# Supplemental materials: The illusion of declining temperature sensitivity with warming

the lab & friends

## 1 Models of leafout timing

Here we consider how a common process model of leafout day ($n$) varies as a function of average daily temperature ($X_i$). In this model leafout occurs after accumulated daily temperatures ($\sum_{X_1}^{X_n} X_i$) cross a critical threshold ($S_i$). Ideally, researchers would know the date that temperatures generally start to accumulate, and accumulate from that zero point to $n$. In practice researches often accumulate over a fixed window ($[a, b]$ such as March 1 to April 30) that they apply to all species or sites. Thus, let:

$$
\begin{aligned}
i &= \text{index the days, } i = 0, 1, ..., N \\
X_i &= \text{temperature on day } i\text{, assume } X_0 = 0 \\
\mu &= \text{average temperature on day } i = 1; X_i \sim \mathcal{N}(\mu * i, \sigma), i > 0 \\
S_i &= \sum_{X_1}^{X_n} X_i, \text{(for example, GDD)} \\
M_i &= \text{cumulative mean, } S_i/i \text{ and thus } M_n = S_n/n \\
\beta &= \text{a threshold of interest, } \beta > 0, \text{ (for example, F* or required GDD)} \\
n &= \text{the first day such that } \beta < S_n, \text{ (for example, day of year (doy) of budburst)} \\
[a, b] &= \text{temporal window over which temperature is measured, thus } S_a^b = \sum_{i=a}^{b} X_i
\end{aligned}
$$

Q1: Does $X_i \sim \mathcal{N}(\mu * i, \sigma), i > 0$ mean that each day temperature increases by $\mu$? (If yes, what if it is not so linear?)

Here, $X_i$ is a gaussian random walk with drift and $n$ is a hitting time. While the mean and variance are difficult to work out in closed form, the continuous time generalization would be a Brownian motion with instantaneous variance $\sigma$ and drift $\mu$. The time, $t$, the process hits some threshold $\beta$ is distributed Inverse Gaussian Distribution, having mean $\mu/\beta$ and variance $\mu * \sigma/\beta^3$. Q2: Are we still happy with this hitting time model given a=0 and b=n?

Assuming $n$ is large, we can replace this sum with its average:

$\sum_{i=a}^{n} X_i = \sum_{i=a}^{n} \mu * i$

So,

$\beta \approx \sum_{i=a}^{n} \mu * i = \mu * \sum_{i=a}^{n} *i$

and,

$S_a^b = \mu * \sum_{i=a}^{b} *i = \mu * \sum_{i=a}^{n} *i + \mu * \sum_{i=n}^{b} *i = \beta + \mu * \sum_{i=n}^{b} *i$

But $\sum_{i=n}^{b} *i$ is just the sum of the integers from $n$ to $b$ which is the sum of integers from 1 to b minus the sum of integers from 1 to n

$b*(b+1)/2 - n*(n+1)/2$ Q3: I follow the math generally until here, is this some special known solution (that I don't know)?

So,

$S_a^b = \beta + \mu * (b * (b+1)/2 - n * (n+1)/2)$

Letting $\alpha_1 = \beta + \mu * b * (b+1)/2$ and $\alpha_2 = \mu/2$

$S_a^b = \alpha_1 - \alpha_2 * n * (n+1)$

which is clearly not a straight line, and OLS (either $S_a^b$ on $n$ or $n$ on $S_a^b$) is not a good idea.

## 2 Results using long-term empirical data from PEP725

To examine how estimated sensitivities shift over time, we selected sites of two common European tree species (*Betula pendula* (silver birch) and *Fagus sylvatica* (European beech) ) that have long-term observational data of leafout, through the Pan European Phenology Project (PEP725, **?**). We used a European-wide gridded climate dataset *(*E-OBS, **?***)* to extract daily minimum and maximum temperature for the grid cells where observations of leafout for these two species were available.

## 3 Tables & Figures

Table S1: Climate and phenology statistics for two species (*Betula pendula, Fagus sylvatica*) from the PEP725 data across all sites with continuous data from 1950-1960 and 2000-2010. MST is spring temperature from March 1 to April 30, MST.LO is temperature 30 days before leafout, LO is leafout and GDD is growing degree days 30 days before leafout. We calculated all metrics for for each species x site x 10 year period before taking mean or variance estimates.

| when | species | mean(MST) | mean(MST.LO) | var(MST) | var(LO) | mean(GDD) |
|---|---|---|---|---|---|---|
| 1950-1960 | Betpen | 5.60 | 7.04 | 3.38 | 110.51 | 71.71 |
| 2000-2010 | Betpen | 6.58 | 6.80 | 1.21 | 46.96 | 64.63 |
| 1950-1960 | Fagsyl | 5.62 | 7.54 | 3.33 | 71.89 | 83.81 |
| 2000-2010 | Fagsyl | 6.66 | 7.75 | 1.23 | 38.29 | 86.69 |

Table S2: Climate and phenology statistics for two species (*Betula pendula, Fagus sylvatica*) from the PEP725 data across all sites with continuous data from 1950-2010. MST is spring temperature from March 1 to April 30, MST.LO is temperature 30 days before leafout, LO is leafout and GDD is growing degree days 30 days before leafout. We calculated all metrics for for each species x site x 20 year period before taking mean or variance estimates. **Side note: Why is my hline.after command not working?**

| when | species | mean(MST) | mean(MST.LO) | var(MST) | var(LO) | mean(GDD) |
|---|---|---|---|---|---|---|
| 1950-1970 | Betpen | 7.91 | 7.30 | 1.23 | 79.89 | 77.77 |
| 1970-1990 | Betpen | 8.12 | 7.16 | 0.84 | 104.83 | 72.15 |
| 1990-2010 | Betpen | 9.04 | 6.77 | 0.89 | 36.22 | 62.17 |
| 1950-1970 | Fagsyl | 7.66 | 7.61 | 1.24 | 63.35 | 86.02 |
| 1970-1990 | Fagsyl | 7.86 | 7.52 | 0.87 | 56.19 | 81.31 |
| 1990-2010 | Fagsyl | 8.87 | 7.47 | 0.90 | 32.82 | 79.92 |

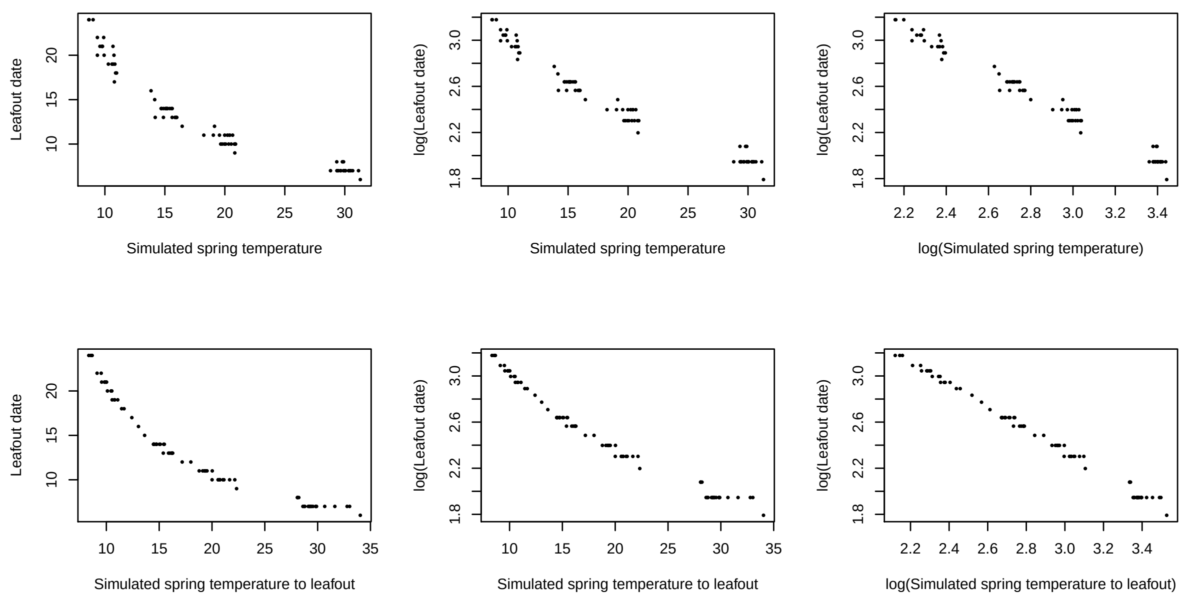



Figure S1: **Simulated leafout as a function of temperature across different temperatures highlights non-linearity of process.** Here we simulated sets of data where leafout constantly occurs at 200 growing degree days across mean temperatures of 0, 5, 10 and 20C (constant SD of 4), we calculated estimated mean temperature across a fixed window (top row, similar to estimates of 'spring temperature') or until leafout date (bottom row). While within any small temperature range the relationship may appear linear, is non-linear relationship becomes clear across the range shown here (left). Taking the log of leafout (middle) reduces this some, but taking the log of both leafout and temperature (right) linearized the relationship.

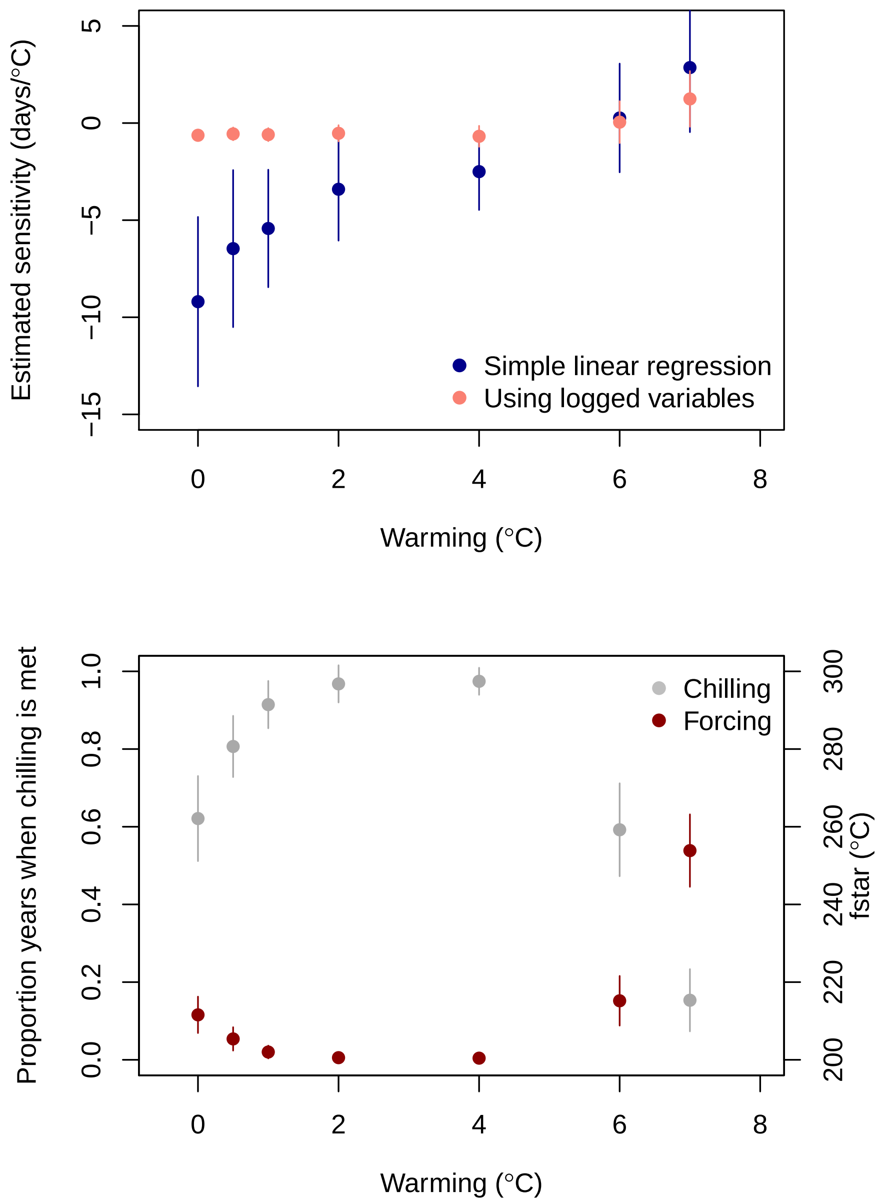
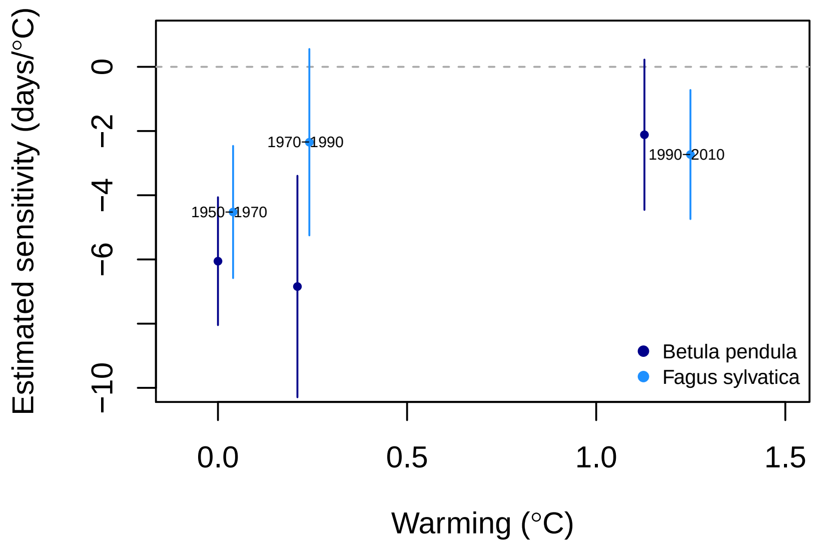



Figure S2: **Simulated leafout as a function of temperature across different temperatures with shifts in underlying cues.** Here we simulated sets of data where leafout occurs at 200 growing degree days ('fstar') when chilling is met, and requires additional growing degree days when chilling is not met. We show estimates sensitivities in the top panel, and the shifting cues on the bottom panel.



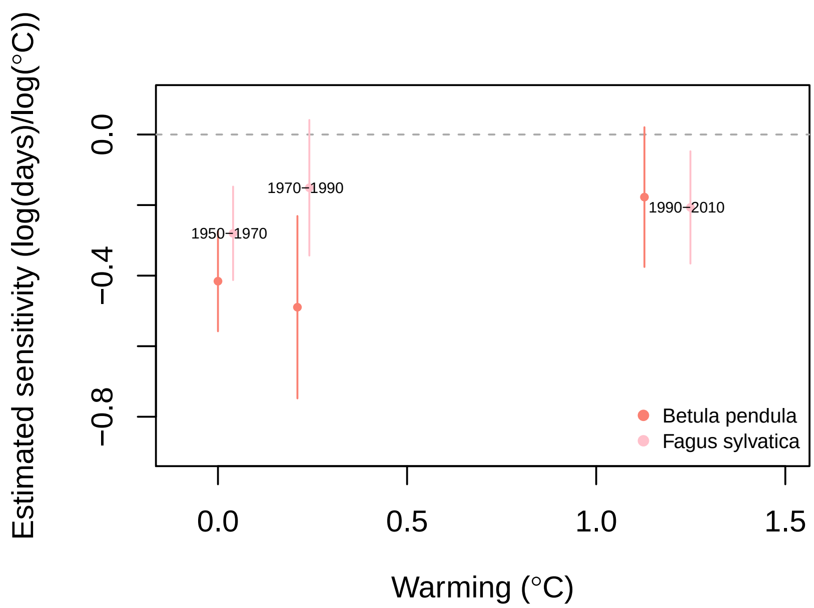




Figure S3: **Sensitivities for two species from PEP725 data** using raw data (top) or logged variables (bottom) using 20 year windows of data (lines show 78% confidence intervals). Amounts of warming are calculated relative to 1950-1970 and we used only sites with leafout data in all years shown here. Both approaches show variation in sensitivity across time. To aid visualization *Fagus sylvatica* are jittered slightly to the right, but warming is approximatly 0.08 C greater for *Fagus sylvatica* in 1990-2010 relative to *Betula pendula*.