Assignment 3 (deadline: May 19, 2014)

Send the solutions in the form a complete report to: mpp@eit.lth.se

Problem 1: Simplex

Solve the following LPs by simplex. In each case start with a graphical illustration, transform the considered integer program to the standard form, and solve it by giving consecutive simplex tables in the canonical form. Please follow the solution pattern used in the lecture slides. In each case use the slack variables for the initial basis if possible.

LP 1:

$$\begin{aligned}
&\textit{maximize} && z = x_1 + x_2 && \text{(1a)}\\
&\textit{subject to} && 2x_1 + x_2 \leq 10 && \text{(1b)}\\
& && x_1 + 4x_2 \leq 12 && \text{(1c)}\\
& && x_1, x_2 \geq 0. && \text{(1d)}
\end{aligned}$$

LP 2:

$$\begin{aligned}
&\textit{maximize} && z = x_1 + 3x_2 && \text{(2a)}\\
&\textit{subject to} && 2x_1 + x_2 \leq 7 && \text{(2b)}\\
& && x_1 \leq 3 && \text{(2c)}\\
& && x_2 \leq 3 && \text{(2d)}\\
& && x_1, x_2 \geq 0. && \text{(2e)}
\end{aligned}$$

LP 3:

$$\begin{aligned}
&\textit{maximize} && z = x_1 + 2x_2 && \text{(3a)}\\
&\textit{subject to} && x_1 \leq 1 && \text{(3b)}\\
& && x_2 \leq 1 && \text{(3c)}\\
& && x_1 + x_2 \geq \tfrac{1}{2} && \text{(3d)}\\
& && x_1, x_2 \geq 0. && \text{(3e)}
\end{aligned}$$

Remark: Note that in the last problem the slack variable s_3 of the third inequality cannot be used as an initial basic variable (since it would be negative, equal to $-\frac{1}{2}$). Therefore, use x_1, s_1 and s_2 as initial basic variables (you will see that this corresponds to the vertex $x_1 = \frac{1}{2}, x_2 = 0$ of the original polyhedron in the graphical illustration. So you first have to find the canonical form with respect to these assumed basic variables.

Problem 2: Convexity

Prove that function $F(x) = x^2$ is strictly convex in its domain R using definition of convexity given in the lecture slides. Give a graphical illustration of the quantities involved proof.

Problem 3: Manhattan network

Derive the formula for the number of all shortest-hop paths between the two opposite nodes in a $n \times n$ Manhattan network (an example for $n = 5$ is given in Figure 1). Compute this number for $n = 3, 4, 5, 6$, and, approximately, for $n = 50$ using the Stirling formula for $n!$.

Hint: note that each paths in question correspond to certain combination with repetitions.

Problem 4: Solving an LP with Gurobi

For the directed network showed in Figure 2 below, solve problem FAP specified in the lecture slides, both in the link-path and the node-link formulation. Solve all combinations of the following variants:

- objective function
  - $\min z$
  - $\min \sum_e \xi_e \sum_d \sum_p \delta_{edp} x_{dp}$
  - no objective function
- demand volumes increased (with respect to the traffic demands given in the figure) by
  - 0%
  - 10%
  - 50%
  - 100%

Please number the links, nodes, and arcs according to Figure 2. In the report, please attach the Gurobi output files for all the considered variants, together with appropriate tables summarizing the optimal Gurobi solutions. Give the graphs illustrating how the optimal value of the first objective function changes with traffic demand growth. Please attach also the problem in both formulations in the LP format but only for the first objective function.

Remark: Note that the resulting problem may not be feasible in some cases.

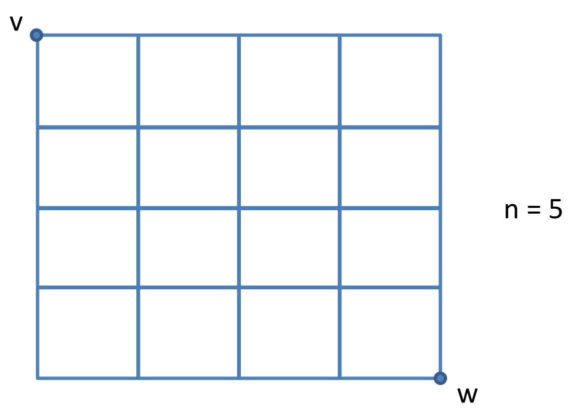
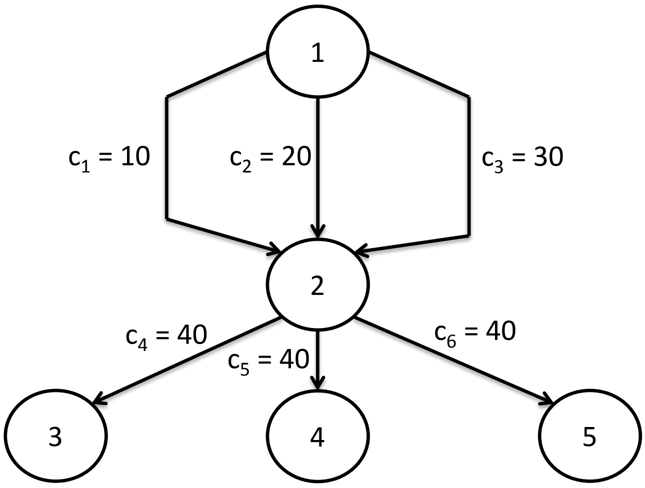


Figure 1: Manhattan network



demands:	$1 \to 3$	$h_1 = 10$
	$1 \to 4$	$h_2 = 15$
	$1 \to 5$	$h_3 = 20$
link costs:	$\xi_1 = 30$, $\xi_2 = 20$, $\xi_3 = 10$	
	$\xi_4 = 0$, $\xi_5 = 0$, $\xi_6 = 0$	

Figure 2: Network to consider