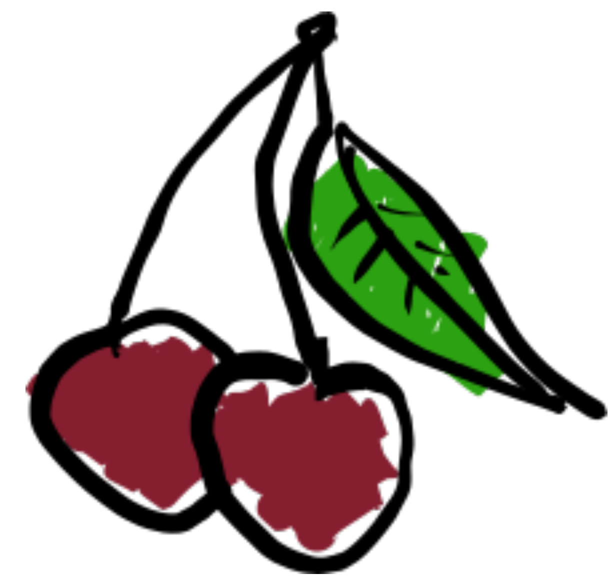
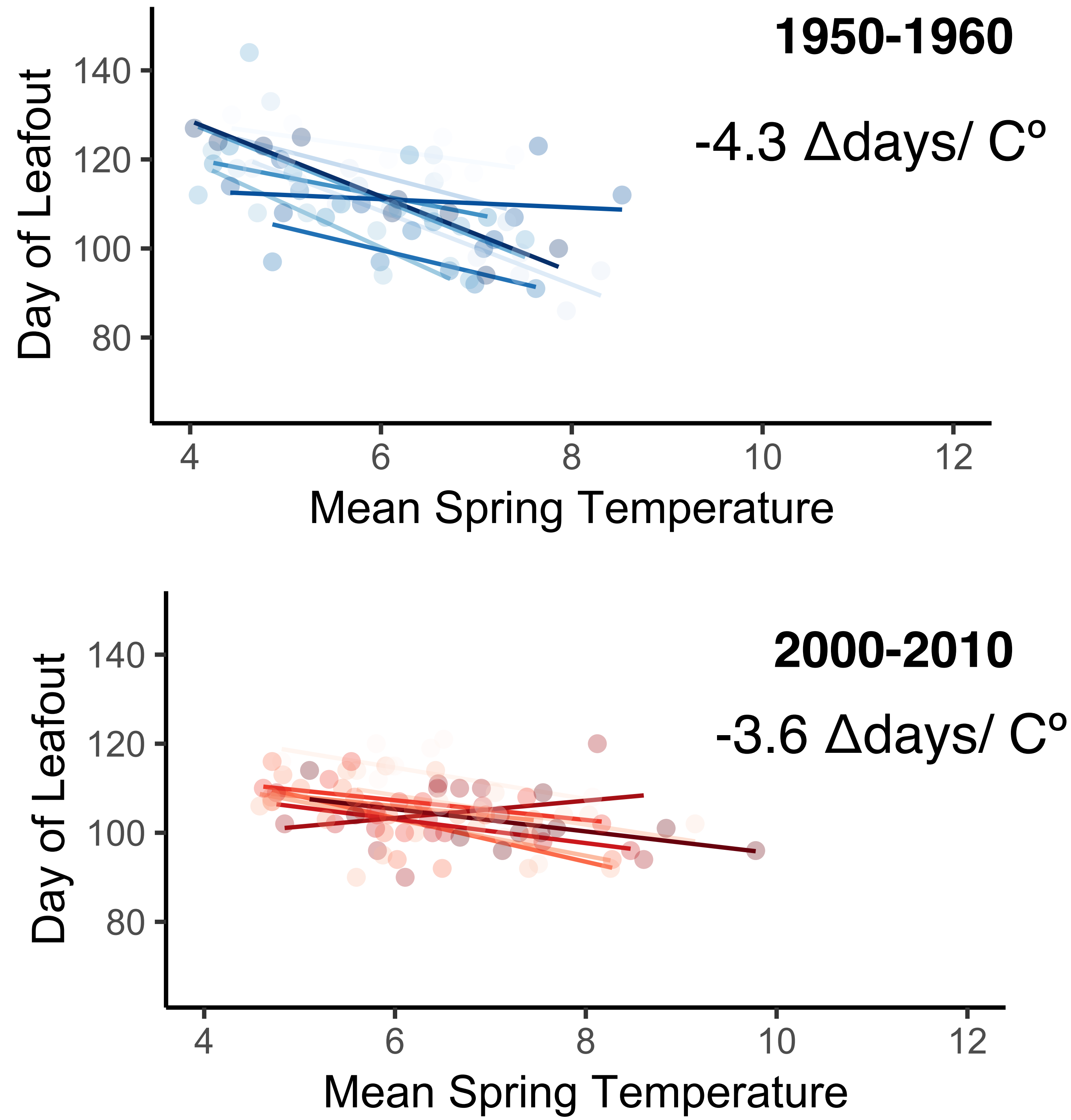
Cherry sandbox 

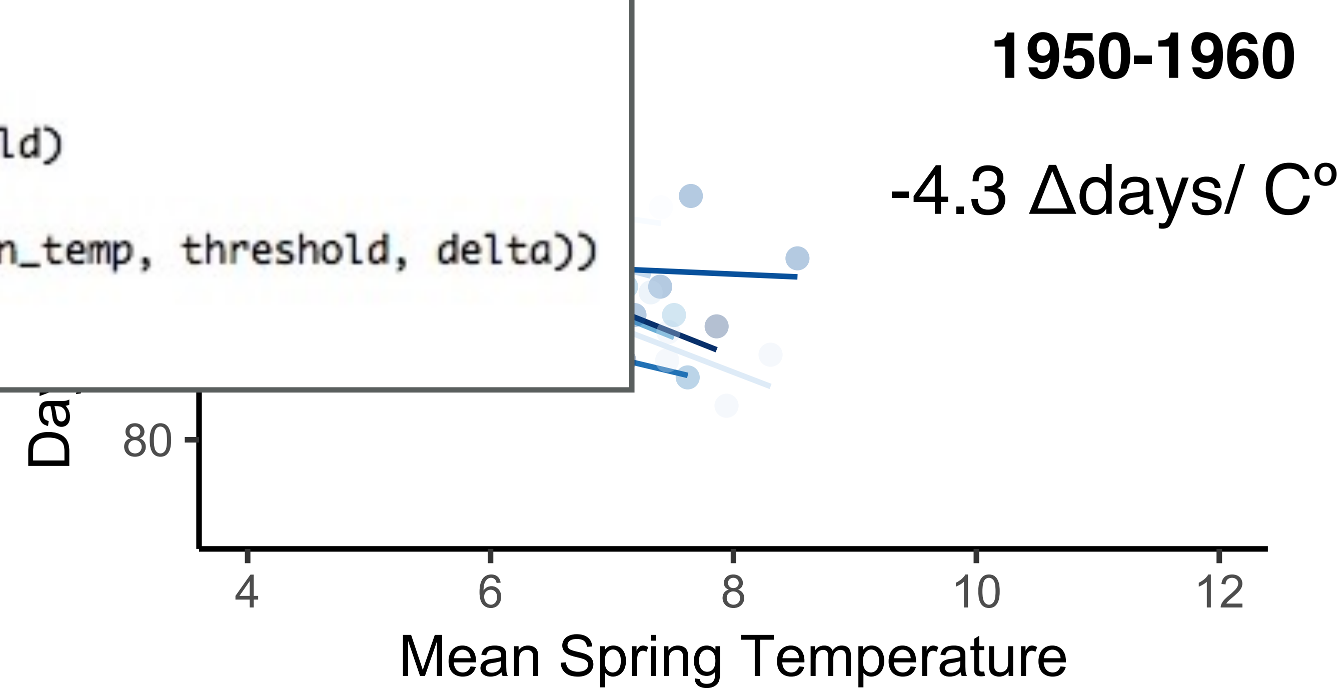
24 November 2020 Bayes Group

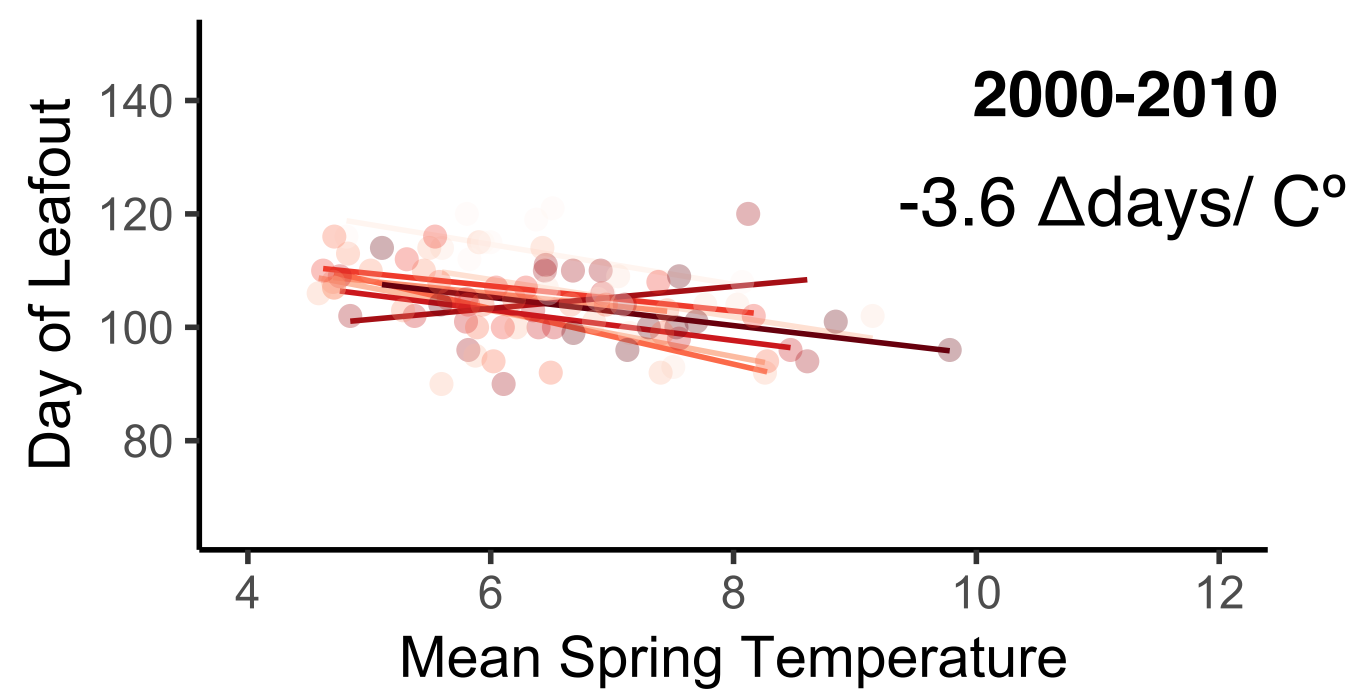
Simple thermal sum model



Simple thermal sum model

```r
threshold <- 500
for(delta in c(5, 10, 15, 20)) {
  for(sim in 1:1000) {
    temp <- delta * (1:100) + rnorm(100, 0, 50)
    leaf_date <- which.min(cumsum(temp) < threshold)
    mean_temp <- mean(temp[1:leaf_date])
    data <- rbind(data, data.frame(leaf_date, mean_temp, threshold, delta))
  }
}
```





Simple thermal sum model

```
threshold <- 500
for(delta in c(5, 10, 15, 20)) {
  for(sim in 1:1000) {
    temp <- delta * (1:100) + rnorm(100, 0, 50)
    leaf_
    mean_
    data
  }
}
```

1950-1960

/ C°

2

10

s/ C°

2

$$n = \text{day since temperatures start to accumulate, } n = 0, 1, ..., N$$

$$S_0^n = \sum_{i=0}^{n} X_i, \text{the cumulative daily temperature from day 0 to day } n$$

$$M_0^n = \frac{S_0^n}{n}, \text{the average daily temperature from day 0 to day } n$$

$$\beta = \text{the threshold of interest, } \beta > 0, \text{ (thermal sum required for leafout)}$$

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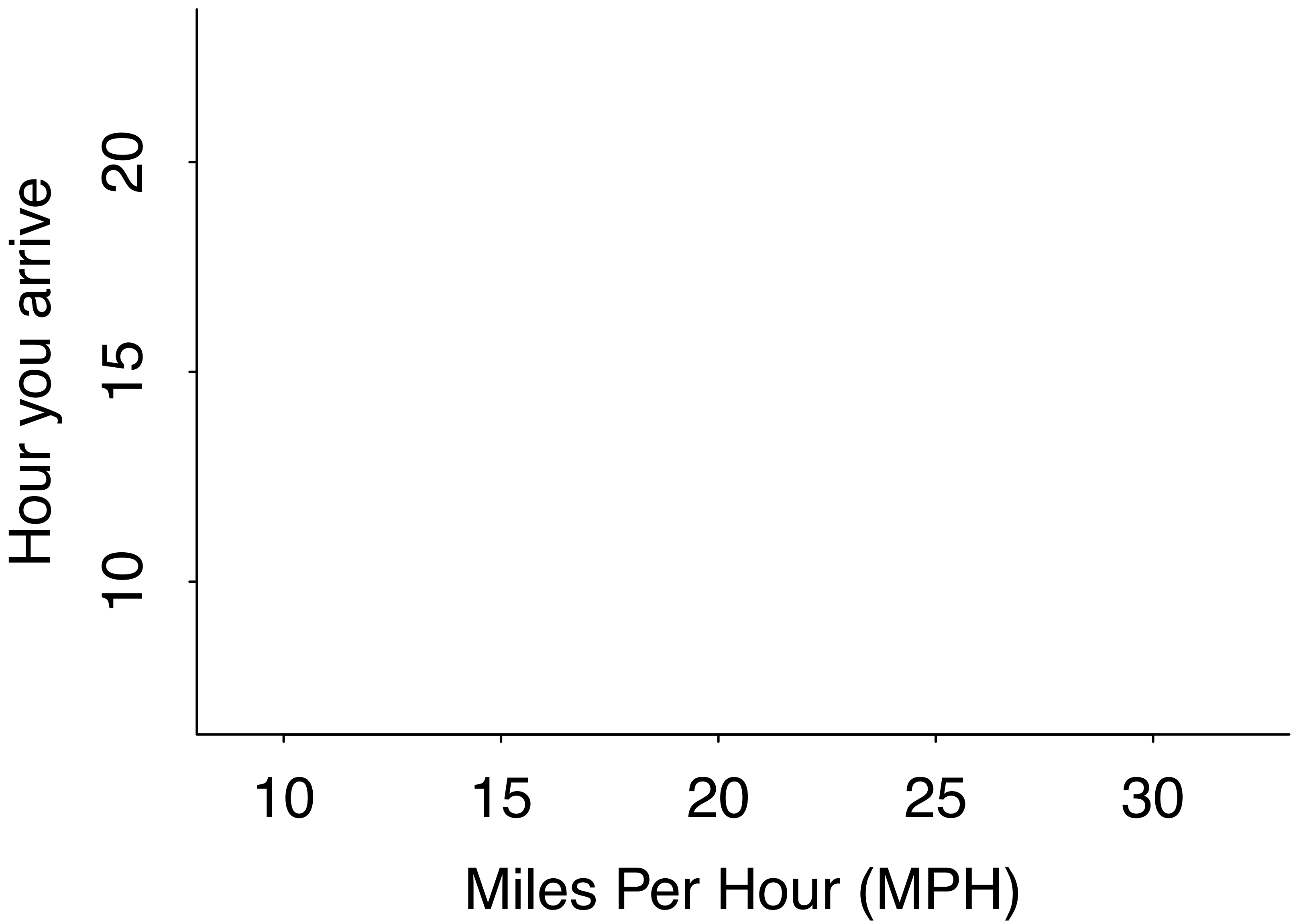
Thus,

$$n_\beta = \frac{\beta}{M_0^{n_\beta}}$$

We model X_n as a Gaussian random walk, $X_n \overset{\text{i.i.d}}{\sim} \text{normal}\,(\alpha_0 + \alpha_1 n, \sigma)$, where $\alpha_0 > 0$ is the average temperature on day $n = 0$, $\alpha_1 > 0$ is the day-over-day increase in average temperatures, and σ is the standard deviation. This model differs from the traditional Gaussian random walk because of the factor n.

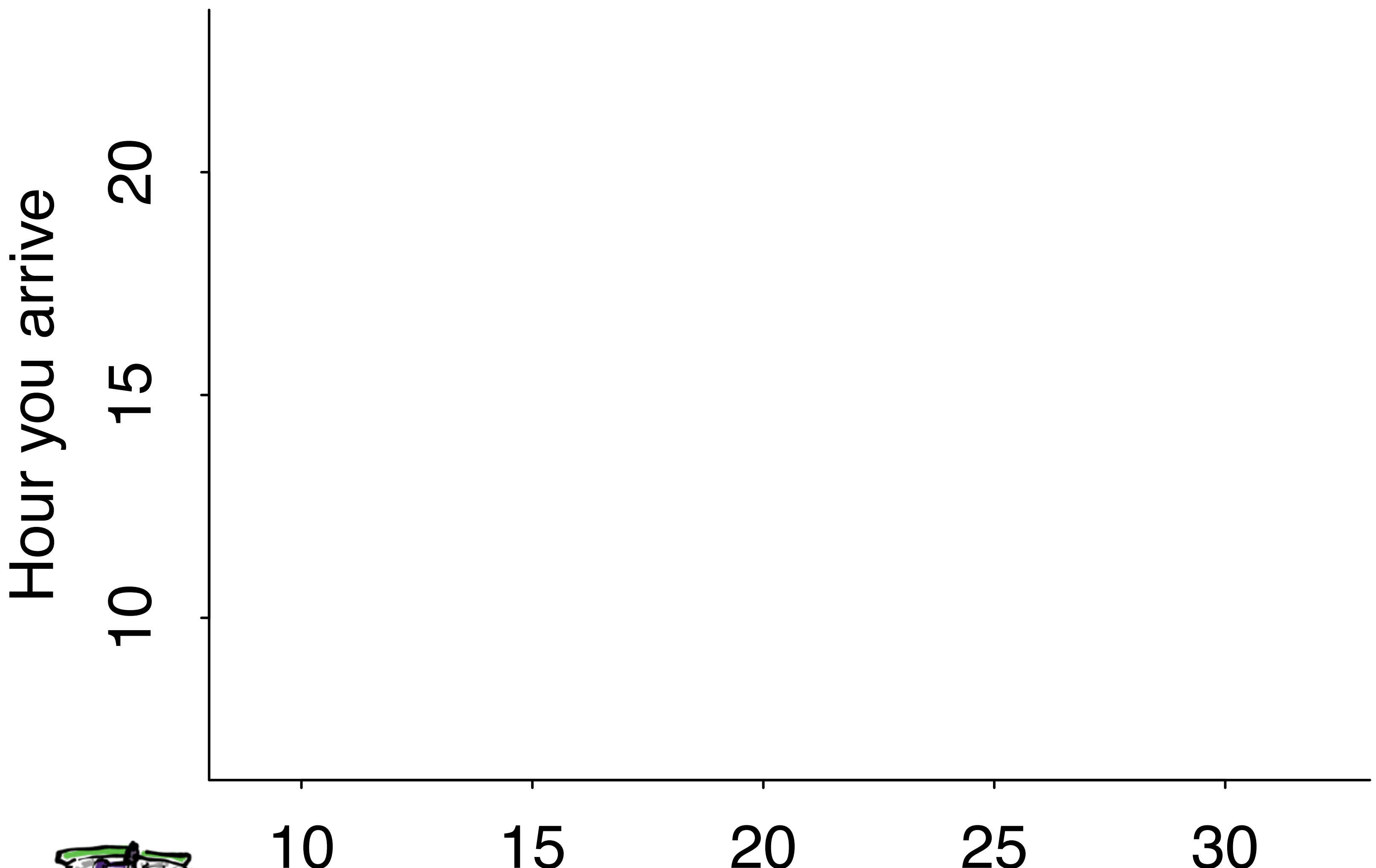
Rate dependent process



Grandma lives 200 miles away

Art by Faith Jones

Rate dependent process

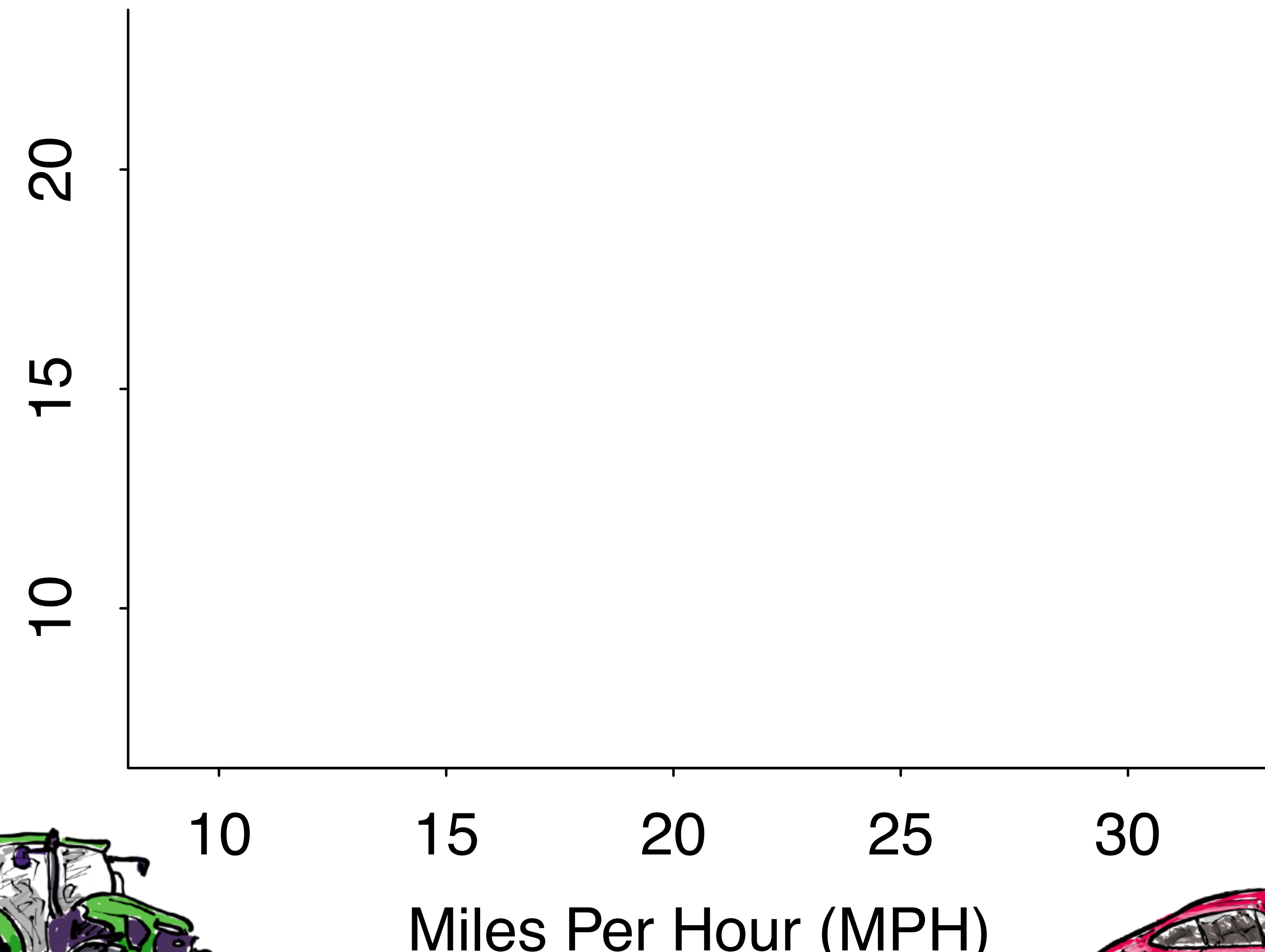


Grandma lives
200 miles away

slow

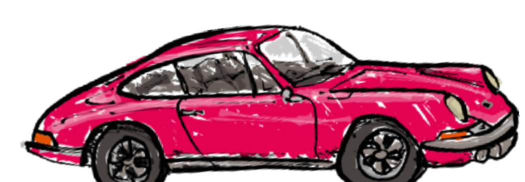
Art by Faith Jones

Rate dependent process



Grandma lives
200 miles away

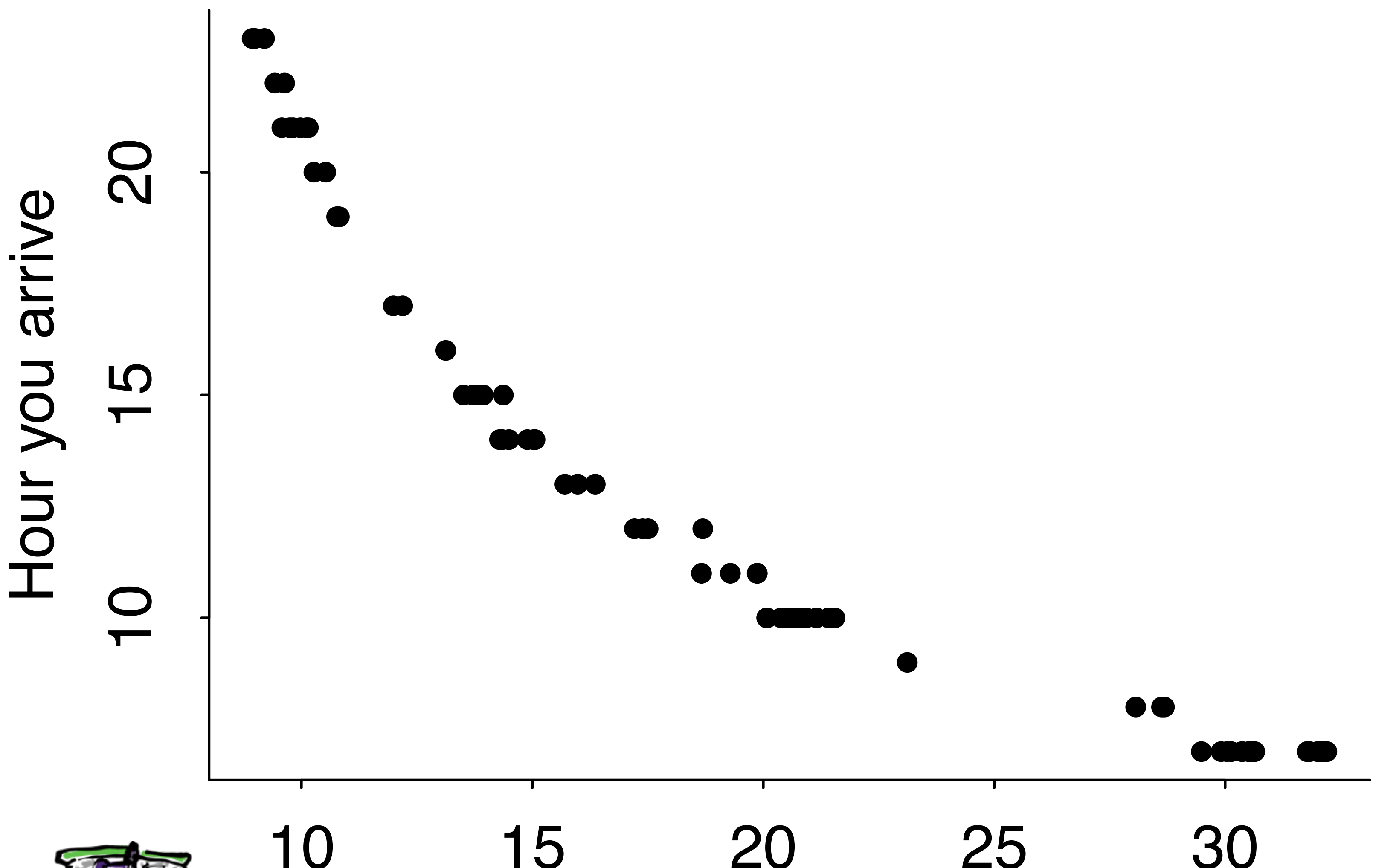
slow



fast!

Art by Faith Jones

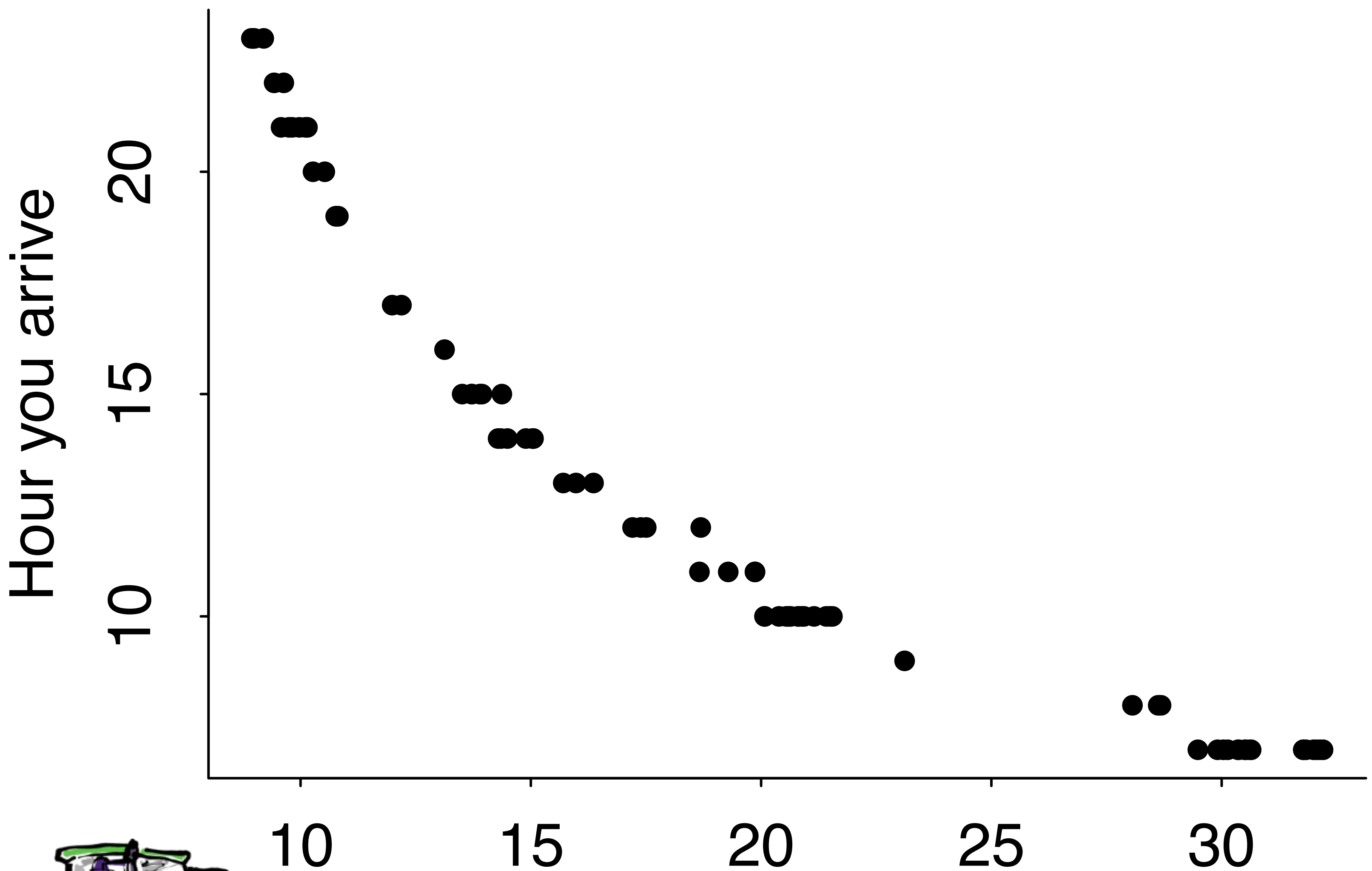
Rate dependent process



Grandma lives
200 miles away

fast!

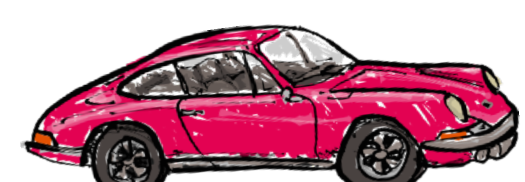
Rate dependent process



Grandma lives
200 miles away

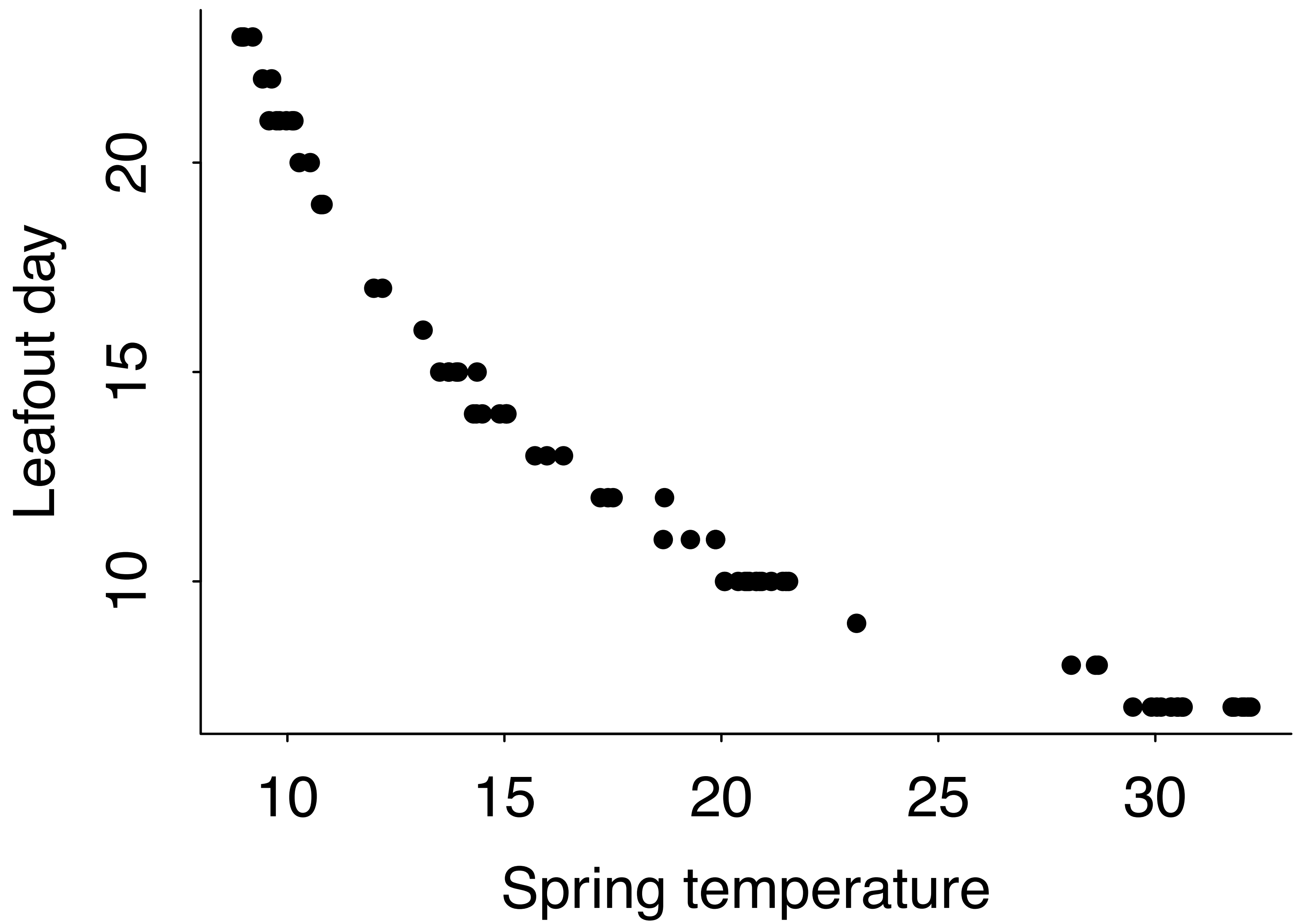
slow

Warming as accelerator



fast!

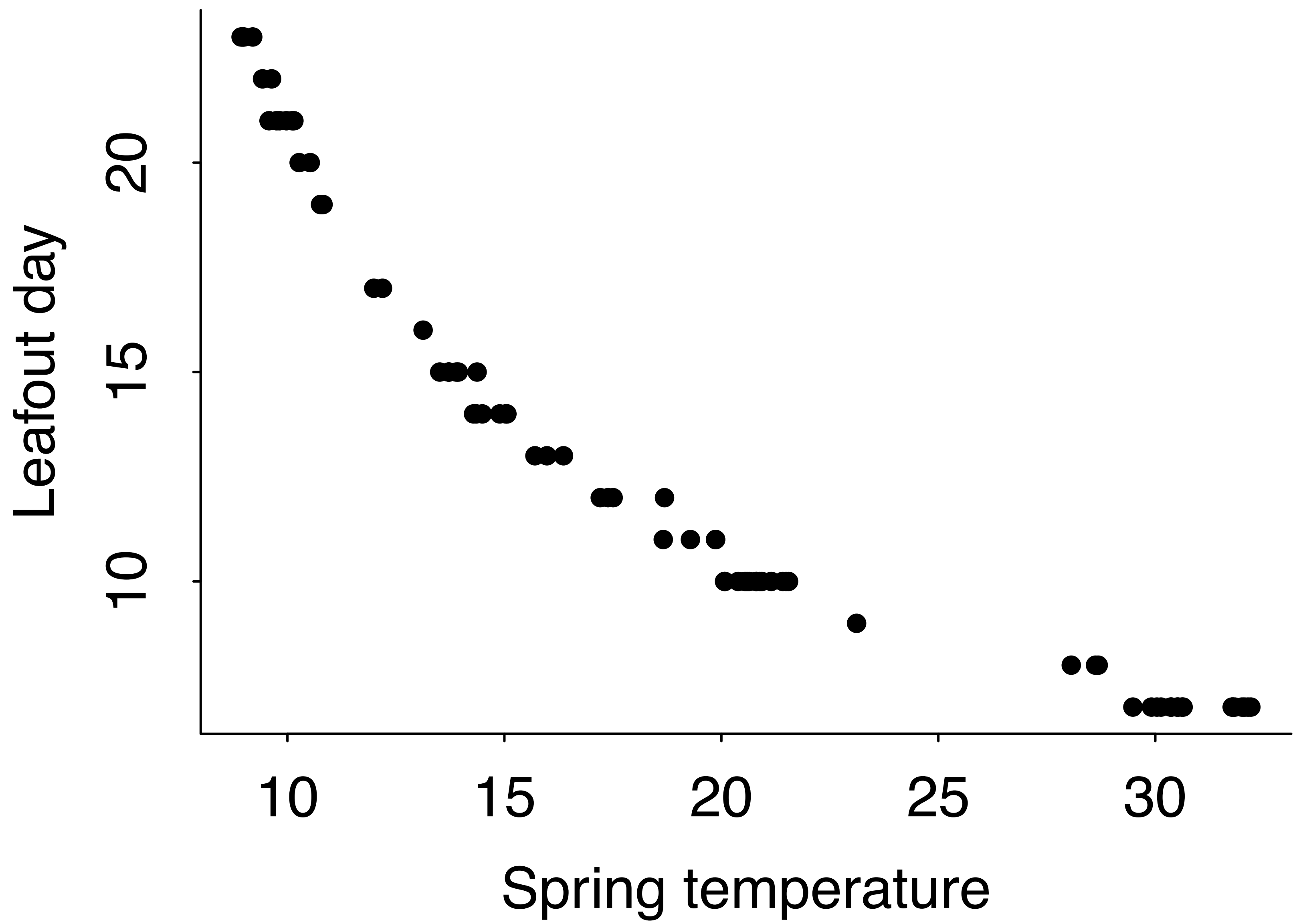
Leafout is rate dependent



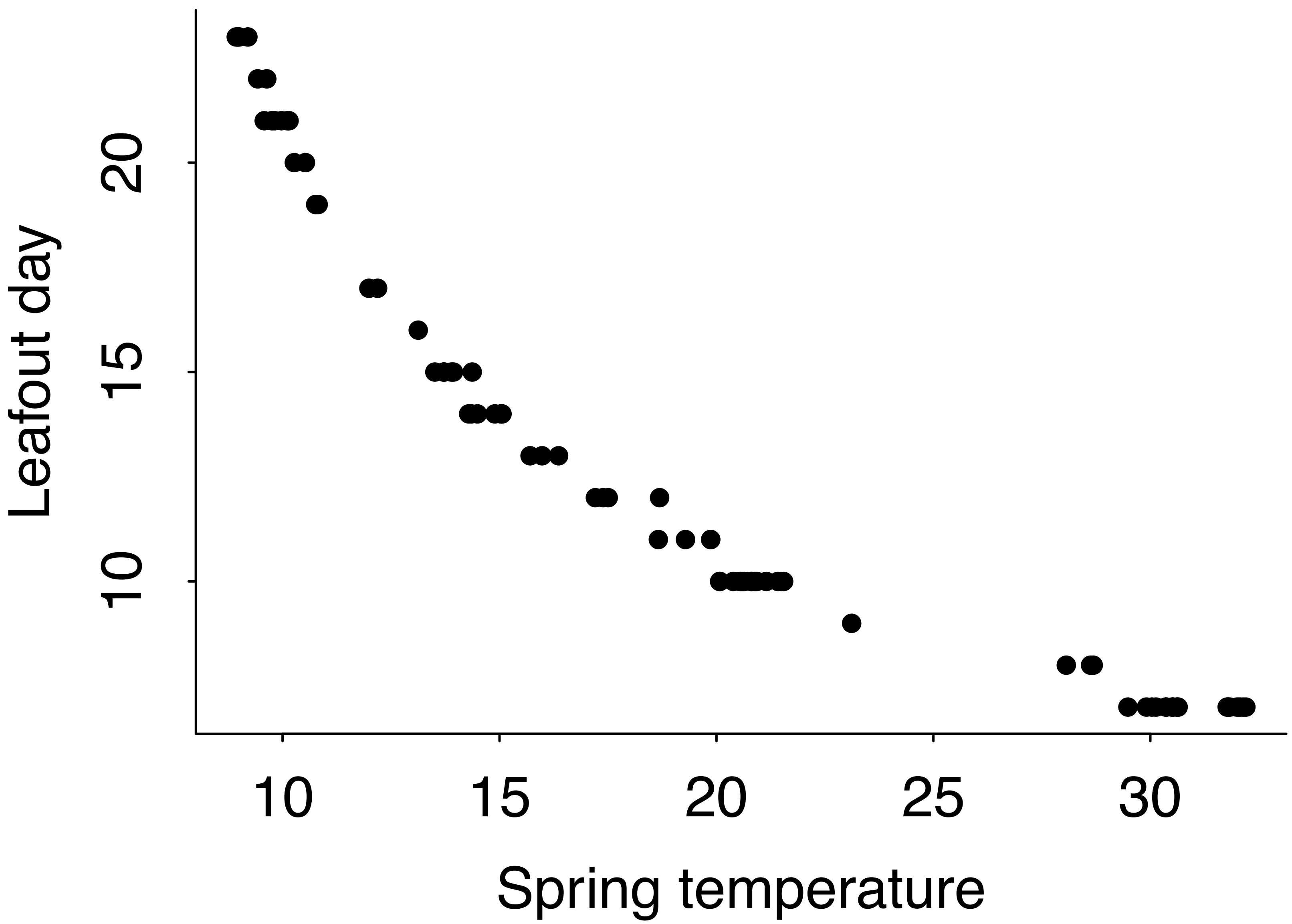
Warming as accelerator

Leafout takes
200 thermal units

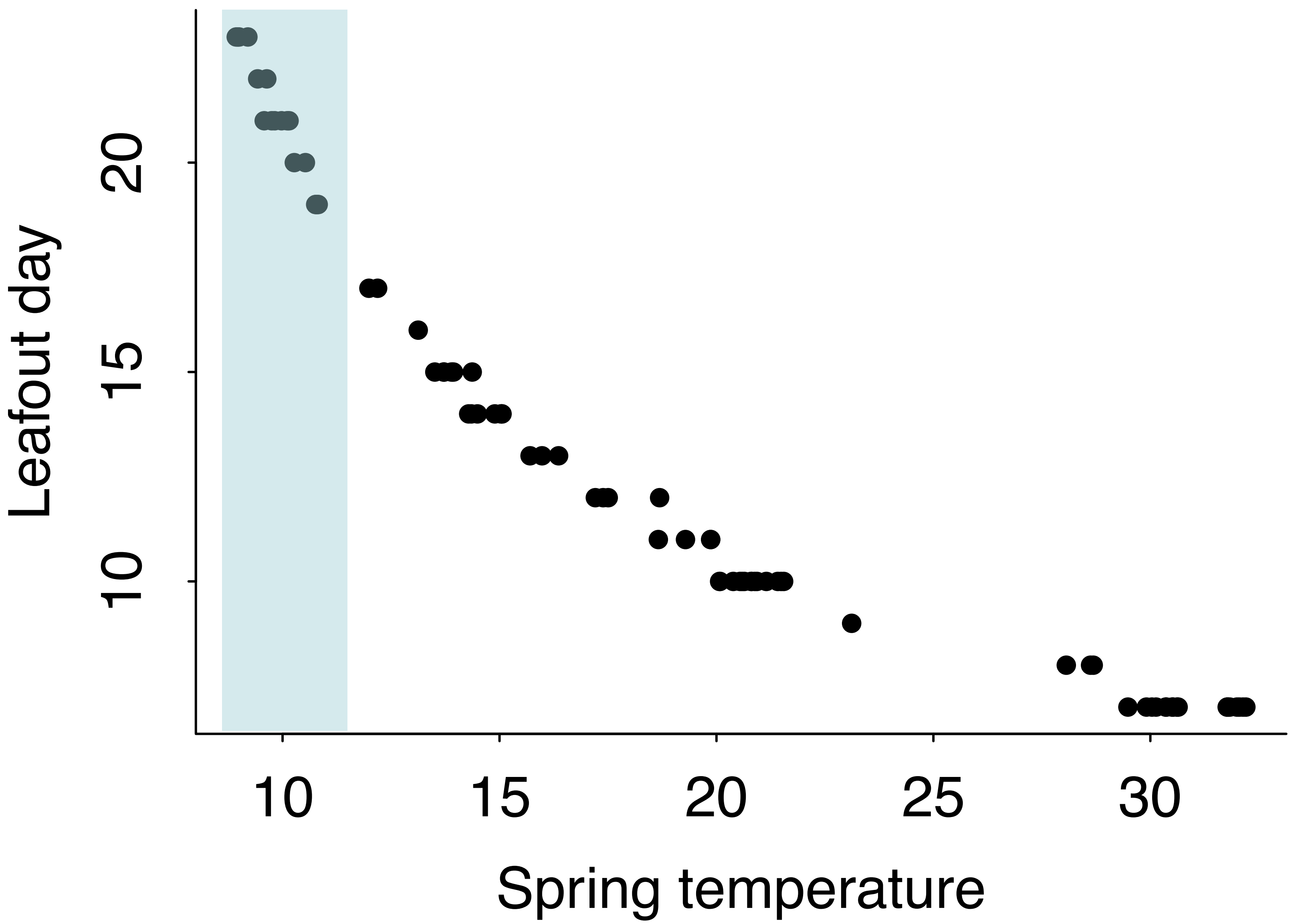
Thermal sum = inverse



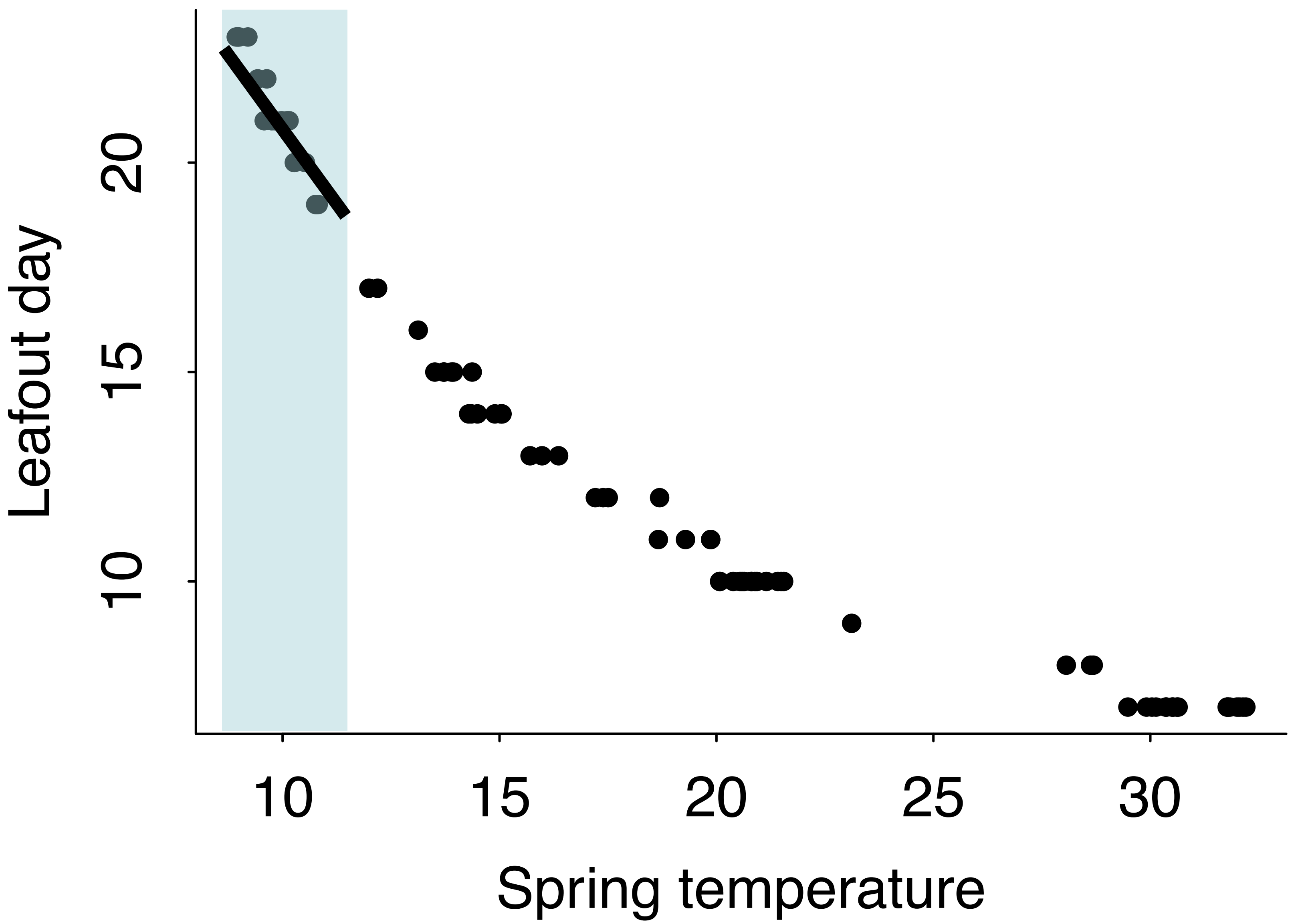
Thermal sum = inverse
= non-linear



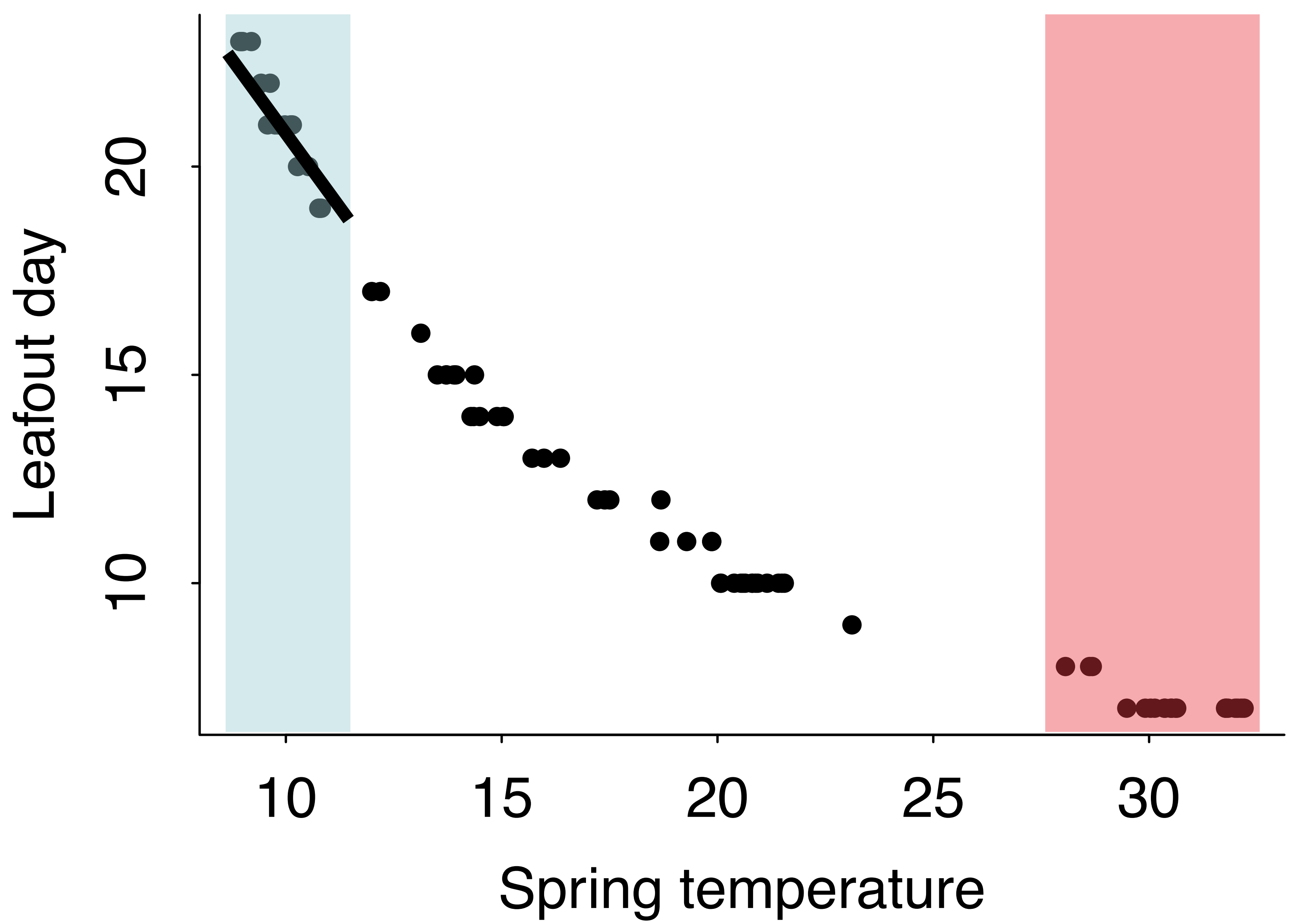
Linear approximation works only within a range



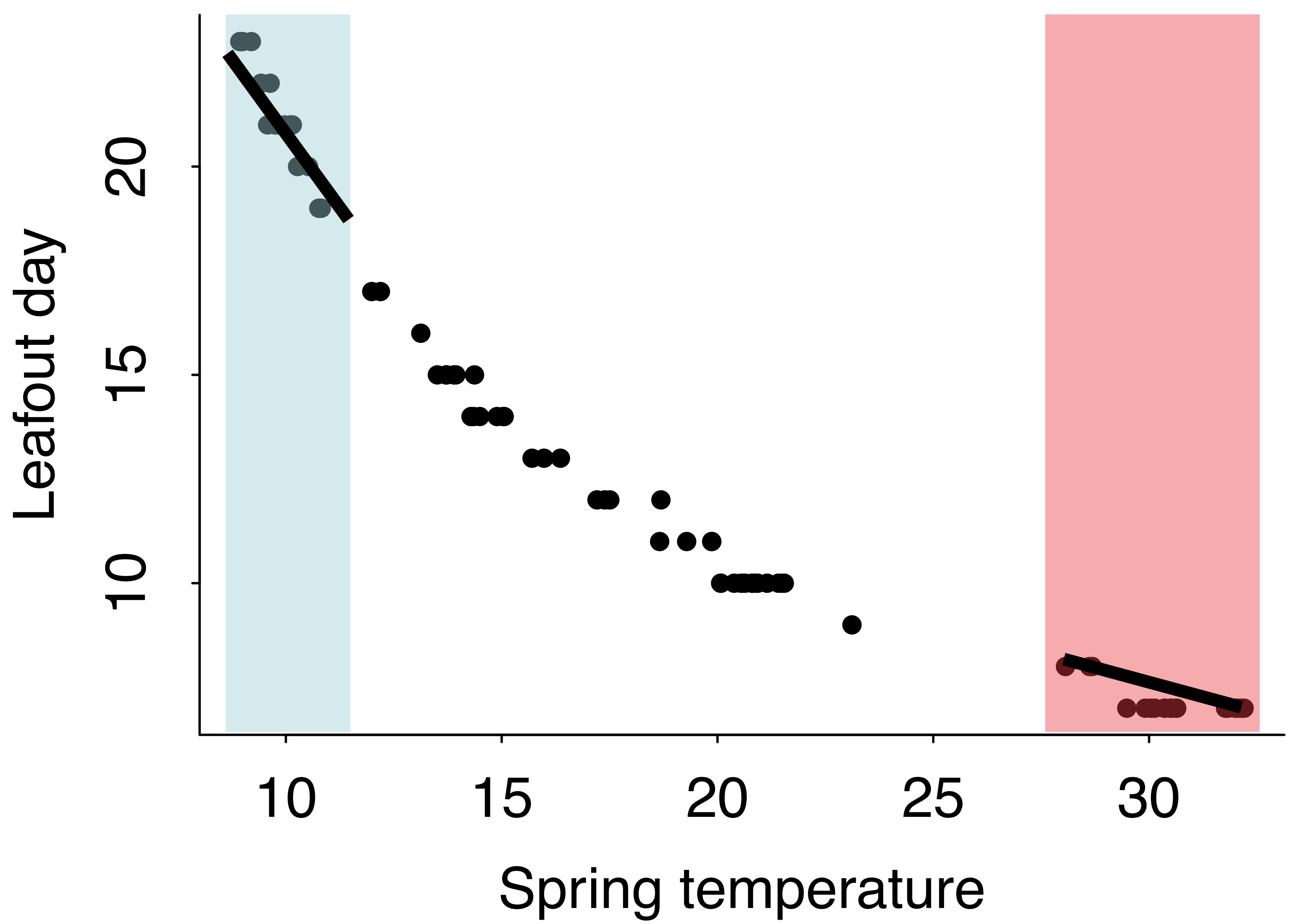
Linear approximation works only within a range



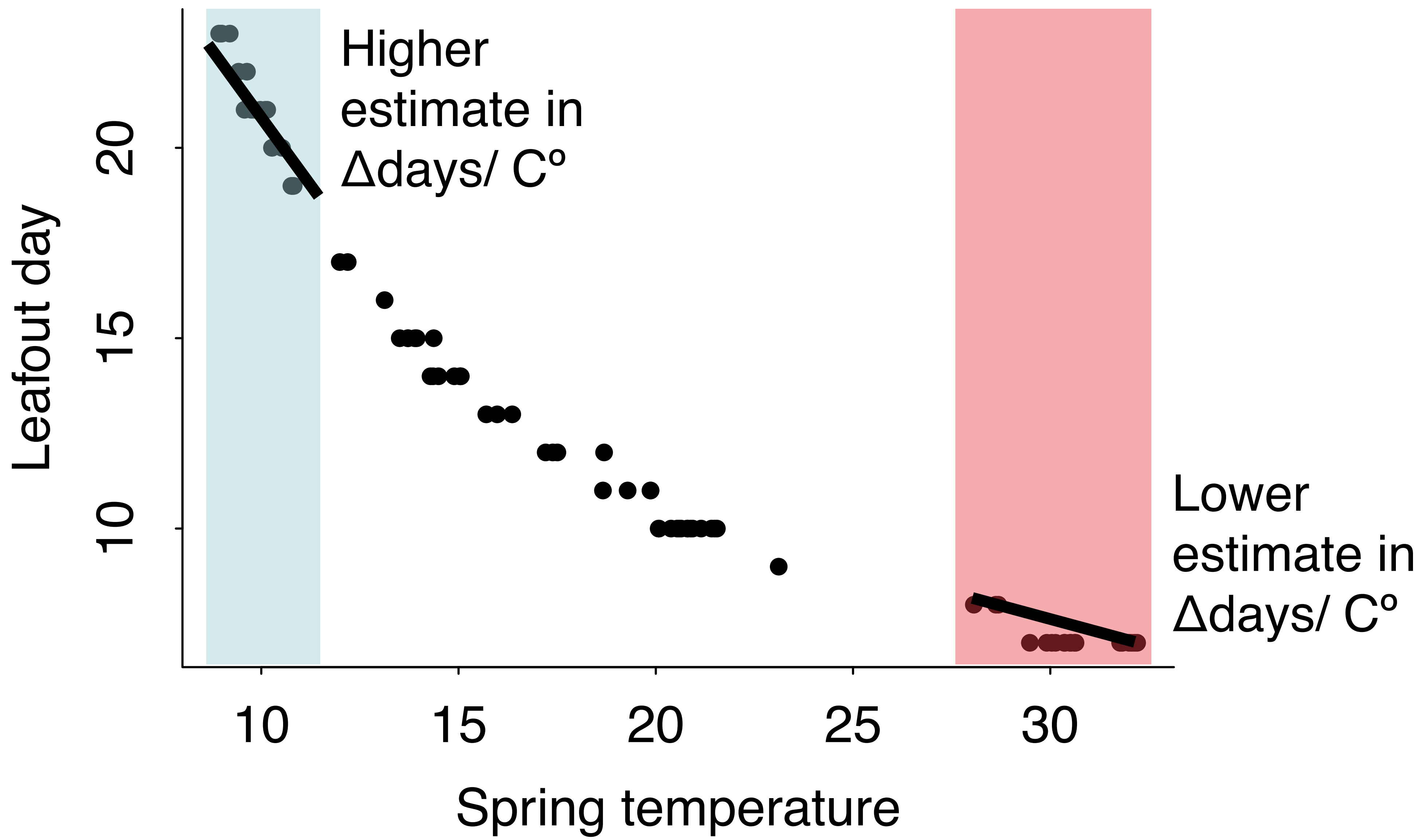
Linear approximation works only within a range



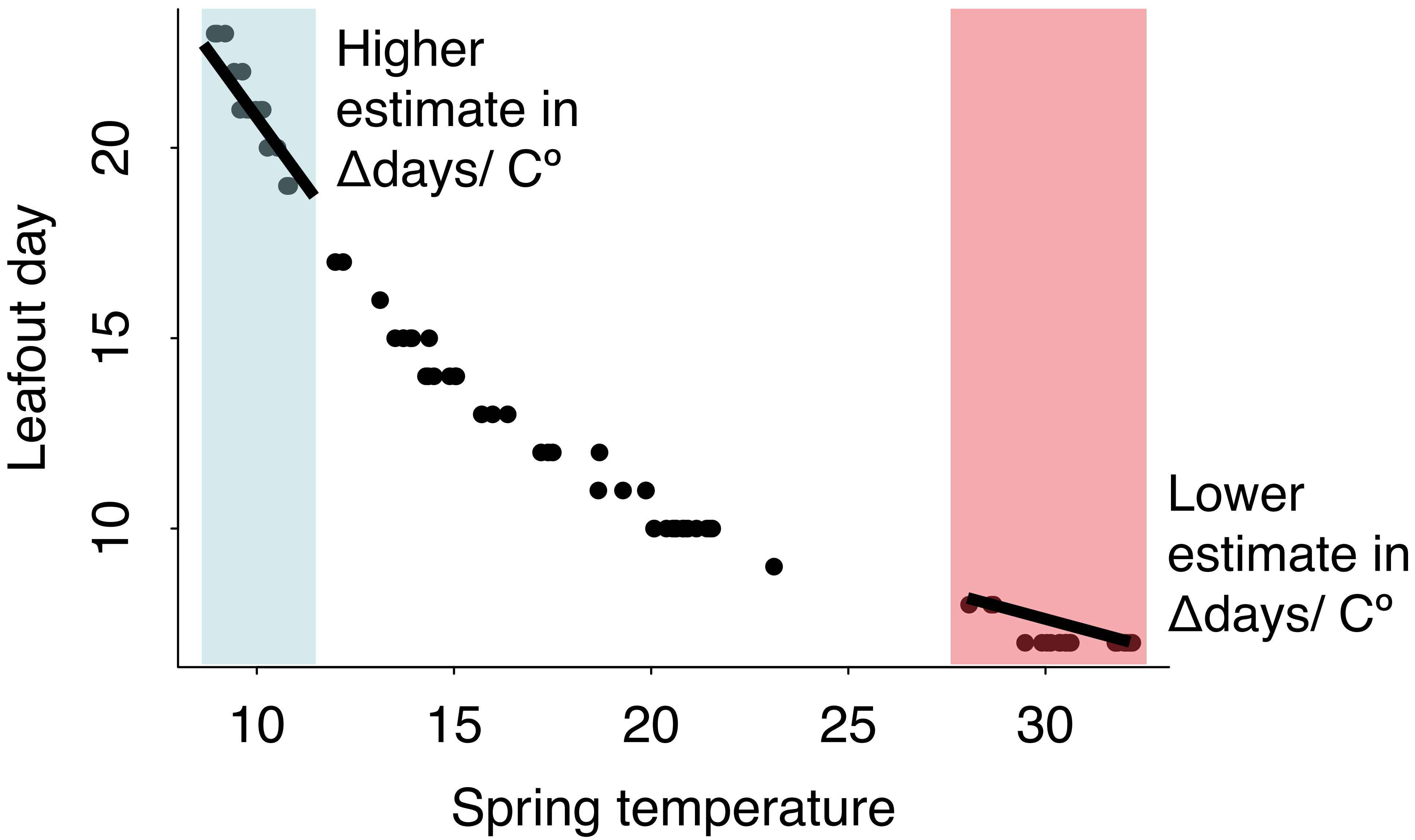
Linear approximation works only within a range



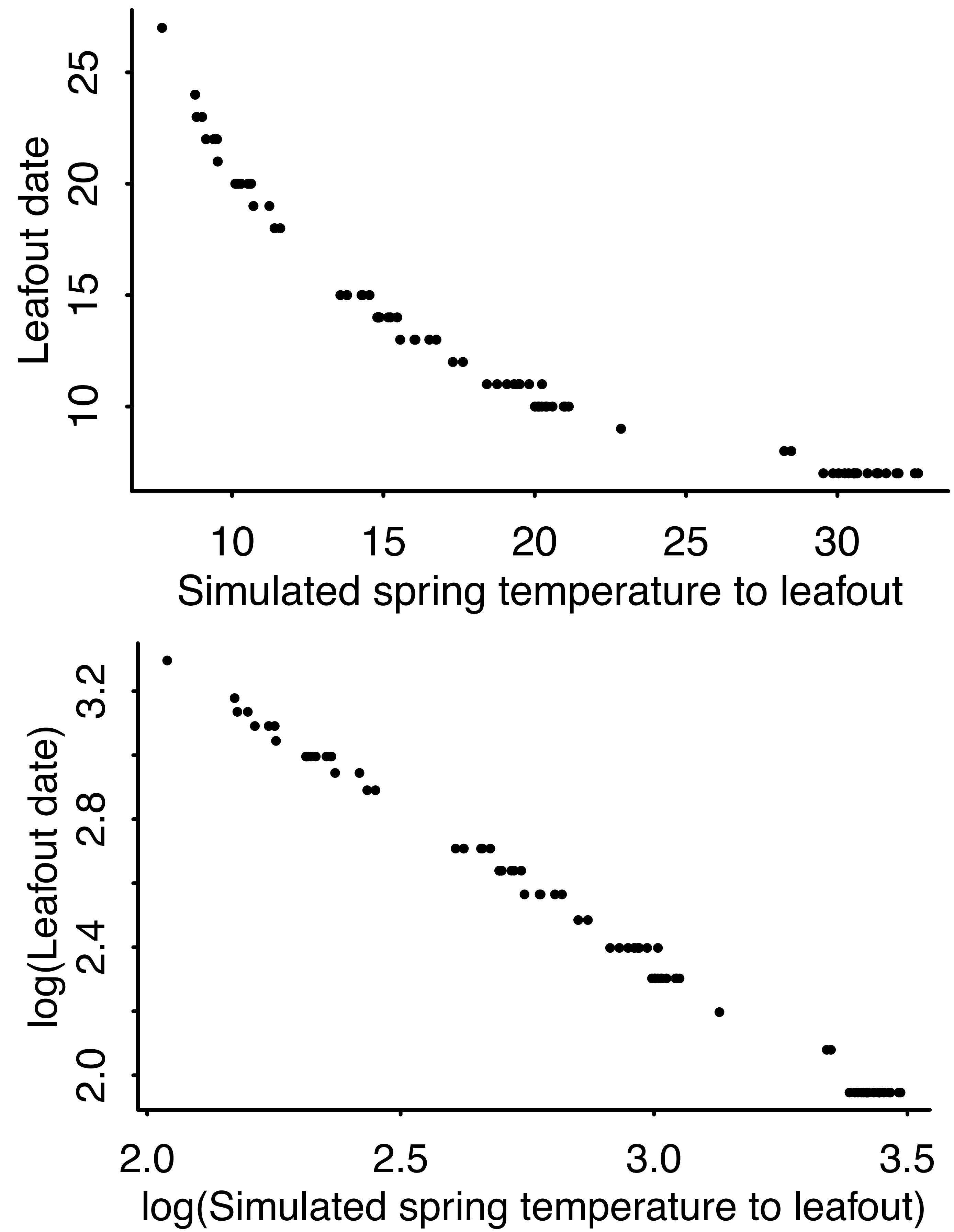
Linear approximation works only within a range

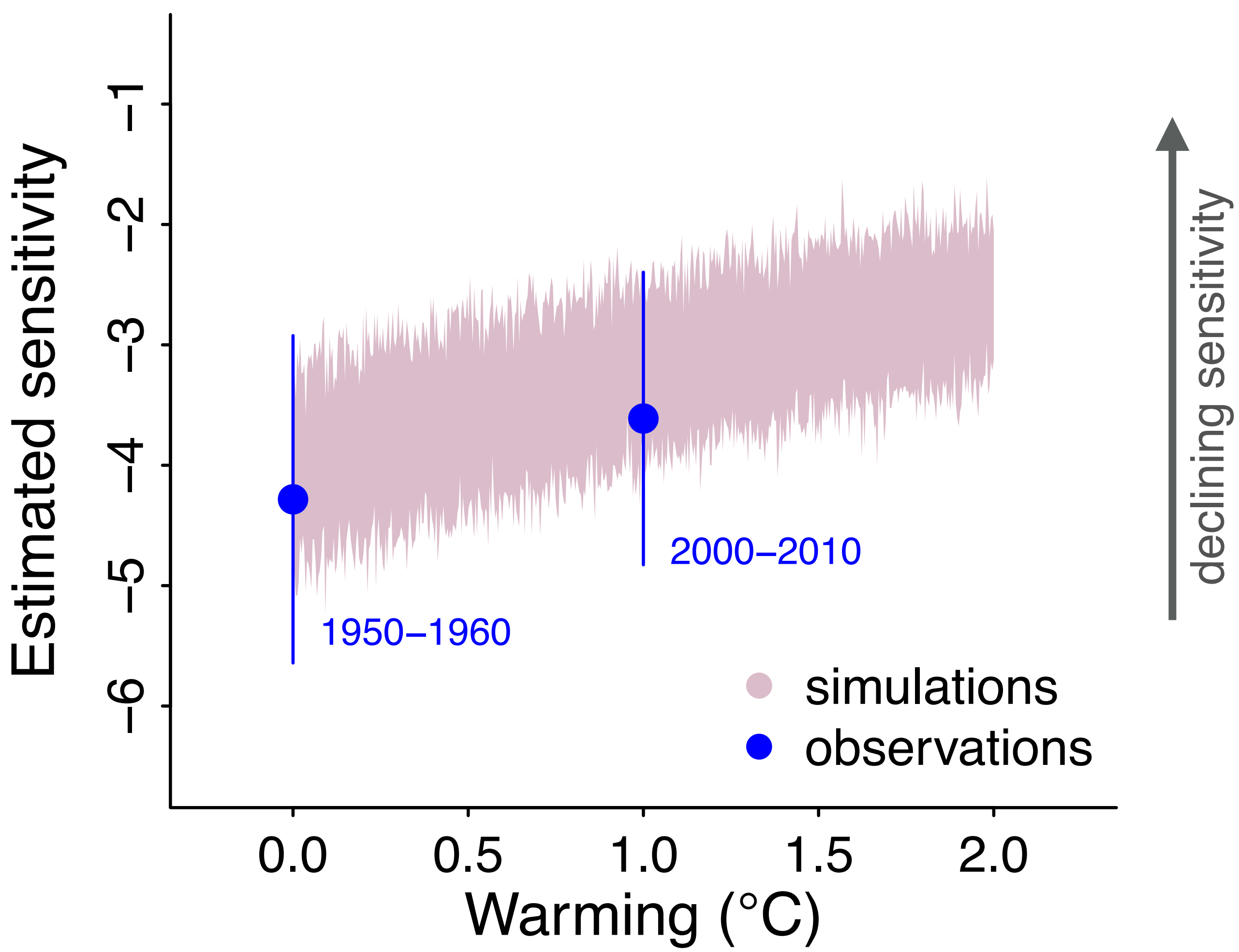


How do we address this?

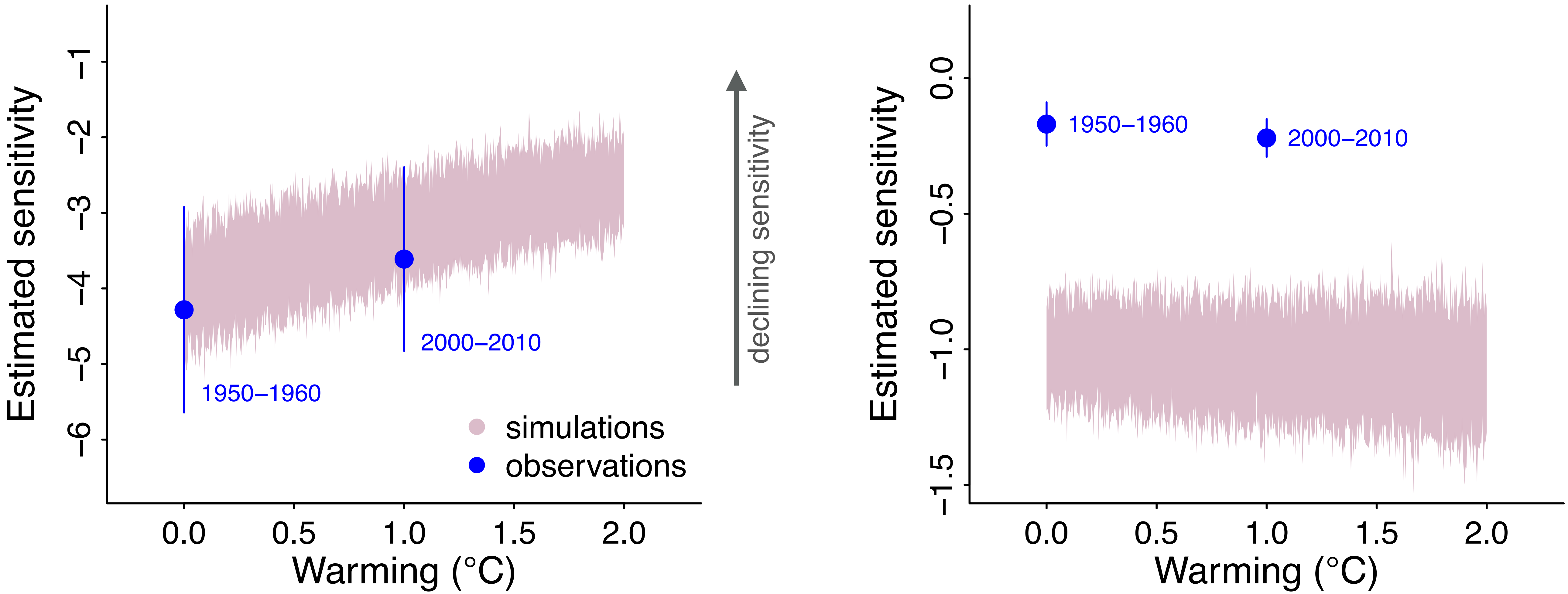


Option 1: Log linearizes inverse





Estimated sensitivity
−1
−2
−3
−4
−5
−6
0.0
0.5
1.0
1.5
2.0
Warming (°C)
1950−1960
2000−2010
simulations
observations
declining sensitivity



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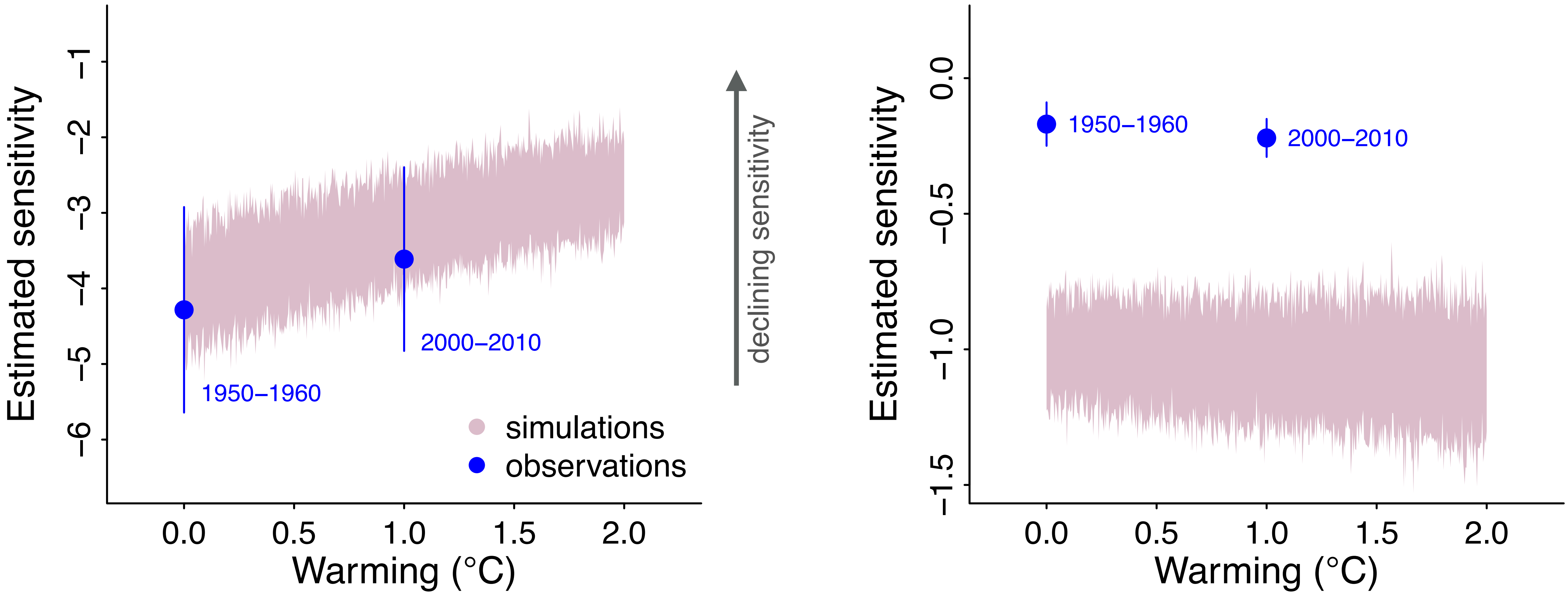
0.5
1.0
1.5
2.0
Warming (°C)
1950−1960
2000−2010

No evidence of decline



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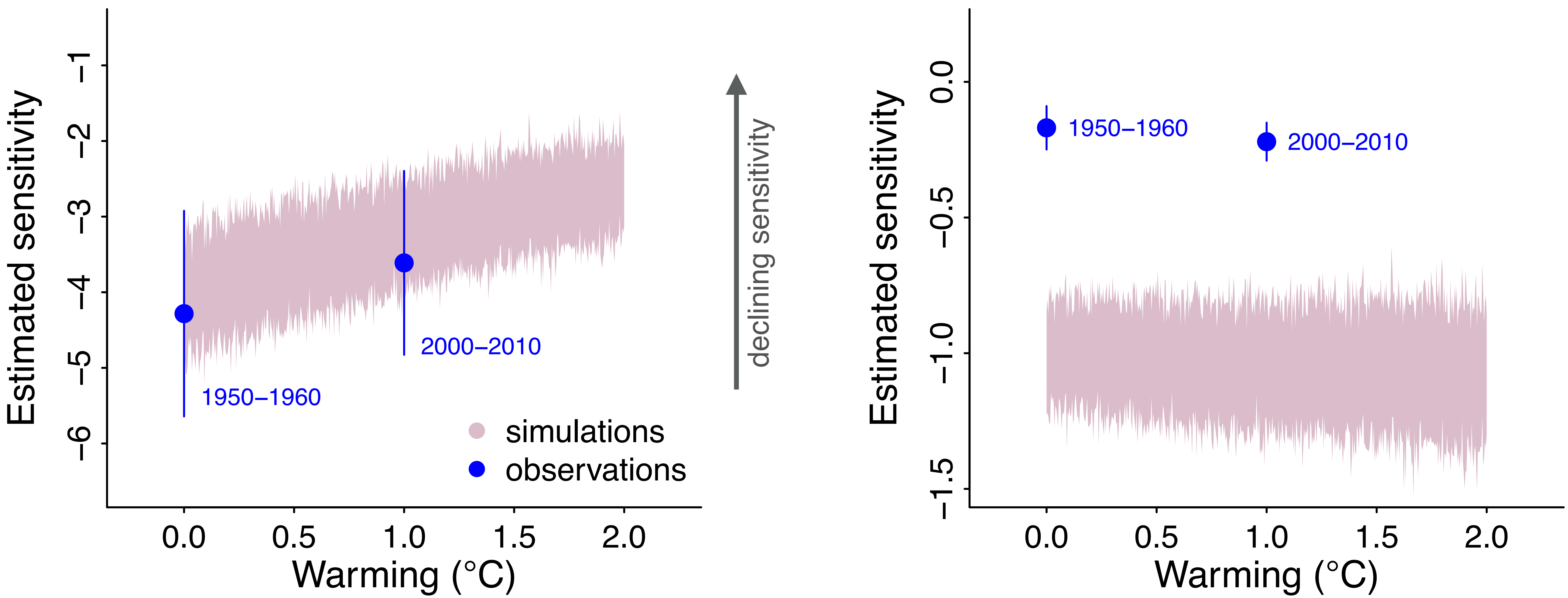


We have found — and removed — 'declining sensitivities' in every long-term phenology dataset we have looked at

Option 2:

Build a generative model that includes climate change

... in progress

$$
\begin{aligned}
n &= \text{day since temperatures start to accumulate, } n = 0, 1, ..., N \\
S_0^n &= \sum_{i=0}^{n} X_i, \text{the cumulative daily temperature from day 0 to day } n \\
M_0^n &= \frac{S_0^n}{n}, \text{the average daily temperature from day 0 to day } n \\
\beta &= \text{the threshold of interest, } \beta > 0, \text{ (thermal sum required for leafout)} \\
n_\beta &= min(\ S_n > \beta), \text{ leafout day}
\end{aligned}
$$

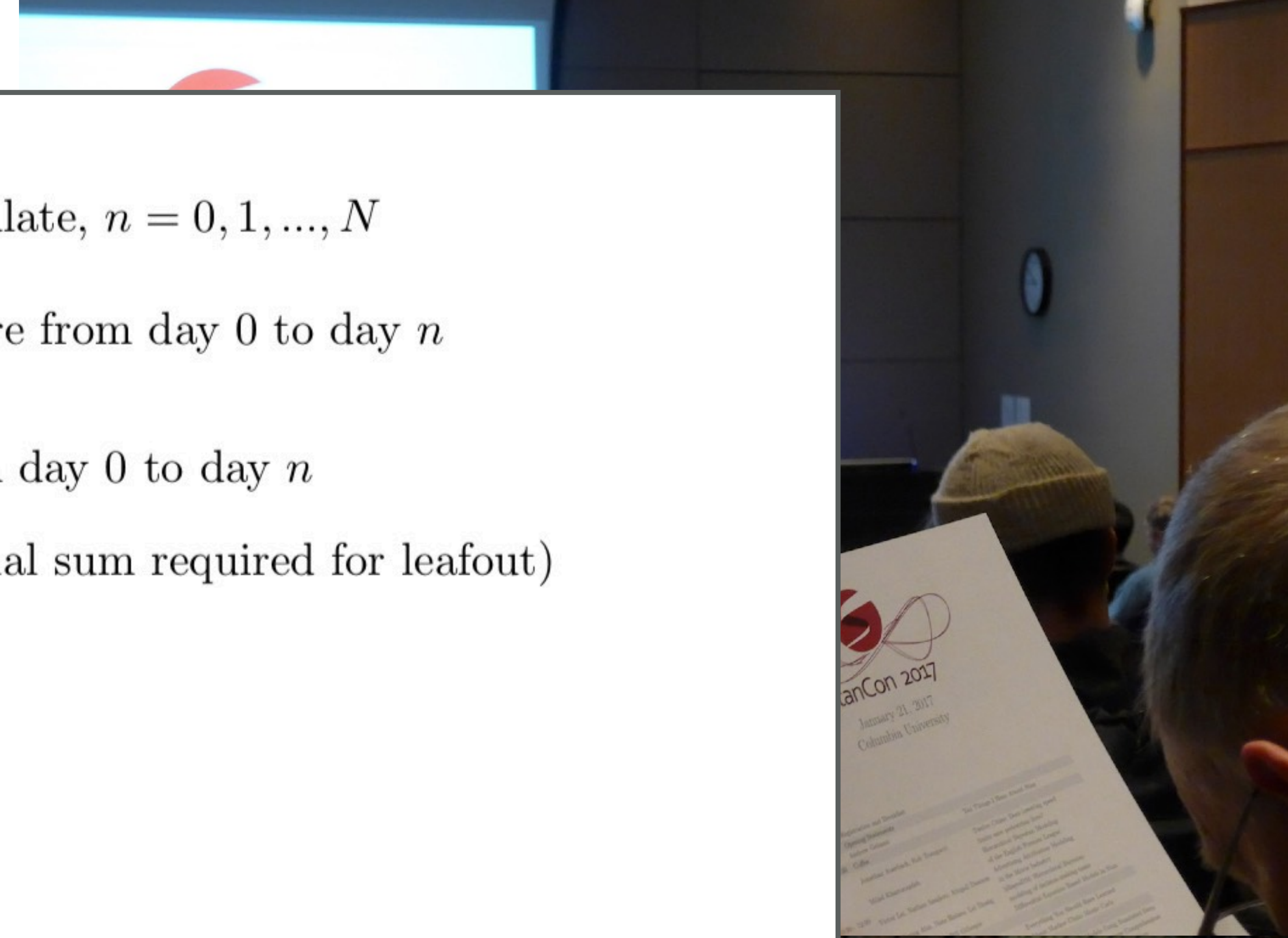
Thus,

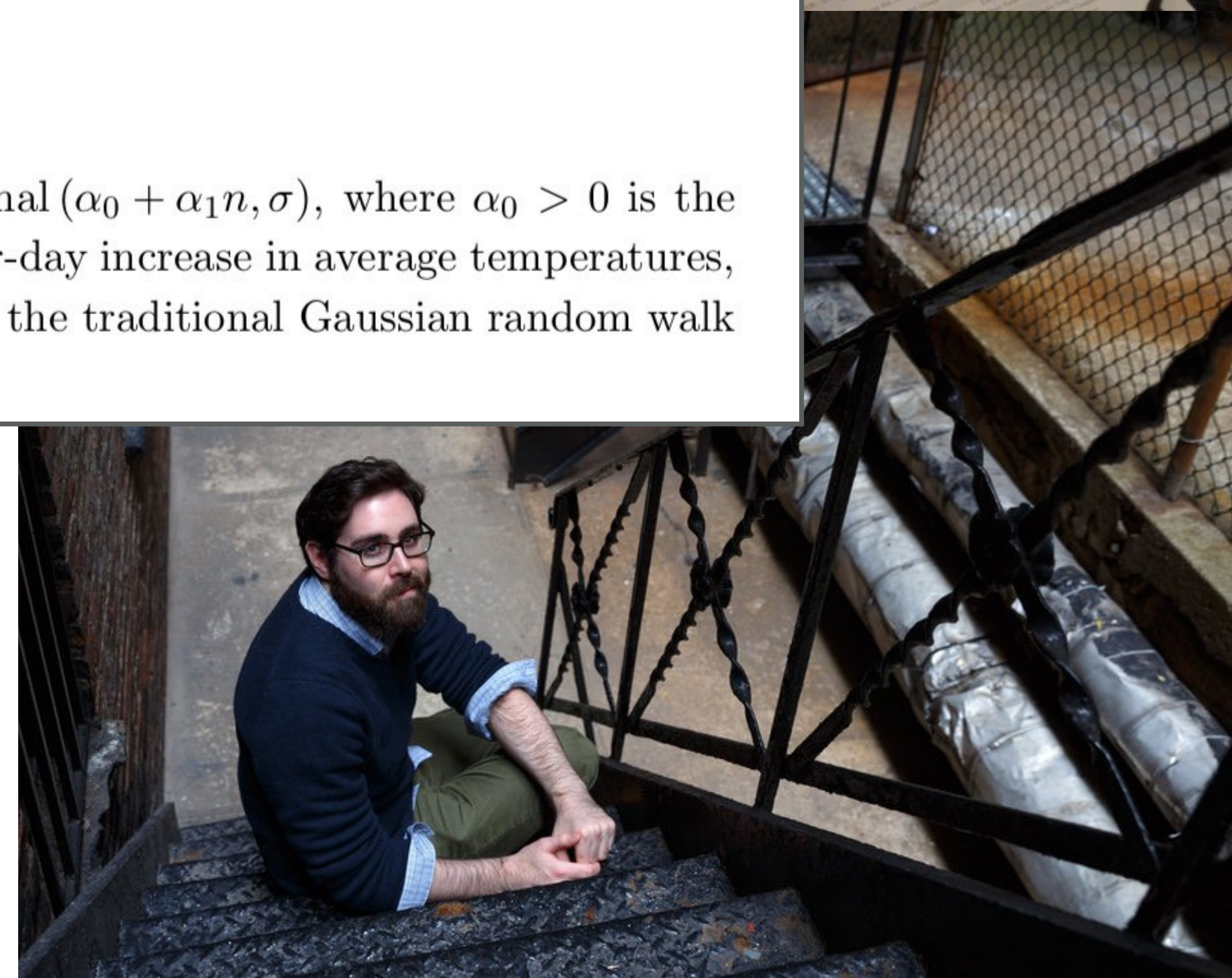
$$n_\beta = \frac{\beta}{M_0^{n_\beta}}$$

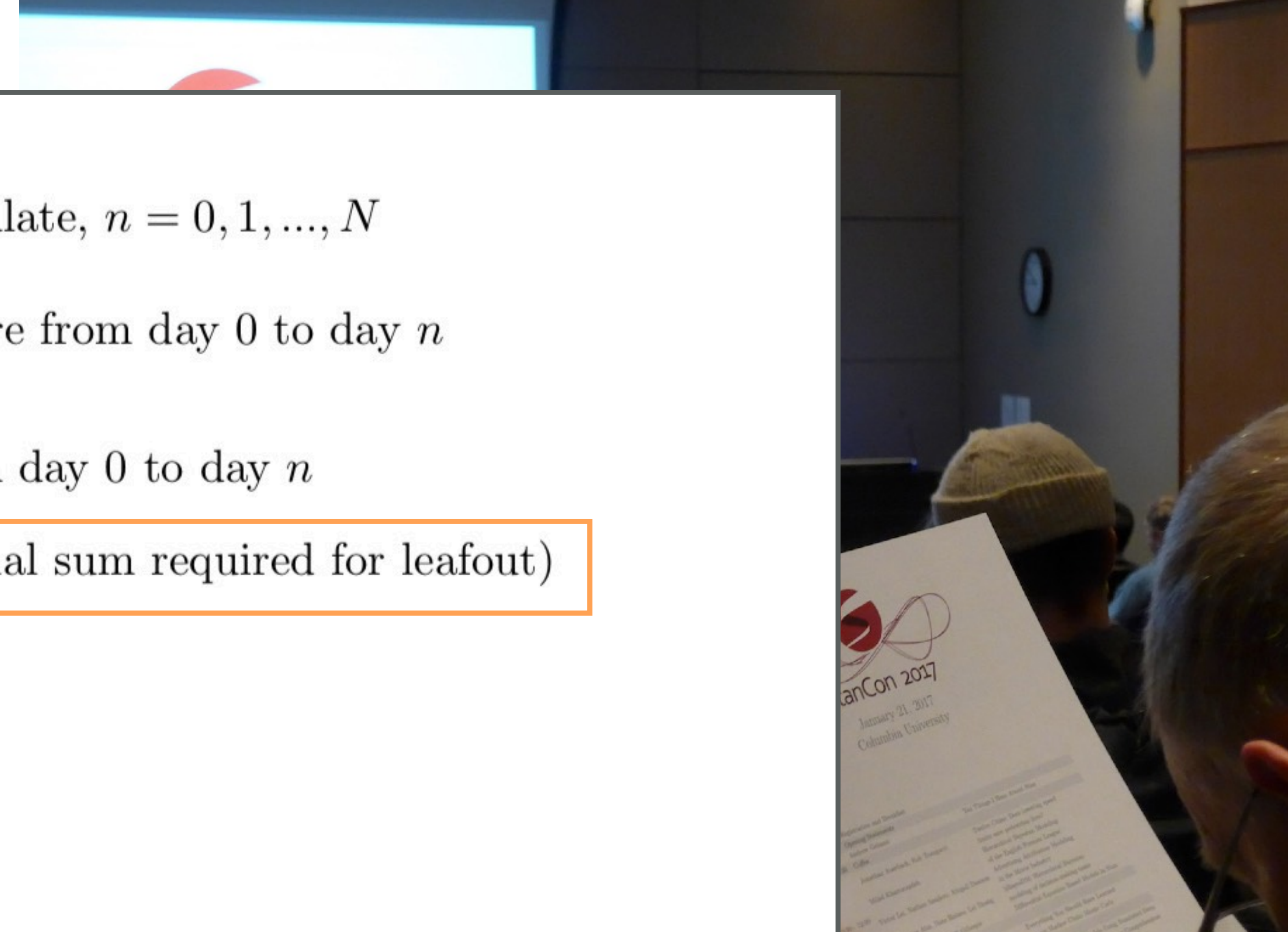
We model X_n as a Gaussian random walk, $X_n \overset{\text{i.i.d}}{\sim} \text{normal}\,(\alpha_0 + \alpha_1 n, \sigma)$, where $\alpha_0 > 0$ is the average temperature on day $n = 0$, $\alpha_1 > 0$ is the day-over-day increase in average temperatures, and σ is the standard deviation. This model differs from the traditional Gaussian random walk because of the factor n.

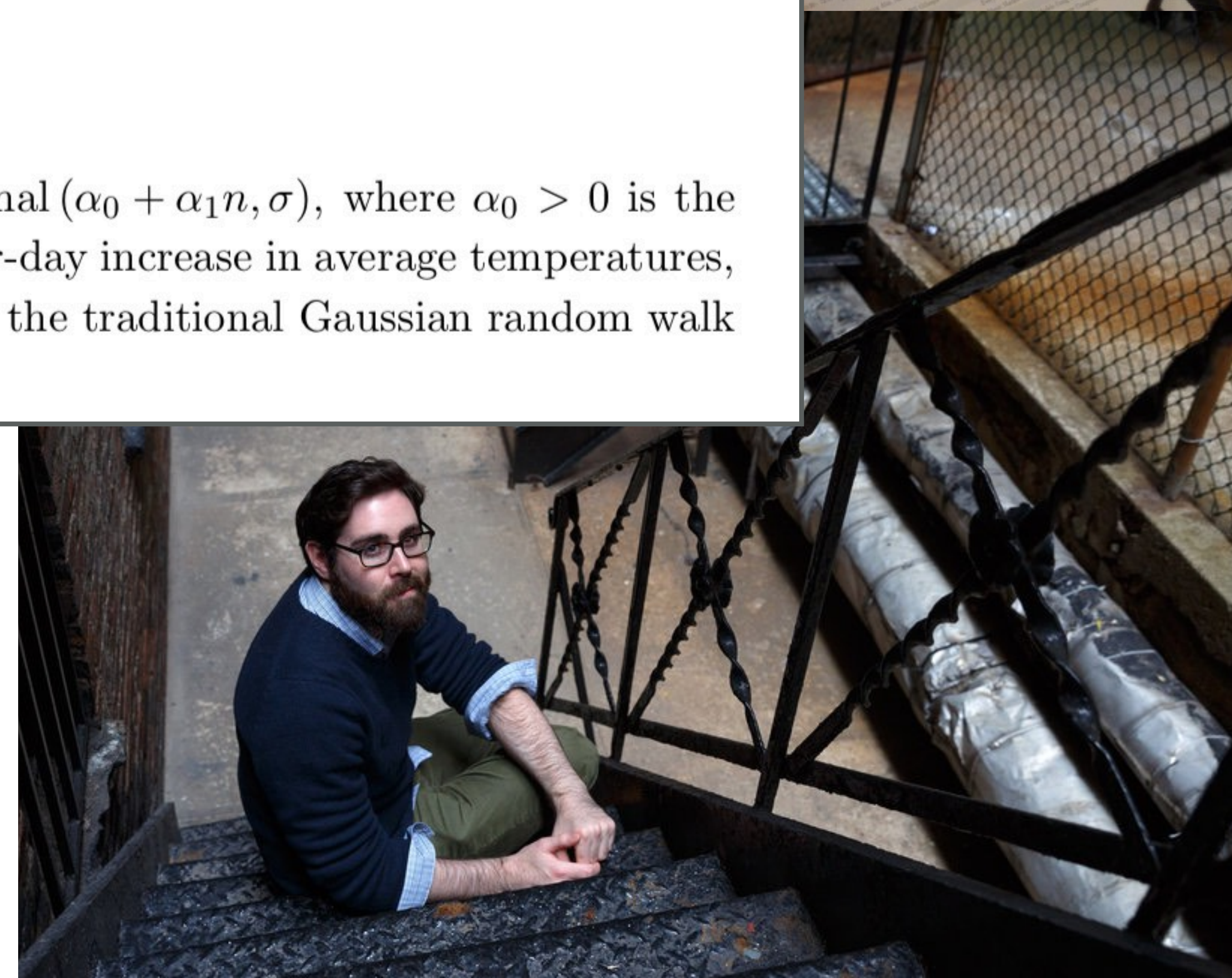
climate change

… *in progress*









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climate change

… *in progress*

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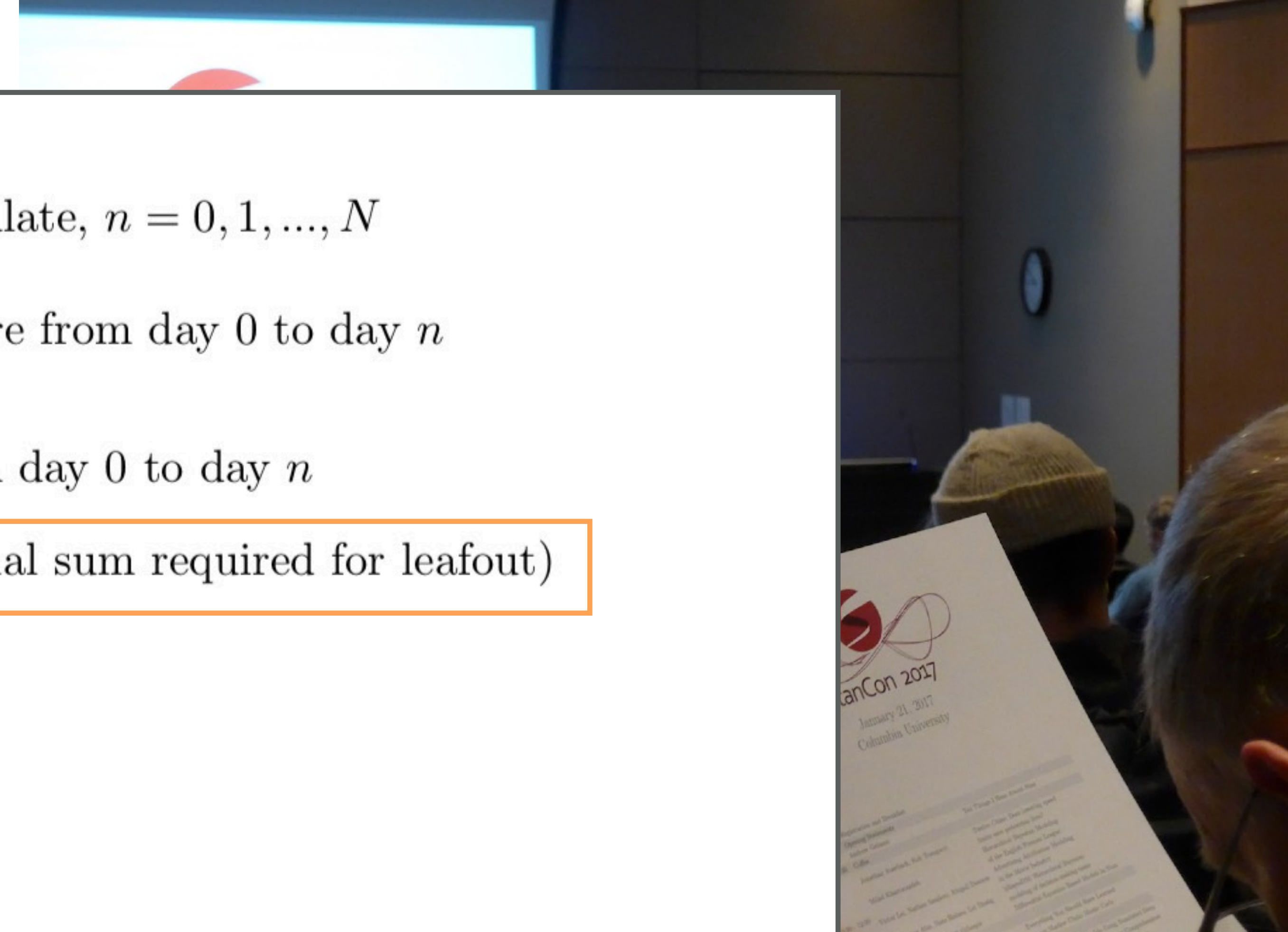
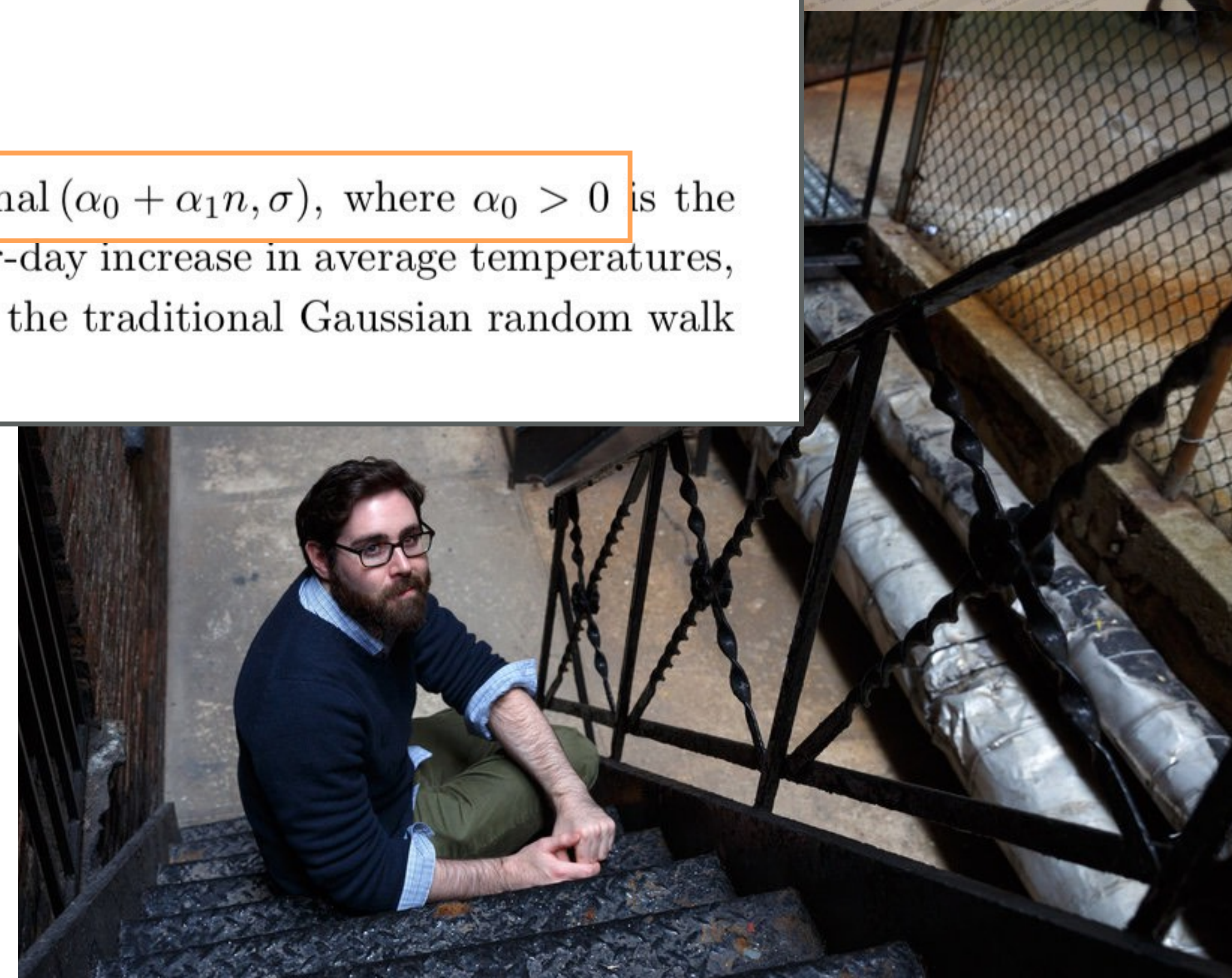
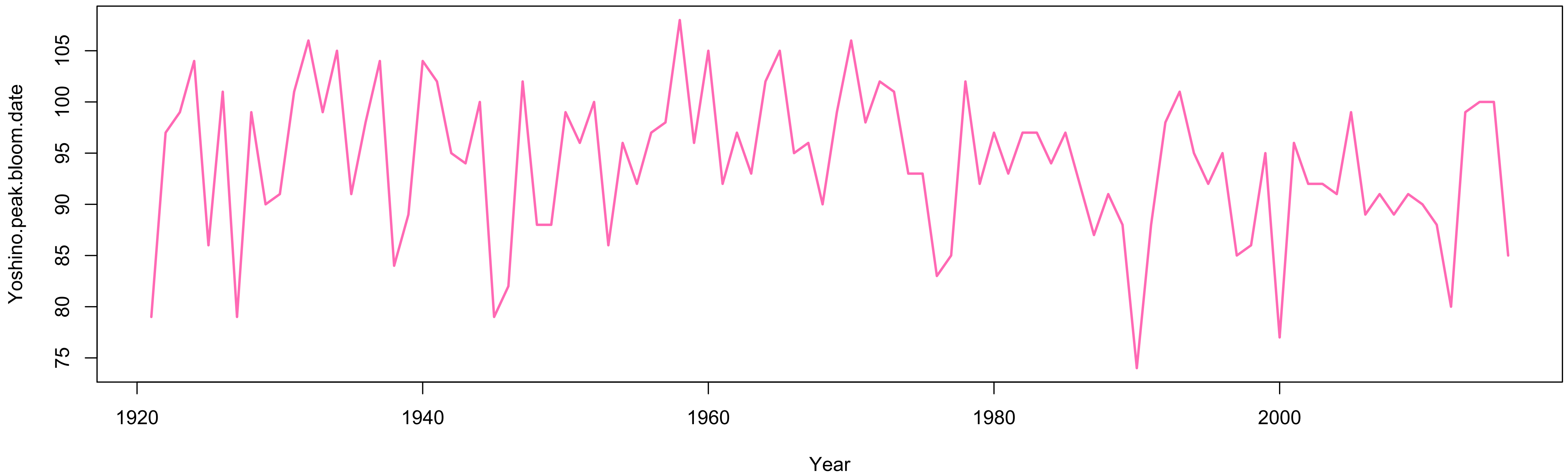




climate change

… *in progress*

Japanese cherry data



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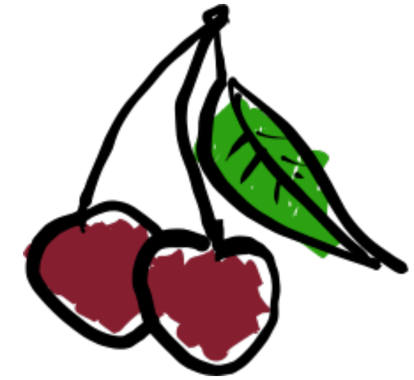

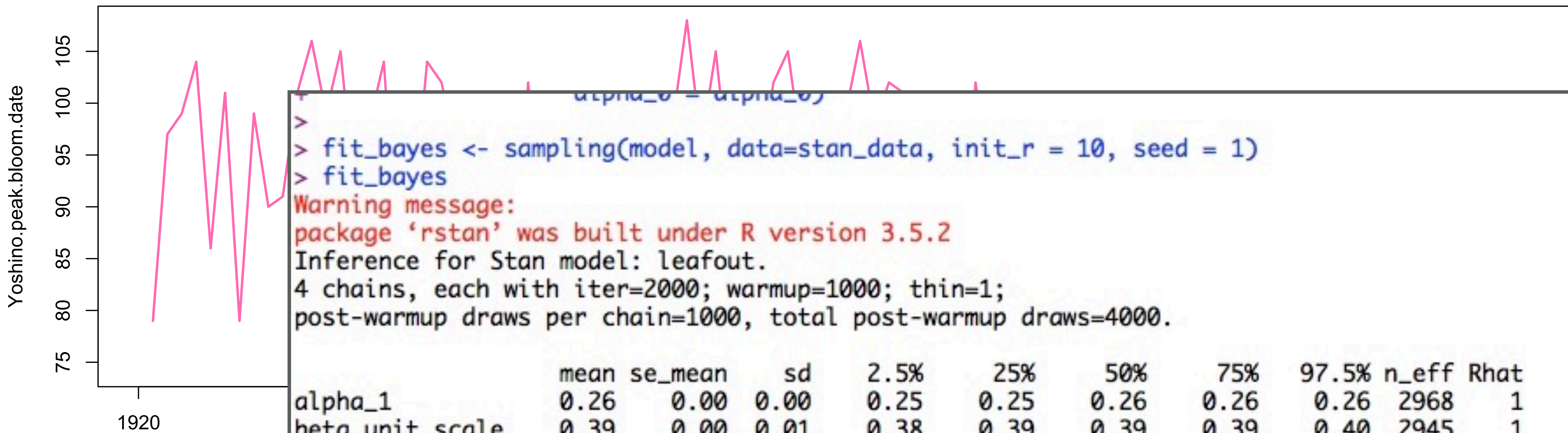


```
+            alpha_0 = alpha_0)
>
> fit_bayes <- sampling(model, data=stan_data, init_r = 10, seed = 1)
> fit_bayes
Warning message:
package 'rstan' was built under R version 3.5.2
Inference for Stan model: leafout.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

                   mean se_mean    sd    2.5%     25%     50%     75%   97.5% n_eff Rhat
alpha_1            0.26    0.00  0.00    0.25    0.25    0.26    0.26    0.26  2968    1
beta_unit_scale    0.39    0.00  0.01    0.38    0.39    0.39    0.39    0.40  2945    1
sigma             31.15    0.03  1.59   28.12   30.04   31.09   32.22   34.31  2824    1
beta            4169.31    0.48 26.10 4118.00 4151.55 4169.10 4187.32 4220.21  2945    1
lp__            -522.81    0.03  1.19 -525.93 -523.32 -522.51 -521.95 -521.44  1939    1

Samples were drawn using NUTS(diag_e) at Mon Nov 23 13:34:28 2020.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
>
> if(FALSE){
+ ## check with equinox correction (does little)
+ library(timeDate)
+ cherry$verneq <- as.numeric(format(JPVernalEquinox(cherry$Year), "%j"))
+ cherry$Yoshino.peak.bloom.date.adj <- cherry$Yoshino.peak.bloom.date-cherry$verneq
```

Applying to *Betula pendula*