To understand how experimental warming and precipitation treatments affect soil moisture (y_i) I want to fit a hierachical model with 3 levels: one level for site, represented here by j; one level for year, represented here by k; and one level for day of year, represented here by l. β_1 is the warming treatment and β_2 is the precipitation treatment. I am also interested in their interactive effects. Each observation is i, and partial pooling on the intercept (α) only:

$$y_i = \alpha_{j[i]1} + \alpha_{k[i]2} + \alpha_{l[i]3} + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_1 \beta_2 X_{i1} X_{i2} + \epsilon_i \tag{1}$$

The multilevel parts of the model are:

$$\alpha_{j1} \sim N(\mu_{\alpha_1}, \sigma^2_{\alpha_1}) \tag{2}$$

$$\alpha_{k2} \sim N(\mu_{\alpha_2}, \sigma^2_{\alpha_2}) \tag{3}$$

$$\alpha_{l3} \sim N(\mu_{\alpha_3}, \sigma^2_{\alpha_3}) \tag{4}$$