

Edited by Lizzie ...

Let:

i = index the days, $i = 0, 1, \dots, N$

X_i = temperature on day i , assume $X_0 = 0$

μ = average temperature on day $i = 1$; $X_i \sim \mathcal{N}(\mu * i, \sigma), i > 0$

$S_i = \sum_{X_1}^{X_n} X_i$, (for example, GDD)

M_i = cumulative mean, S_i/n

β = a threshold of interest, $\beta > 0$, (for example, F^* or required GDD)

n = the first day such that $\beta < S_n$, (for example, day of budburst)

l = the last date over which temperature is integrated

‘Sensitivity’ is generally measured by taking the slope—I am calling it m here—of a linear regression of n against M_l .

It is not surprising that n decreases with μ (and thus M_l) under the null model (this is equivalent to saying it is not surprising that plants leafout earlier with warmer spring temperatures, we all are fine with this). Researchers are surprised that the decrease of m as μ increases and suggest this is evidence of declining sensitivities; n is “reacting” or “changing” to M_l .

So I say (I think, assuming the math still holds): Plotting n against M_l is similar to plotting n against S_n/n , which is approximately the same as plotting n against β/n , which is a hyperbola. The data is consistent with the null model; no need for n to react to M_l .

Original from J. Auerbach on 15 January 2020 ...

Let:

i = index the days, $i = 0, 1, \dots, N$

X_i = temperature on day i , assume $X_0 = 0$

μ = average temperature on day $i = 1$; $X_i \sim \mathcal{N}(\mu * i, \sigma), i > 0$

$$S_i = \sum_{X_1}^{X_n} X_i$$

M_i = cumulative mean, S_i/n

β = a threshold of interest, $\beta > 0$

n = the first day such that $\beta < S_n$

Note: a time n , we have $M_n = S_n/n \approx \beta/n$.

Some say: It is NOT surprising that n decreases in μ (and thus M_n) under the null model. It IS surprising that the decrease of n in M_n slows under the null model. This may be evidence of declining sensitivities; n is “reacting” or ”changing” to M_n .

You say: Plotting n against M_n is the same as plotting n against S_n/n , which is approximately the same as plotting n against β/n , which is a hyperbola. The data is consistent with the null model; no need for n to react to M_n .

If this is the correct framing of the problem, X_i is a gaussian random walk with drift and n is a hitting time. I believe the mean and variance are difficult to work out in closed form since you would have to integrate S_n over n . However, the continuous time generalization can be worked out: a Brownian motion with instantaneous variance sigma and drift mu. The time, t , the process hits some threshold beta is distributed Inverse Gaussian Distribution, having mean μ/β and variance $\mu * \sigma/\beta^3$. This seems like a more accurate description of the problem anyway since the relationship between temperature and time is not discrete.