1 Model

For our example, we adapt the model from ...

[Add an overview of model here.]

Across years, for a community of n species, the seedbank (N) of species i at time $t + 1$ is determined by the survival (s) of seeds that did not germinate in previous season $(1 - g_i(t))$ plus new biomass (B_i) produced during the length of the growing season (δ) converted to seeds at rate ϕ:

$$N_i(t + 1) = sN_i(t)(1 - g_i(t)) + \phi B_i(t + \delta) \tag{1}$$

The production of new biomass each season follows a basic R^* competition model: new biomass production depends on its resource uptake ($f(R)$ converted into biomass at rate c_i) less maintenance costs (m), with resource uptake controlled by species-level parameters that define how the uptake increases as R increases (a), the inverse of the maximum uptake (u), and the shape of the uptake (θ):

$$\frac{\mathrm{d}B_i}{\mathrm{d}t} = (c_i f(R) - m)B_i + G_i(t) \tag{2}$$

$$f(R) = \frac{aR^\theta}{1 + auR^\theta} \tag{3}$$

Where $G_i(t)$ is germination as a time dependent forcing function that adds daily biomass to the ODE according to a Hill equation:

$$G_i(t) = N_0 s_i b_i g_i(t) \tag{4}$$

$$g_i(t) = g_{max,i}\left(\frac{\text{within-season-time}^{nH}}{\tau_{g50}^{nH} + \text{within-season-time}^{nH}}\right) \tag{5}$$

where b_i is the biomass per seed and where τ_{g50} is a Poisson random variable for the days of delay due to chilling, ξ, according to:

$$\tau_{g50} \sim \text{Poisson}(\tau_{delay}) \tag{6}$$

$$\tau_{delay} = \tau_{max} e^{-\tau_\xi \xi} \tag{7}$$

The resource (R) itself declines across a growing season due to uptake by all species and abiotic loss (ϵ):

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -\sum_{i=1}^{n} f_i(R)B_i - \epsilon R \tag{8}$$

Germination depends both on the traits of the species and on the environment that year. The fraction of seeds germinating for a species each year is determined by the distance between τ_i,

a species characteristic, and $\tau_p(t)$, an attribute of the environment, which varies year-to-year. Germination fraction declines according to a Gaussian curve as the distance between τ_i and $\tau_p(t)$ grows, with the rate of decline determined by h (we refer to this distribution as the 'germination curve').

$$g_i(t) = g_{max}e^{-h(\tau_p(t)-\tau_i)^2} \tag{9}$$

The model is designed for multiple conceptualizations (**?**); given our focus here, we consider $\tau_p(t)$ to represent the environmental (abiotic) start of the growing season that varies from year-to-year and refer to it as the 'environmental start time.' τ_i represents the 'intrinsic biological start time' for species i. How well matched a species is to its environment each year can be measured as τ_i - $\tau_p(t)$, or the distance between the intrinsic (biological) start time and the environmental start time.

Adding XXX to model:
We adjust the biological start time, τ_i so that it can respond to the environment dynamically through what we refer to as tracking. Tracking (α, which can vary between 0 to 1) decreases the distance between τ_i and $\tau_p(t)$, i.e. moving the intrinsic start time closer to the environmental start time in that year, resulting in a higher germination fraction (e.g. species B in Fig. **??**B-C).

$$\alpha_i \in [0, 1]$$

$$\hat{\tau}_i = \alpha_i\tau_p(t) + (1 - \alpha_i)\tau_i \tag{10}$$

Thus,

$$\text{when } \alpha_i = 0, \ \hat{\tau}_i = \tau_i$$
$$\text{when } \alpha_i = 1, \ \hat{\tau}_i = \tau_p(t)$$

ADD: While explaining the model, will need to reference (and include in figures), the plots that are made in `Prieff_Species.R` to visualize.
We are drawing weeks of chilling from a normal distribution and that distribution is always the same.
Simulations:
Given an overview of simulations performed, be sure to mention: two-species communities, species vary in *what* and explain a little what that does (for example: We varied species' resource use efficiency (*via* c_i), yielding species with different R^* (a metric of resource competition where a lower R^* means a species can draw the resource down to a lower level and is thus considered the superior competitor)) and then state something like "All other parameters were identical between species (Table S1)." Also need: starting conditions (for example: both species were initialized with a census size of $N(0) = 100$ per unit area, and the temporally varying parameters $R_0(t)$ and $\tau_p(t)$ were generated) and then some re-wording of this:

> Within-year R^* competition dynamics were solved using an ode solver (`ode` in the R package `deSolve`) and ended when the resource was drawn down to $min(R^*)$,

i.e. the R^* value of the better resource competitor. The end-of-season biomass of each species was converted to seeds, and the populations were censused. At each census, a minimum cutoff was applied to define extinction from the model. Note that 'coexistence' in this model is defined by joint persistence through time and not by low density growth rate.

Table S1: Parameter values, definitions, and units.

Parameter	Value(s)	Definition	Unit
N_i	initial conditions $N_i(0) = 100; min(N_i(t)) = 10^{-4}$	census of seedbank of species i	seeds per unit area
s	0.8	survival of species i	unitless
B_i	see Eqn 2	biomass of species i	biomass
R_0	$\sim logN(\mu, \sigma) mu = log(2), \sigma = 0.2$	annually varying initial value of resource at the beginning of the growing season	resource
ξ_0	$\sim$??	chilling	chilling
c_i	$\sim$Unif(8,20)	conversion efficiency of R to biomass of species i	$\frac{biomass}{resource}$
m	0.05	maintenance costs during growth season i	$days^{-1}$
a	20	uptake increase as R increases for species i	$days^{-1}$
u	1	inverse of maximum uptake for species i	$\frac{(days)(biomass)}{resource}$
θ	1	shape of uptake for species i	unitless
ϕ	0.05	conversion of end-of-season biomass to seeds	$biomass^{-1}$, but conceptually $\frac{seeds}{(biomass)(seeds)}$
ϵ	1	abiotic loss of R	$days^{-1}$
g_{max}	0.5	maximum germination rate of species	unitless
h	100	controls the the rate at which germination declines as τ_p deviates from optimum for species i	$days^{-2}$
g_i	see Eqn 6	germination rate	unitless
τ_g	$\sim \beta(?,?)$	days of delay (given on weeks of chilling)	days
$\tau_{g_{50}}$	$\sim \text{Pois}(\tau_g)$	??	days
ξ_τ	$\sim$?,?	rate of delay (given weeks of chilling) i	days/week
b_i	1	biomass of a seedling	$\frac{biomass}{seeds}$
$f(R)$	see Eqn 3	resource uptake rate for species i	$\frac{resource}{(days)(biomass)}$
t	1	annual timestep	years
$0 \rightarrow \delta$	determined by rate of resource depletion	time during the growing season	days