Cat's Ranges Model
A 3 level Hierarchical model with ncp

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Contents

1 Data and question

This model takes what I assume is OSPREE forcing and photoperiod data for lots of studies and species. There is an additional level of variation in this data, which is the population the species comes from. I think it is possible for different species to come from the same population, but Cat you should correct me if I'm wrong. The question we focus on is whether there is greater variation within a species (in terms of variation between populations) than between species.

2 Model

This model differs a bit from Cat's original one. Maybe we will get time to discuss how they differ at the end of the meeting. The original code is in *stan /nointer_3levelwpop_force&photo_ncp.stan*, and the modified code I refer to in this documents is in *stan /nointer_3levelwpop_force&photo_ncp_FaithExample.stan*

$$\tilde{y}_i \sim normal(\mu_i, \sigma_y) \tag{1}$$

$$\mu_i = \alpha + \alpha_{study} + \alpha_{sppop} + force_i * \beta_{force,sppop} + photo_i * \beta_{photo,sppop} \tag{2}$$

Study intercept

$$\alpha_{study} = \sigma_{\alpha,study} * \alpha raw_{study} \tag{3}$$

Species intercept

$$\alpha_{species} = \sigma_{\alpha,species} + \alpha raw_{species} \tag{4}$$

Subpopulation intercept

$$\alpha_{sppop} = \alpha_{species} + \sigma_{\alpha,sppop} * \alpha raw_{sppop} \tag{5}$$

Species forcing slope

$$\beta_{force,species} = \beta_{force} + \sigma_{betaforce,species} * \beta raw_{force} \tag{6}$$

Subpopulation forcing slope

$$\beta_{force,sppop} = \beta_{force,species} + \sigma_{\beta force,sppop} * \beta raw_{force,sppop} \tag{7}$$

Species photoperiod slope

$$\beta_{photo,species} = \beta_{photo} + \sigma_{betaphoto,species} * \beta raw_{photo} \tag{8}$$

Subpopulation photoperiod slope

$$\beta_{photo,sppop} = \beta_{photo,species} + \sigma_{\beta photo,sppop} * \beta raw_{photo,sppop} \tag{9}$$

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Grand intercept

$$\sigma_{\alpha,study} \sim normal(0, 20) \tag{10}$$

Study intercept

$$\sigma_{\alpha,study} \sim normal(0, 20) \tag{11}$$

$$\alpha raw_{study} \sim normal(0, 1) \tag{12}$$

Species intercept

$$\alpha raw_{species} \sim normal(0, 1) \tag{13}$$

$$\sigma_{\alpha,species} \sim normal(0, 20) \tag{14}$$

Subpopulation intercept

$$\sigma_{\alpha,sppop} \sim normal(0, 20) \tag{15}$$

$$\alpha raw_{sppop} \sim normal(0, 1) \tag{16}$$

Grand forcing slope

$$\beta_{force} \sim normal(0, 20) \tag{17}$$

Species forcing slope

$$\beta raw_{force} \sim normal(0, 1) \tag{18}$$

$$\sigma_{\beta force,species} \tag{19}$$

Subpopulation forcing slope

$$\sigma_{\beta force,sppop} \sim normal(0, 20) \tag{20}$$

$$\sigma_{\beta force,sppop} \tag{21}$$

Grand photoperiod slope

$$\beta_{photo} \sim normal(0, 20) \tag{22}$$

Species photoperiod slope

$$\sigma_{\beta photo,species} \tag{23}$$

$$\beta raw_{photo} \sim normal(0, 1) \tag{24}$$

Subpopulation photoperiod slope

$$\sigma_{\beta photo,sppop} \tag{25}$$

$$\beta raw_{photo,sppop} \sim normal(0, 1) \tag{26}$$

General variance

$$\sigma_y \sim normal(0, 10) \tag{27}$$

4 Model explained in words

$$\tilde{y}_i \sim normal(\mu_i, \sigma_y) \tag{28}$$

Every $\tilde{y}_i$ value is centered around a mean predicted value μ_i with a normal distribution of width σ_y

$$\mu_i = \alpha + \alpha_{study} + \alpha_{sppop} + force_i * \beta_{force,sppop} + photo_i * \beta_{photo,sppop} \tag{29}$$

Each mean predicted value μ_i has intercept which is a combination of a grand intercept (α), an intercept based on the study of value i (α_{study}), and intercept variation due to the species and population combination (α_{sppop}). There are also two slopes that relate forcing ($\beta_{force,sppop}$) and photoperiod ($\beta_{photo,sppop}$) of value i to it's predicted μ_i value. These slope values depend on the species and subpopulation of value i, meaning a different species and subpopulation combination may react faster or slower to forcing and photoperiod.

$$\alpha_{study} = \sigma_{\alpha,study} * \alpha raw_{study} \tag{30}$$

Each study deviates somewhat from the grand mean intercept α; we call this value α_{study}. Sometimes you might see this written as $\alpha_{study} \sim normal(\alpha, \sigma_{apha,study})$.

$$\alpha_{sppop} = \alpha_{species} + \sigma_{\alpha force,sppop} * \alpha raw_{force,sppop} \tag{31}$$

Each population deviates somewhat from it's species mean value ($\alpha_{species}$). The amount they deviate a normal distribution with a width of $\sigma_{\alpha force,sppop}$.

$$\alpha_{species} = \sigma_{\alpha,species} * \alpha raw_{species} \tag{32}$$

How much each species deviates from the grand mean α value is called $\alpha_{species}$, and is drawn from a normal distribution of width $\sigma_{\alpha,species}$.

$$\beta_{force,sppop} = \beta_{force,species} + \sigma_{\beta force,sppop} * \beta raw_{force,sppop} \tag{33}$$

Each population's rate of change due to an amount of forcing, that is the slope of forcing, is somewhat different. We describe this as the population values being centered around it's species mean slope $\beta_{force,species}$ and has a distribution of width $\sigma_{\beta force,sppop}$.

$$\beta_{force,species} = \beta_{force} + \sigma_{betaforce,species} * \beta raw_{force} \tag{34}$$

Each species's slope value for teh effect of forcing is drawn from a normal distribution centred around the grand beta slope β_{force} and a width of $\sigma_{betaforce,species}$.

$$\beta_{photo,sppop} = \beta_{photo,species} + \sigma_{\beta photo,sppop} * \beta raw_{photo} \tag{35}$$

Each population's rate of change due to an amount of photoperiod, that is the slope of photoperiod, is somewhat different. We describe this as the population values being centered around it's species mean slope $\beta_{force,species}$ and has a distribution of width $\sigma_{\beta force,sppop}$.

$$\beta_{photo,species} = \mu_{\beta photo,species} + \sigma_{betaphoto,species} * \beta raw_{photo,sppop} \tag{36}$$

Each species's slope value for the effect of photoperiod is drawn from a normal distribution centred around the grand beta slope β_{force} and a width of $\sigma_{betaforce,species}$.