Bayes Class: Wrap up Crossed vs. Nested

Finishing hierarchical models:

(1) Review notation: no pooling: $\hat{y}_{[sp]i} \sim \alpha_{[sp]} + \beta_{[sp]}$

w/ sp. as grouping factor

$y_{[sp]i} \sim normal(\hat{y}_{[sp]i}, \sigma_{[sp]i})$

Reminder?
$\hat{\sigma}_{sp} \to \infty$

NOTE! $sp \to j$ notation

Complete pooling: as above — no sp.

(PP)

Partial pooling on α & β (see also Gelman blog post last week? Jan 23?)
$\hookrightarrow$ add notation

$\sigma_{sp} \to 0$
(& PP is in between?)

Many ways to write/complexify these models, we'll review 2 ways:

(1) Crossed effects
(2) Nested effects
} Put on diff. sides of board.

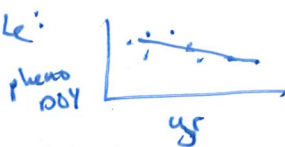Before we start.... let's re-parameterize <u>PP</u> model!

$\alpha_{sp} + \beta_{[sp]} X \implies \alpha + \alpha_{sp} + \beta_{sp}$

$\alpha_{sp} \sim N(\mu, \sigma)$

α ← grand mean

Ask them?

$\alpha_{sp} \sim N(?, \sigma_{sp})$

Now, motivating example:

pheno DOY | | ys

} meta-analysis w/ species in diff <u>studies</u>

↓

Next draw the <u>columns</u>

See $[\underline{C \text{ vs } N}]$

& do notation

Then → Do nested, start w/ colum from $[C \underline{vs} N]$ paper.

Next: (Code) w/ 2 examples ⟹ fit the best model given the <u>aims</u>.

⟹ Discuss fitting only spp.
& <u>not</u> study ID

Simple mixed effects notation including NESTED & CROSSED:

22 January 2025
Copied (somewhat) from
IMG_20220308...jpg
(still in pictures/sort)

Regular w/ hier on slopes & intercepts:

$$y \sim (1|sp) + (x|sp)$$

$$\hat{y}_i = \alpha_{sp[i]} + \beta_{sp[i]} x_i$$

$$\alpha_{sp} \sim normal(\mu_\alpha, \sigma_\alpha) \leftarrow \text{hyper-prior}$$

$$\begin{bmatrix} 0.35 \\ 0.30 \\ 0.25 \\ \dots \\ n \end{bmatrix} \Rightarrow \beta_{sp} \sim normal(\mu_\beta, \sigma_\beta)$$

$$y_i \sim normal(\hat{y}_i, \sigma_y)$$

CROSSED

$$\boxed{y \sim (1|sp) + (1|site) + x}$$

Site	SP
1	1
1	2
1	2
1	3
1	1
2	1
2	1
2	2
2	2
2	3

$$\hat{y}_i = \alpha + \alpha_{sp[i]} + \alpha_{site[i]} + \beta x_i$$

$$\alpha_{sp} \sim normal(0, \sigma_{sp})$$

$$\alpha_{site} \sim normal(0, \sigma_{site})$$

$$y_i \sim normal(\hat{y}_i, \sigma_y)$$

NESTED

Site	SP
1	1
1	1
1	2
2	3
2	3
2	4
2	4

At some pt. discuss
Why α (grand mean)
is critical
here (non-identifiability)

$$y \sim (1|site/sp)$$

$$\hat{y}_i = \alpha + \alpha_{sp[site]} + \beta x_i$$

$$\alpha_{sp[site]} \sim normal(\mu_{site}, \sigma_{site})$$

$$\alpha_{sp} \sim normal(\mu_{sp}, \sigma_{sp})$$

$$y_i \sim normal(\hat{y}_i, \sigma_y)$$