To understand how experimental warming and precipitation treatments affect soil moisture (y_i) I want to fit a hierachical model with 3 levels: one level for site, represented here by j; one level for year, represented here by k; and one level for day of year, represented here by l. β_1 is the warming treatment and β_2 is the precipitation treatment. I am also interested in their interactive effects. Each observation is i, and partial pooling on the intercept (α) only:

$$y_i = \alpha_{j[i]1} + \alpha_{k[i]2} + \alpha_{l[i]3} + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_1 \beta_2 X_{i1} X_{i2} + \epsilon_i \tag{1}$$

The multilevel parts of the model are:

$$\alpha_{j1} \sim N(\mu_{\alpha_1}, \sigma^2_{\alpha_1}) \tag{2}$$

$$\alpha_{k2} \sim N(\mu_{\alpha_2}, \sigma^2_{\alpha_2}) \tag{3}$$

$$\alpha_{l3} \sim N(\mu_{\alpha_3}, \sigma^2_{\alpha_3}) \tag{4}$$

To understand how soil moisture affects GDDcrit(y_i) (the accumulated growing degree days on the day of a phenoogical event such as bud burst), I want to fit a hierachical model with 2 levels: one level for site, represented here by j; and one level for species, represented here by k; β_1 is soil moisture (averaged over the whole year- not sure if this is best). Each observation is i, and partial pooling could be on the intercept (α) only or on the slope and intercept. I'm not sure which is best....shown here for pooling on both slope and intercept for species, and intercept-only for site:

$$y_i = \alpha_{j[i]1} + \alpha_{k[i]2} + \beta_1 X_{k[i]1} + \epsilon_i \tag{5}$$

The multilevel parts of the model are:

$$\alpha_{j1} \sim N(\mu_{\alpha_1}, \sigma^2_{\alpha_1}) \tag{6}$$

$$\alpha_{k2} \sim N(\mu_{\alpha_2}, \sigma^2_{\alpha_2}) \tag{7}$$

$$\beta_{k1} \sim N(\mu_{\beta_1}, \sigma^2_{\beta_1}) \tag{8}$$