Advances in Stochastic Models for Animal Movement and Assessment of Attitudes Toward Probability

link to slides

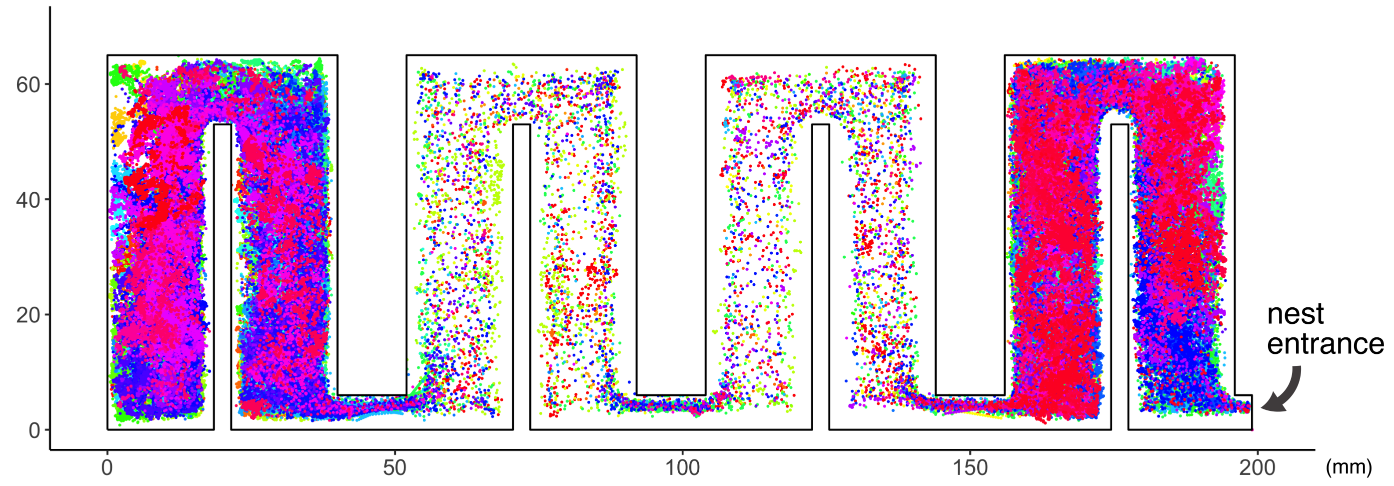
Elizabeth Eisenhauer

Co-advisors: Ephraim Hanks & Matthew Beckman

Outline

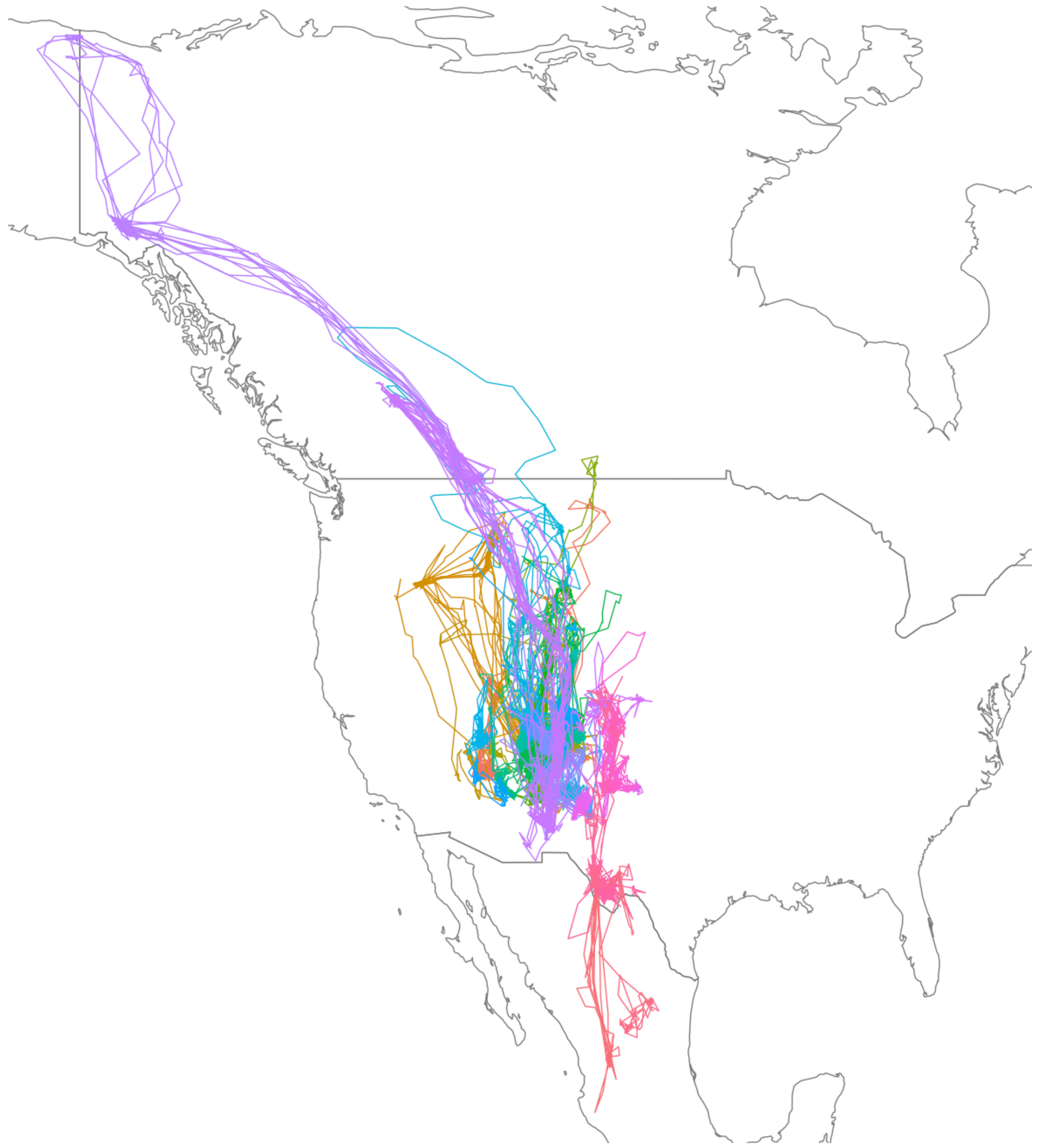
1

A Lattice and Random Intermediate Point Sampling Design for Animal Movement



2

Modeling Yearly Patterns in Golden Eagle Movement



3

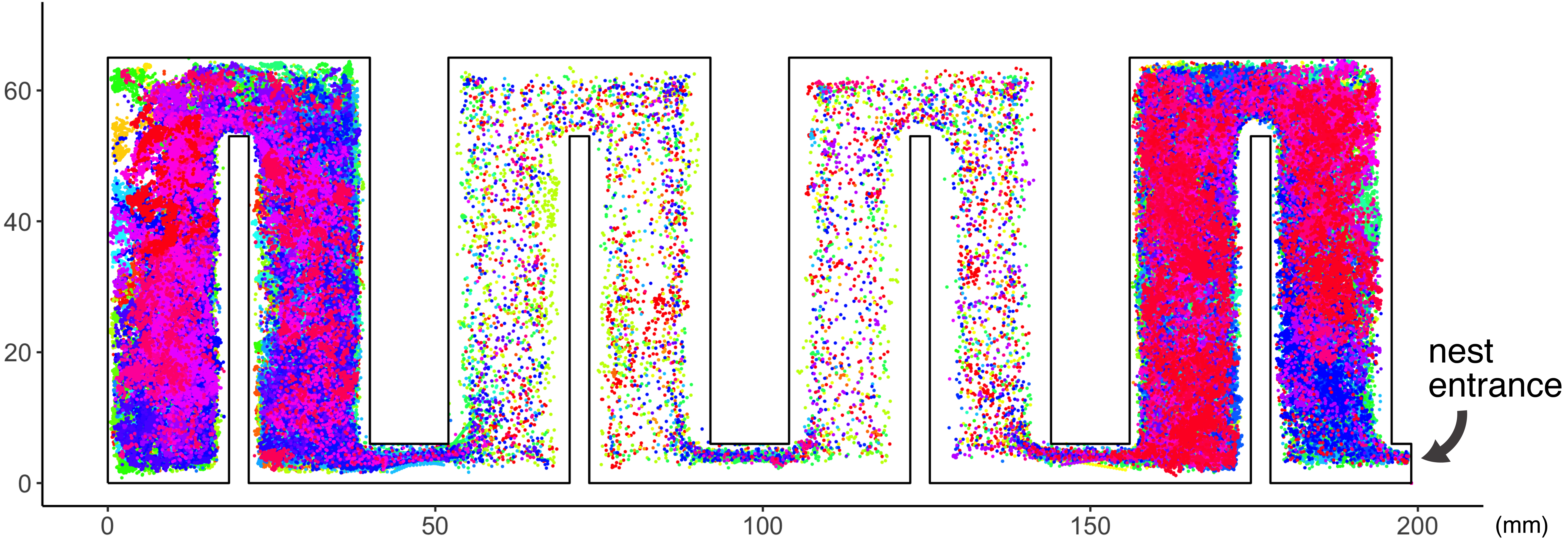
Survey of Attitudes toward Probability

A Lattice and Random Intermediate Point (LARI) Sampling Design for Animal Movement

Hughes lab at Penn State collected **high resolution ant data,** but data collection was **time-consuming** (1000s of student-hours).

4 hours of movement data
78 ants
1 second intervals

The resulting dataset consists of **4 hours** of movement data for **78 ants** at 1 second intervals (14,401 observations per ant).



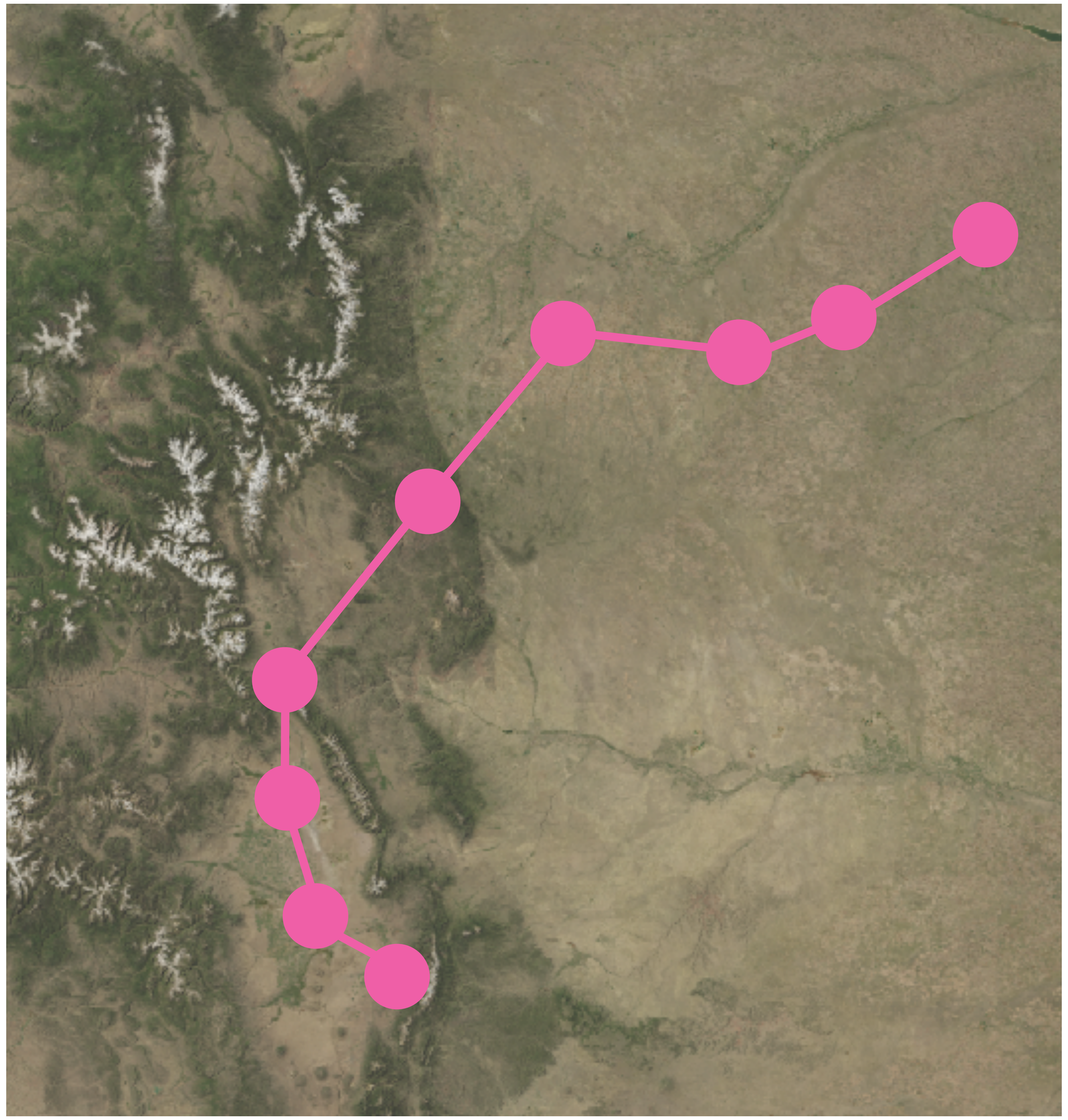
Each color is one individual.

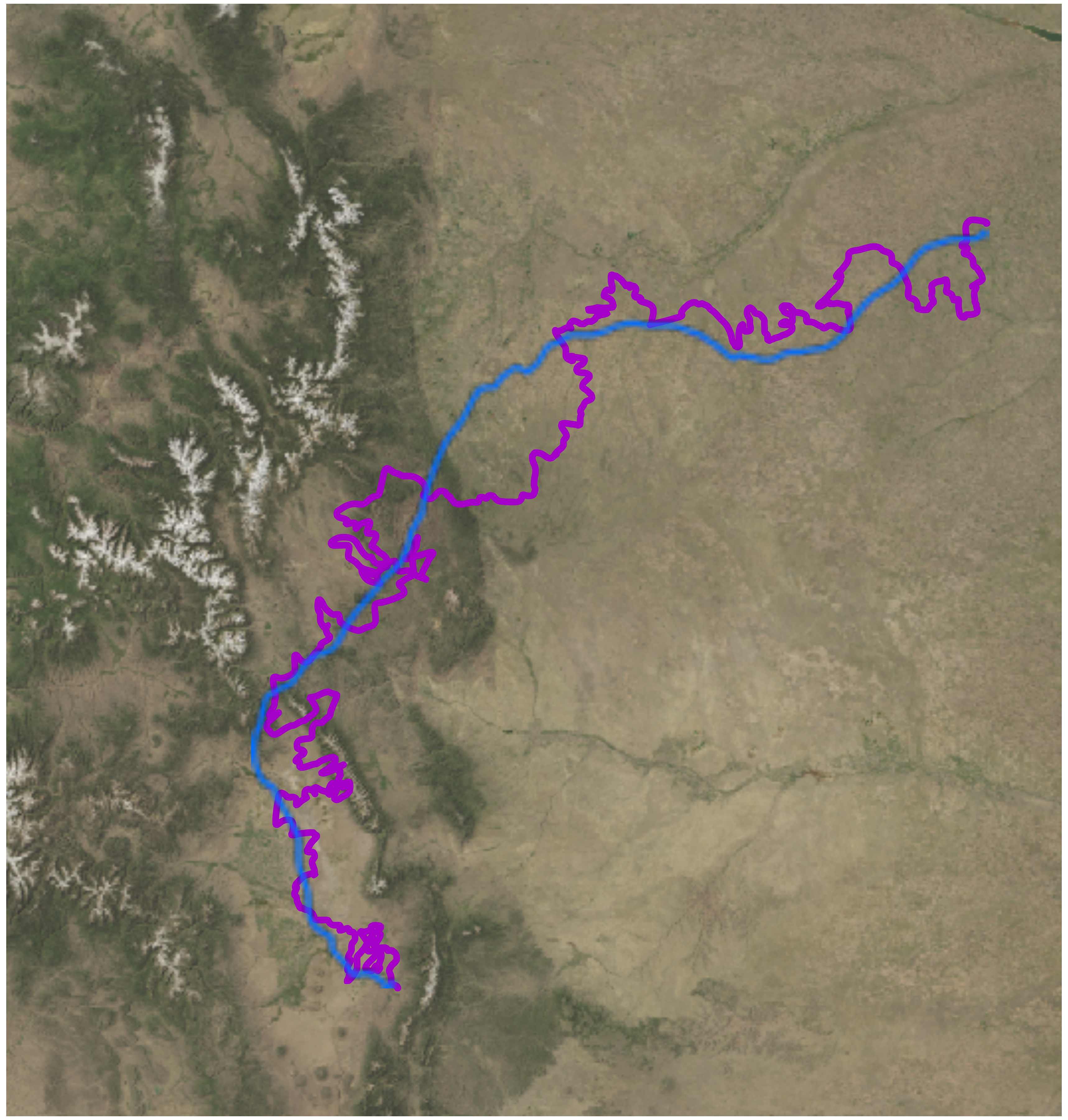
We were approached by the researchers with the **scenario**:

> Next time, we will collect **lower resolution** data.
>
> We want to know how best to collect this data.

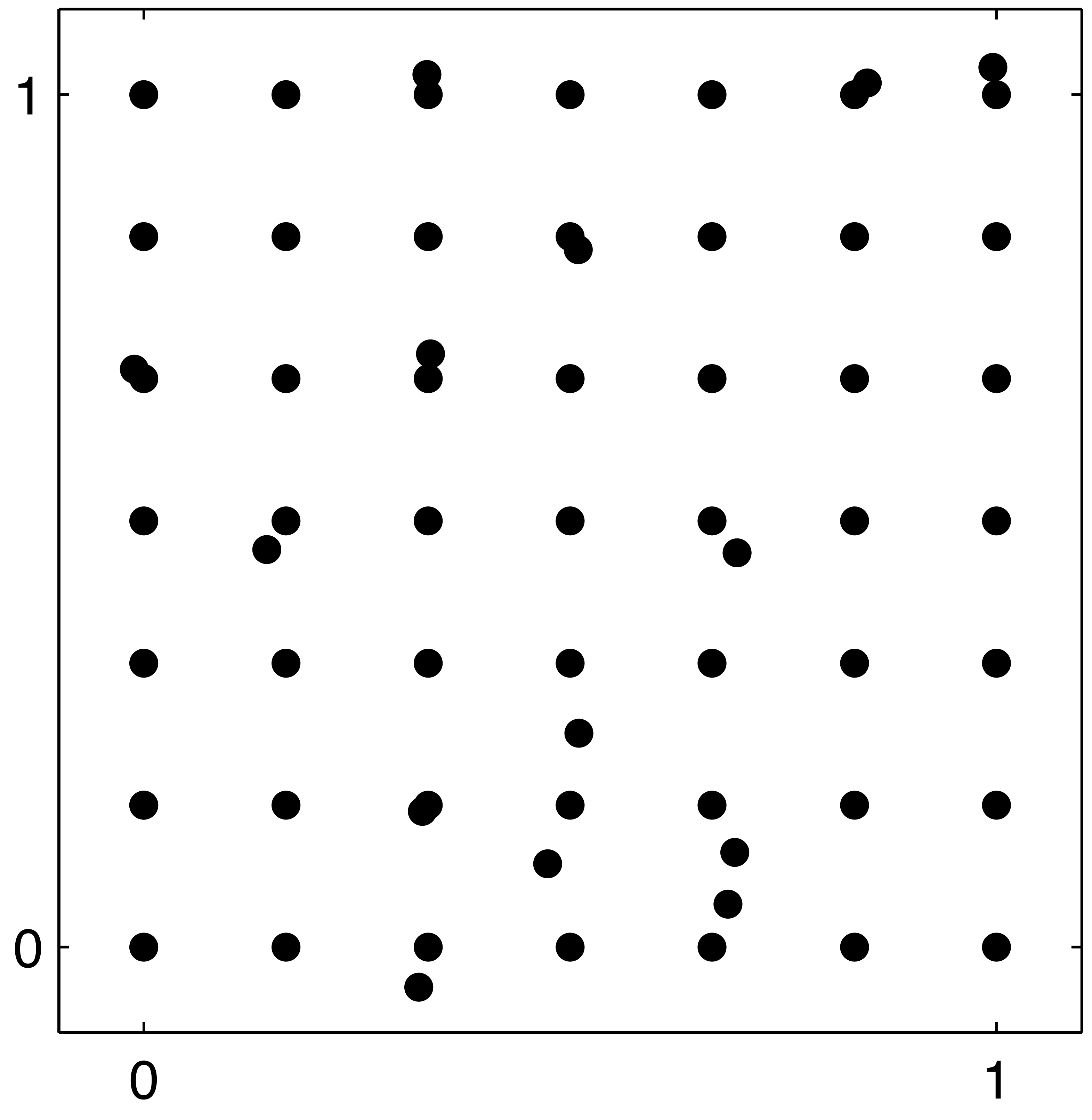
This question is **relevant to many researchers** collecting animal movement data.

Sampling at **regular time intervals can hide important information** about the speed and tortuosity of the path.





In geostatistics, researchers often adopt a **lattice plus close pairs** design over a lattice alone or a lattice and infill approach.

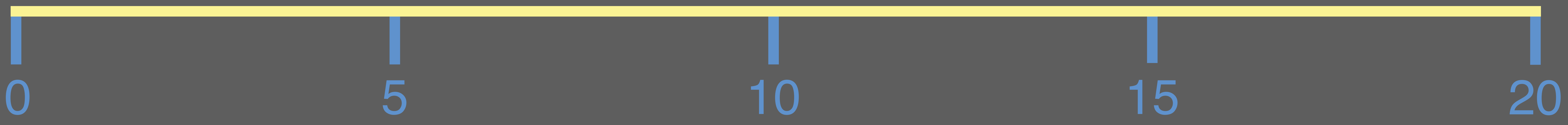


(Diggle and Lophaven, 2006)

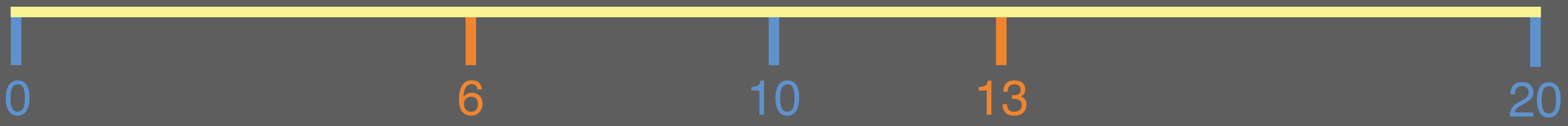
We **propose a sampling scheme** for animal telemetry data inspired by the lattice plus close pairs geostatistical design.

2 sampling designs:

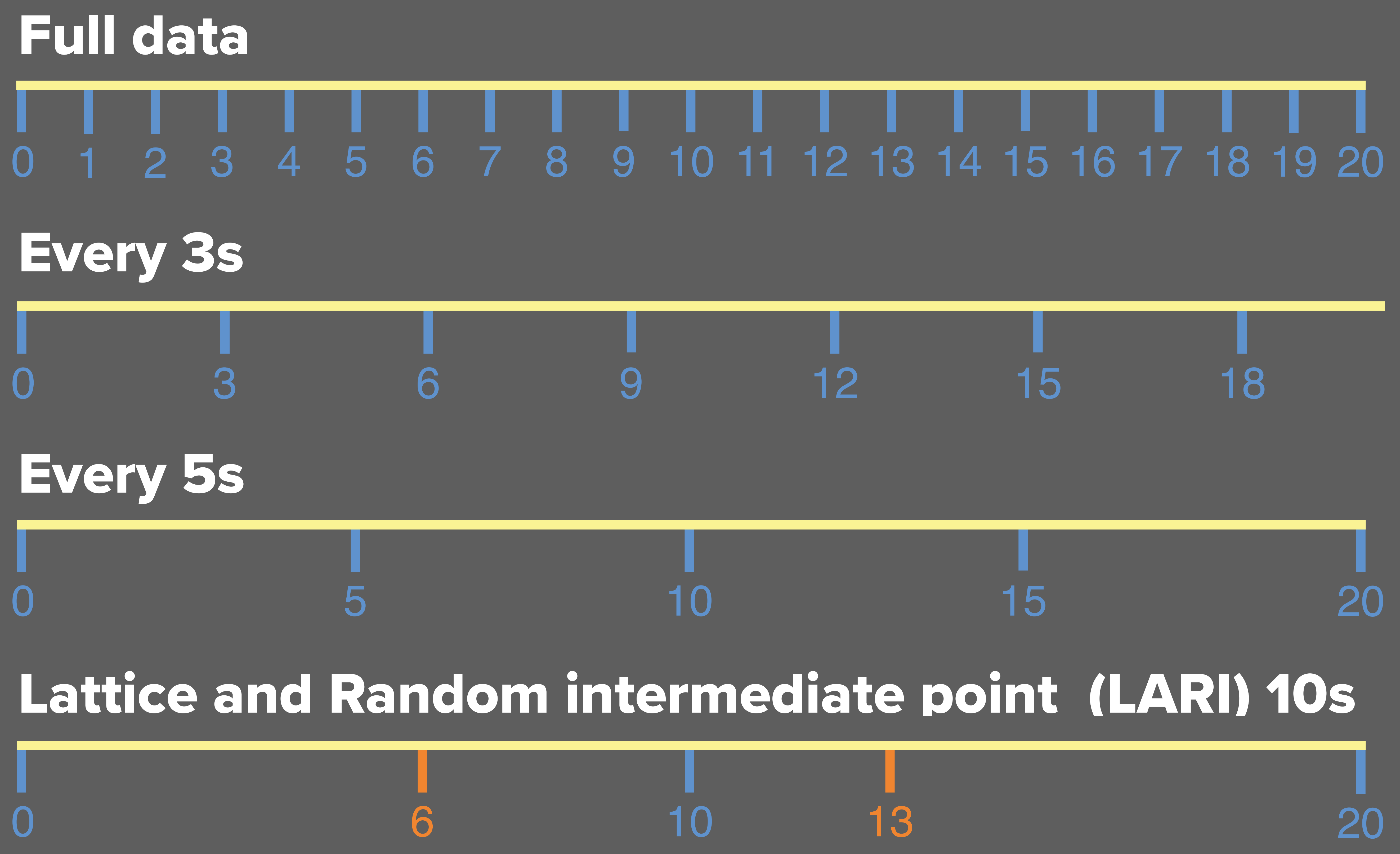
REGULAR



LATTICE AND RANDOM INTERMEDIATE POINT (LARI)



To compare regular and LARI sampling designs, we look at **4 subsamples** of the ant data.



Every 5s and LARI 10s have the same number of data points

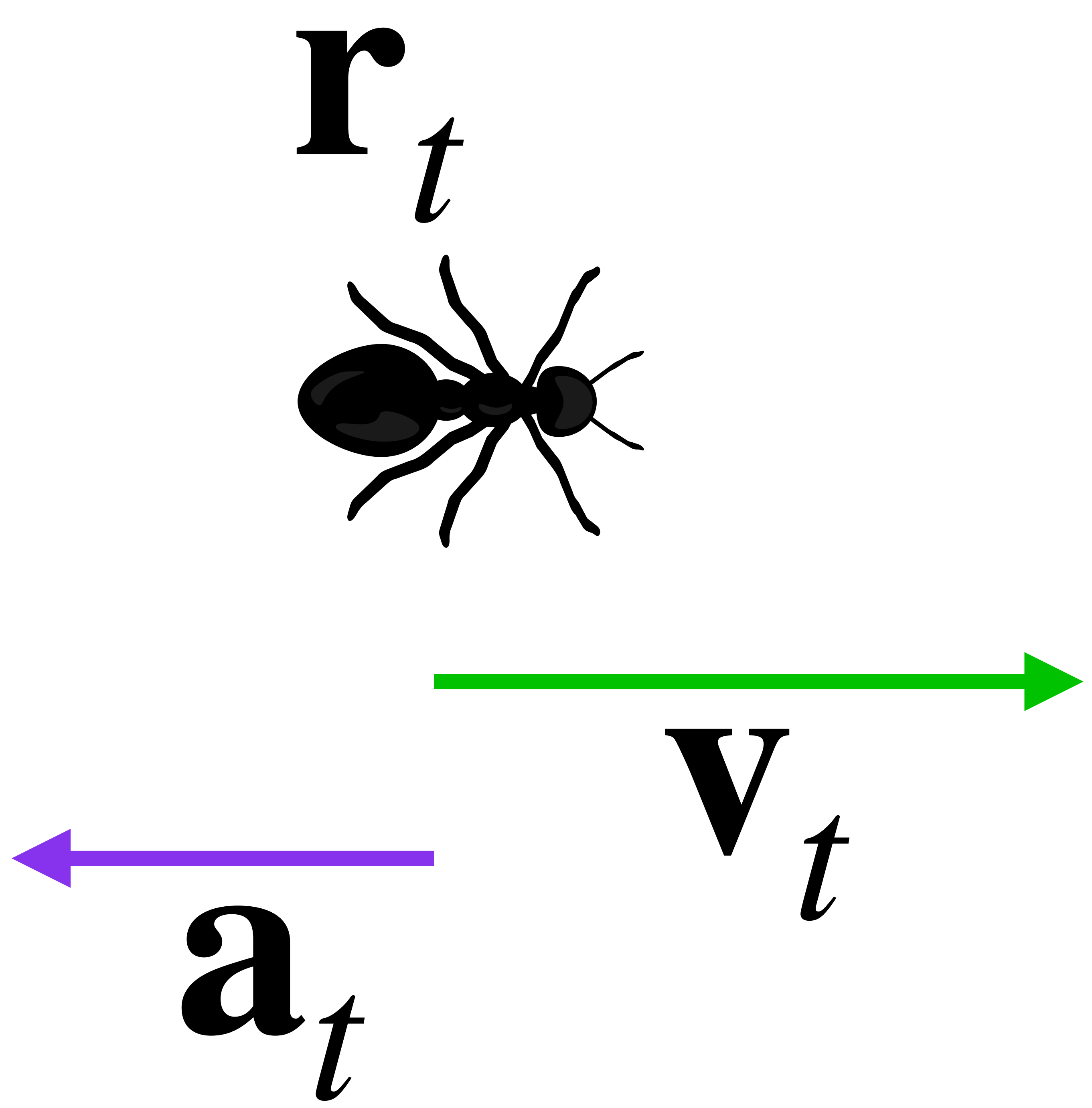
To **model movement at an individual level**, we use basic concepts from physics.

$\mathbf{r}_t =$ position

$\mathbf{v}_t =$ velocity

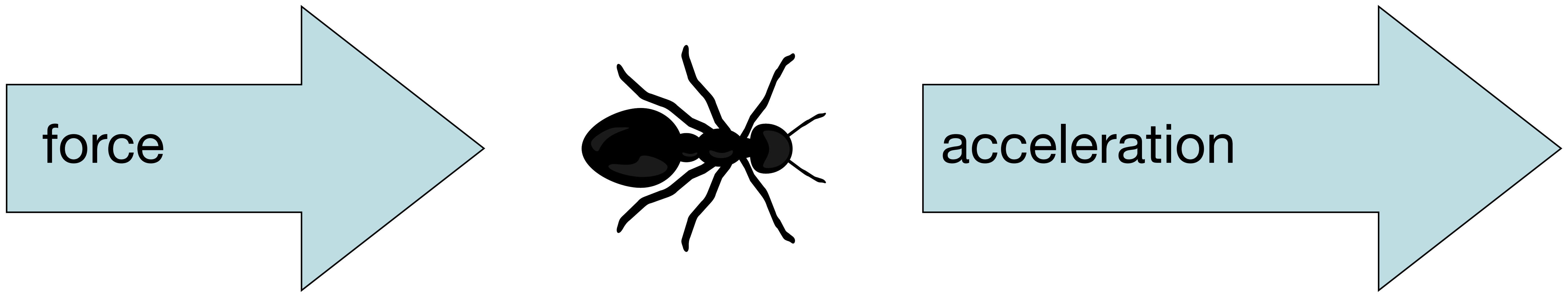
$\mathbf{a}_t =$ acceleration

This ant is slowing down



$$\mathbf{F}_t = m\mathbf{a}_t$$

So modeling acceleration is the same as **modeling “force”** acting on an animal.



The **2 main equations** for this model

The derivative of position with respect to time is velocity.

$$\frac{d\mathbf{r}_t}{dt} = \mathbf{v}_t \quad \longleftrightarrow \quad d\mathbf{r}_t = \mathbf{v}_t dt$$

The derivative of velocity with respect to time is acceleration.

$$\frac{d\mathbf{v}_t}{dt} = \mathbf{a}_t \quad \longleftrightarrow \quad d\mathbf{v}_t = \mathbf{a}_t dt$$

The **2 main equations** for this model

To model animal movement, we use

$$\boxed{d\mathbf{r}_t = \mathbf{v}_t \, dt}$$

and rewrite acceleration as a sum of forces

$$d\mathbf{v}_t = \underbrace{\boxed{\beta \, (\boldsymbol{\mu}(\mathbf{r}_t) - \mathbf{v}_t) \, dt}}_{\text{mean-reverting force}} + \underbrace{\boxed{c(\mathbf{r}_t) \, \mathbf{I} d\mathbf{w}_t}}_{\text{random force}}$$

Stochastic differential equation (SDE) model for animal movement

Data: $\mathbf{r}_t,\ t = 1, 2, \ldots, 14401$ for each ant

SDE model framework:

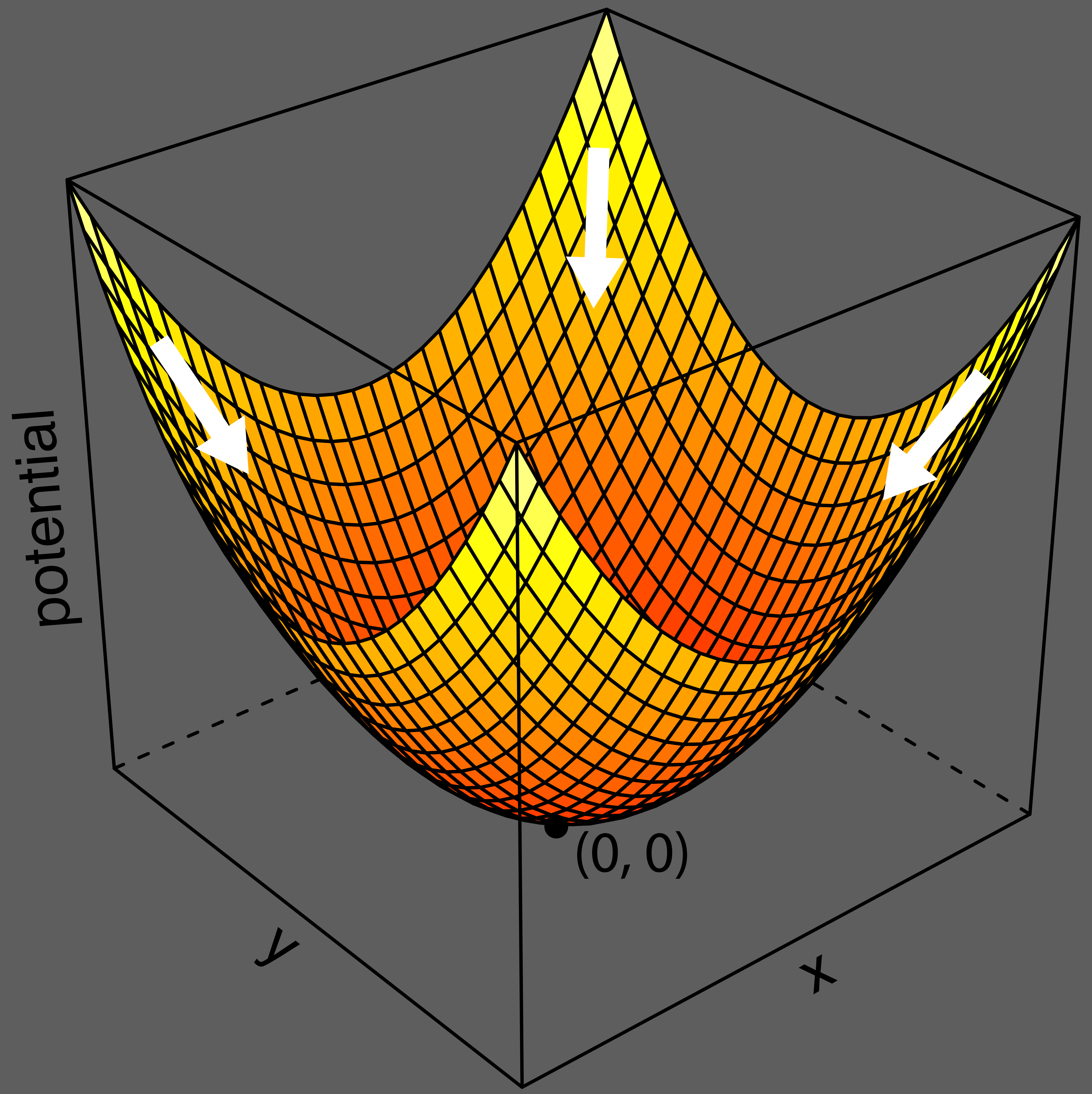
$$d\mathbf{r}_t = \mathbf{v}_t\ dt$$
$$d\mathbf{v}_t = \beta\ (\boldsymbol{\mu}(\mathbf{r}_t) - \mathbf{v}_t)\ dt + c(\mathbf{r}_t)\ \mathbf{I}d\mathbf{w}_t$$

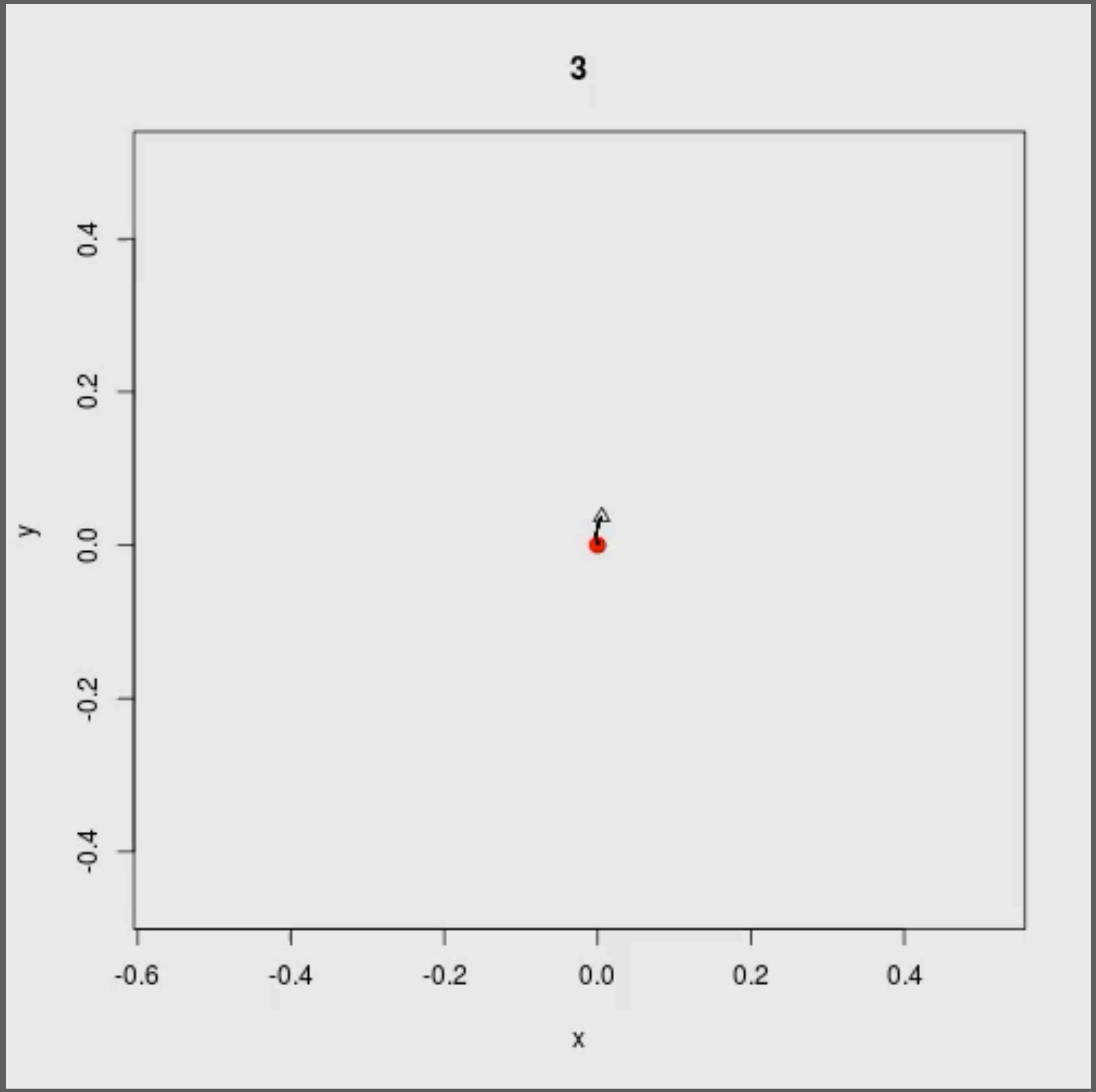
Utilizing motility and potential surfaces, define:

$$\boldsymbol{\mu}(\mathbf{r}_t) = m(\mathbf{r}_t)\ [- \nabla p(\mathbf{r}_t)] \qquad \text{(mean drift)}$$
$$c(\mathbf{r}_t) = \sigma\ m(\mathbf{r}_t) \qquad \text{(magnitude of stochasticity)}$$

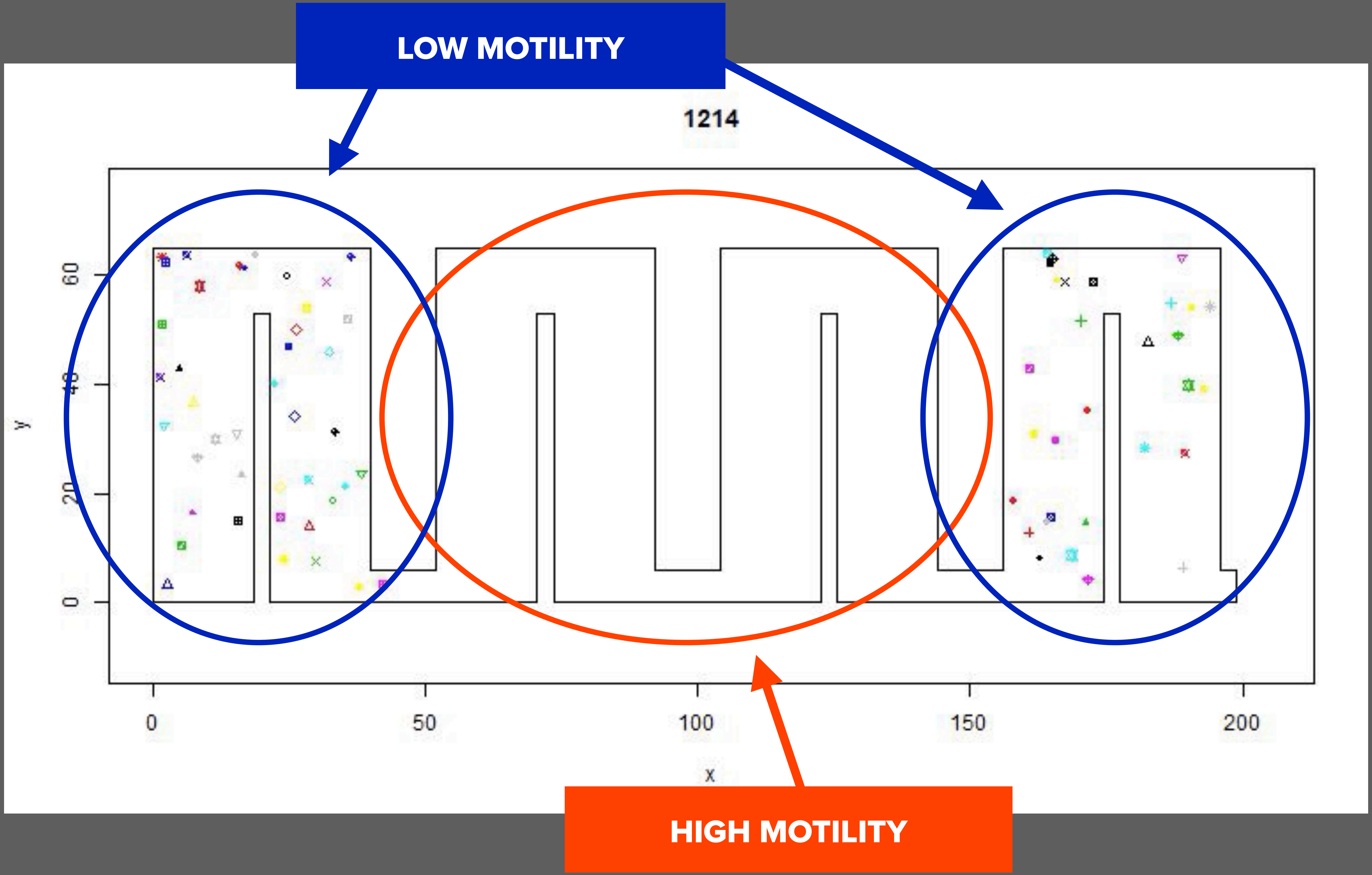
(James Russell et al., 2018)

We describe animal movement using a stochastic differential equation model with 2 parameters: **potential** and motility





We describe animal movement using a stochastic differential equation model with 2 parameters: potential and **motility**



Since we don't observe animal movement in continuous time, we **numerically approximate derivatives** (Euler-Maruyama method).

$$\frac{d\mathbf{r}_\tau}{dt} \approx \frac{\mathbf{r}_{\tau+1} - \mathbf{r}_\tau}{h_\tau}$$

$$\frac{d\mathbf{v}_\tau}{dt} \approx \frac{\mathbf{v}_{\tau+1} - \mathbf{v}_\tau}{h_\tau} \approx \frac{\mathbf{r}_{\tau+2} - \mathbf{r}_{\tau+1}}{h_\tau h_{\tau+1}} - \frac{\mathbf{r}_{\tau+1} - \mathbf{r}_\tau}{h_\tau^2}$$

Note that this works for data that is irregular in time.

where

- $\mathbf{r}_\tau = [x_\tau \;\; y_\tau]'$ is the position of ordered observation τ
- $\mathbf{v}_\tau$ is the (unobserved) velocity of observation τ
- h_τ is the change in time from observation τ to $\tau + 1$

Resulting in the **model equation**

$$\mathbf{r}_{\tau+2} = \left(1 + \frac{h_{\tau+1}}{h_\tau} - \beta h_{\tau+1}\right)\mathbf{r}_{\tau+1} + \left(\beta h_{\tau+1} - \frac{h_{\tau+1}}{h_\tau}\right)\mathbf{r}_\tau + \beta\ h_\tau h_{\tau+1}\ m(\mathbf{r}_\tau)\ [-\nabla p(\mathbf{r}_\tau)] + \sigma\ m(\mathbf{r}_\tau)\ h_{\tau+1}h_\tau^2\ N(\mathbf{0}, I)$$

Recall:

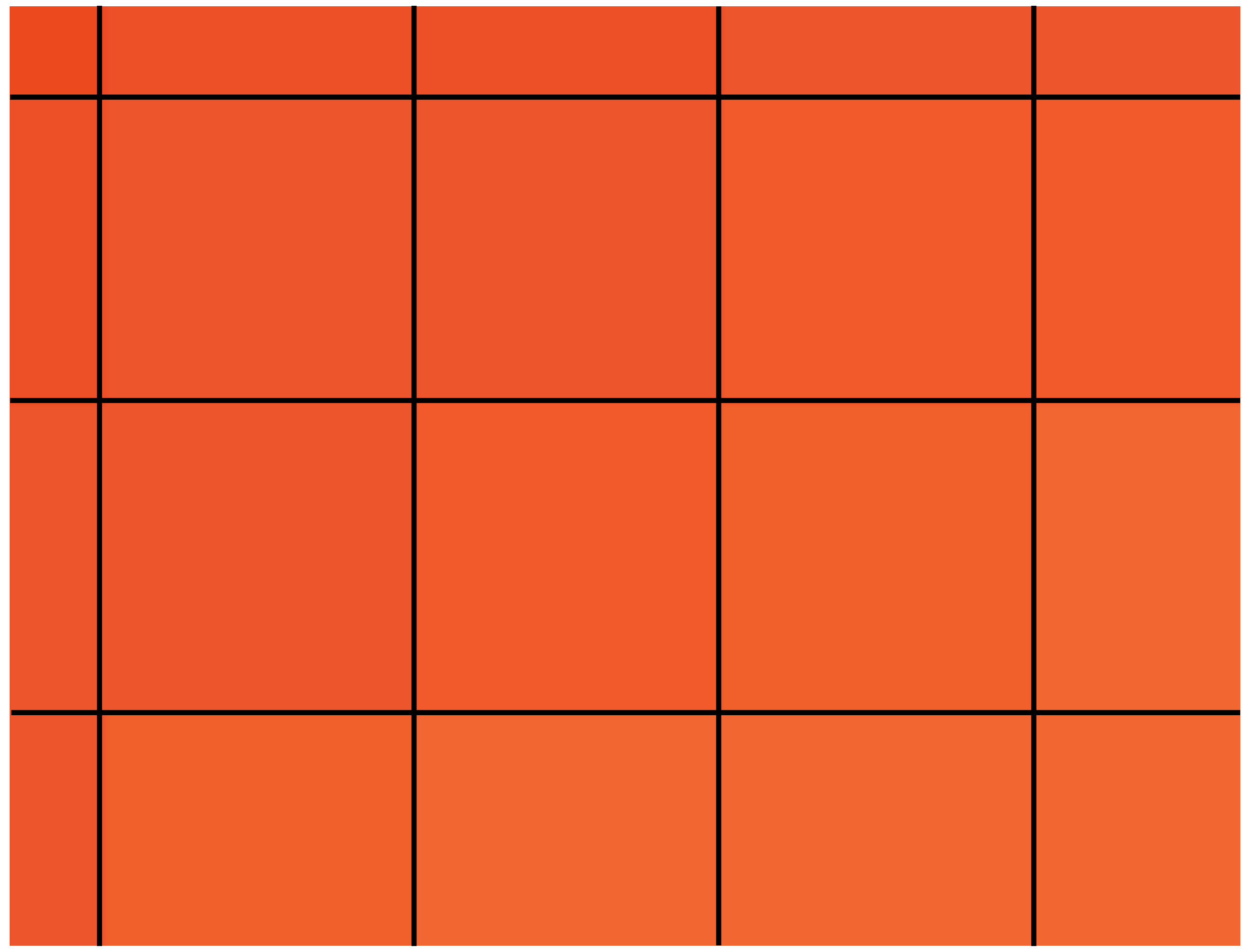
- $\mathbf{r}_\tau = [x_\tau\ \ y_\tau]'$ is the position of ordered observation τ
- $\mathbf{v}_\tau$ is the (unobserved) velocity of observation τ
- h_τ is the change in time from observation τ to $\tau + 1$

Spline expansion (degree 0, piecewise constant) of the motility and potential surfaces

$$m(\mathbf{r}_t) = \sum_{j=1}^{J} m_j s_j(\mathbf{r}_\tau)$$

$$p(\mathbf{r}_t) = \sum_{j=1}^{J} p_j s_j(\mathbf{r}_\tau)$$

$$s_j(\mathbf{r}_\tau) \equiv \begin{cases} 1, & \mathbf{r}_\tau \textbf{ in } j^{th} \textbf{ grid cell} \\ 0, & \textbf{otherwise} \end{cases}$$



Penalize the roughness of ***m*** and ***p***

Smoothness parameters are chosen with a holdout set.

$$\mathbf{r}_{\tau+2} = \left(1 + \frac{h_{\tau+1}}{h_\tau} - \beta h_{\tau+1}\right)\mathbf{r}_{\tau+1} + \left(\beta h_{\tau+1} - \frac{h_{\tau+1}}{h_\tau}\right)\mathbf{r}_\tau + \beta\ h_\tau h_{\tau+1}\ m(\mathbf{r}_\tau)\ [-\triangledown p(\mathbf{r}_\tau)] + \sigma\ m(\mathbf{r}_\tau)\ h_{\tau+1} h_\tau^2\ N(\mathbf{0}, I)$$

Goal – Estimate $\mathbf{m} \equiv [m_1\ \ldots\ m_J]'$ and $\mathbf{p} \equiv [p_1\ \ldots\ p_J]'$ for J grid cells with an iterative approach:

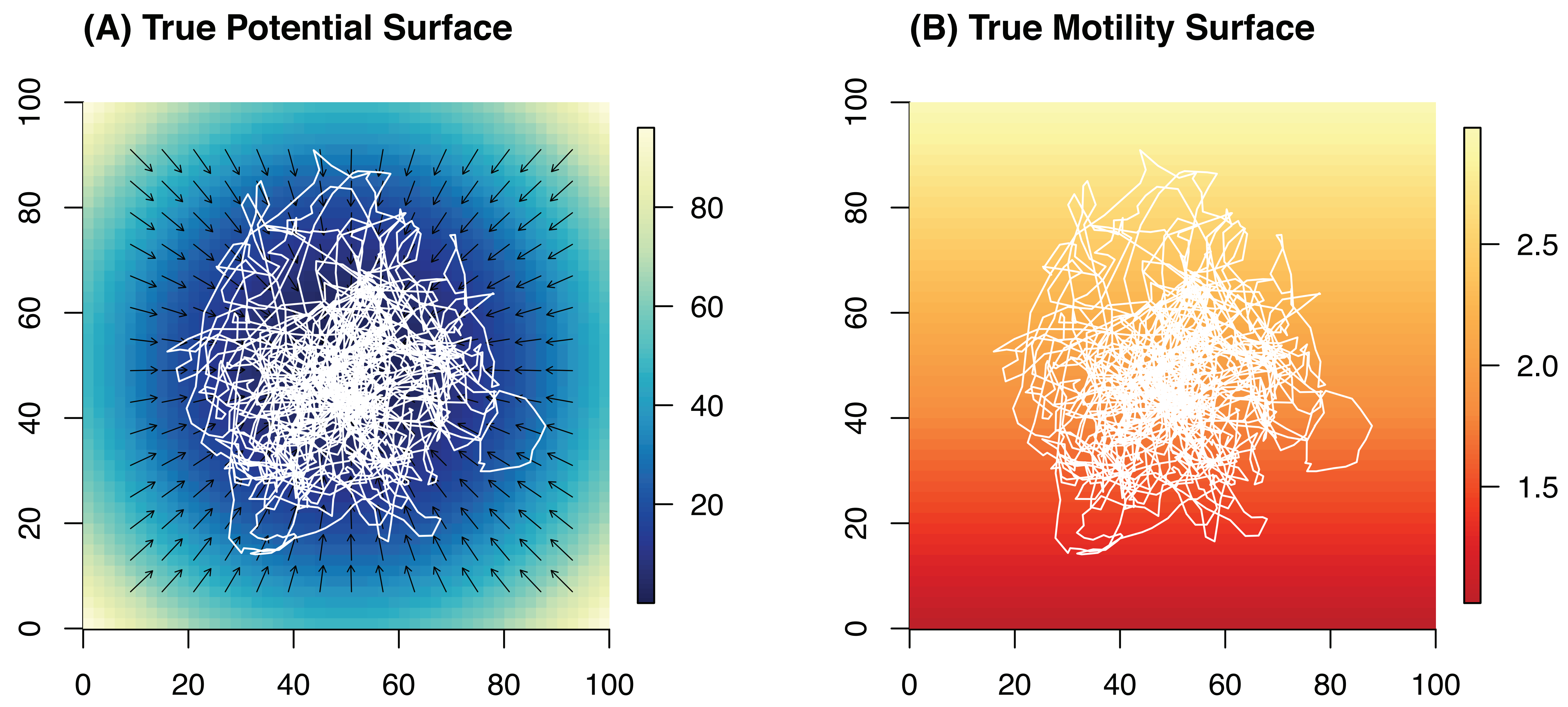
1. Obtain a preliminary estimate of mean parameters (β and $\mathbf{p}$) assuming the motility surface is constant (model errors are i.i.d.).
2. Estimate the variance parameters ($\mathbf{m}$) using residuals from step 1.
3. Estimate mean parameters (β and $\mathbf{p}$) conditioned on the variance estimates from step 2.

Computing time **~20 minutes** (single core)

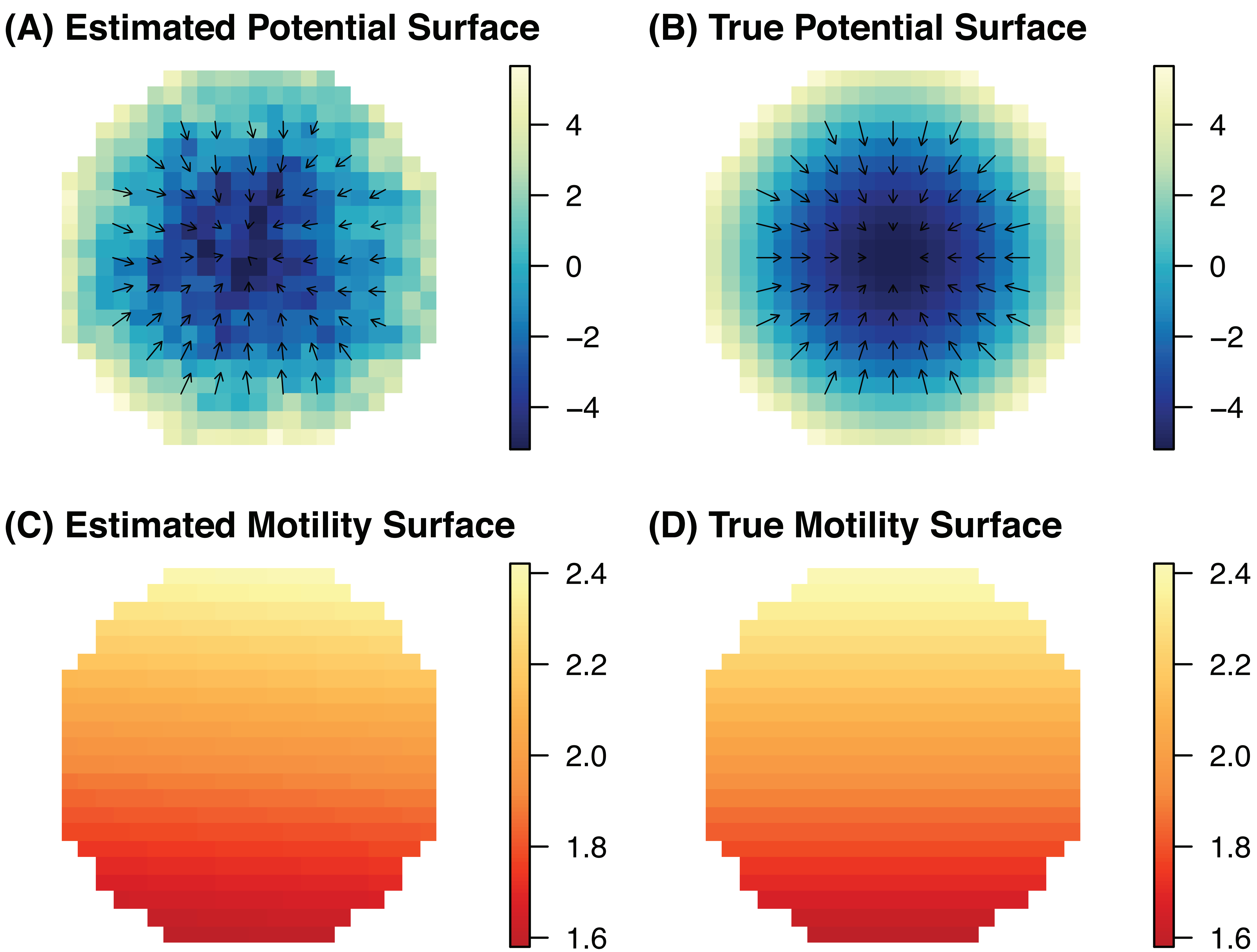
- 14,401 x 78 data points

We conducted a **simulation study** to quantify the bias in the estimation procedure.

One of 500 simulations

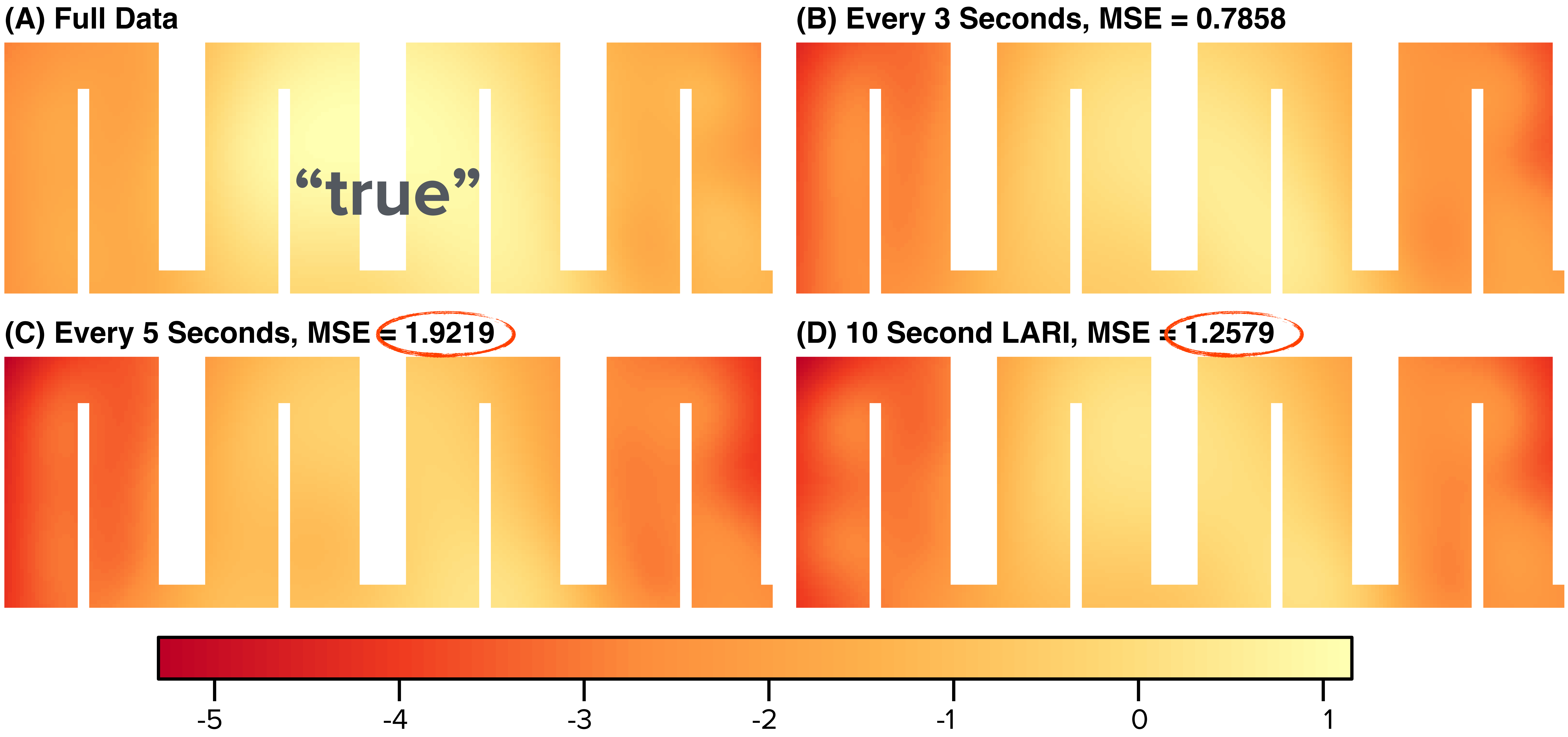


True and estimated surfaces for one randomly selected simulation

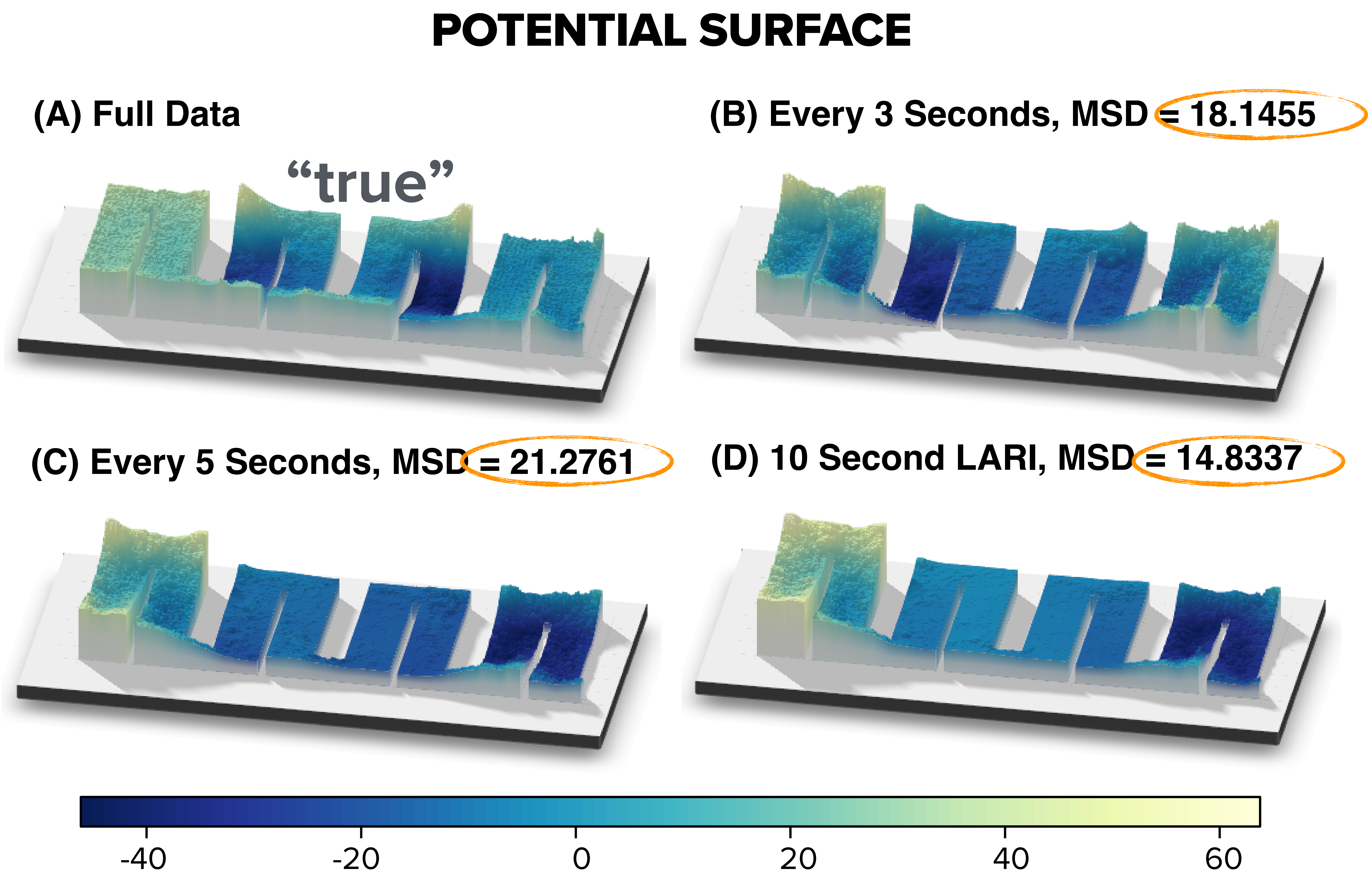


We **compared true and estimated motility and potential** surfaces using multiple metrics.

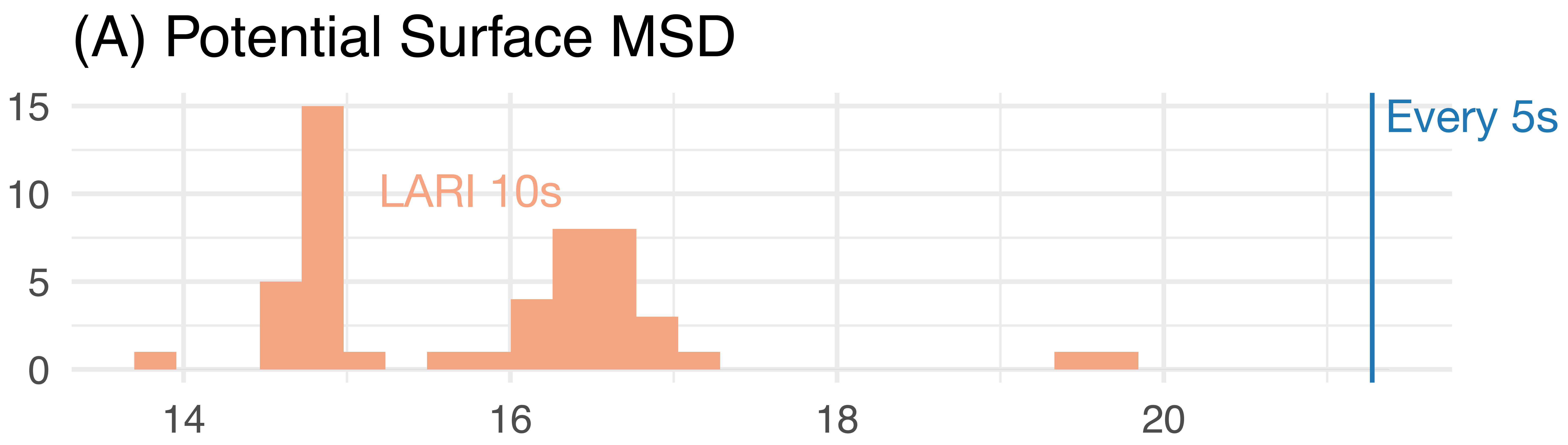
We **compare motility and potential** surfaces estimated with the 4 subsamples using multiple metrics.

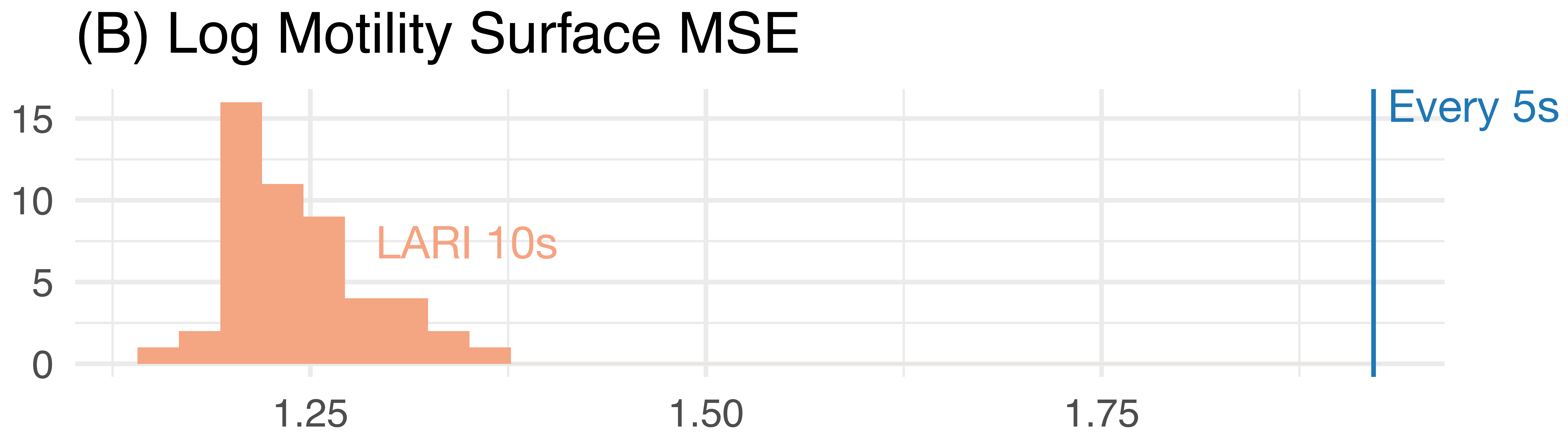


We **compare motility and potential** surfaces estimated with the 4 subsamples using multiple metrics.



We fit 50 different LARI subsamples to understand random variation.

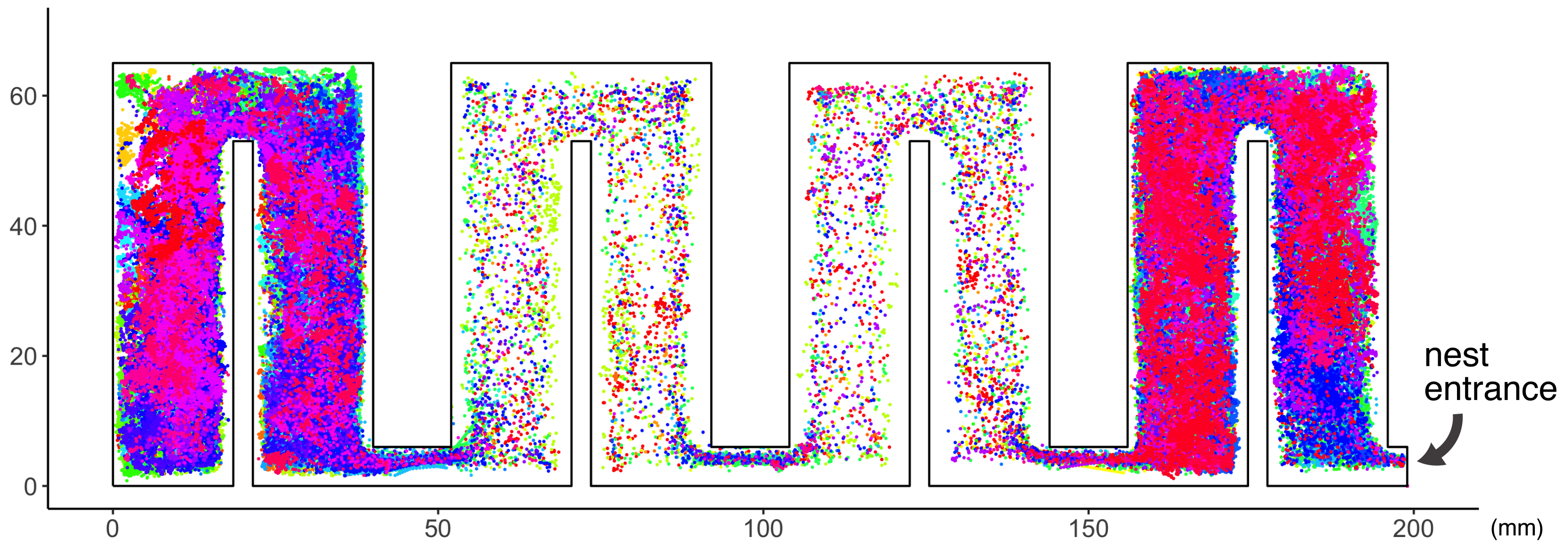




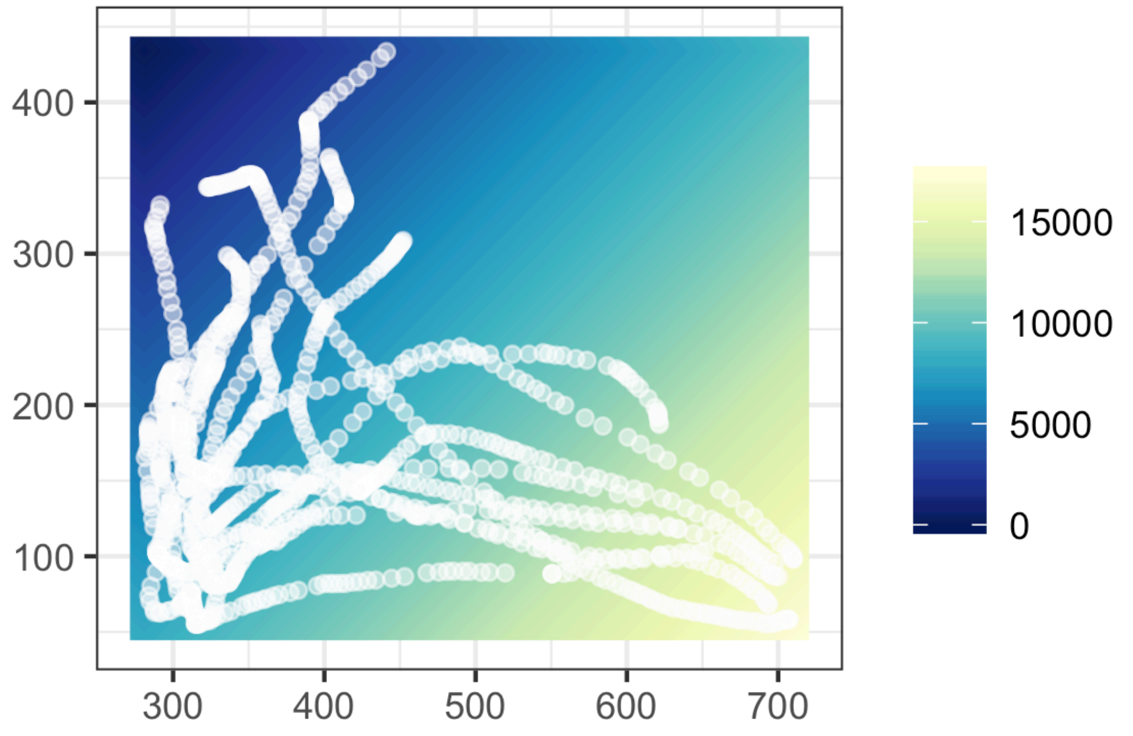
Result: **LARI sampling was better** than regular sampling overall for understanding movement behavior. A simulation study and additional data example support this conclusion. It may also be better for estimating missing data.

Conclusion: Regular sampling may not always be the best choice.

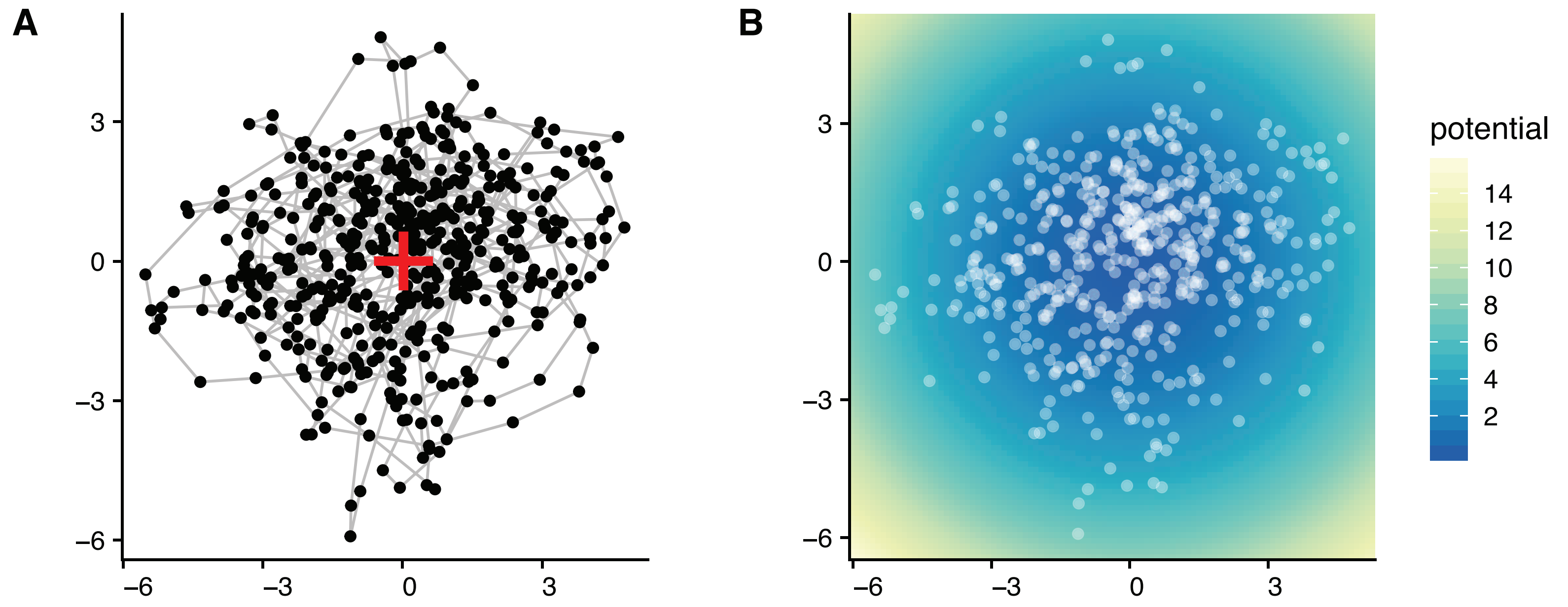
Ant data



Guppy data



Simulated data



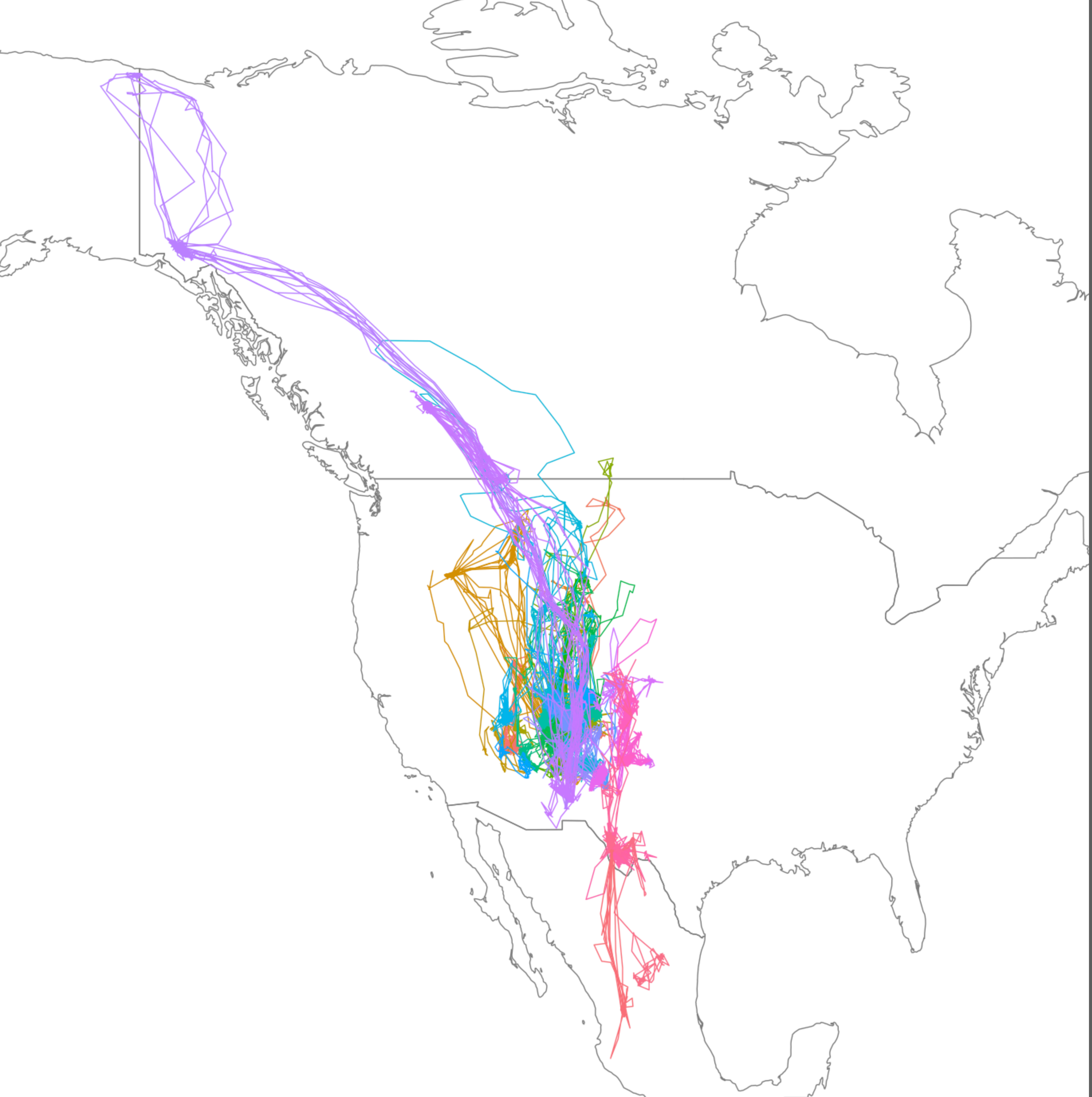
- **Eisenhauer**, Elizabeth, and Ephraim Hanks. "A lattice and random intermediate point sampling design for animal movement." *Environmetrics* (2020): e2618.
- Wijeyakulasuriya, D. A., **Eisenhauer**, E. W., Shaby, B. A., & Hanks, E. M. "Machine learning for modeling animal movement." *Plos one* 15.7 (2020): e0235750.


Modeling Yearly Patterns in
Golden Eagle Movement

Golden eagles, like many species, display **partial migration**, meaning only some individuals in the population migrate.

Due to climate change, we can expect migration to change or become less common. Thus it is important that we can **identify** and **quantify** different movement strategies.

Current methods to classify movement strategies work best on the most stereotypical cases and are often in disagreement (Cagnacci et al., 2016).



Data collection funded by the USFWS

68 eagles with at least 1 year of data

Large **variability** in individual movement behaviors

Each color is one individual

Big picture goals:

1. Relatively **simple** model (few parameters)
2. Capture the full range of movement behavior from **resident** to **migrant** to **dispersal**.
3. Use to **classify individuals** and better understand boundary individuals

We describe animal movement using a **stochastic differential equation (SDE)** model with a constant motility surface.

Data: $\mathbf{r}_t,\ t = 1, 2, \ldots, T$ for each eagle

SDE model framework:

$$d\mathbf{r}_t = \mathbf{v}_t\ dt$$
$$d\mathbf{v}_t = \beta\ (\boldsymbol{\mu}(\mathbf{r}_t, t) - \mathbf{v}_t)\ dt + \sigma\ d\mathbf{w}_t$$

Utilizing a potential function, define:

$$\boldsymbol{\mu}(\mathbf{r}_t, t) = -\nabla\, p(\mathbf{r}_t, t) \qquad \text{(mean drift)}$$

(James Russell et al., 2018)

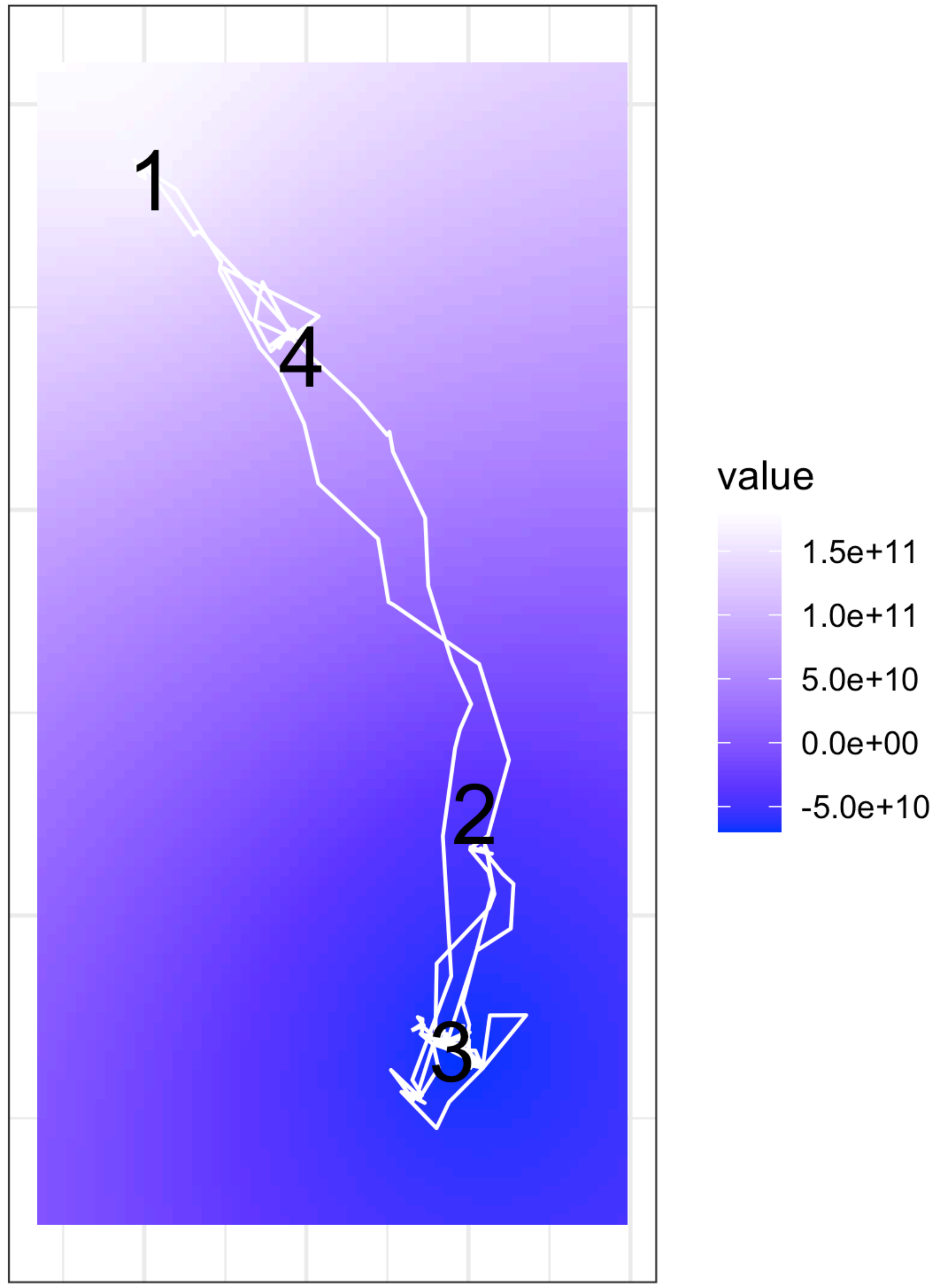
We again use **Euler-Maruyama** approximations.
We assume **regular time steps**.

Resulting model equation:

$$\mathbf{r}_{t+2} - 2\mathbf{r}_{t+1} + \mathbf{r}_t = \beta\left(-\nabla p(\mathbf{r}_t) - \mathbf{r}_{t+1} + \mathbf{r}_t\right) + \sigma\,\epsilon_t$$

where $p(\mathbf{r}_t) = \sum_{i=1}^{4} k_{it}\sqrt{(x_t - a_{xi})^2 + (y_t - a_{yi})^2}$,

a weighted sum of distances to fixed k means attractors.



We want the coefficient of attraction k_{it} for attractor i to **change over time.**

Methods:

1. **Varying coefficient model** allows k_{it} to change smoothly over time.
2. **Latent-state model** allows k_{it} to switch between discrete values over time.
   - Note: The term latent-state model is used over HMM because of the feedback inherent in the model, i.e., $\mathbf{r}_t$ depends on the previous 2 time points as well as the state at time t

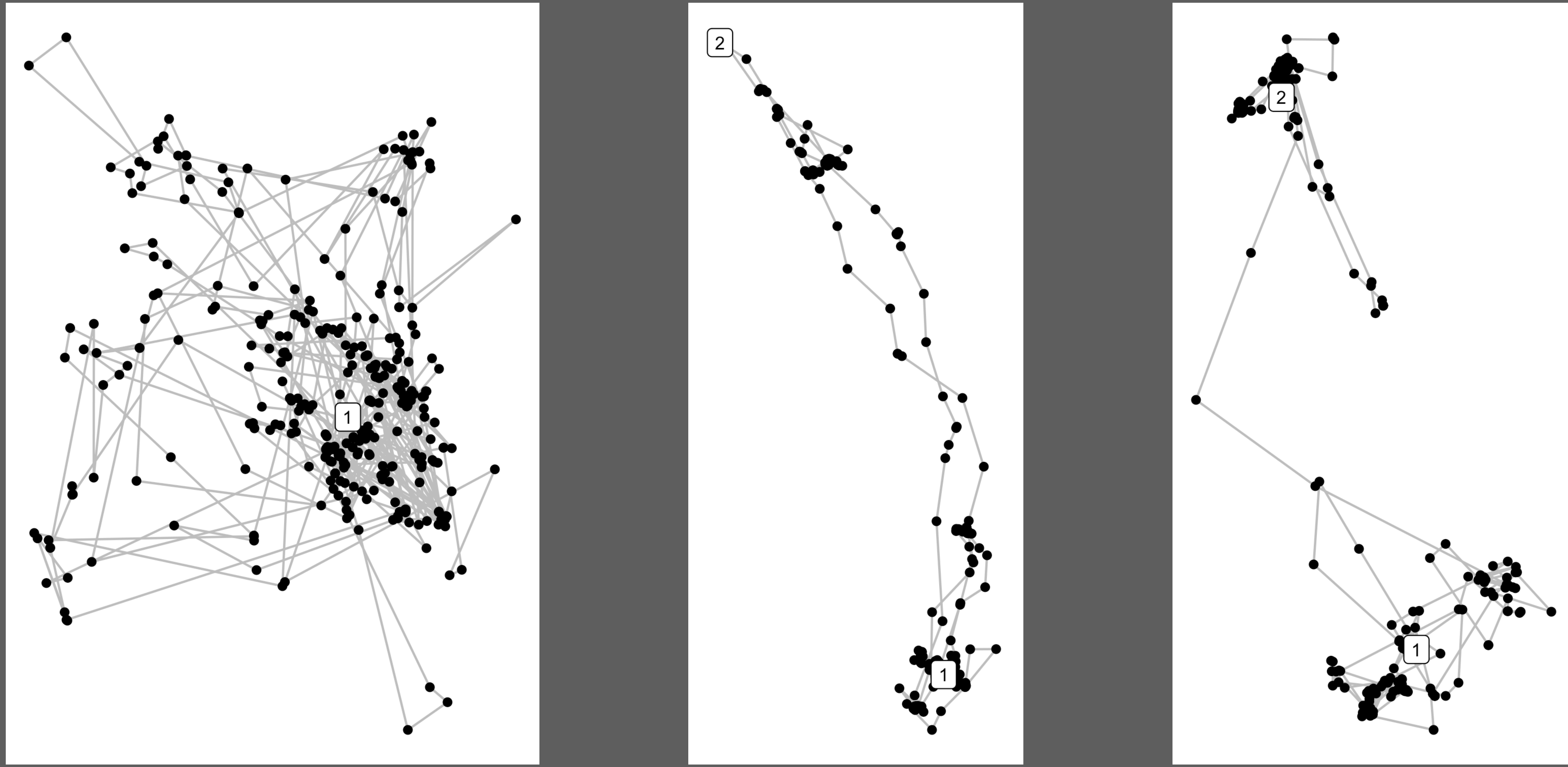
Varying coefficient model:

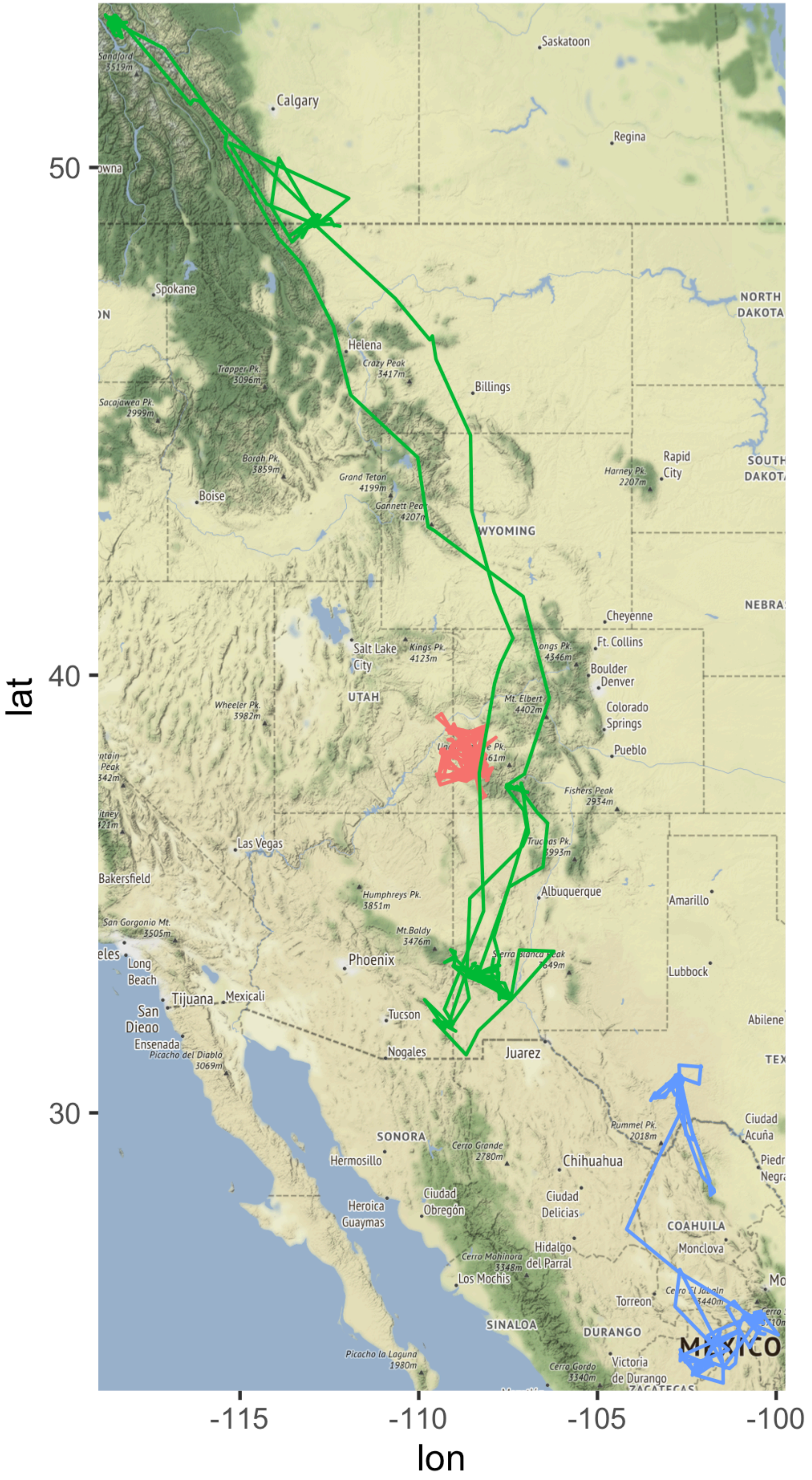
1. k_{it} changes smoothly over time
2. Fit using **GAM** function in R
3. The **same model** for resident, migrant, and disperser

Latent-state model:

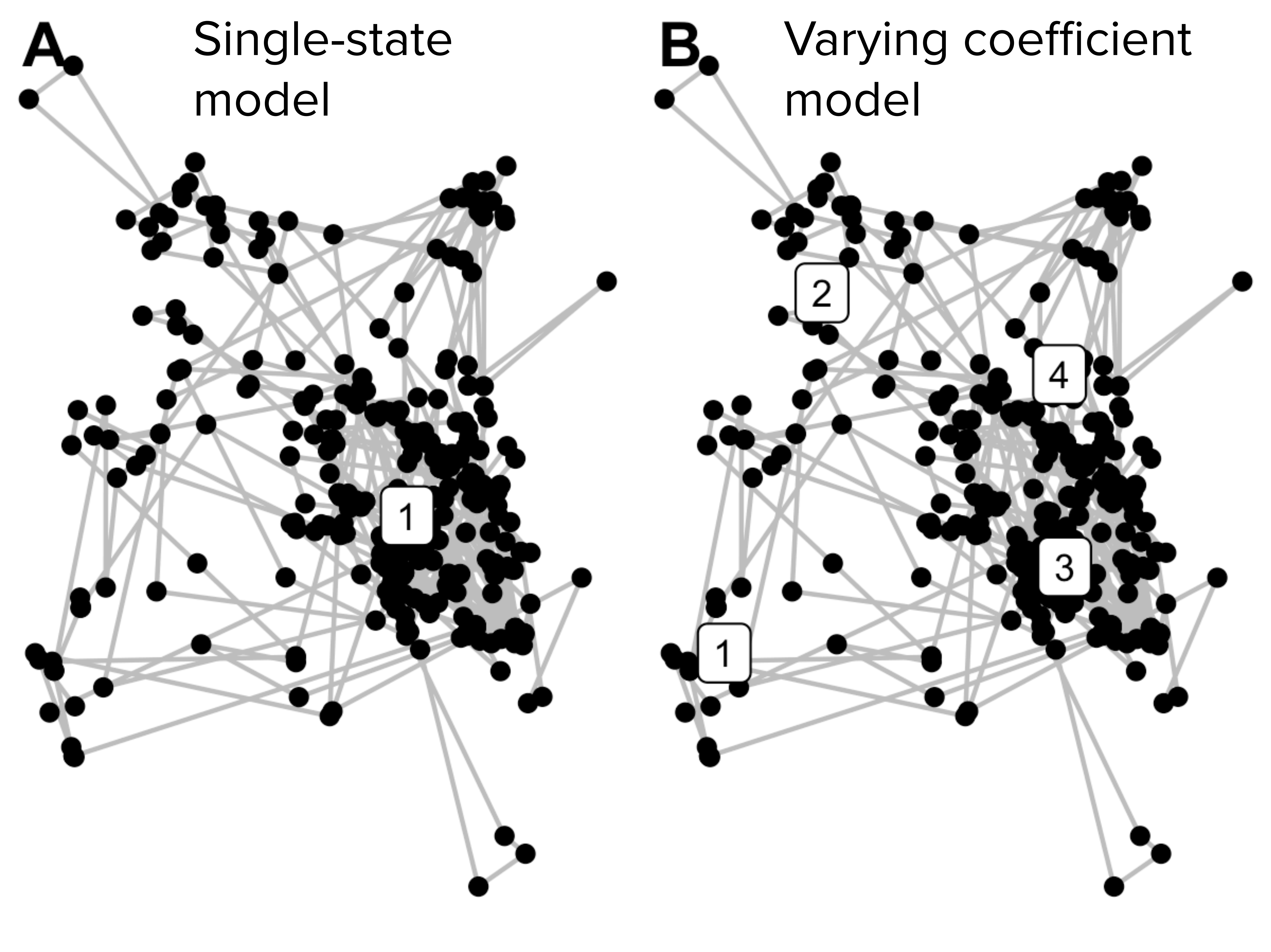
1. k_{it} switches between discrete values for each state
2. We fit each model in a **Bayesian framework**
   - We sample using the No U-Turn Sampler implemented in Stan

We will fit these models for **3 representative individuals**





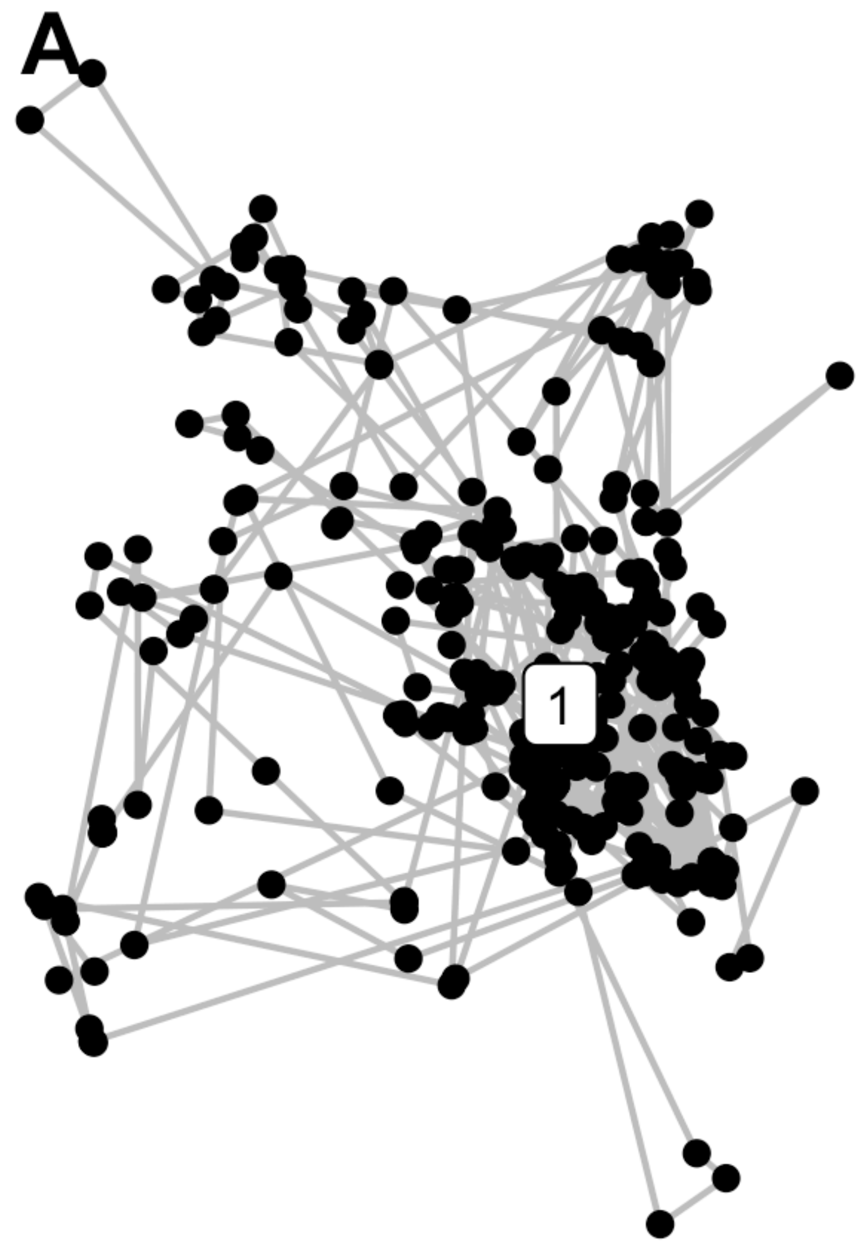
Resident: 4C.Cahone in 2015

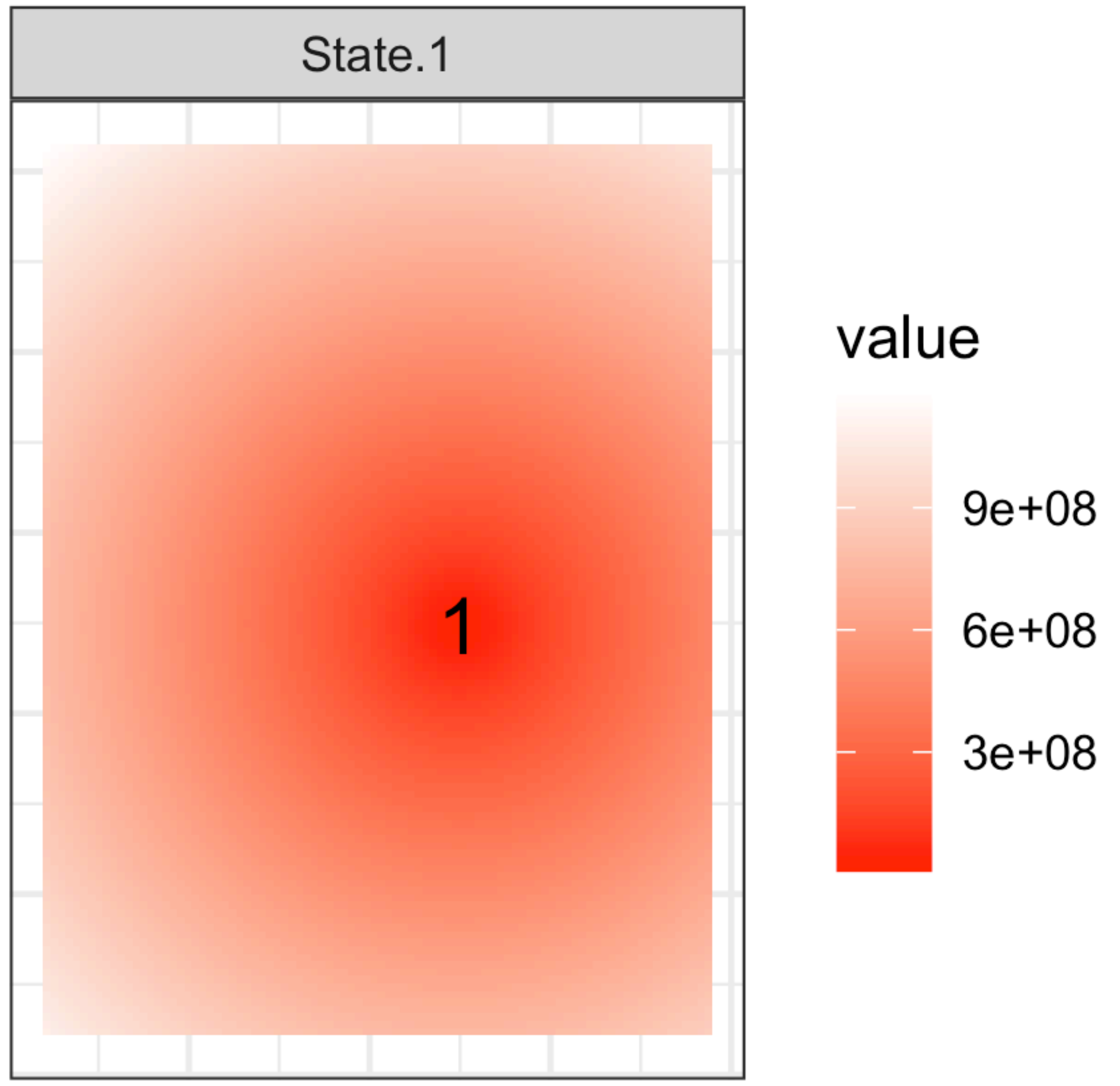


Single-State Model for Resident

$m = 1$ attractor with coefficient of attraction $k_{1_t} = k$

$$p(\mathbf{r}_t) = k\sqrt{(x_t - a_{x1})^2 + (y_t - a_{y1})^2}$$



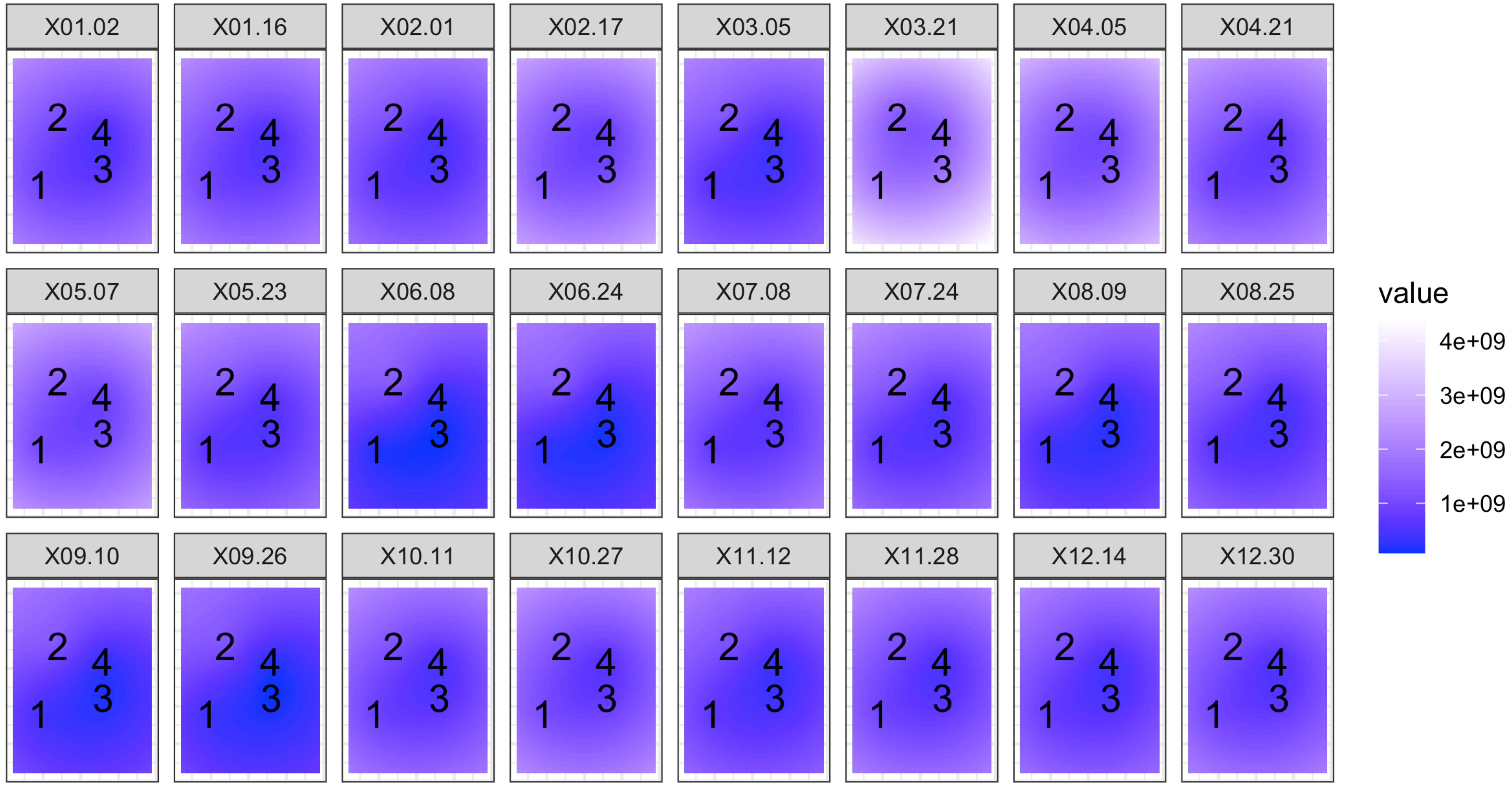


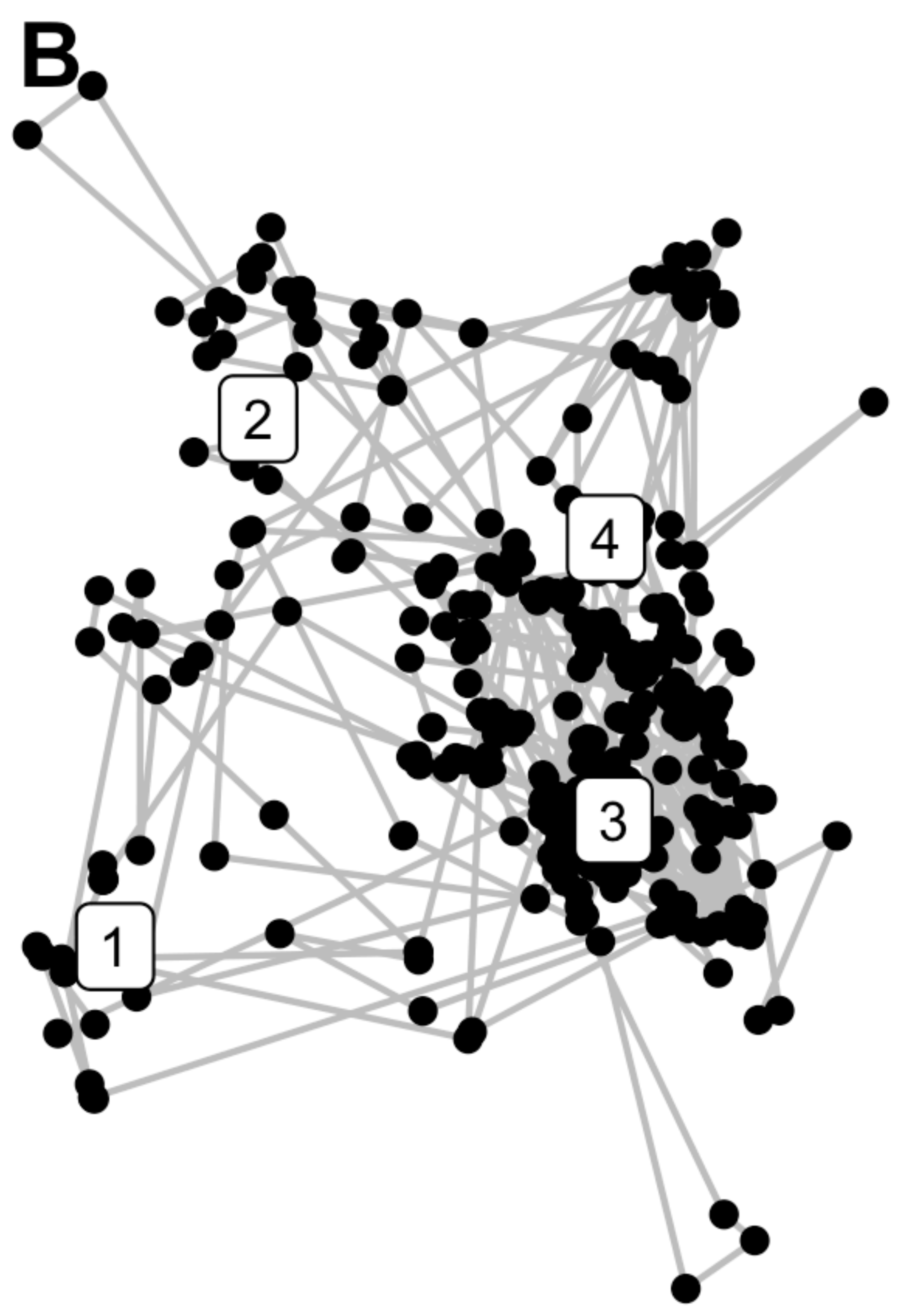
Varying Coefficient Model for Resident

Fixed $m = 4$ attractors and k_{it} for $i = 1,2,3,4$ changes smoothly over time.

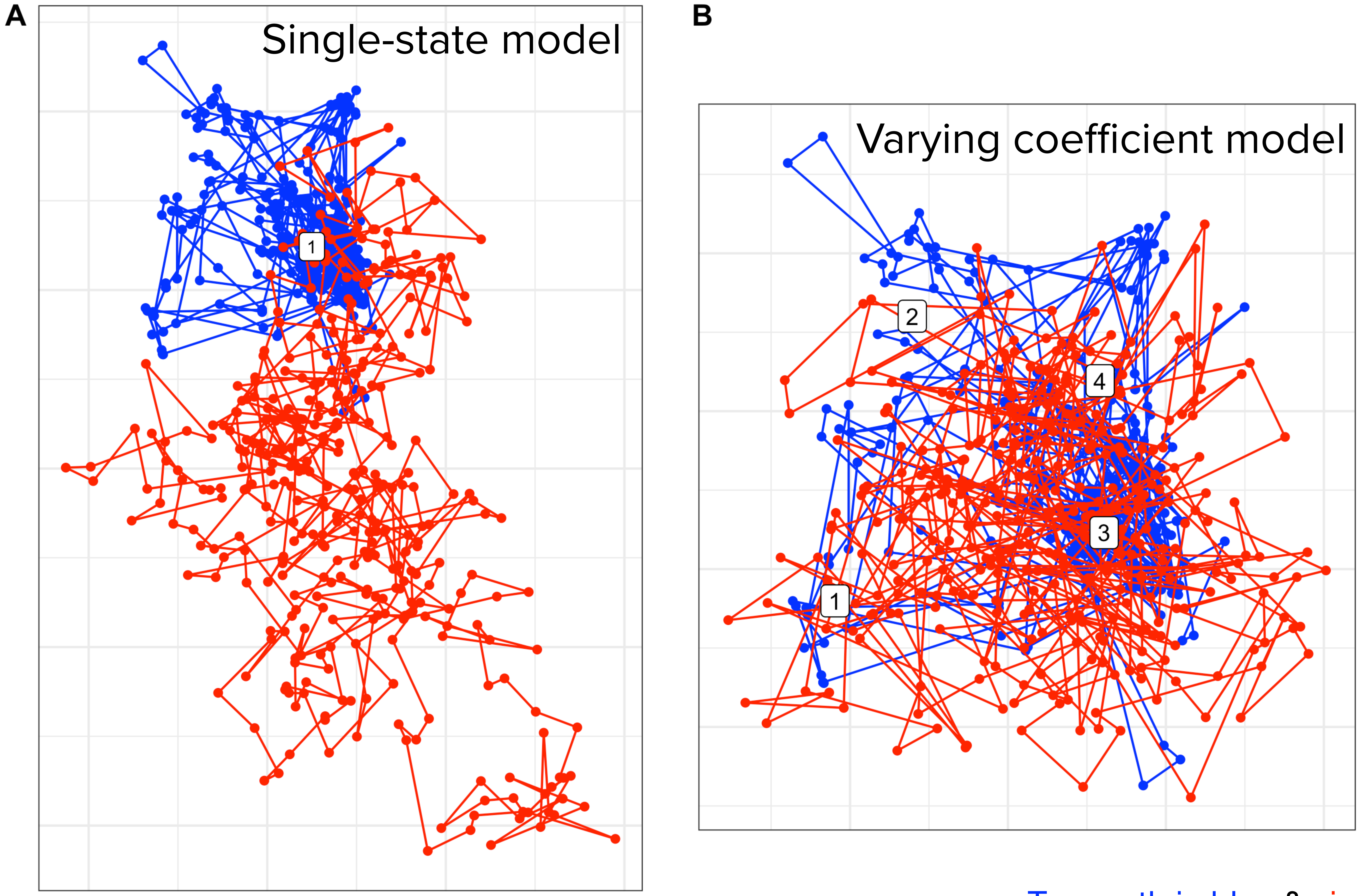
$$p(\mathbf{r}_t) = \sum_{i=1}^{m} \sum_{j=1}^{J} \alpha_{ij} B_j(t) \sqrt{(x_t - a_{xi})^2 + (y_t - a_{yi})^2}$$

where α_{ij} is the coefficient of the j^{th} cyclic cubic basis function $B_j(t)$ for attractor i



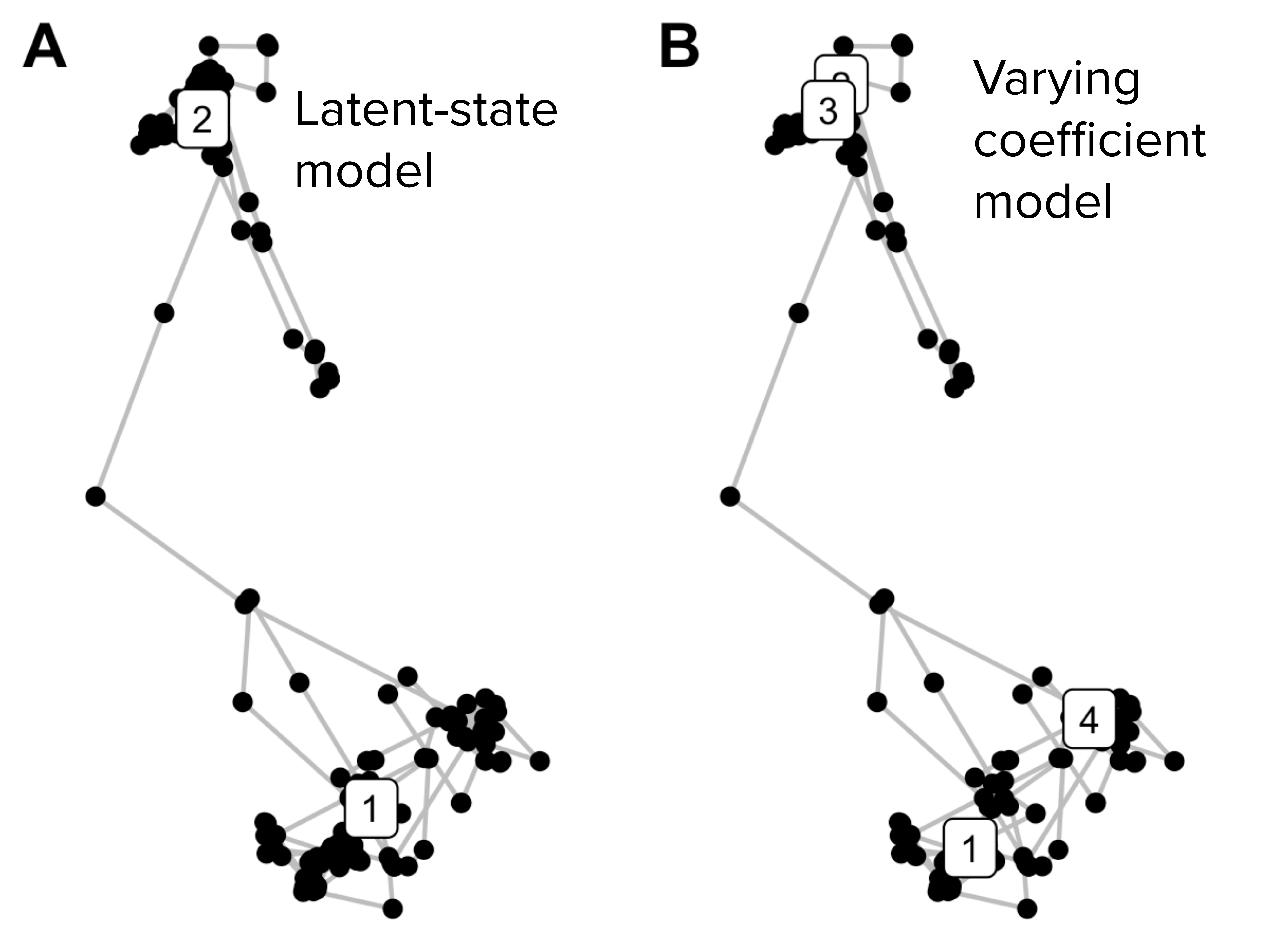


Simulations for **resident**



True path in blue & simulation in red

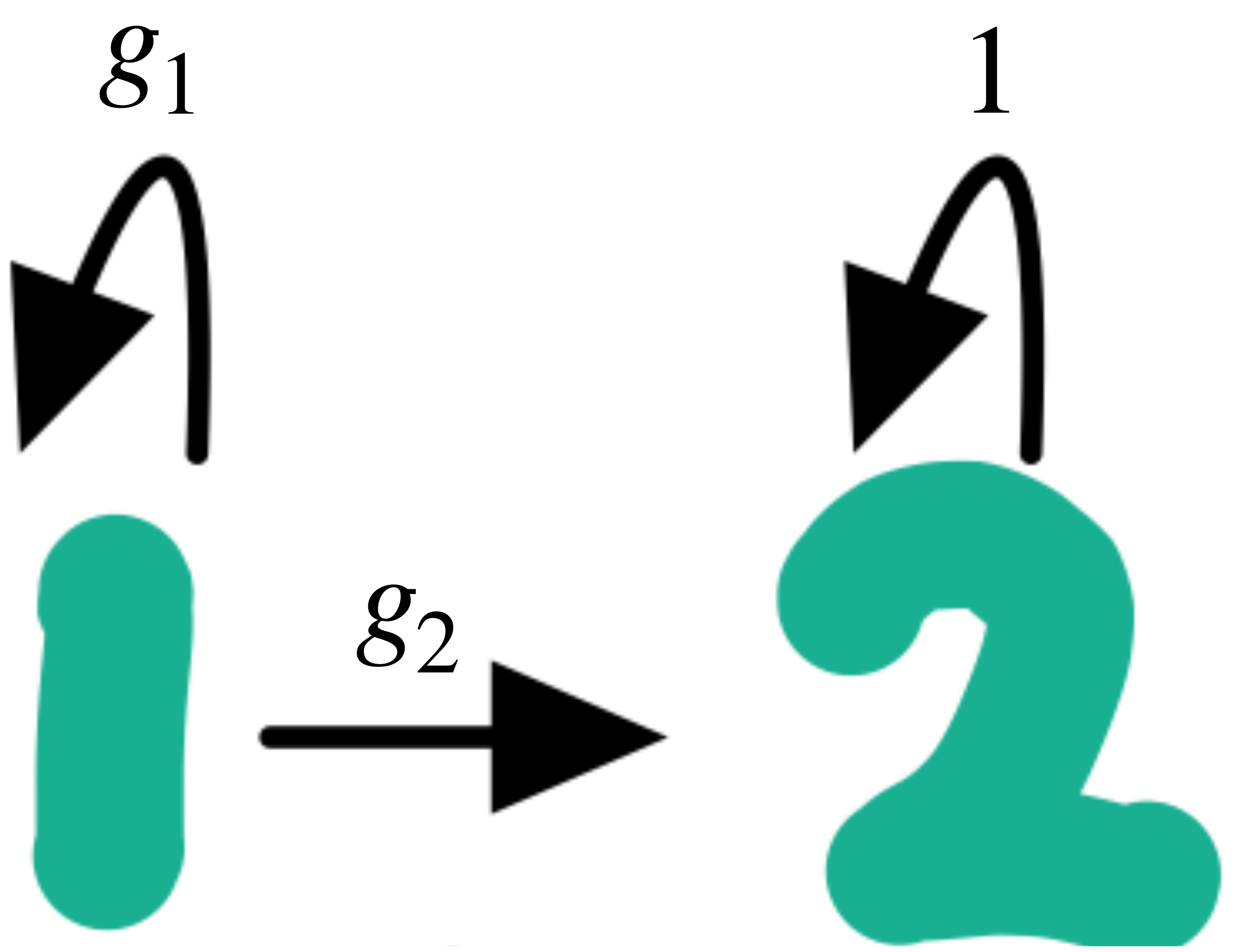
Dispersal: TP.N Zacatecas in 2018



Latent-State Model for Dispersal

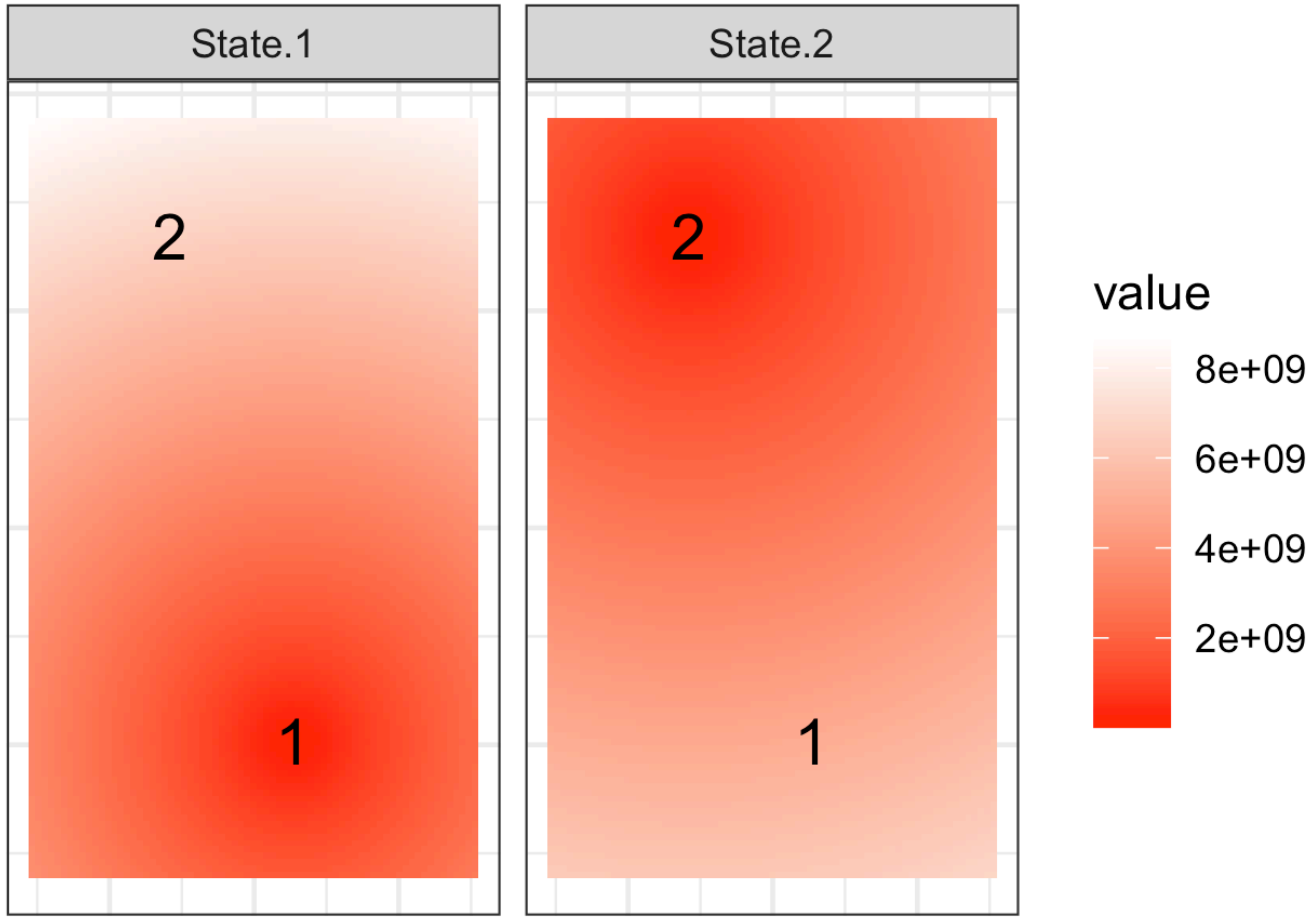
$$p(\mathbf{r}_t, s_t) = k_{s_t}\sqrt{(x_t - a_{xs_t})^2 + (y_t - a_{ys_t})^2}$$

Latent states:



Attractor 1 and coefficient k_1

Attractor 2 and coefficient k_2

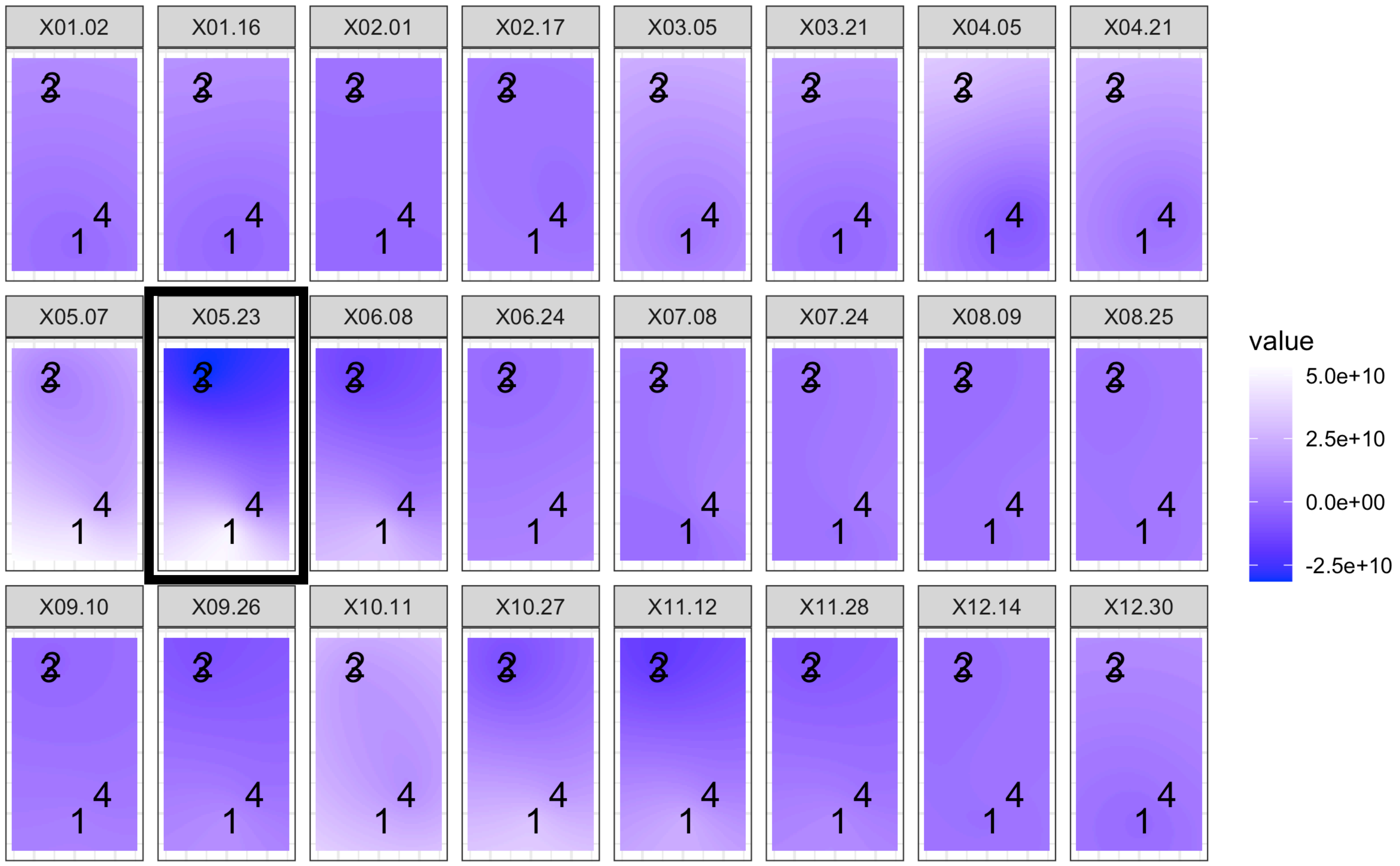


Varying Coefficient Model for Dispersal
(identical to varying coefficient model for resident)

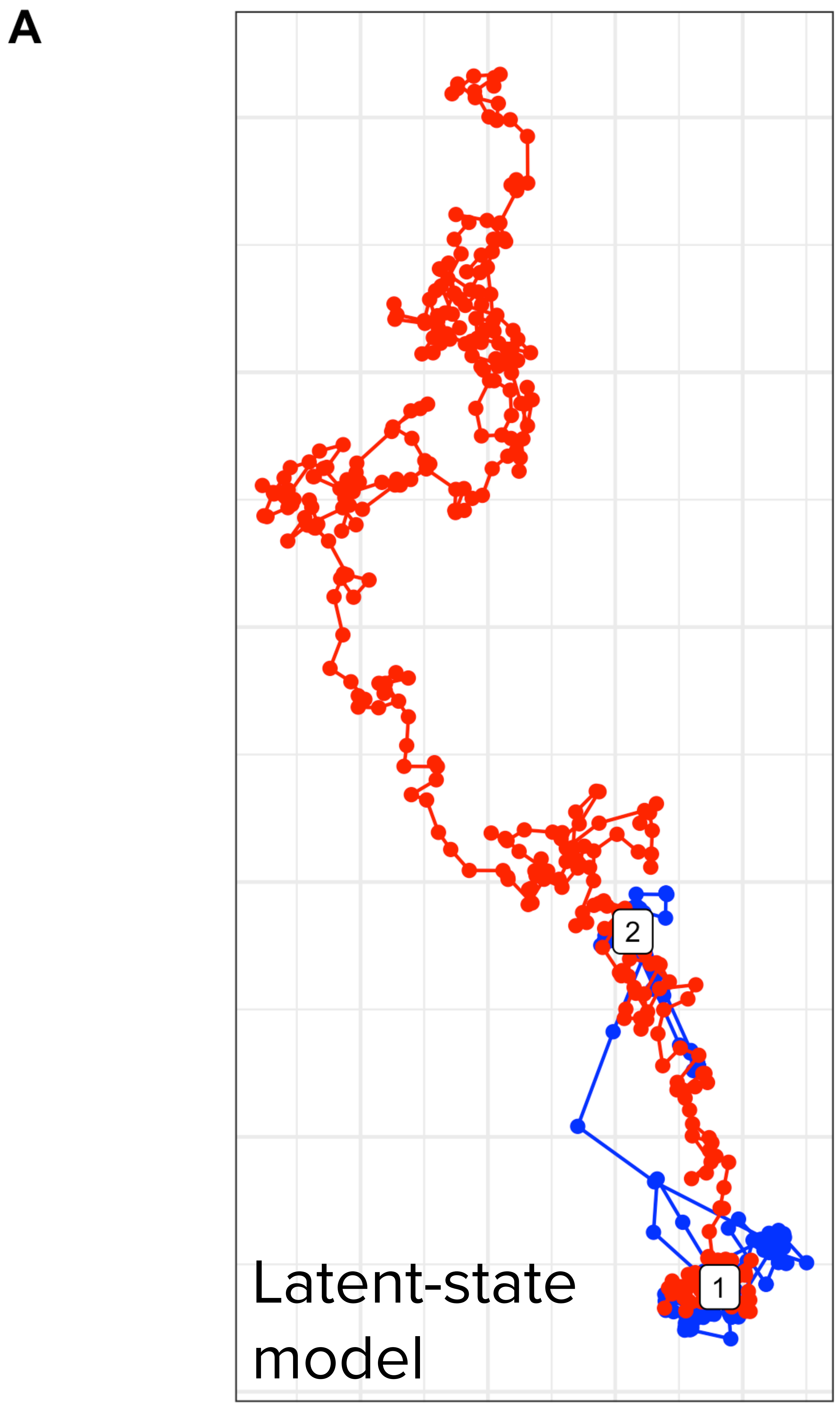
Fixed $m = 4$ attractors and k_{it} for $i = 1,2,3,4$ changes smoothly over time.

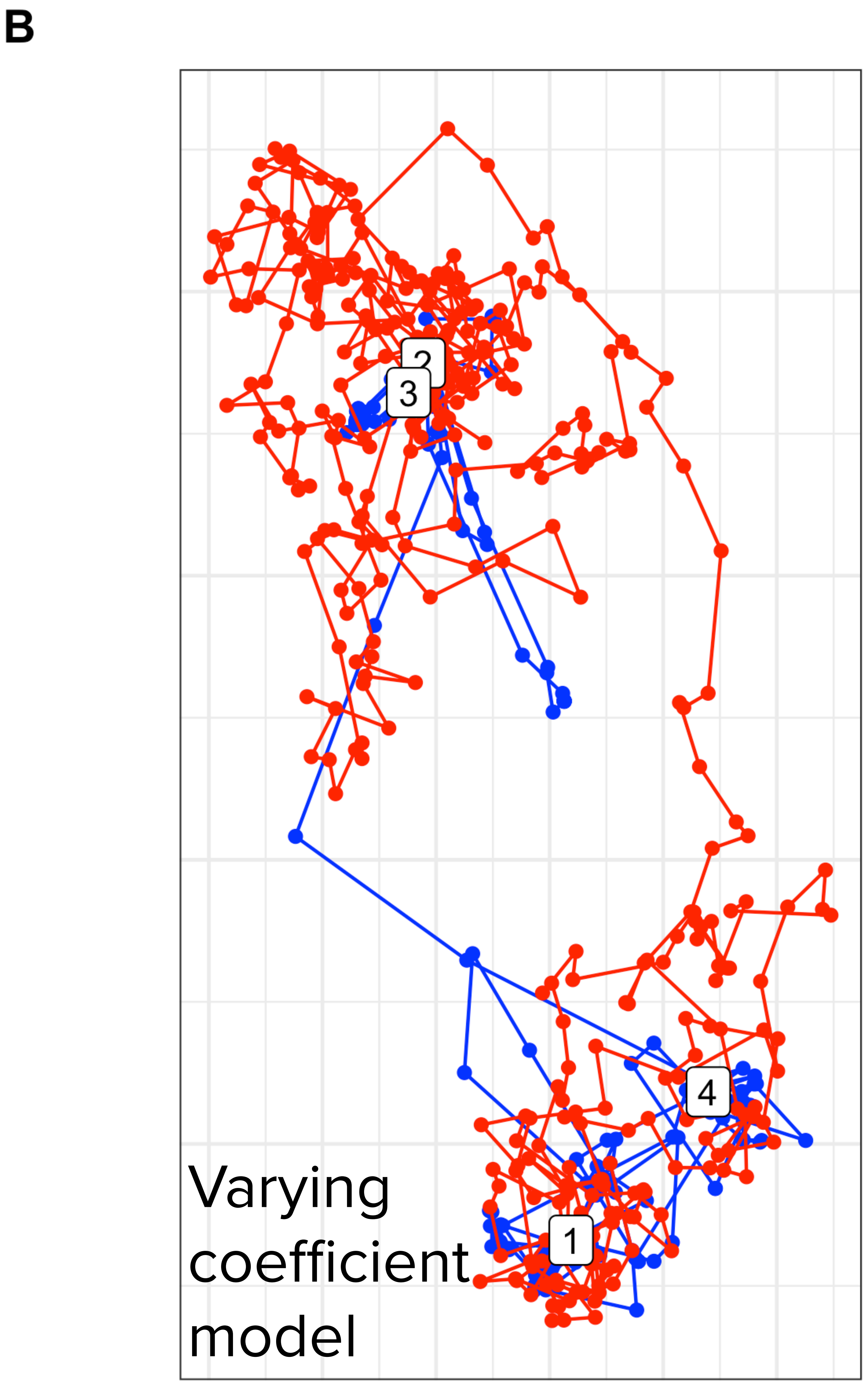
$$p(\mathbf{r}_t) = \sum_{i=1}^{m} \sum_{j=1}^{J} \alpha_{ij} B_j(t) \sqrt{(x_t - a_{xi})^2 + (y_t - a_{yi})^2}$$

where α_{ij} is the coefficient of the j^{th} cyclic cubic basis function $B_j(t)$ for attractor i

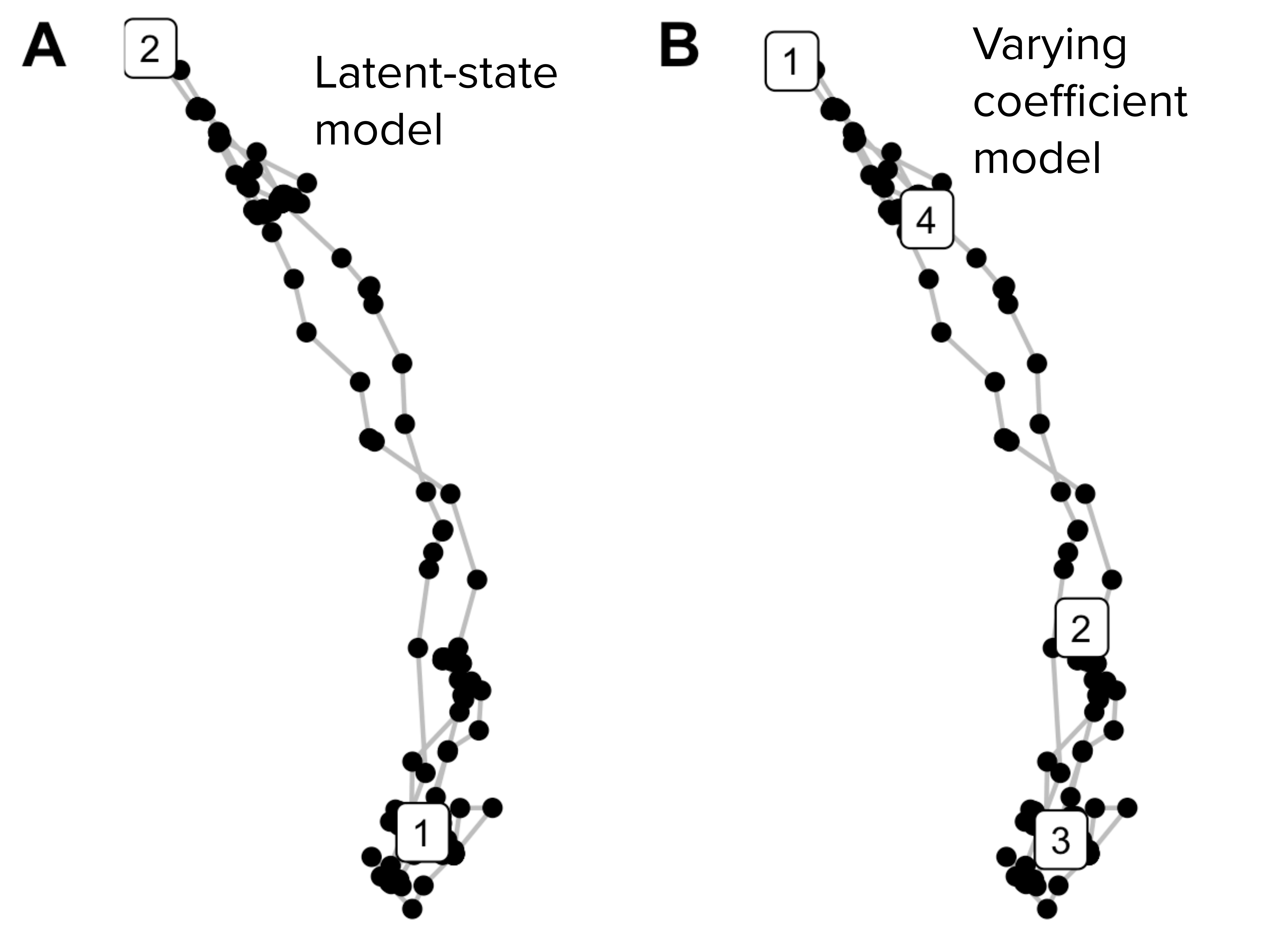


Simulations for **dispersal**



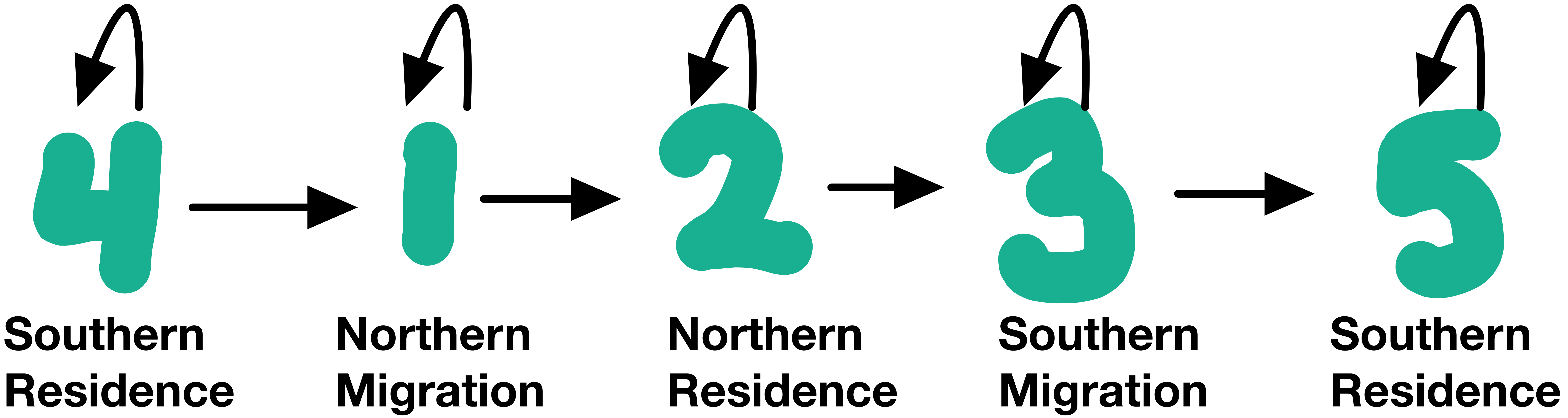


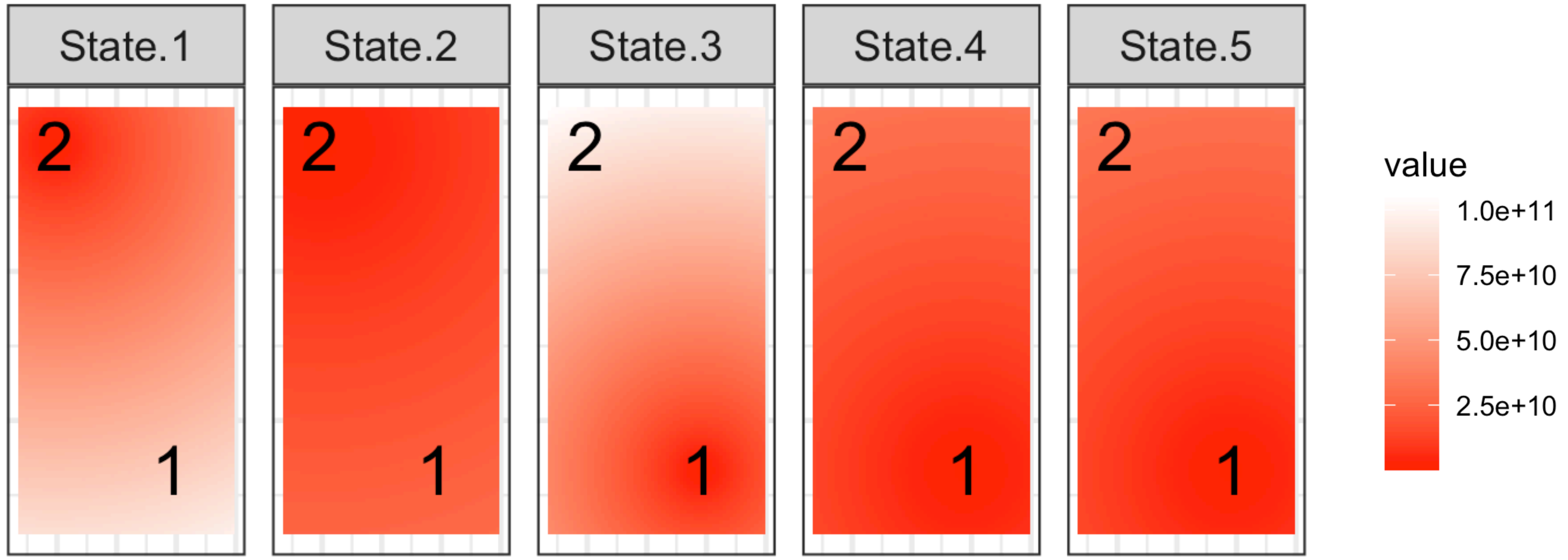
Migrant: NM.Tredwell in 2012



Latent-State Model for Migration

Latent states:





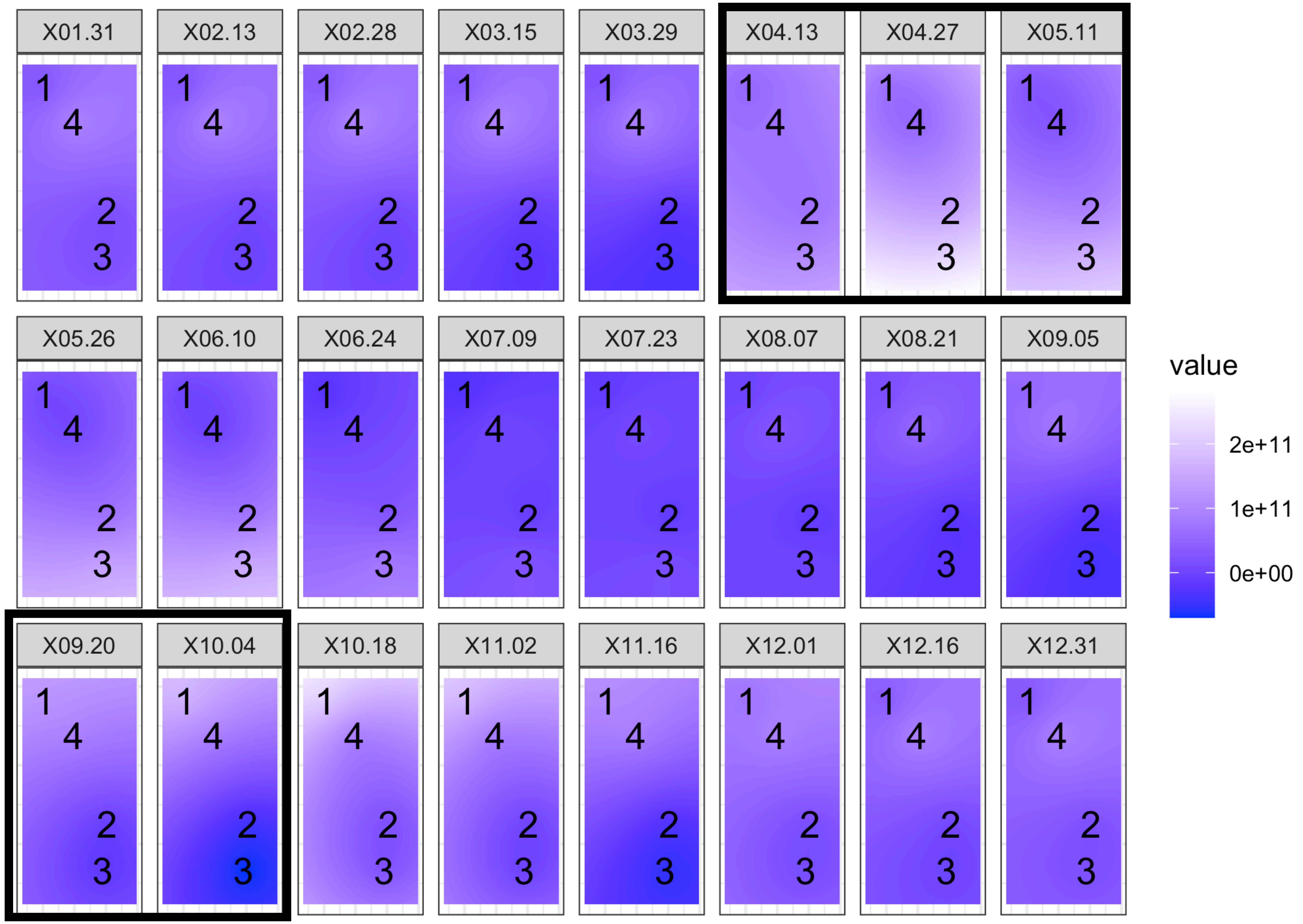
Varying Coefficient Model for Migrant
(identical to varying coefficient model for resident)

Fixed $m = 4$ attractors and k_{it} for $i = 1,2,3,4$ changes smoothly over time.

$$p(\mathbf{r}_t) = \sum_{i=1}^{m} \sum_{j=1}^{J} \alpha_{ij} B_j(t) \sqrt{(x_t - a_{xi})^2 + (y_t - a_{yi})^2}$$

where α_{ij} is the coefficient of the j^{th} cyclic cubic basis function $B_j(t)$ for attractor i

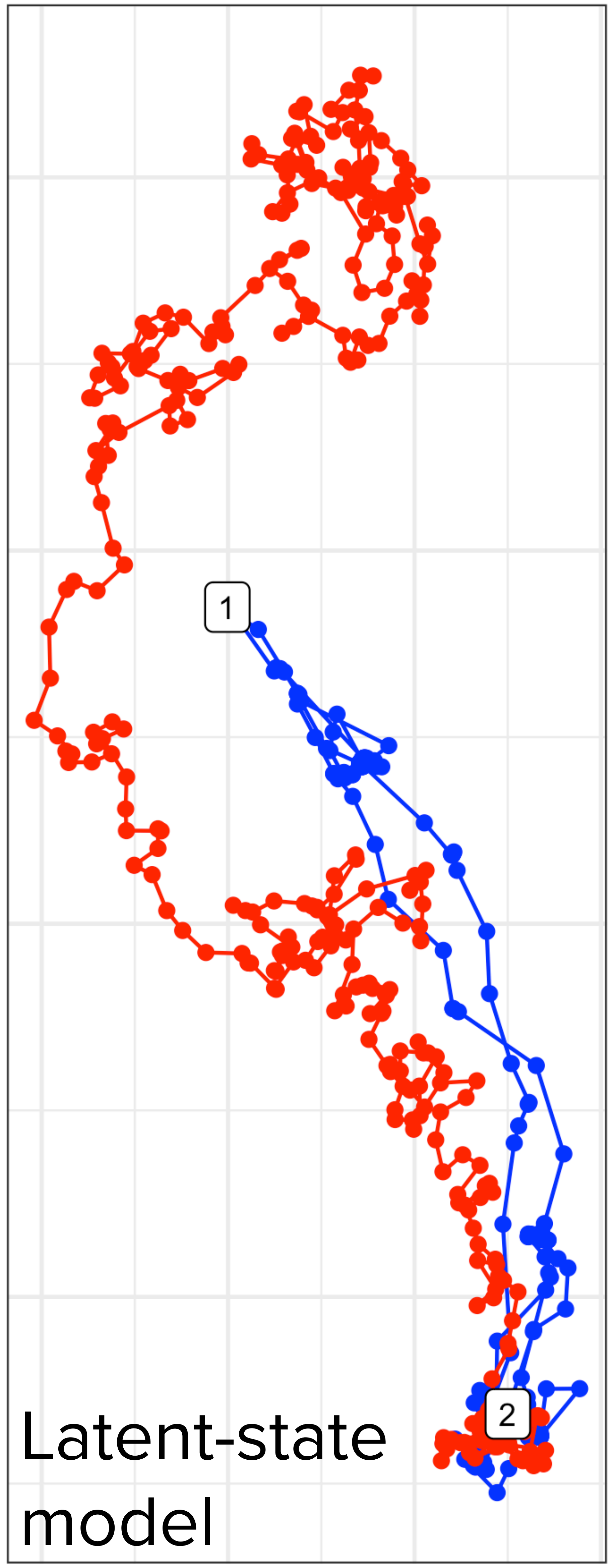
Northern migration



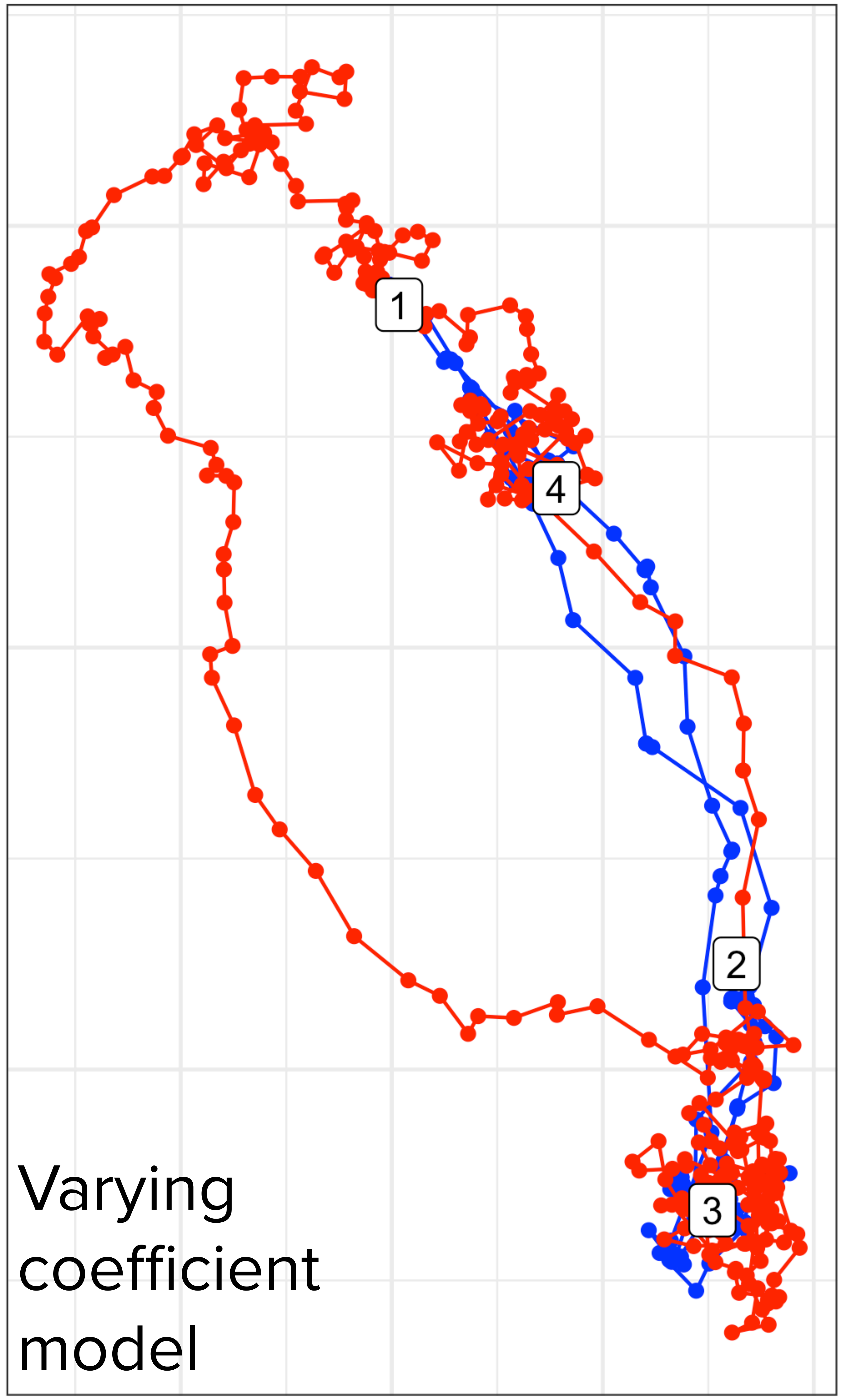
Southern migration

Simulations for **migrant**

A



B



Future work:

1. **Classify** paths as migrant, resident, or disperser.
2. Make varying coefficient model more interpretable by **restricting attractor coefficients** to be positive
3. Fit varying coefficient model in a **Bayesian framework**
4. Fit **more individuals**, including some boundary individuals

Survey of Attitudes Toward Probability

We provide one of the first assessments of students' attitudes about the subject of probability, called the **Survey of Attitudes toward Probability (SAP)**.

The SAP could be a **tool for researchers** to inform changes in probability courses.

Why do we care about students' attitudes toward probability?

- Probability is a **foundational course** for students interested in pursuing research or a degree in a quantitative field.
- Often the first or second course taken in the statistics department, and it can be a "**make or break**" course.
- **Not as well studied** as introductory statistics.

- ✓ Developed the SAP as an adaptation of the well-studied Survey of Attitudes Toward Statistics (**SATS-36**), informed by 3 think-alouds with **former students** and input from probability **instructors**.
- ✓ Obtained **IRB approval** in summer 2020.
- ✓ Coordinated survey distribution in **15 different sections** in the first and last 3 weeks of the fall 2020 semester.
  - ✓ Ensured comparable incentive structures within all sections and worked with instructors to provide **extra credit** to students whether they consented to allowing their data to be used or not.
  - ✓ Students who did not consent to their data being used completed an **alternative assignment**.

Each Likert survey question is associated with an **attitude component**, following Candice Schau’s SATS-36:

- **Affect** (6 items) – e.g. “I will like probability.”
- **Cognitive Competence** (6 items) – e.g. “I can learn probability.”
- **Value** (9 items) – e.g. “Probability is worthless.”
- **Difficulty** (7 items) – e.g. “Probability is a complicated subject.”
- **Interest** (4 items) – e.g. “I am interested in learning probability.”
- **Effort** (3 items) – e.g. “I plan to work hard in my probability course.”

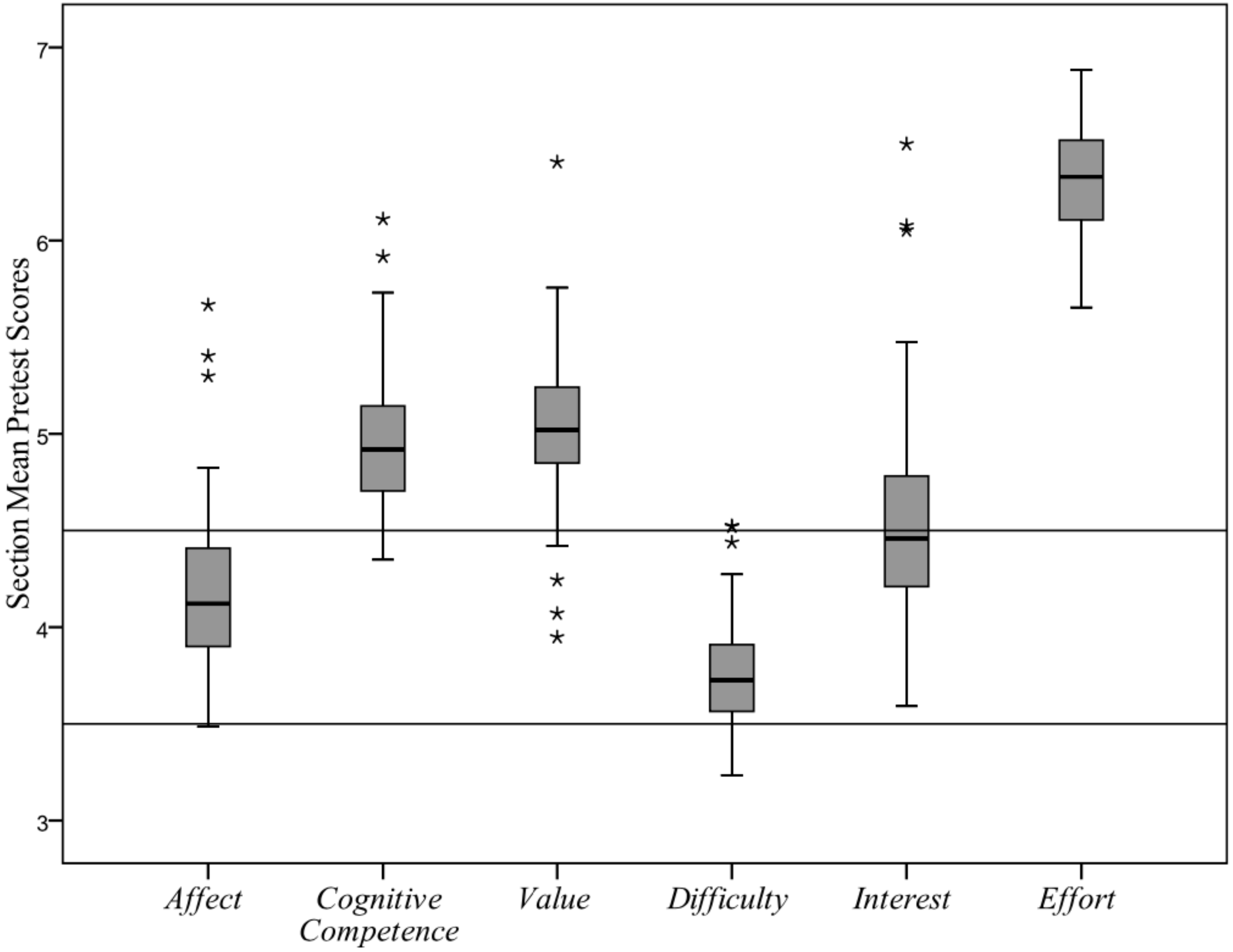
We **compare our results to the Schau (2012)** analysis of the SATS-36.

(2200 students enrolled in 101 sections of post-secondary introductory statistics service courses across the US)

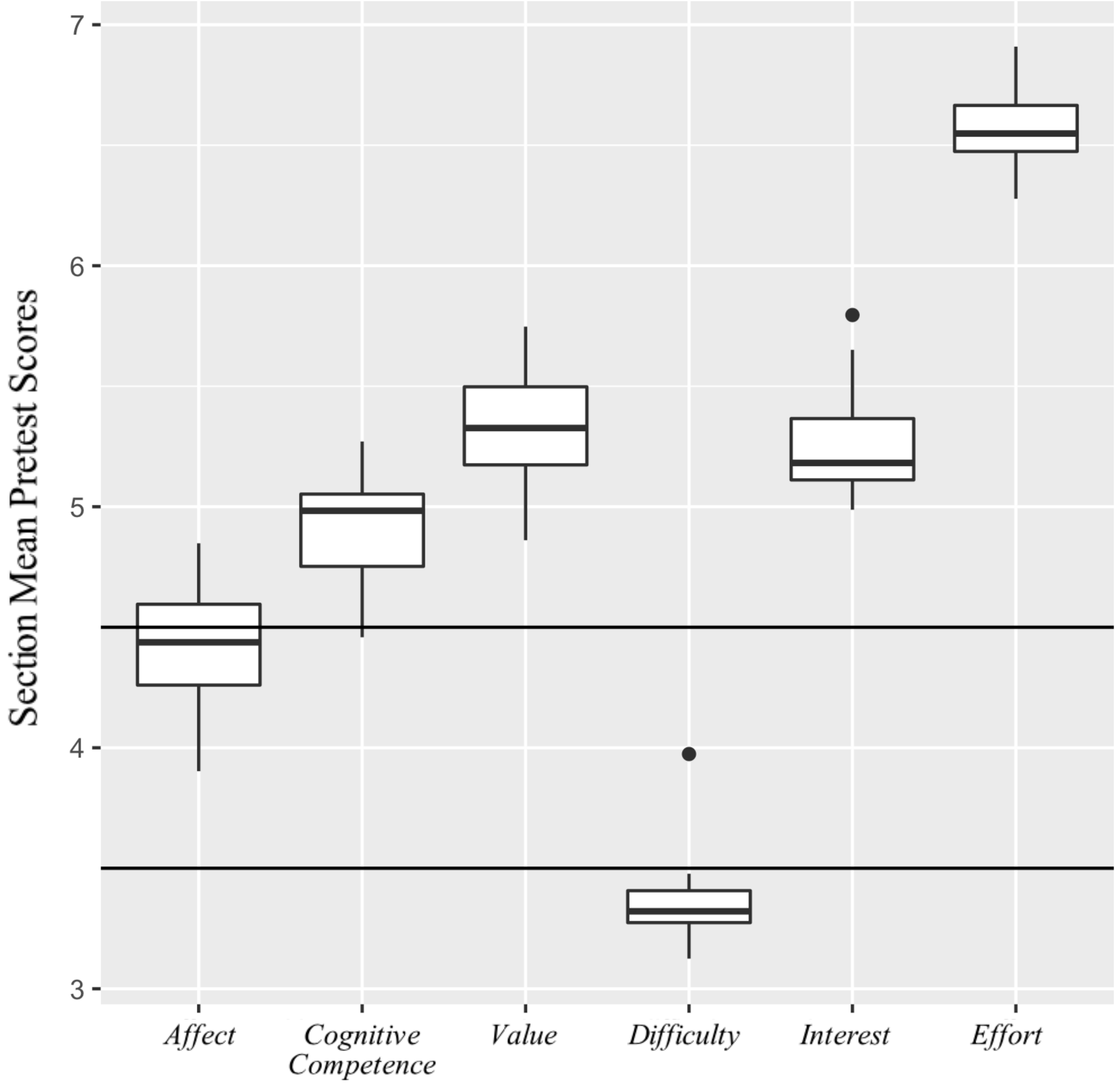
Our sample: **343 Penn State students in 15 sections** completed both pre and post surveys.

Pre scores (section means are plotted)

Schau (2012) - Attitudes toward statistics



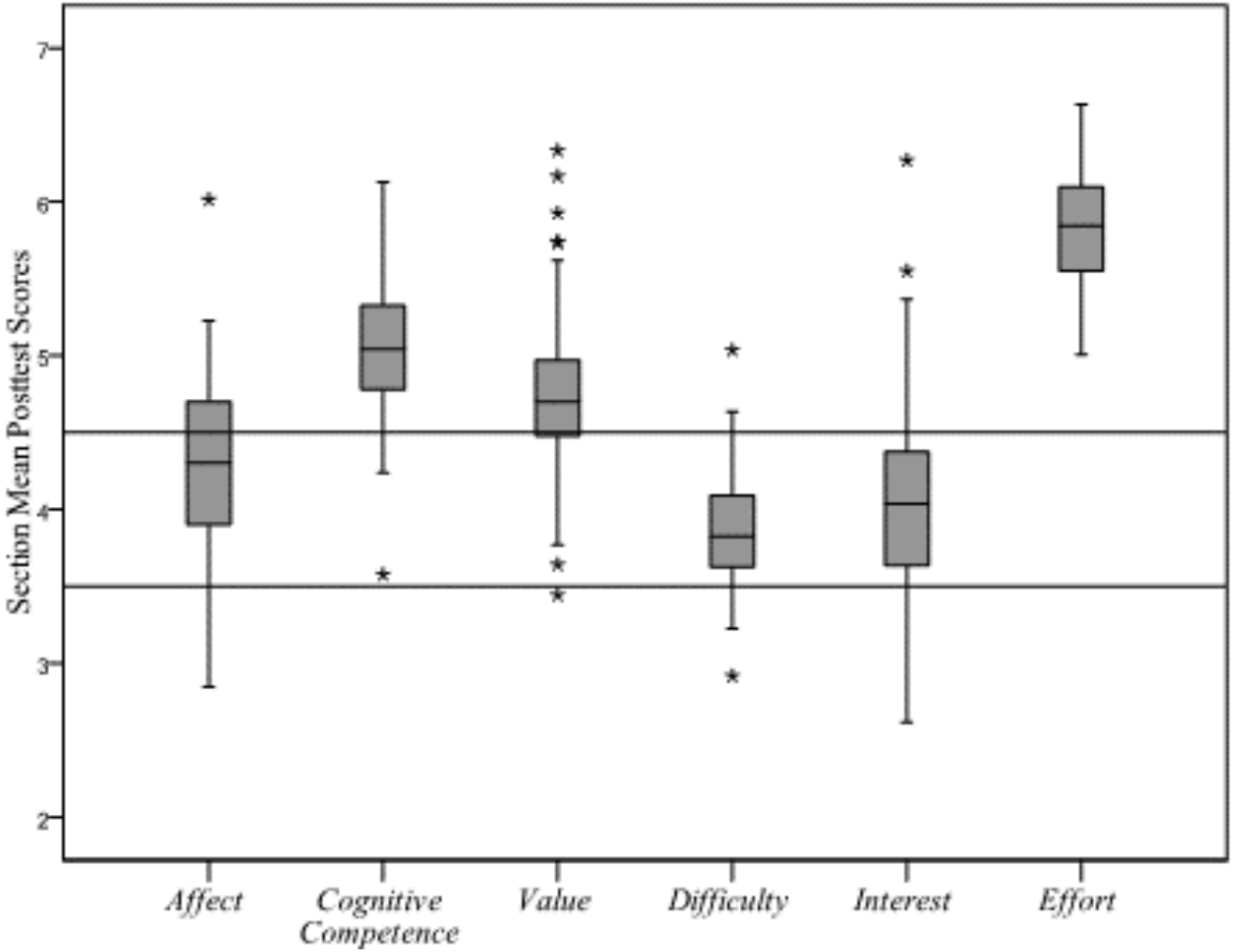
Us - Attitudes toward probability



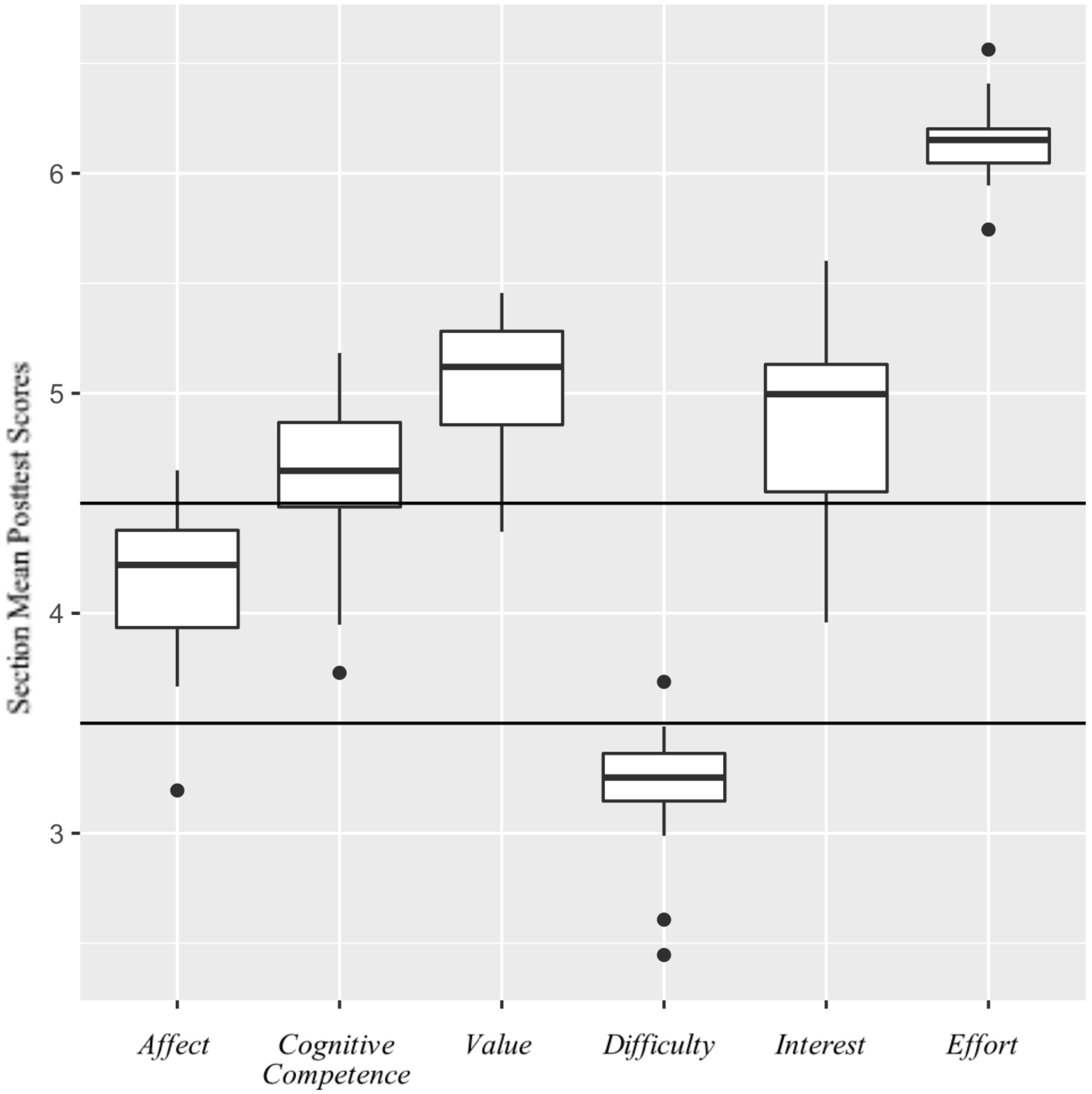
Students who took our pre survey thought probability was **more difficult** and were **more interested** in the subject. This is compared to the introductory statistics students Schau surveyed on their attitudes toward statistics.

Post scores (section means are plotted)

Schau (2012)



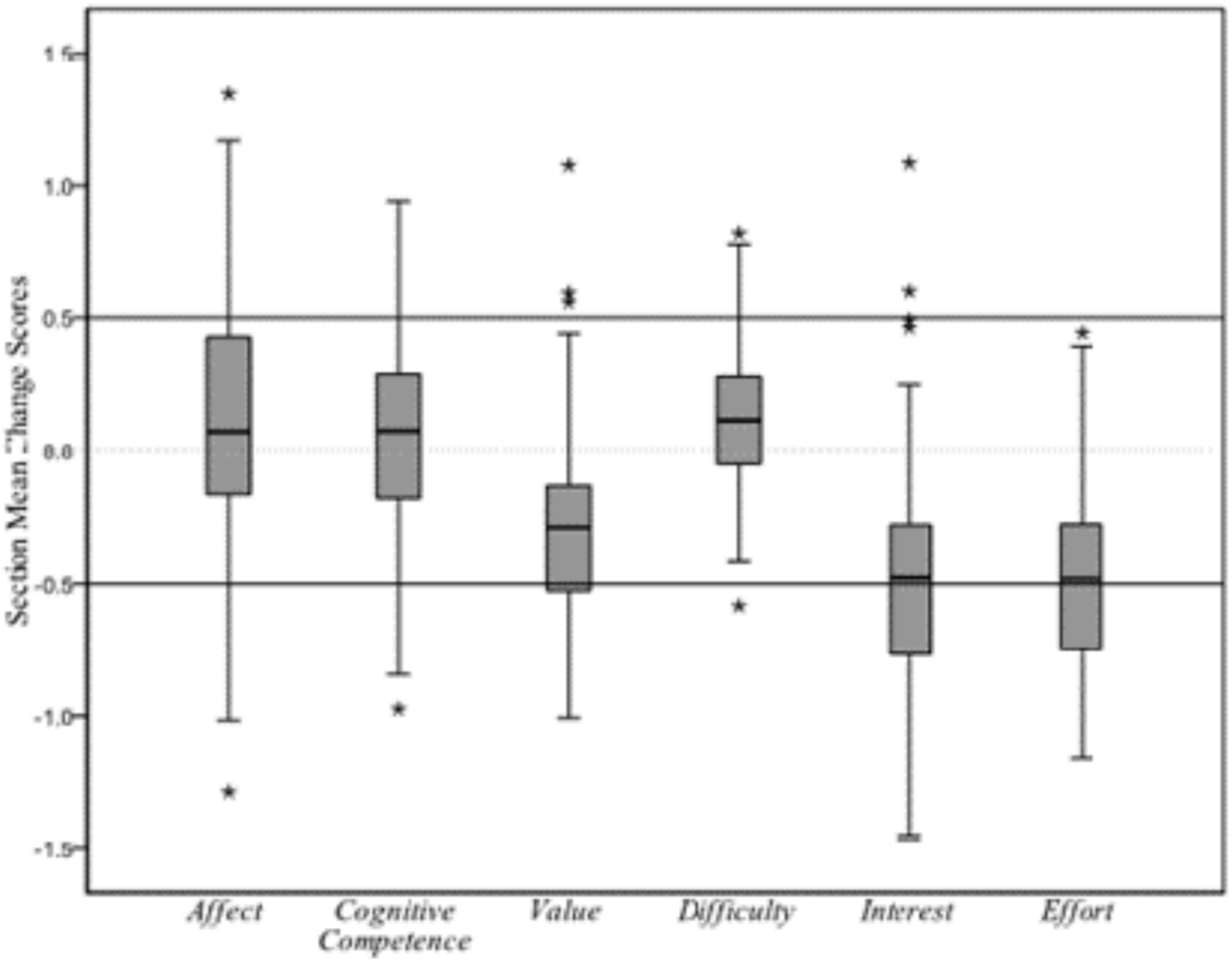
Us



The trend described on the previous slide is **also present in the post survey**.

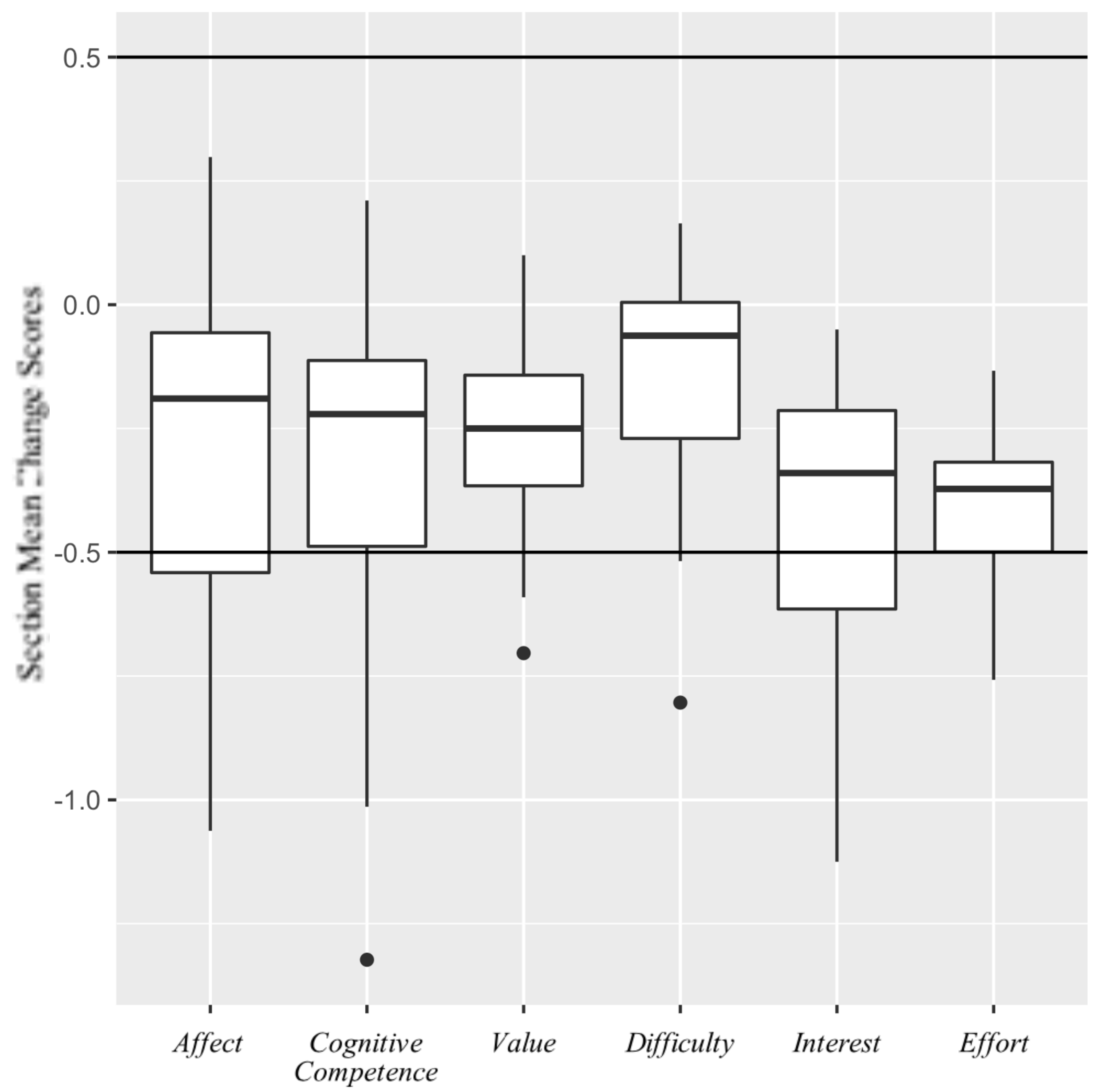
Change in scores (section means are plotted)

Schau (2012)



All attitude components had a slight mean **negative shift** from pre to post surveys in our data. This was true even for components with mean positive shifts in Schau's data.

Us



Future work:

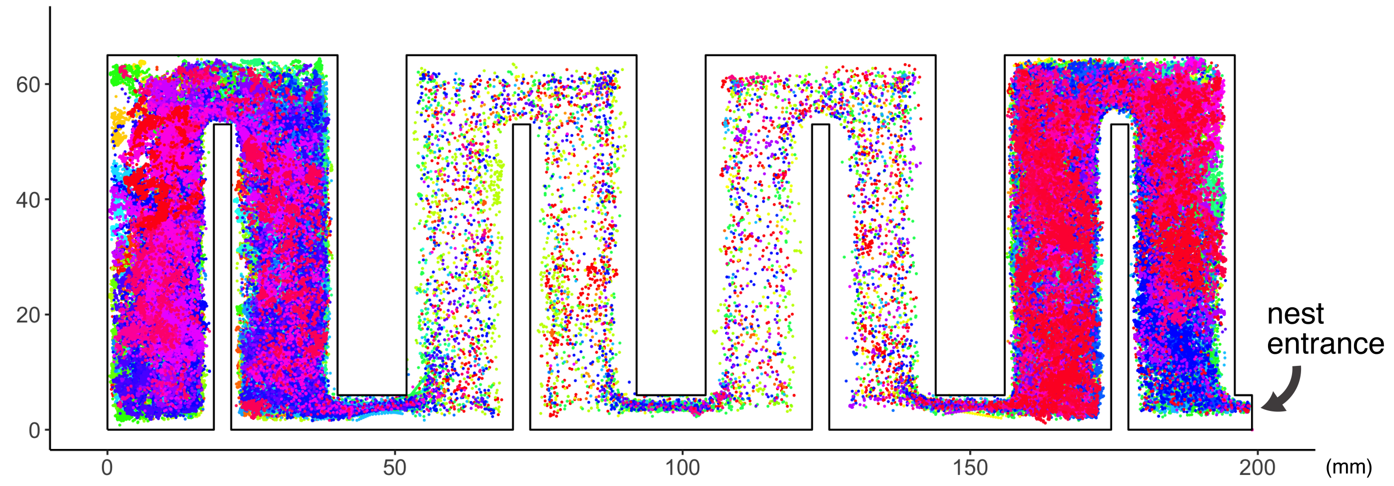
1. **More detailed analyses** of this data.
   - **Demographic information** and math background could help explain variability.
   - Examine **open-ended questions**.
   - Assess the **internal consistency** of the SAP.
2. Collecting more data with an updated survey in **Spring 2021** from 20 sections, including 7 non-probability sections.
   - >600 responses to the pre SAP from probability sections

Thanks to all the instructors for their help!

Projects

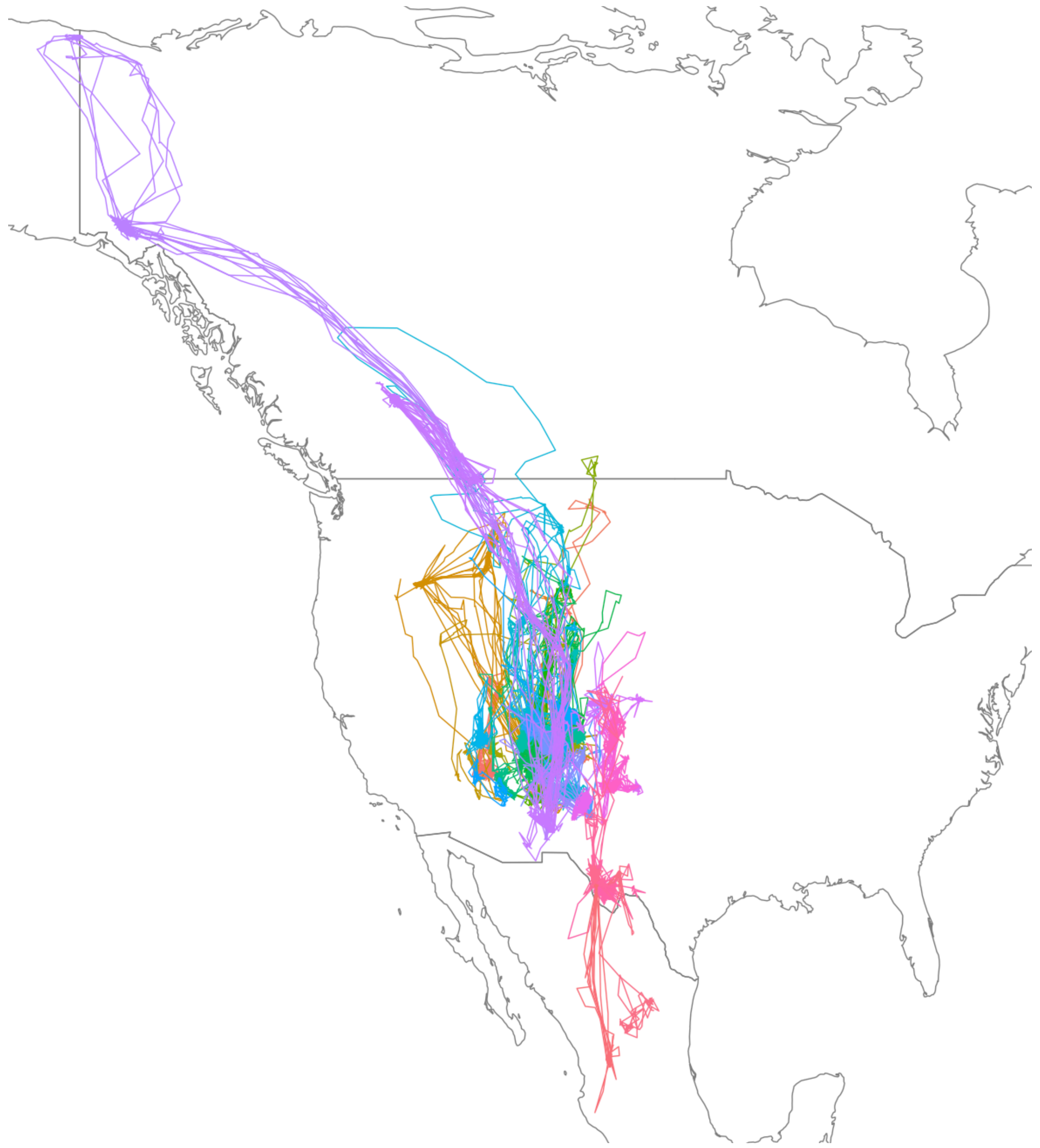
1

A Lattice and Random Intermediate Point Sampling Design for Animal Movement



2

Modeling Yearly Patterns in Golden Eagle Movement



3

Survey of Attitudes toward Probability

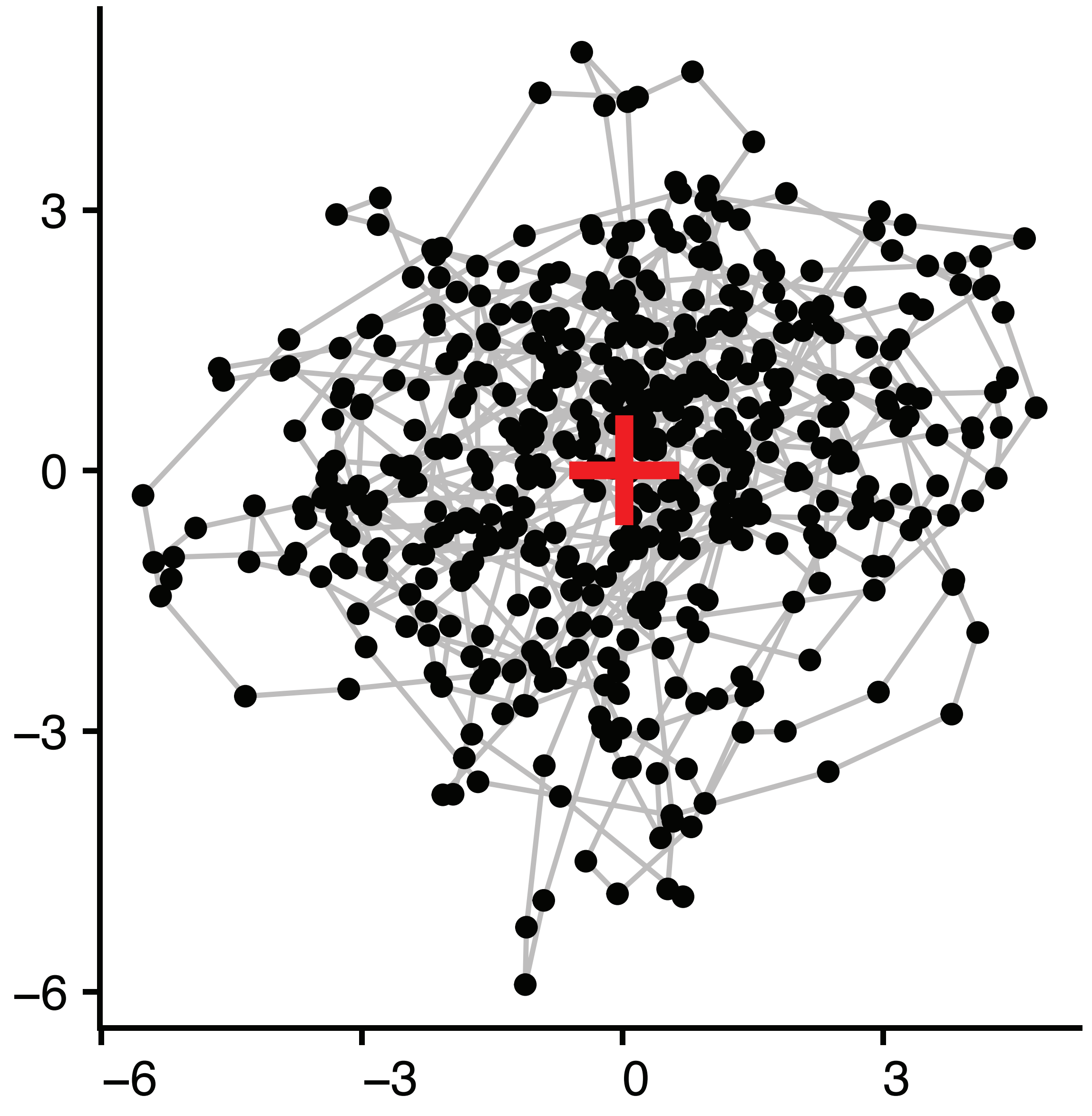
Additional slides

Project 1: Simulation example

Our goal:

- Simulate movement paths from an SDE model and **compare parameter estimates and predictions** using different subsamples of the data.

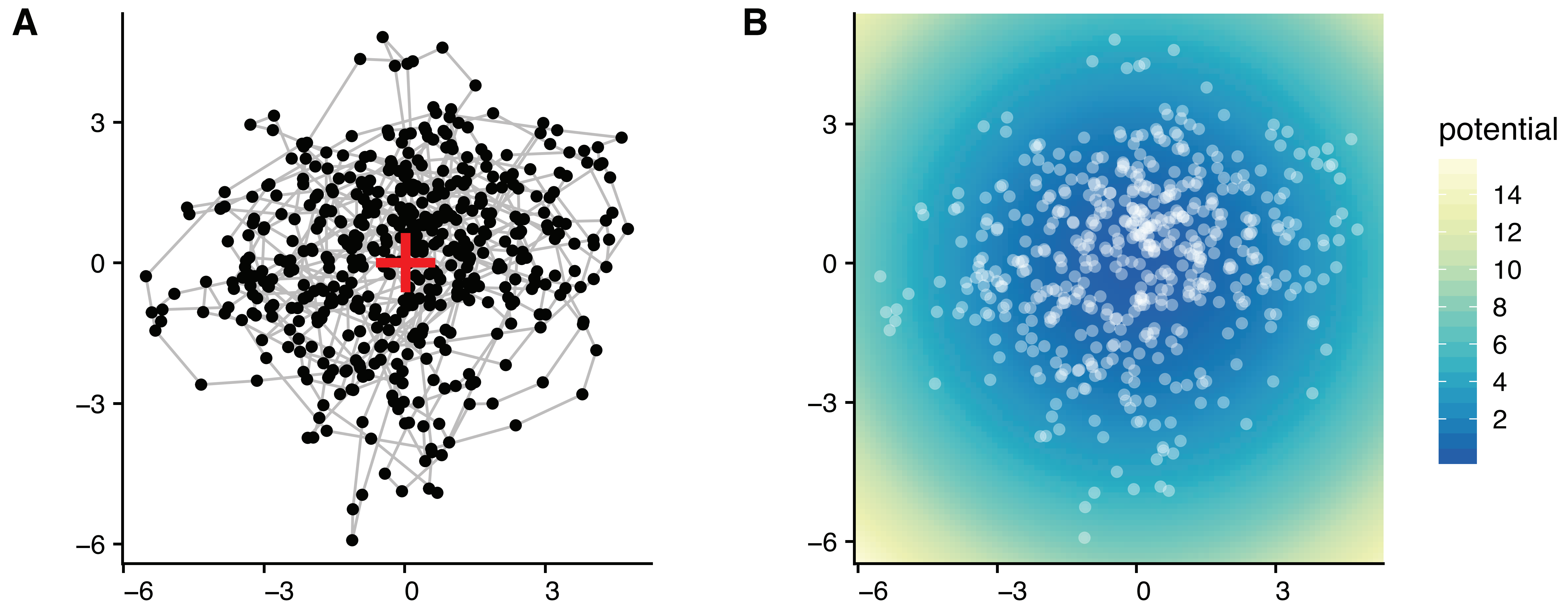
When we subsample the data, we pretend that we originally collected just the data in the subsample.



We simulated paths at a fine temporal scale from an SDE model with **quadratic potential function** and **constant motility** function.

$$p(\mathbf{x}_t) = k\mathbf{x}_t'\mathbf{x}_t$$

where $k \in \mathbb{R}$ controls strength of attraction to the origin.



Plugging in the quadratic potential surface results in an equation where the animal's position is a function of the two previous positions.

$p(\mathbf{x}_t) = k\mathbf{x}_t'\mathbf{x}_t$ implies

$$\boldsymbol{\mu}(\mathbf{x}_t) = -\nabla p(\mathbf{x}_t) = \boxed{-2k\mathbf{x}_t}$$

External force

Resulting SDEs:

$$d\mathbf{x}_t = \mathbf{v}_t dt$$

$$d\mathbf{v}_t = -\beta\mathbf{v}_t dt + \beta\boxed{(-2k\mathbf{x}_t)}dt + \sigma_t dW_t$$

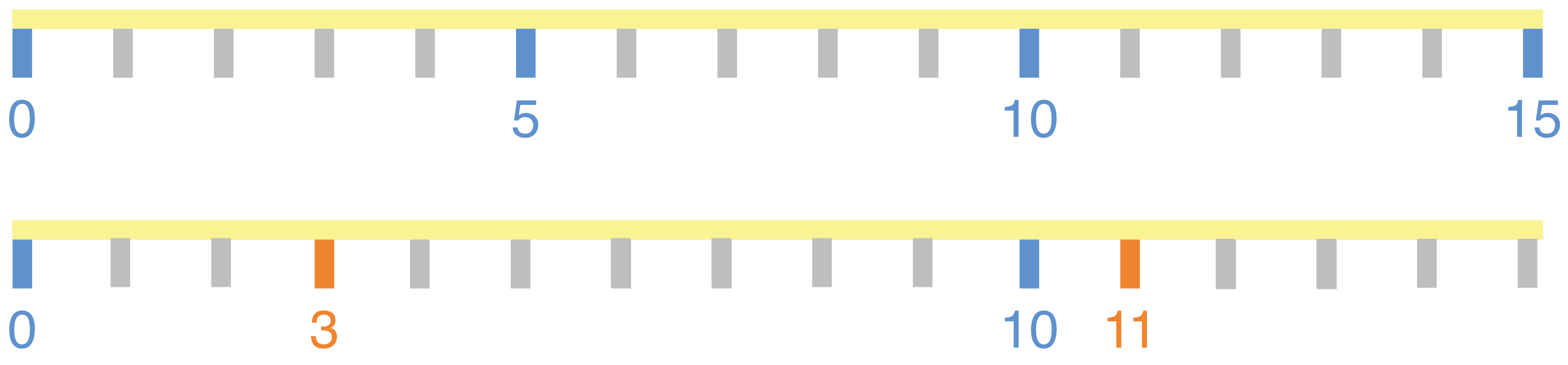
Using numerical approximations results in

$$\mathbf{x}_{t+2h} = \mathbf{x}_{t+h}(2 - \beta h) + \mathbf{x}_t(\beta h - 1 - 2\beta k h^2) + N(\mathbf{0}, h^3\sigma^2\mathbf{I})$$

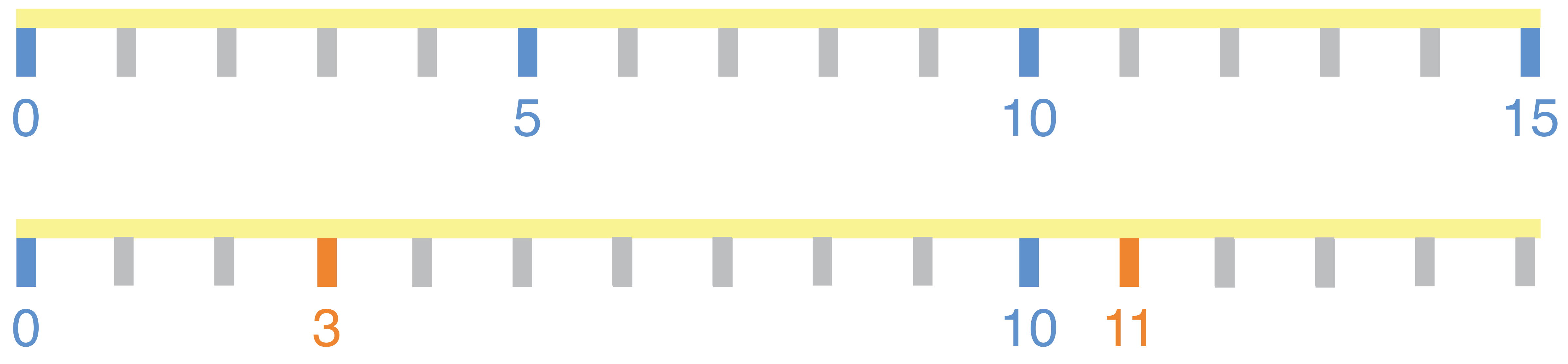
where $h = \Delta t$.

We simulated **150 paths**,
each of length 500,
with time step $h = 1$,
and model parameters
$\beta = 0.4$,
$\alpha \equiv k\beta = 0.08$,
and $\sigma = 0.5$.

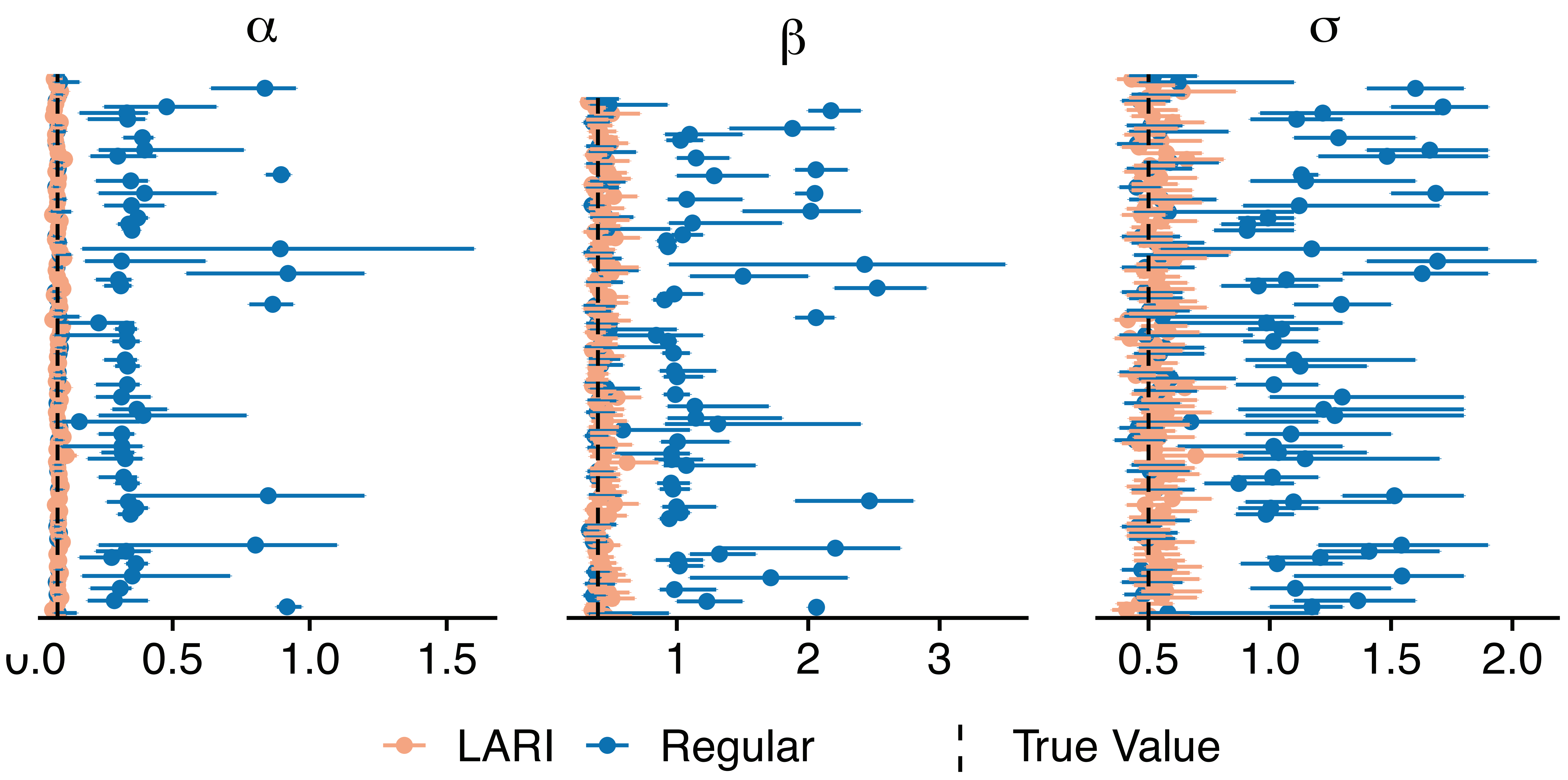
We took **subsamples** using regular and LARI sampling designs.



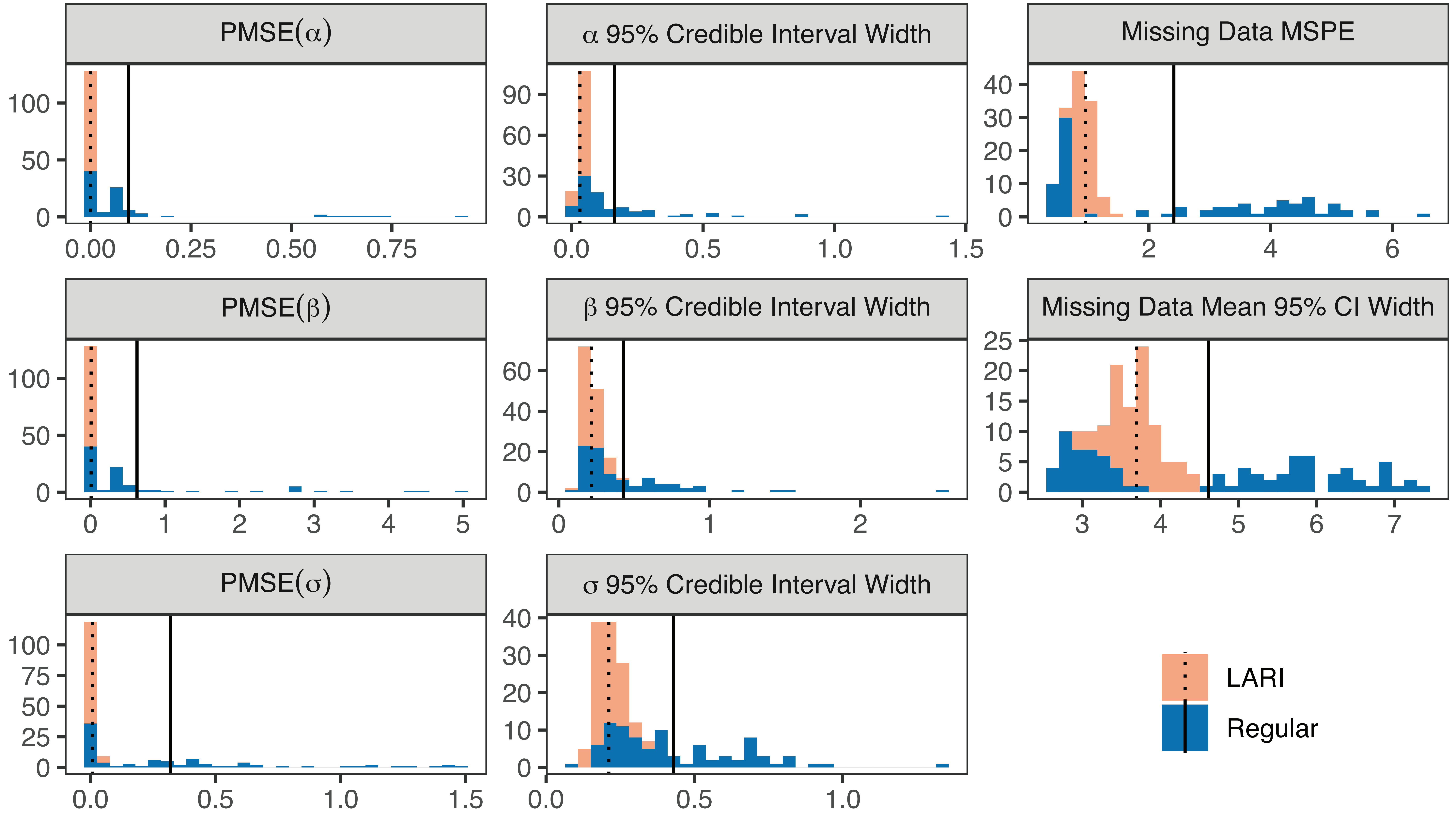
To estimate β, α, σ, and the "missing" locations, we took a **Bayesian approach** and constructed a MCMC algorithm to sample from the joint posterior.



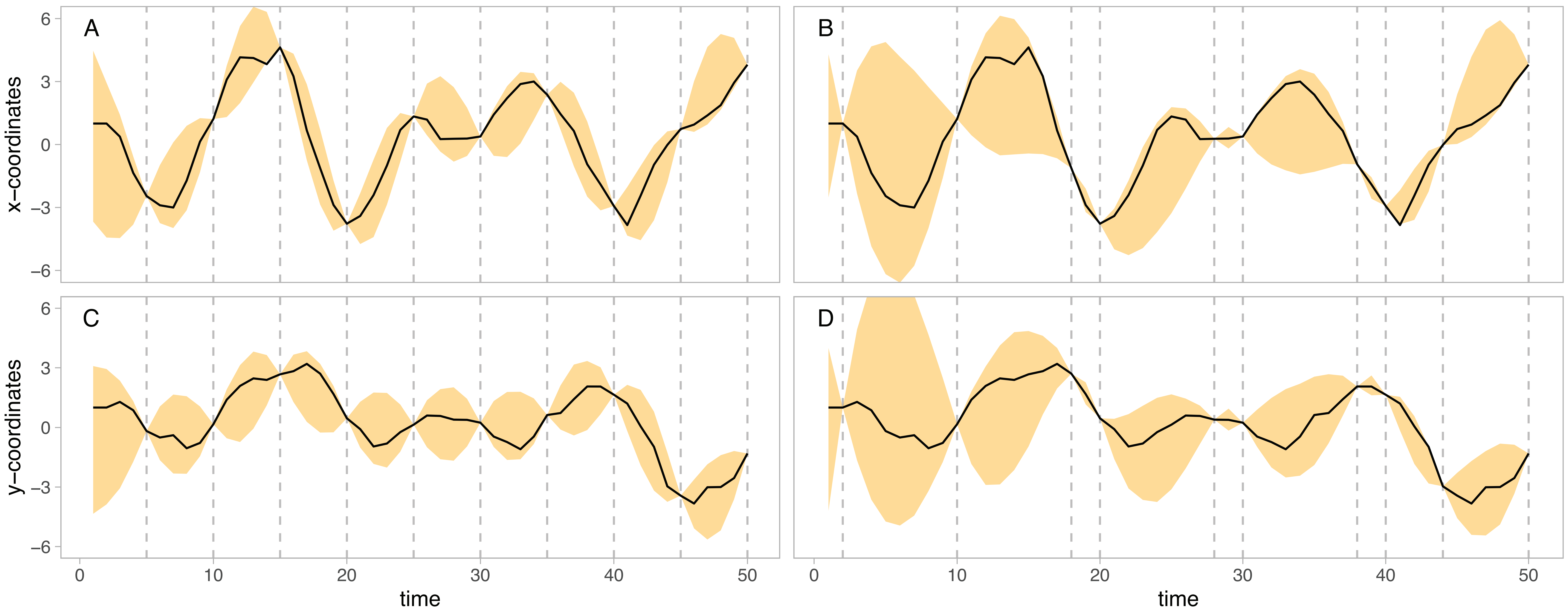
Many regular subsamples result in poor estimation of the model parameters.



The regular subsample also performs poorly with regard to estimation of missing data, compared to LARI.



As long as both accurately capture the model parameters, the regular sample outperforms LARI with regard to prediction of missing data.



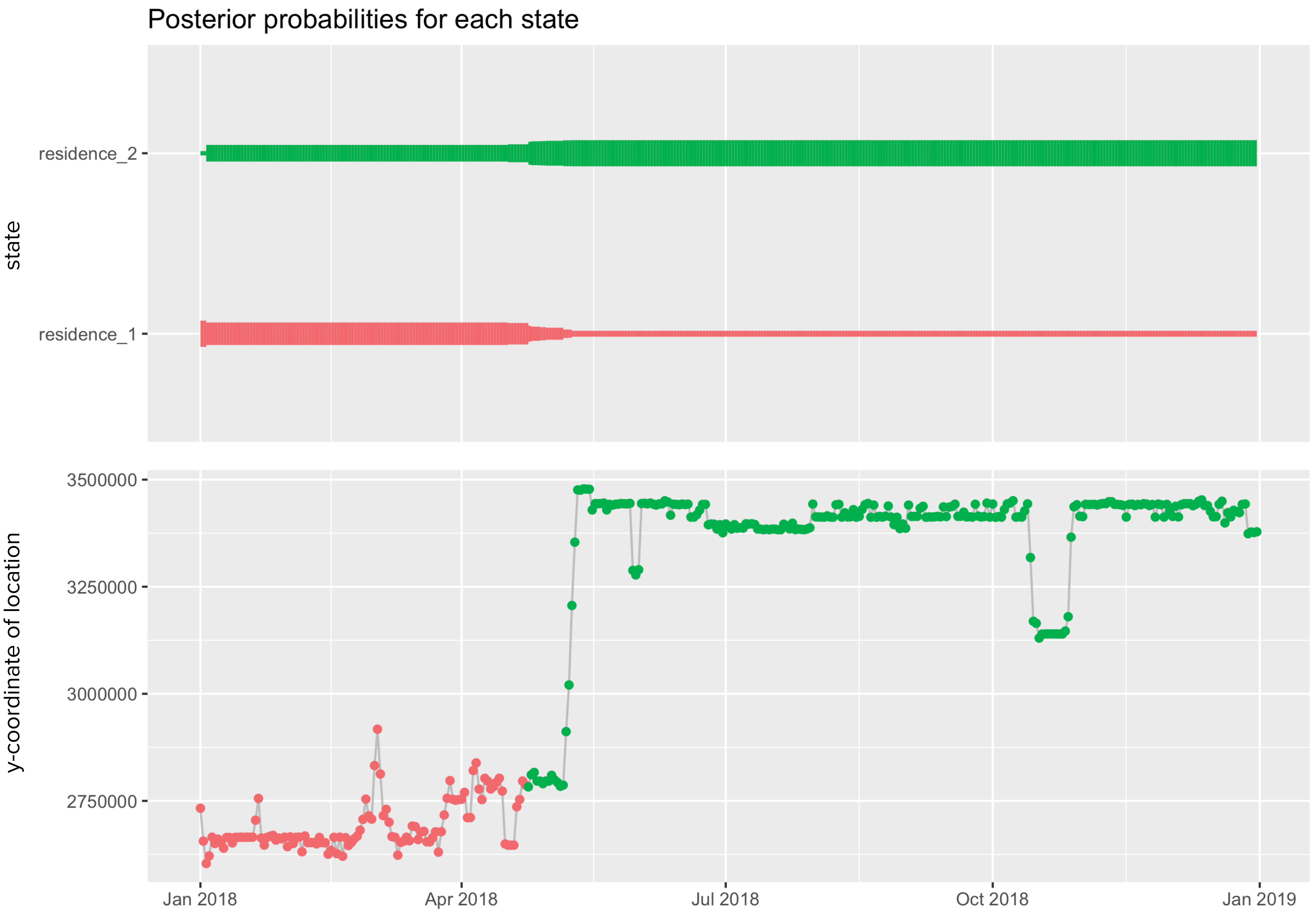
In conclusion:

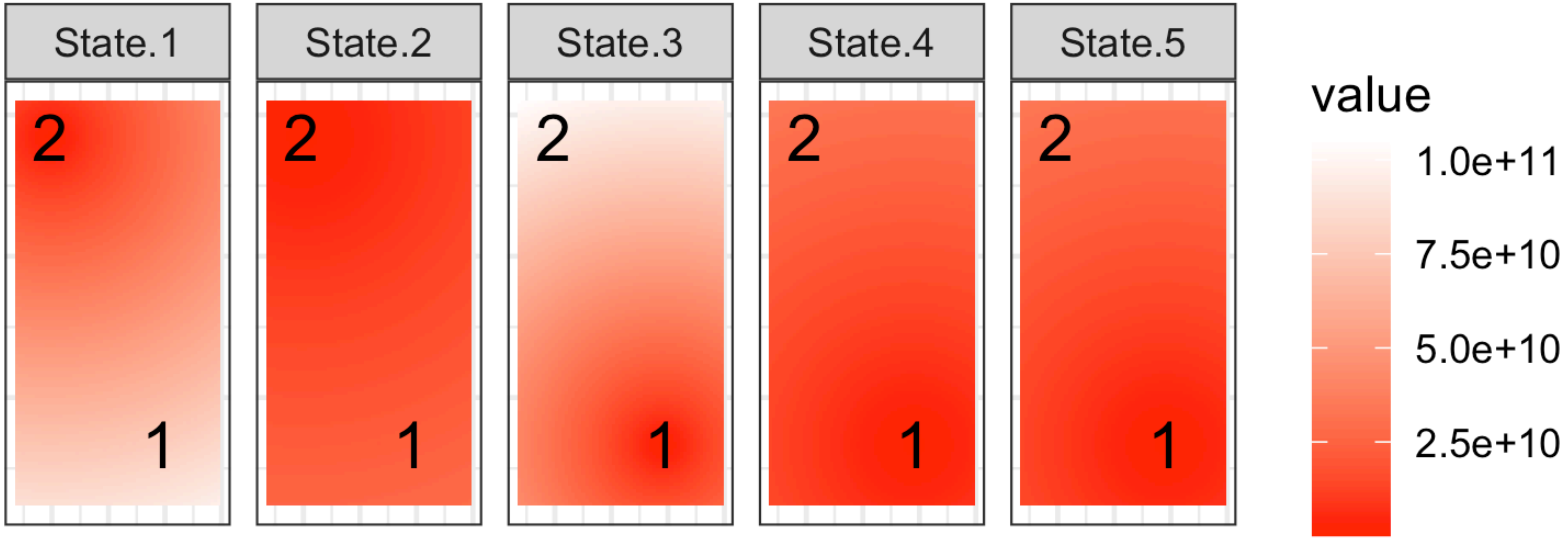
We performed 150 simulations to compare regular and LARI subsamples.

Result: **LARI sampling led to better accuracy and precision** in estimation of model parameters. Thus, LARI also better predicted missing data. However, in the cases where both subsamples led to accurate estimation of model parameters, the regular subsample led to better prediction of missing data.

Golden Eagle data cleaning:

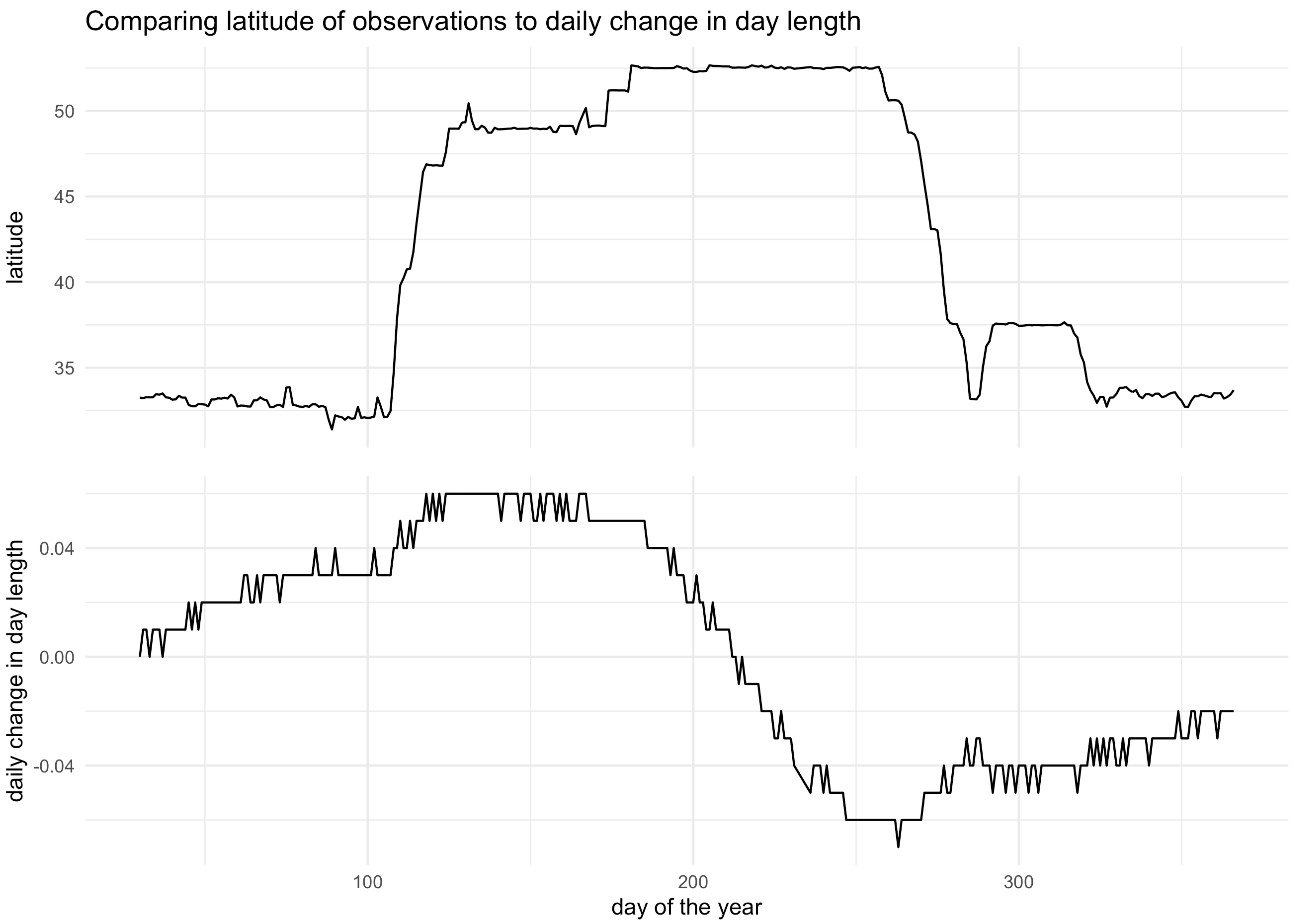
1. Subset **1 observation per day**
2. **Linearly interpolate** missing intervals smaller than 30 days
3. Each **“path”** is one year for one individual and is analyzed separately


Posterior probabilities for each state
residence_2
residence_1
state
3500000
3250000
3000000
2750000
y-coordinate of location
Jan 2018
Apr 2018
Jul 2018
Oct 2018
Jan 2019

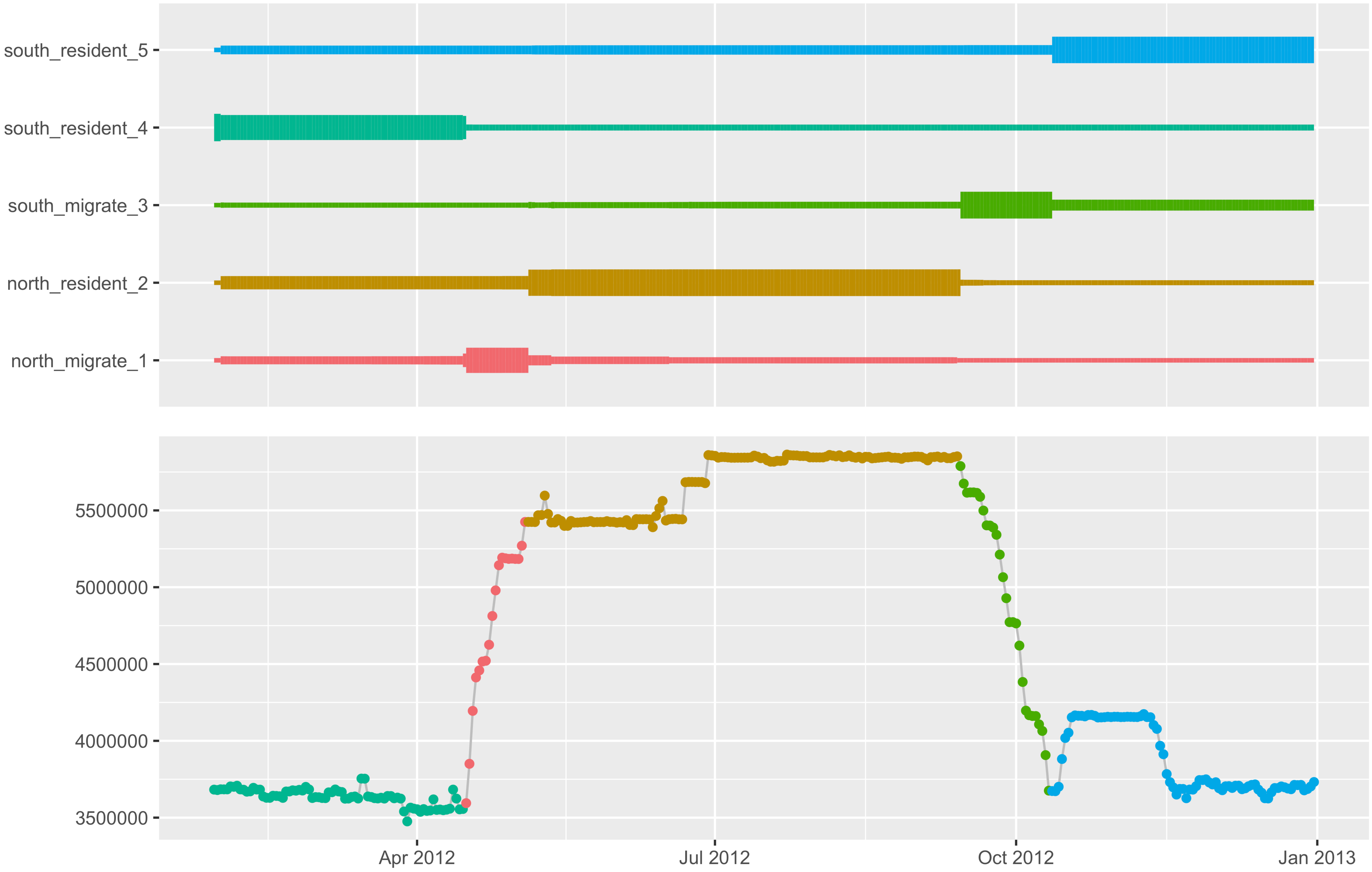


$$p(\mathbf{r}_t, s_t) = \begin{cases} k_{s_t}\sqrt{(x_t - a_{x1})^2 + (y_t - a_{y1})^2}, & s_t \in \{3,4,5\} \\ k_{s_t}\sqrt{(x_t - a_{x2})^2 + (y_t - a_{y2})^2}, & s_t \in \{1,2\} \end{cases}$$

Transition probabilities are functions of a covariate.



Posterior probabilities for each state



Conclusions:

1. We described **2 classes of models** (varying coefficient and latent-state) for fitting resident, migrant, and dispersal movement strategies.
2. The varying coefficient model is more **flexible** and seems to **better explain movement behavior**.

SAP – We replace words related to statistics with words related to probability.

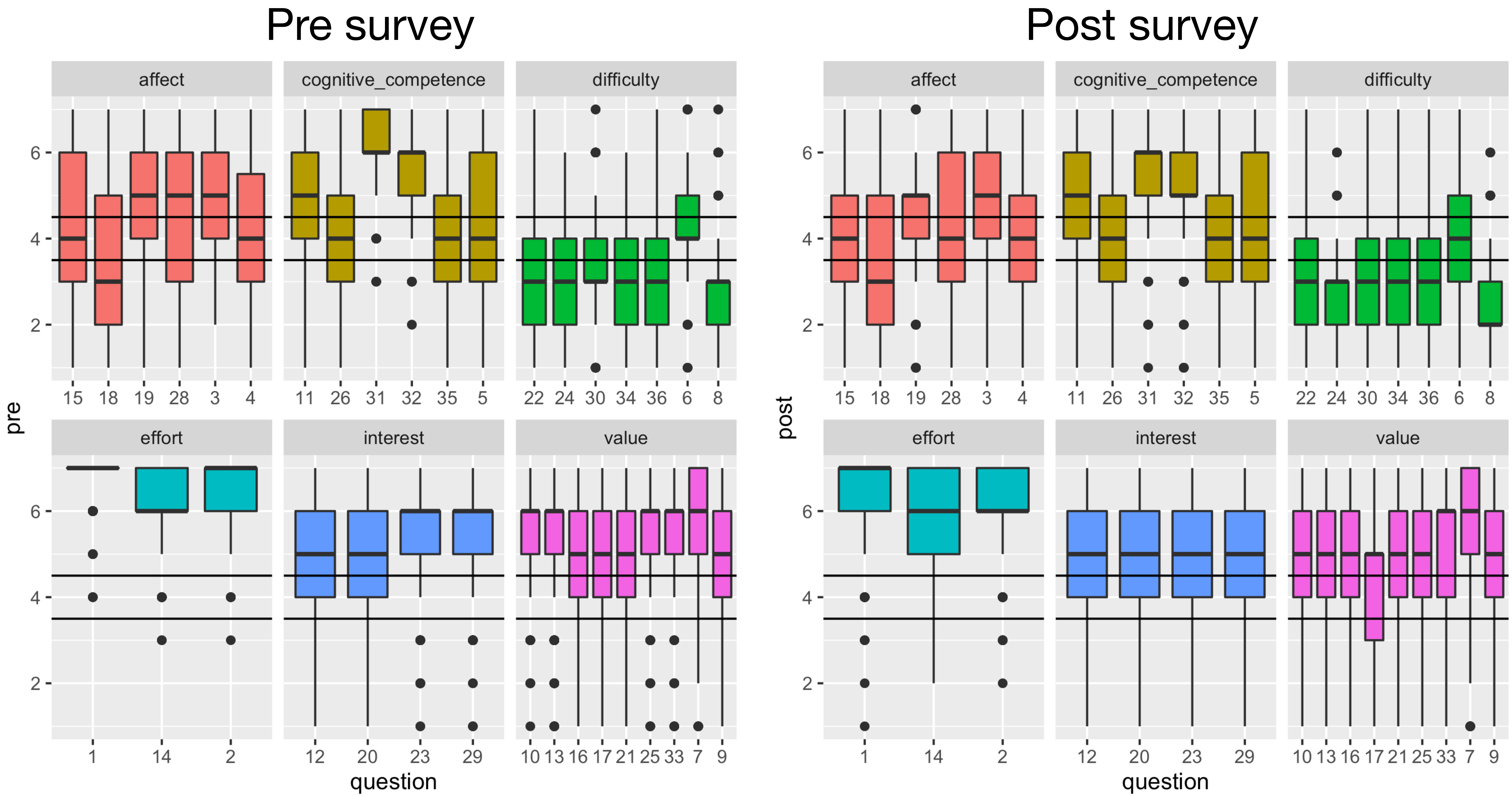
I will like **statistics**.	I will like **probability**.
I am interested in understanding **statistical information**.	I am interested in understanding **probabilistic arguments**.

We updated demographic questions and added open ended questions including:

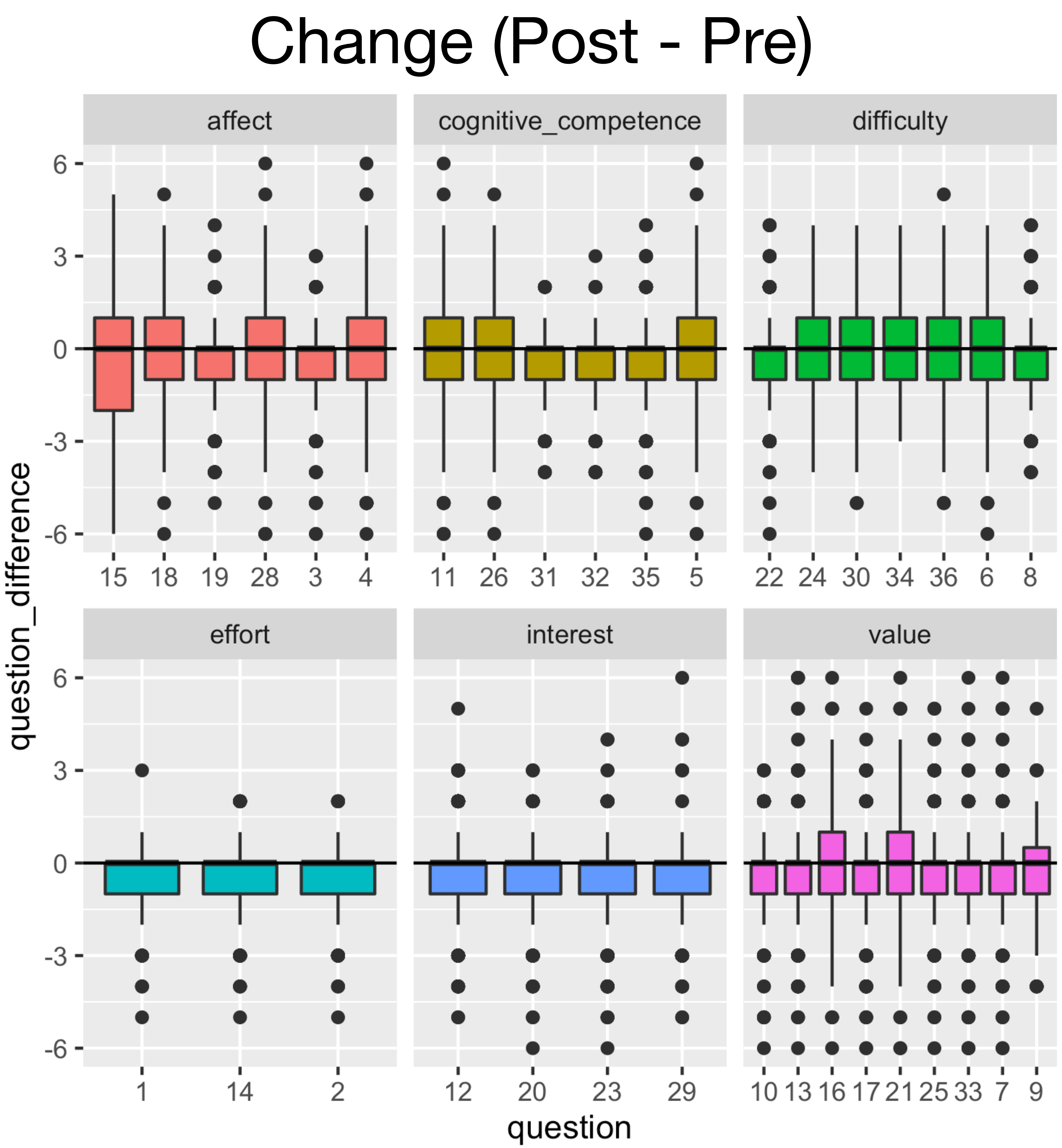
- Describe the **difference between probability and statistics** in your own words.

- What experiences do you believe most *positively* **influenced your current attitude** toward **probability**?

Each Likert question was associated with an attitude component (each data point is one student response)

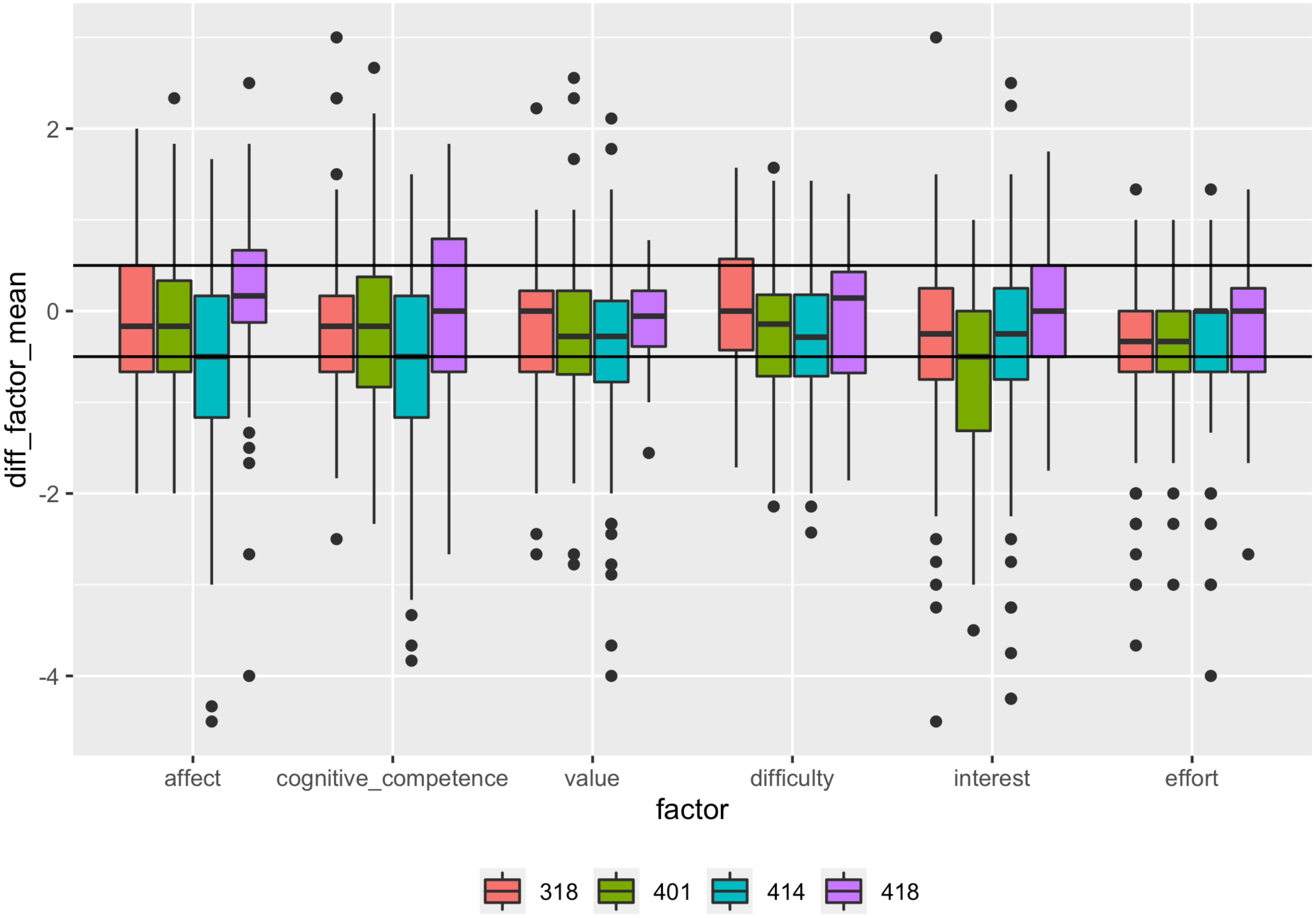


Each question was associated with an attitude component (each data point is one student response)



Change in scores by class (all students separately)

418 often has a more positive change (might have to do with type of student)

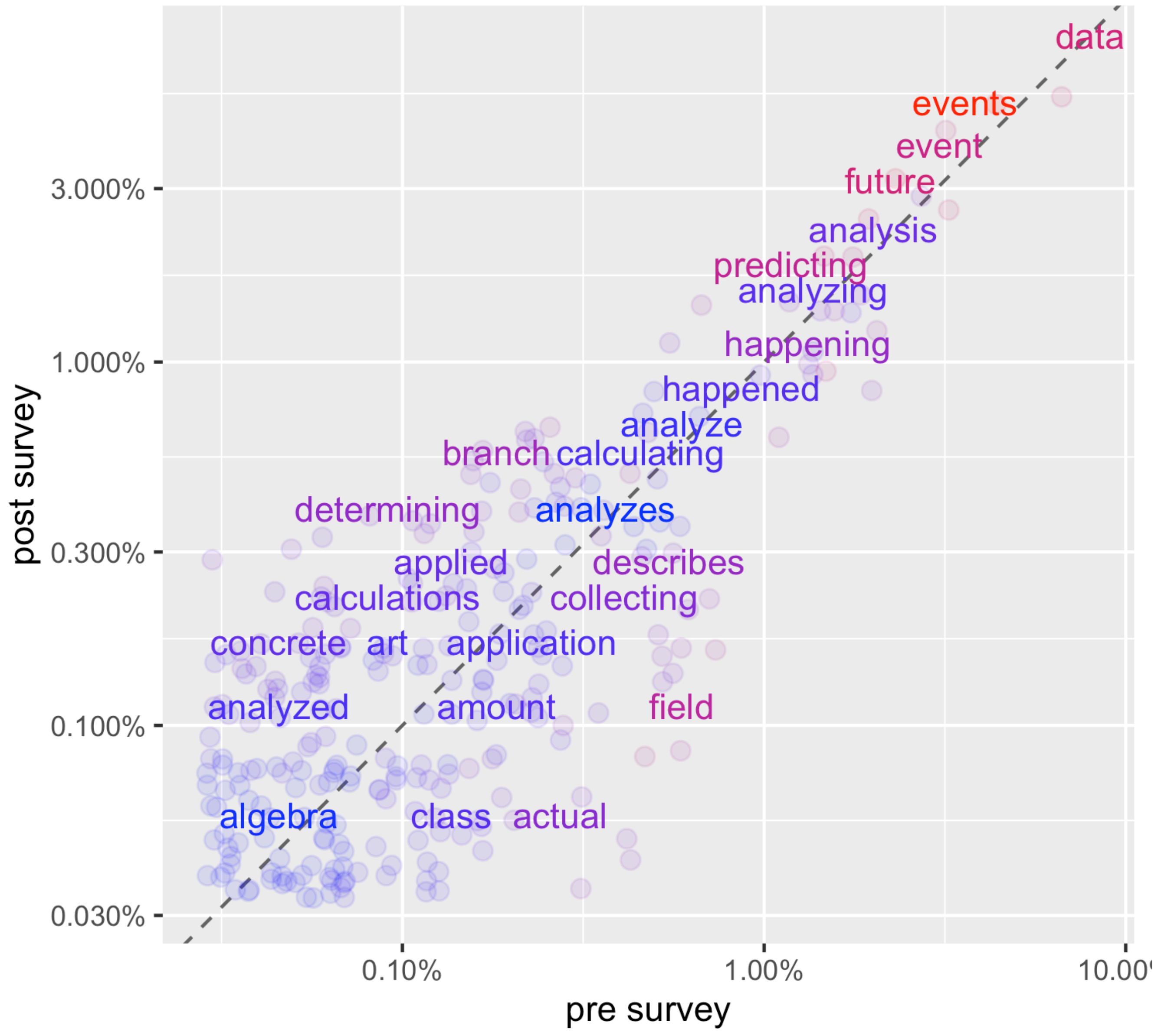


Conclusions:

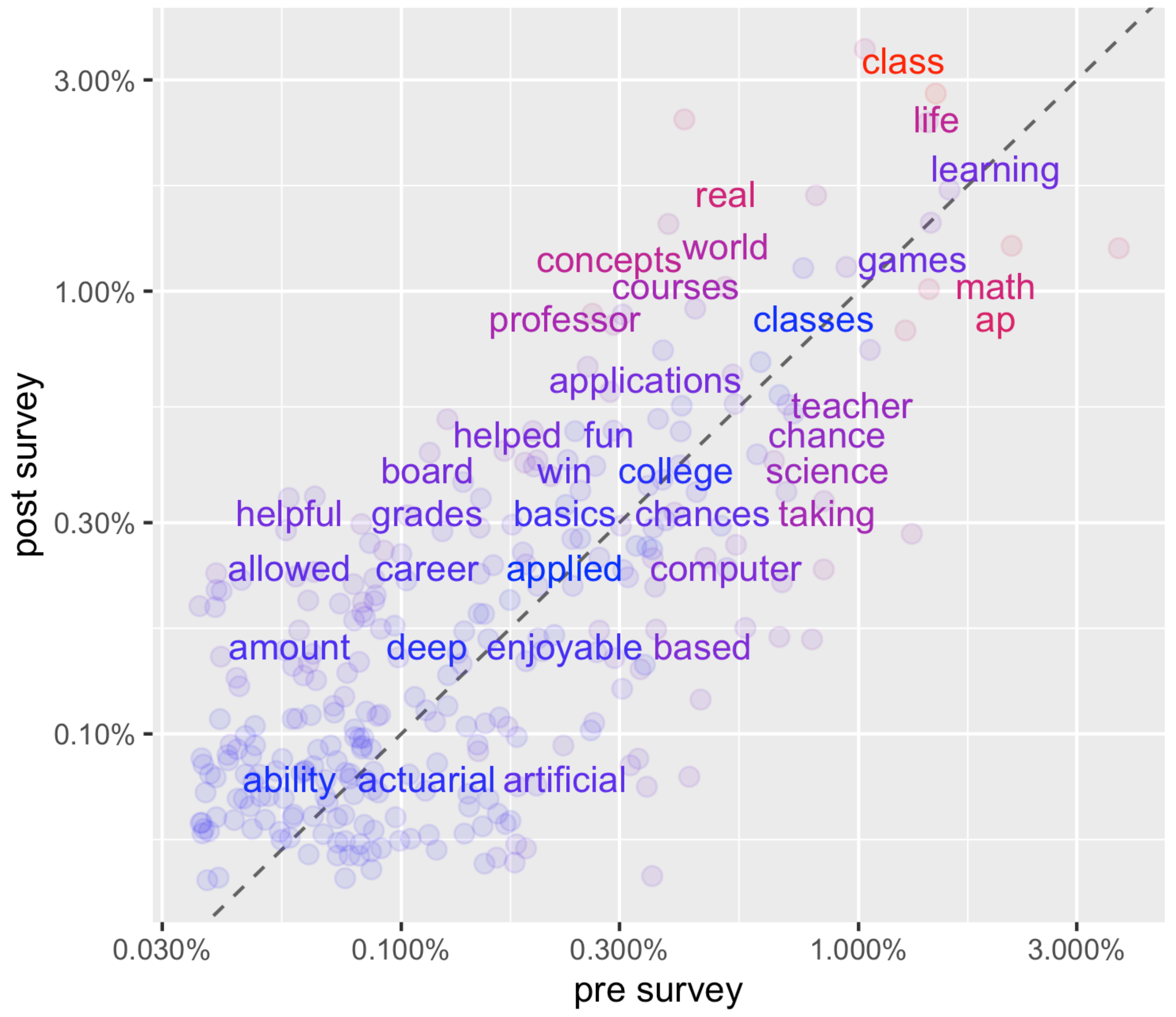
1. We **adapted the SATS-36** to assess attitudes toward probability.
2. In our **preliminary analyses**, we compared SAP results and SATS-36 results.
3. The SAP could be a **tool for researchers interested in improving probability courses** in the same way that the SATS-36 has been for statistics courses.

Common words in pre and post open ended questions

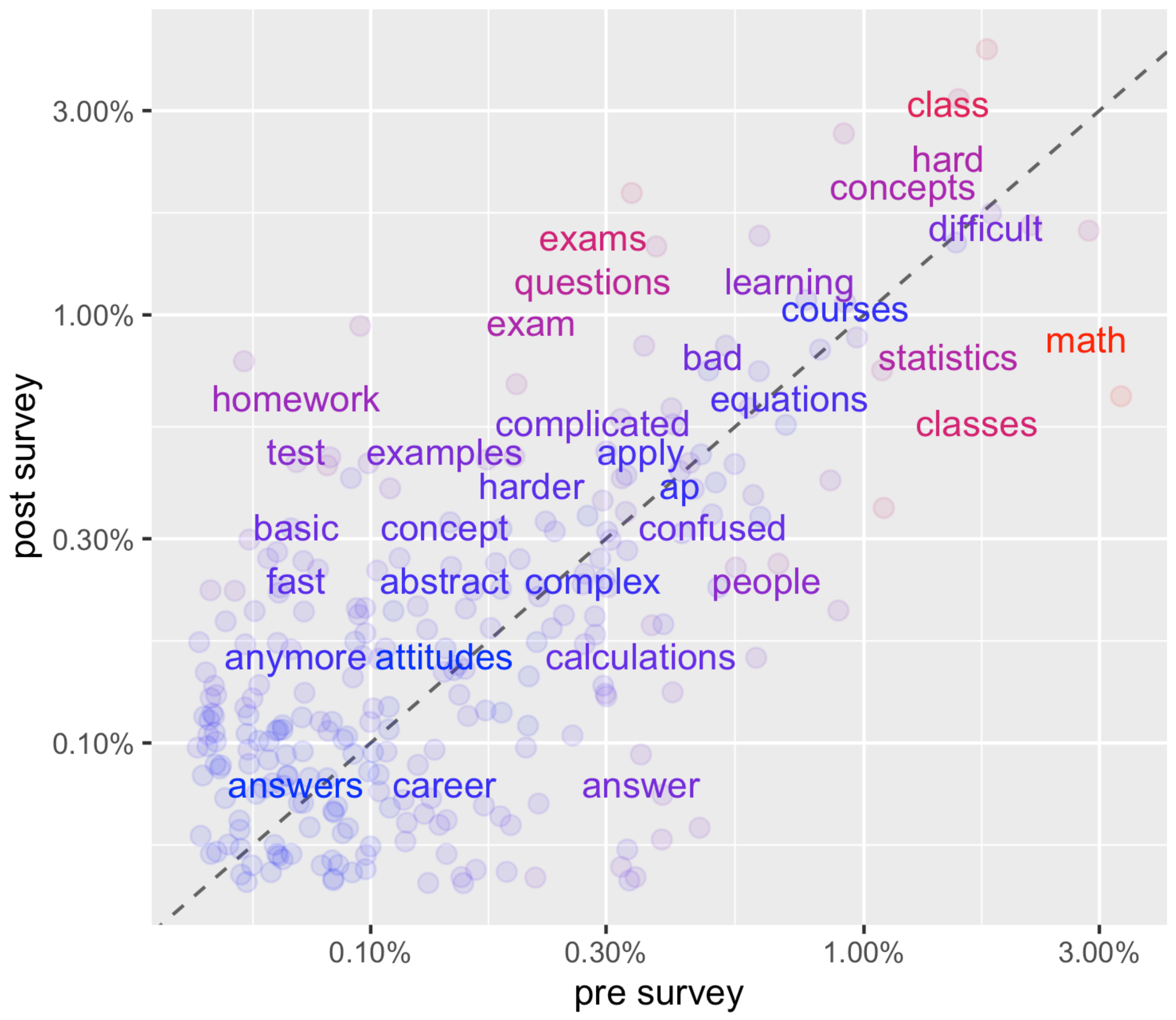
Describe the difference between probability and statistics in your own words.



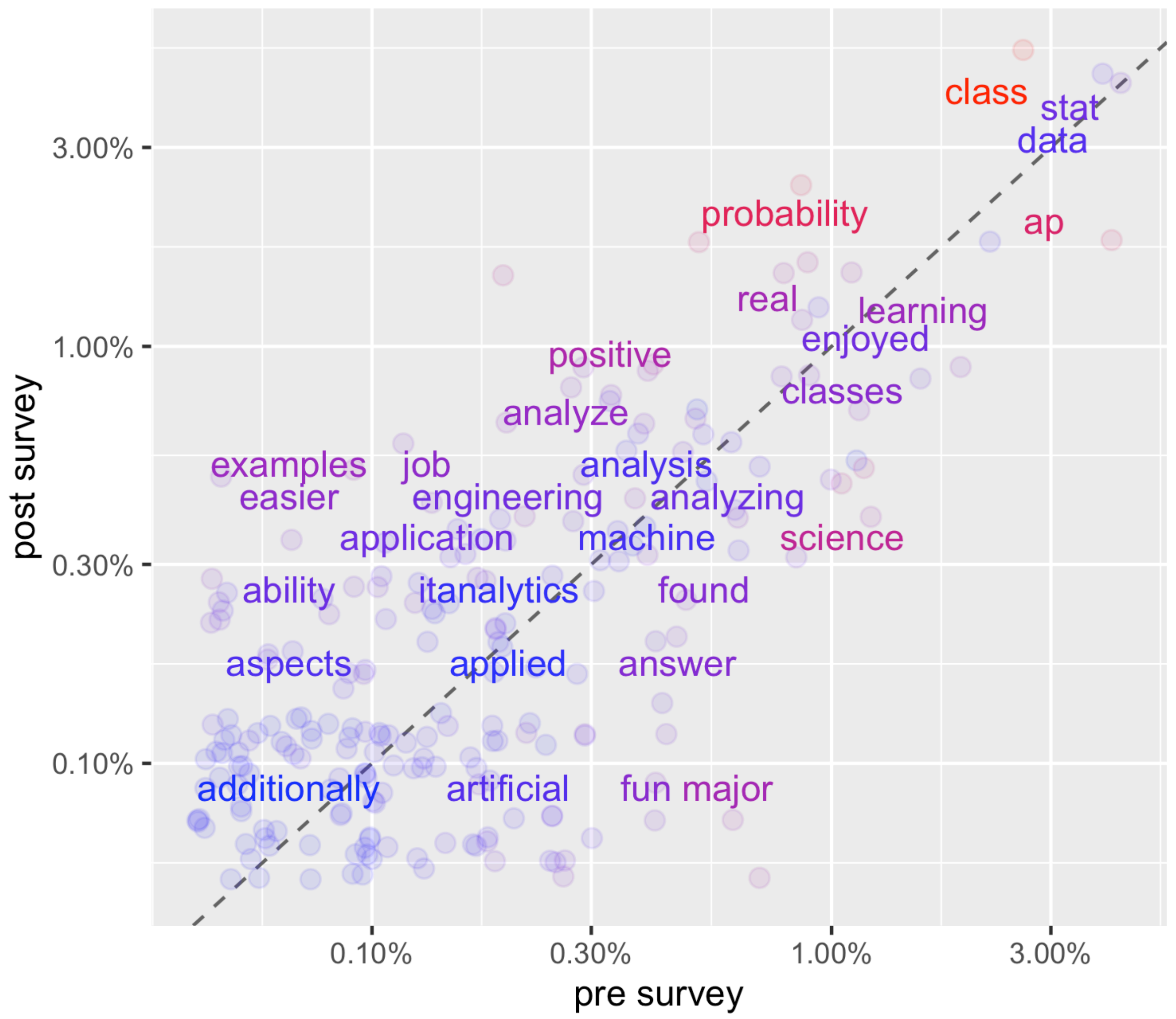
What experiences do you believe most positively influenced your current attitude toward probability?



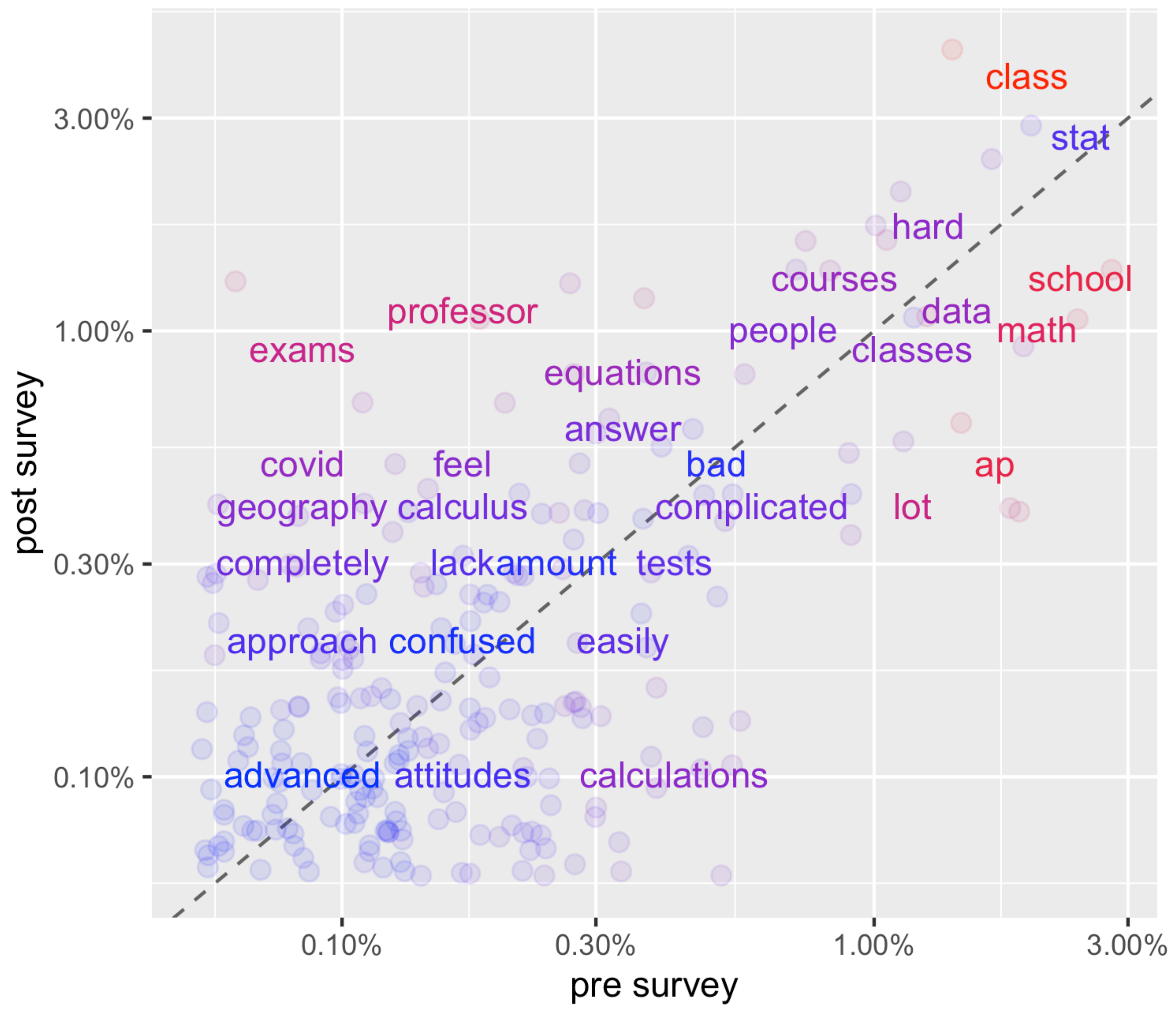
What experiences do you believe most negatively influenced your current attitude toward probability?



What experiences do you believe most positively influenced your current attitude toward statistics?



What experiences do you believe most negatively influenced your current attitude toward statistics?



How would you compare the ethical aspect of probability to the ethical aspect of statistics?