate structural effects using equation (13). In order to identify the effects of changes in the X variables on hours, I make the following three assumptions. These assumptions follow Maurer, Klein and Vella (2011), who apply a similar approach in a semiparametric binary choice model that is estimated by maximum likelihood.

Define $X_i$ as individual i's realization of X for all time periods; $X_i = X_{i,1}, X_{i,2}, ..., X_{i,T}$

.

Assumption 1:

[2] This requires the assumption that any endogeneity between hours and wages is time-invariant.

$h_{i,t}$ depends on $X_i$ and the error term only through contemporaneous components.

$$h_{i,t} \perp X_i, \eta_i | X_{i,t}, \eta_{i,t}$$

This assumption follows directly from life-cycle theory and is the driving intuition behind the Frisch labor supply equation. Past and future wages and taste-shifters affect labor supply for individual i at time t only by changing the value of marginal utility of wealth. After controlling for the unobserved heterogeneity, $\lambda_{i0}$, the variables in $X_{it}$ and the idiosyncratic forecast error $\varepsilon_{it}$ have no effect on hours in other time periods.

Assumption 2:

There exists a single index $X_{i,t}\beta$ such that $h_{i,t}$ and $X_{i,t}$ are conditionally independent given $X_{i,t}\beta$ and $\eta_{it}$

$$h_{it} \perp X_{it} | X_{it}\beta, \eta_{it}$$

This assumption is a dimensionality reduction, or index restriction, which states that the effect of a change in $X_{it}$ on $h_{it}$ can be summarized through a single index. An index restriction is not required theoretically, but is required for the function to be well identified using a reasonable sample size of data. Note that wages and household characteristics have been included in the same index. Ideally, one might estimate a model with three indices, to allow arbitrary interactions of the wage, the characteristics in Z, and the individual effect. This is not feasible given the assumptions needed for the current estimator, however. Thus I assume that the relationship between the individual effect and the X variables can be summarized by the index restriction.

The remaining barrier to estimating the hours equation is that a conventional orthogonality condition is violated. As described above, theory indicates that $X_{it}$ is correlated with the unobserved marginal utility of wealth effect, $\lambda_{i0}$. A control function is therefore employed to restore the desired orthogonality conditions. Once the marginal utility of wealth is controlled for, $X_{i,t}$ is uncorrelated with the remaining error term. The control function assumption is stated as follows.

Assumption 3:

There exists a control function $V_i$ such that $\eta_{it}$ and $X_{i,t}$ are conditionally independent given $V_i$

$$\eta_{it} \perp X_{i,t} | V_i$$

This assumption allows for identification of what are known in the semiparametric literature as structural effects. By requiring $X_{i,t}$ and $\eta_{it}$ to move separately in the data, conditional on the control function, the effects of a change in X while holding $\lambda$ constant can be identified.

The choice of an appropriate control function is guided by the theory of life-cycle labor supply. Marginal utility of wealth depends on wages, taste-shifters, and any non-wage income that contributes to wealth, in all periods of an individual's life. A linear combination, specifically the average, of these observed variables is employed here to control for the part of the error term that is correlated with $X_{it}$. Conditioning on this function, $X_{it}$ and $\lambda_{i0}$ are independent.

Let $V_i$ be a vector of the time means of each X variable for individual i, as well as the mean of household income other than his own wage, for the individual over time. Other household income provides a natural exclusion restriction, entering the control function but not $X_{i,t}$. Income obviously affects wealth, and thus marginal utility, but does not enter the hours equation once marginal utility has been controlled for. In addition, age and the age of children are left out of the control function, assuming for example that the men in the sample have the same expected life span and thus the same average age over time. The control function and hours equation to be estimated become:

$$\widetilde{X}_i = \frac{1}{T}\sum_{t=1}^{T}\widetilde{X}_{it}$$

$$V_i = \widetilde{X}_i\gamma \tag{14}$$

$$h_{i,t} = h(X_{i,t}\beta, \widetilde{X}_i\gamma) + e_{i,t} \tag{15}$$

This approach is similar to Chamberlain's (1982) "correlated random effects" model, in which the dependence of the individual effect on the X variables is modeled as a combination of past and future Xs. It is also related to the identification strategy of Altonji and Matzkin (2005), who use additional external variables as controls in a nonlinear panel data model. The approach used here is much more practical to implement, however, because of the index restrictions.

The final estimator is the double-index semiparametric least squares estimator of Ichimura and Lee (1988). Let $\theta = [\beta \;\; \gamma]$. The estimate of the parameters is

$$\widehat{\theta} = \min_{\theta}\sum_{i,t}\tau_{it}\left(h_{it} - \widehat{E}\left[h|X_{it}\beta, \widetilde{X}_i\gamma\right]\right)^2 \tag{16}$$

Here, $\widehat{E}$ is a nonparametric expectation. The indices are orthogonalized, then the joint density is estimated as the product of two normal kernels. Local smoothing is used as a bias-reduction technique, following Klein and Vella (2009). They find that using local smoothing, rather than Ichimura and Lee's suggestion of higher-order kernels, significantly improves the finite sample performance of the double-index estimator. A trimming function, $\tau_{it}$, is employed to place zero weight on extreme values of the indices.

The semiparametric estimation described above allows estimation of the average structural function (ASF) suggested by Blundell and Powell (2000, 2003). The ASF describes how the structural function, $h^*(X_{it}\beta, \eta_{it})$, averaged over the unobserved individual heterogeneity $\eta_{i,t}$, depends on X. This is an important object to estimate in the present application, as the relationship between the structural index, $X_{it}\beta$, and hours of work depends on the marginal utility of wealth. For example, individuals with a lower level of wealth (and thus higher marginal utility), might be less responsive to a change in the index variables if they need to keep working to maintain a minimum level of income. Households with greater wealth may have the ability to be more flexible when preference variables change. In particular, this means that the Frisch elasticity may depend on on an individual's level of marginal utility, as earlier proposed by Ziliak and Kniesner (1999).

At a fixed realization of X, $X_0$, the ASF is defined as

$$\mu(X_0) = \int h^*(X_0\beta, \eta_{it})dF_{\eta_{it}} \tag{17}$$

This gives the average value of the hours function at $X_0$, with the average taken over the marginal density of the individual heterogeneity. Employing the control function assumption, this expression becomes.

$$\begin{aligned} \mu(X_0) &= \int\int h^*(X_0\beta, \eta_{it})dF_{\eta_{it}|\widetilde{X}\gamma}dF_{\widetilde{X}\gamma} \\ &= \int h(X_0\beta, \widetilde{X}\gamma)dF_{\widetilde{X}\gamma} \end{aligned} \tag{18}$$

The estimates of the index parameters, $\widehat{\beta}$ and $\widehat{\gamma}$, and the function $\widehat{h}()$ above, allow computation of the predicted $\widehat{h}$ at any $X_{it}$ and $\widetilde{X}_i$ combination. The average structural function is computed at a given $X_0$ as

$$\widehat{\mu}(X_0) = \frac{1}{n}\sum_{i=1}^{n}\widehat{h}(X_0\widehat{\beta}, \widetilde{X}_i\widehat{\gamma}) \tag{19}$$

The average is taken over the marginal distribution of the estimated control function.

The estimation above also allows computation of the average partial effects (APE), which give the change in hours with respect to a change in X, averaged over the marginal distribution of the individual effect. In particular, denote the wage elasticity at a given $X_0$ as $h_w$, and the APE as $\delta(X_0)$.

$$\begin{aligned} h_w(X_0\beta, \eta_{it}) &= \frac{\partial h(X_0\beta, \eta_{it})}{\partial w} \\ \delta(X_0) &= E_\eta\left[h_w(X_0\beta, \eta_{it})\right] \end{aligned} \tag{20}$$

Given assumption 3, the control function assumption, this expression can be rewritten as

$$\delta(X_0) = E_{\widetilde{X}_i\gamma}\left[h_w(X_0\beta, \widetilde{X}_i\widehat{\gamma})\right] \tag{21}$$

(Wooldridge 2002). This function gives estimates of the Frisch elasticity for different values of the structural index, averaged over the marginal utility of wealth. To estimate $\delta(X_0)$, first the wage elasticity is estimated for an individual with structural index equal to $X_0\beta$, at each value of the control function index, $\widetilde{X}_i\widehat{\gamma}$, by local linear regression. The wage elasticity at each combination of indices is computed as $\frac{\partial h(X_0\widehat{\beta}, \widetilde{X}_i\widehat{\gamma})}{\partial\left(X_0\widehat{\beta}\right)} * \widehat{\beta}^{wage}$[3]. Next, the estimated APE of the wage for a given $X_0$ is the average of these elasticities over the

[3]Here, the wage coefficient is re-normalized to account for the fact that the wage varaible used in estimation was normalized by dividing by the standard deviation of log wages.

control function.

$$\widehat{\delta}(X_0) = \frac{1}{n}\sum_{i=1}^{n}\widehat{h}_w(X_0\widehat{\beta}, \widetilde{X}_i\widehat{\gamma}) \tag{22}$$

Given the interest in the literature in the variation of the Frisch elasticity with respect to different levels of wealth, I also present the wage elasticities without averaging out the individual effect. Using the local linear regression estimates of the Frisch elasticity for each individual, I examine variation in the elasticity with respect to the control function. This provides an illustration of how different values of marginal utility of wealth impact an individual's responsiveness to changes in the wage.

# 4 Results

The data are from the Michigan Panel Study of Income Dynamics (PSID). The sample was chosen to most closely match the samples used in the standard papers on life-cycle labor supply, and therefore includes prime-age men, aged $25-55$, who were employed during each period in the sample. The hours variable used is an individual's annual hours of work. Characteristics in X include marital status, self-reported health status on a scale of 1 to 5, the numbers of total children and young children (under age 6) present in the household, age, education, and an interaction between age and education.

Although the control function accounts for endogeneity of the wage due to time-invariant heterogeneity, there is still an important potential source of correlation between the wage term and the error. The typical labor-supply equation uses a measure of hourly earnings that is computed by dividing labor income by the number of hours worked. If hours of work are measured with error, a negative correlation is induced between the measurement error of hours and the measurement error of wages (see Altonji 1986). To avoid this problem, I use data only for workers who report an hourly wage rate. This strategy has the disadvantage of limiting the sample to workers who earn an hourly wage, rather than an annual salary. As these workers are not likely to be a random sample of employees, the results and conclusions below can only be interpreted as statements about prime-age men who work for an hourly wage. The resulting sample is a balanced panel of 241 individuals observed over 11 years, from 1984 to 1994. [4]

The first column of Table 1 presents the results of estimating the hours equation by OLS. The wage coefficient is -0.009 and not significantly different from zero. The third column presents the results of fixed effects estimation which provides estimates of the standard Frisch labor supply model, controlling for unobserved marginal utility of wealth with individual fixed effects. The signs and significance levels of several of the coefficients change, indicating that the individual effects are in fact correlated with the regressors. The wage coefficient is still negative, but slightly greater in magnitude than the OLS coefficient, and again not significantly different from zero. Life-cycle theory predicts that the intertemporal elasticity of substitution, captured here by the wage coefficient, must be positive. Thus the simple fixed effects model seems to be insufficient.

Table 2 presents the parameter estimates of the structural index. Semiparametric estimates using kernels are identified only up to location and scale. A constant is therefore excluded from each index. The coefficient on the negative of age is normalized to one in the structural index; since increasing age is found, on average, to decrease the number of hours worked in this estimation, using the negative means that the coefficients will have more intuitive interpretations (a positive coefficient will mean increasing x increases the expected hours of work) The coefficient on other household income is normalized to

[4] Interviews asked for the respondent's hours and wages in the previous year, so data on household characteristics in a given year are matched to hours and wage data collected in the subsequent year's interview. After 1995, the PSID switched to biennial interviewing, making such matching signficiantly less reliable (see Andreski, Stafford, and Yeung 2008), so data from later years were not used.

one in the control function index. Given these normalizations, the remaining coefficients can be interpreted in relative terms. All variables have been standardized to have mean zero and standard deviation of one. The log wage, education, and number of young children all enter the index with positive coefficients, and have t-statistics greater than 2. A one standard deviation increase in education, however, has a more than three times a greater impact on the index than a one standard deviation increase in the log wage. A standard deviation increase in the number of young children present impacts the index twice as much as the log wage. The coefficients on self-reported health and marital status are also positive, but not significantly different from zero. In contrast to the number of young children, which increases the index, the total number of children impacts the index negatively. The coefficient on total children is higher, indicating that the reduction in hours worked associated with having children is partially offset when the children are young. Several of the time dummy variables are significant as well.

A look at the Average Structural Function, however, reveals that, while increasing the structural index increases the expected hours worked on average by a small amount, the relationship is not linear. Figure 1 presents a smoothed graph of the estimate of the ASF, which is the expected log hours of work at each value of the structural index, averaged over the distribution of the marginal utility of wealth. The ASF fluctuates up and down over the support of the structural index, ranging from a value of 7.64 to 7.72. The dependent variable is log hours, so these values correspond to annual hours of work ranging from 2,080 to 2,252. Despite the lack of a clear pattern here, it is possible to examine more closely the elasticity of hours with respect to the individual components of the structural index, in particular the wage elasticity evaluated at different points of the index.

The Frisch elasticity at a given value of the structural index is estimated as the Average Partial Effect of the wage on the structural function. A smoothed graph of the APE is presented in Figure 2. The estimated elasticity ranges from -0.004 to 0.009. The elasticity decreases over the support of the structural index, indicating that hours become slightly less responsive to wages as the index increases: in other words, as the wage increases, years of education increases, the number of young children increases, health increases, or the individual becomes married. At the median of the structural index, the Frisch elasticity is estimated to be 0.002. This number is considerably lower than MaCurdy's estimate of 0.1, but generally in keeping with the findings in panel data that the Frisch elasticity is positive but small. The semiparametric model is an improvement over the fixed effects model, in that the wage elasticity is positive, as predicted by life-cycle theory. This result suggests that the control function is successfully capturing the impact of the unobserved marginal utility of wealth term, and that allowing the individual heterogeneity to enter the hours equation non-additively significantly improves the estimation.

It is also instructive to see how the Frisch elasticity varies at different values of the control function. Table 3 presents the parameter estimates for the control function index. Assuming that increasing income, the reference variable, has a negative effect on marginal utility, positive coefficients here indicate other variables that also decrease marginal utility. The mean of the log wage and the mean number of children have positive and significant impacts on this index, while education is found to have a significantly negative effect. The coefficients on average marital and health status are not significantly different from zero. Standard deviation changes in the means of education and log wages have the greatest impact, with similar magnitudes in different directions. The number of children has about 60% of the impact of the log wage.

Figure 3 presents a smoothed graph of the Frisch elasticity against the control function index. Over most of the support the index, the elasticity increases as the index increases. Assuming, as above, that marginal utility of wealth declines as the index gets larger, wealth must be increasing The interpretation of Figure 3 is therefore that wealthier individuals have a greater wage elasticity, indicating that they are more flexible in responding to changes in the wage. This result is in agreement with the findings of Ziliak and Kniesner, who document a higher Frisch elasticity for wealthier men.

# 5 Conclusion

This paper contributes to the literature on life-cycle labor supply by estimating a Frisch labor supply equation without requiring the fixed effect to be additive. Allowing fixed effects to enter the hours equation nonlinearly allows for a non-separable, non-quasihomothetic utility function. The Frisch elasticity at the median of the structural index is found to be positive, at 0.002, and significantly different from zero. This value is not outside the range of estimates found in the existing literature, although it is quite close to zero. Prime-age men who work for an hourly wage are therefore found to respond little to changes in the wage when making labor supply decisions.

The difference between the linear fixed effects estimate and the semiparametric estimate of the Frisch elasticity suggests that an hours equation with additive fixed effects represents a mispecification of the labor supply function. This result can be interpreted as a rejection of the assumptions on utility that are necessary to generate a labor supply equation with additive fixed effects.

In addition, the Frisch elasticity is found to vary at different points in the data. Men with higher values of the structural index - driven by higher wages, education, or number of young children, or fewer children overall - are less responsive to changes in the wage. Men with lower marginal utility of wealth, however, are more responsive to changes in the wage. Assuming a constant Frisch elasticity could therefore lead to incorrect predictions about the impact of policies such as income tax changes, and mask important distributional differences in behavioral responses.

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**Table 1: OLS and Fixed Effects Estimation**

Dependent variable: log annual hours of work

| OLS | Coeff. | T-stat | Fixed Effects | Coeff. | T-stat |
|---|---|---|---|---|---|
| ln wage | -0.009 | (-0.81) | ln wage | -0.011 | (-0.54) |
| married | 0.044 | (3.71) | married | 0.038 | (2.45) |
| educ | 0.001 | (0.68) | educ | -0.004 | (-0.46) |
| kids | 0.006 | (2.01) | kids | 0.001 | (0.1) |
| young kids | -0.017 | (-1.93) | young kids | -0.003 | (-0.38) |
| health | -0.009 | (-2.12) | health | 0.004 | (1.03) |
| age | 0.000 | (0.51) | age | 0.005 | (0.56) |

N=2,651 observations on prime-age men who were continuously employed during the sample period of 11 years. Regressions include a set of time dummies.

**Table 2: Structural Index**

Dependent variable: log annual hours of work

| | Index Coeff. | T-Stat |
|---|---|---|
| ln wage | 0.114 | (2.09) |
| married | 0.042 | (1.25) |
| educ | 0.373 | (6.83) |
| kids | -0.324 | (-4.90) |
| young kids | 0.244 | (4.65) |
| health | 0.021 | (0.56) |
| t1 | 0.045 | (1.01) |
| t2 | 0.072 | (1.42) |
| t3 | 0.107 | (2.29) |
| t4 | 0.192 | (3.79) |
| t5 | 0.267 | (5.17) |
| t6 | 0.306 | (5.95) |
| t7 | 0.353 | (6.35) |
| t8 | 0.416 | (7.87) |
| t9 | 0.506 | (9.08) |
| t10 | 0.524 | (9.76) |

n=2,651

**Table 3: Control Function Index**

| | Index Coeff. | T-stat |
|---|---|---|
| mean ln wage | 0.732 | (16.21) |
| mean married | 0.035 | (0.76) |
| mean educ | -0.853 | (-18.33) |
| mean kids | 0.439 | (11.34) |
| mean health | -0.079 | (-1.26) |

n=2,651

Figure 1

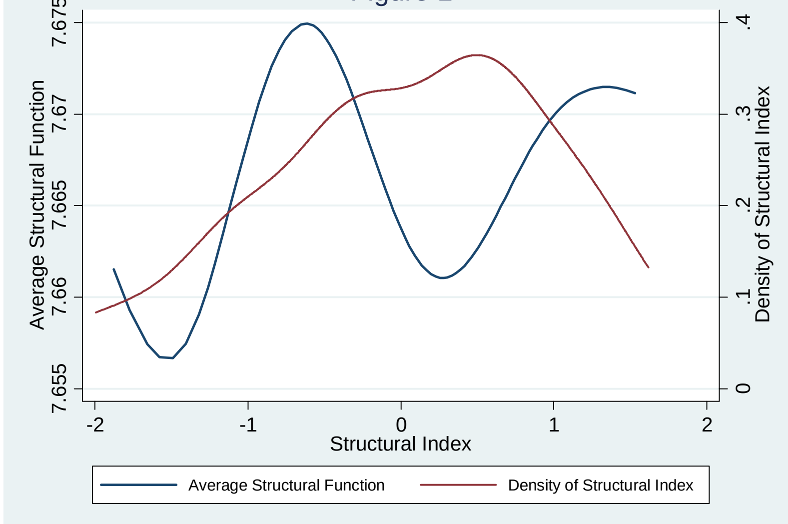
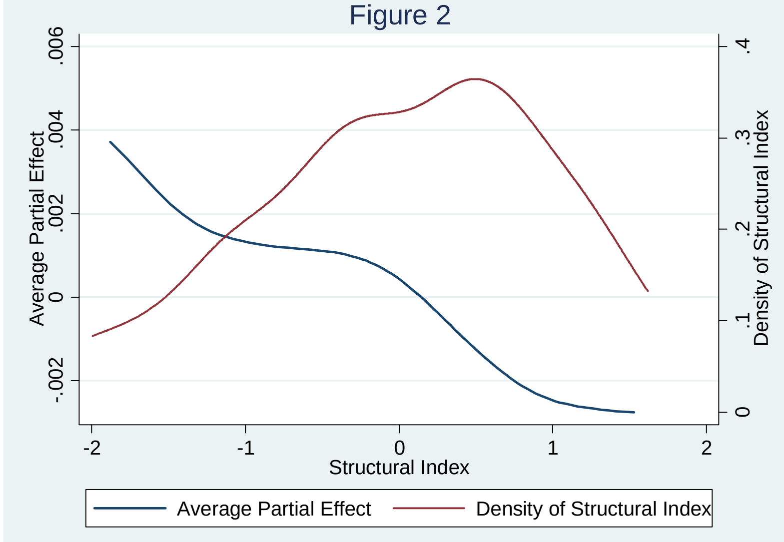



Figure 2



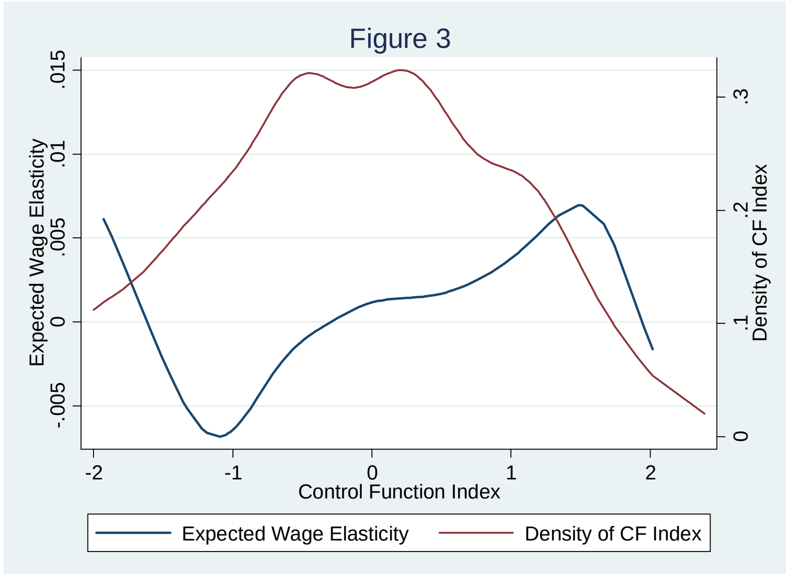


Figure 3
Expected Wage Elasticity
-.005
0
.005
.01
.015
Density of CF Index
0
.1
.2
.3
-2
-1
0
1
2
Control Function Index
Expected Wage Elasticity
Density of CF Index