Subject: Re: Declining sensitivities
From: Jonathan Auerbach <jla2167@columbia.edu>
Date: 1/15/20, 10:43 AM
To: <e.wolkovich@ubc.ca>

Hi Lizzie,

Happy to help if I can—although I'm having trouble with the setup. Is this the correct "null model"?

Let i = index the days, i = 0, 1, ..., N
    X_i = temperature on day i, assume X_0 = 0,
    mu = average temperature on day i = 1; X_i ~ Normal(mu * i, sigma), i > 0
    S_i = cumulative sum, X_1 + ... + X_i
    M_i = cumulative mean, S_i / n
    beta = a threshold of interest, beta > 0
    n = the first day such that beta < S_n

Note: a time n, we have M_n = S_n / n ≈ beta / n

Some say: It is NOT surprising that n decreases in mu (and thus M_n) under the null model. It IS surprising that the decrease of n in M_n slows under the null model. This may be evidence of declining sensitivities; n is "reacting"/"changing" to M_n.

You say: Plotting n against M_n is the same as plotting n against S_n / n, which is approximately the same as plotting n against beta/n, which is a hyperbola. The data is consistent with the null model; no need for n to react to M_n.

If this is the correct framing of the problem, X_i is a gaussian random walk with drift and n is a hitting time. I believe the mean and variance are difficult to work out in closed form since you would have to integrate S_n over n. However, the continuous time generalization can be worked ot: a Brownian motion with instantaneous variance sigma and drift mu. The time, t, the process hits some threshold beta is distributed Inverse Gaussian Distribution, having mean mu/beta and variance mu * sigma / beta^3. This seems like a more accurate description of the problem anyway since the relationship between temperature and time is not discrete.

a. I noticed the slopes of all the log-log regressions (or proportion approach regressions) are always -1

That makes sense. The relationship is y = a/x so log(y) = log(a) - log(x) and regression should return beta_0 = log(a), beta_1 = -1

b. I simulated one biological explanation given for these lower sensitivities with warming

You'll have to walk me through the code more carefully. But if I'm reading it correctly, the model doesn't appear identified. Different thermal/cold effects could produce the same data, including the data observed.

c. Lots of people have taken to various ways to calculate temperature sensitivity (all as change in days per degree C).

Again, not sure I totally get it. But first taking out the differences (translation) should line up the curves and solve the problem. But why not just plot n over S_i? Under the null model, n over S_i should be constant.

On Fri, Jan 10, 2020 at 2:37 PM Elizabeth M Wolkovich <wolkovic@mail.ubc.ca> wrote:

> Hi Jonathan,
>
> Happy new year! And thanks for your help back in December on my 'declining sensitivities' issue. Over the break I pulled together a draft of the paper (not completed) and did some additional simulations. All here: https://github.com/lizzieinvancouver/decsens
>
> I am hoping you were serious about being willing to be an author as I have a few more queries for you. If yes, please read on. If not, is there anything I can do to convince you this would be a fun project to be involved in (and quick, I am hoping)?
>
> My queries:
> (a) In working on the simulations, I noticed the slopes of all the log-log regressions (or proportion approach regressions) are always -1 (figures at the end here: https://github.com/lizzieinvancouver/decsens/blob/master/docs/decsens.pdf). I have a guess on why this is which I wrote in the text, but am wondering if you know?
>
> (b) I simulated one biological explanation given for these lower sensitivities with warming (at least in my field), which is that the required thermal sum (which I call forcing, Fstar or GDD often) can go up when the winter required cold sum ('chilling' or Cstar) is not met. https://github.com/lizzieinvancouver/decsens/blob/master/analyses/decsensSimsMo.R (sorry, my code is not pretty!)
> The way I simulated it there is just a date where the cold sum is evaluated and the thermal sum starts no matter what but various biological explanations suggest you would accumulate the thermal sum *on the date* Cstar is met and later if not (we don't really know when that 'later' is, which is why I did not model it that way).
>
> Either way I am wondering if there is an a priori prediction from the math about whether you have to know this date to calculate a robust sensitivity? Or, really, just any other insights you have here
>
> (c) Lots of people have taken to various ways to calculate temperature sensitivity (all as change in days per degree C). I attach two papers to show this. The absolute sliding time window (SWA), for example (Simmonds pg 1431), is gaining popularity and I have had many colleagues say sensitivities calculated from this should be okay. I don't believe this is true, but is there any good way to show this beyond simulations?

And with real data -- for example, we have used this SWA approach with European data and found the estimated slope was -5.5 days/C for pre climate change with 74 d window and a mean daily temp of 7.9 C, and -3.6 days/C post climate change with a 56 d window and mean daily temp of 5.9C. When we take out that difference in daily temps -- the slopes are effectively the same. Is this another way to try to correct the issue?
(Or am I going in a circle here, I feel sometimes I am.)

I realize you're busy so let me know any thoughts when you can. Happy to video chat if easier, and of course happy to give you push access to the repo.

All the best,
Lizzie

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