Following Chesson 2003 & Godoy & Levine 2014 Appx

1/27/15 ①

$$N_j(t+1) = \underbrace{(1-d_j)}_{\substack{\text{survival in seedbank} \\ = s_i(1-g_i)}} N_j(t) + \underbrace{R_j(t)}_{\substack{\text{germination \& death in seedbank}}} N_j(t)$$

see p. 5 this is a problem

$0^*(1-g_i)(1-s_i) + g_i F_i$

$= g_i F_i$

$$g_i F_i = \frac{\lambda_i}{1+\sum_l \alpha_{il} g_{l,t} N_{l,t}}$$

operational def'n
Let $E_j(t) = \ln R_j^0(t)$ ← recruitment w/o comp
$C_j(t) = \ln(R_j^0(t)/R_j(t))$ ← recruitment w/ comp

$$R_j(t) = g_j F_j = g_j \lambda_j / (1+\sum_l \alpha_{il} g_{l,t} N_{l,t})$$

$$= \underbrace{\ln(g_j \lambda_j)}_{E_j(t)} - \underbrace{\ln(1+\sum_l \alpha_{il} g_{l,t} N_{l,t})}_{-C_j(t)}$$

C2003 Eqn 6

$$C_j(t) = \sum_{l=1}^{n} \underbrace{a_l}_{\substack{\text{competitive effect} \\ \text{of a sp } l \text{ individual}}} \underbrace{e^{E_l(t)} N_l(t)}_{g_l N}$$

$g_l N$] without competition $g_l N_l$ indivs of sp l would recruit each year

Assume that the C_j does not depend on j →

Can I write C in a general form like A.5

A.5 $C = f\left(\sum_l a_l(N) e^{E_l} N_l\right)$

f is increasing fxn
a_l competitive effects of juveniles of sp l as a function of adult density N

$$-C_j = \ln F_j = \ln\left[\frac{\lambda_j}{1+\sum_l \alpha_{jl} g_{l,t} N_{l,t}}\right]$$

$$= \ln \lambda_j - \ln\left(1+\sum_l \alpha_{jl} \underbrace{e^{E_l}}_{=g_l(t)} N_{l,t}\right)$$

* Operational Definition of E, C

$$E_j(t) = \ln(g_{j,t} \lambda_j)$$

$$C_j(t) = \ln\left[\frac{g_{j,t}\lambda_j}{g_{j,t}\lambda_j/(1+\sum_l \alpha_{il} g_{l,t} N_{l,t})}\right]$$

$$= \ln\left(1+\sum_l \alpha_{il} g_{l,t} N_{l,t}\right)$$

Question:

Should λ_j be a part of E_j or C_j?

It seems to fall out naturally as part of E_j

↳ see p. 5 this changes when germination is removed from ~~Eq~~ $(1-d_j)$

C_t vs $C_{j,t}$

Assumption: all species experience the same amount of competition $\rightarrow C_j(t) = C(t)$

More general situation: C_j are proportional between species (not sure what this means)

$\rightarrow$ in both situations, equilibrial coexistence is excluded: in the absence of fluctuation, no coexist

We don't really want to have to make this assumption, I don't think, which may make the calculation more challenging

This comes down to an assumption that all α_{il} are equal. i.e. that intra & inter-specific competition coeffs are equal.

We could make this assumption to start, and then break from that assumption.

Relative Recruitment Variation

$$\begin{aligned}
Var(\ln R_j(t)) &= Var(E_j(t)) + Var(C_j(t)) - 2\,Cov(E_{j,t}, C_{j,t}) \\
&= var(\ln \underset{var(\ln g_{j,t} + \ln \lambda_j)}{g_{j,t} \lambda_j}) + var\left(\ln\left(1 + \sum_l \alpha_{il}\, g_{l,t} N_{l,t}\right)\right) - 2\,Cov(E, C) \\
&= Var(\ln g_{j,t}) + Var\left(\ln\left(1 + \sum_l \alpha_{j,l}\, g_{l,t} N_{l,t}\right)\right) - 2\,cov(\ln g_{j,t}, \ln(1 + \Sigma \ldots))
\end{aligned}$$

Storage effect depends on how $cov(E, C)$ depends on density

$$\bar{r}_i \approx \underbrace{\bar{r}_i'}_{\text{equilibrium measures of coexistence}} + \overbrace{\Delta N_i}^{\text{rel nonlin}} + \overbrace{\Delta I_i}^{\text{storage effect}}$$

Can we calculate the values in Table 1?

$$\bar{r}_i' / d_i = \delta \mu_i + \delta V_i$$

Chesson 2003
Table 1

→ revised post page 5 ←

Two Species

$$\frac{\bar{r}_i'}{d_i} = \delta\mu_i + \delta V_i$$

$\mu_j = \langle E_j \rangle - \ln d_j$

$\delta\mu_i = \mu_i - \overline{\mu_s}^{(s \neq i)}$

$\sigma_j^2 = V(E_j)$

$\delta V_i = \frac{1}{2}(1-d_i)\sigma_i^2 - \overline{\frac{1}{2}(1-d_s)\sigma_s^2}^{(s \neq i)}$

$\delta\mu_i = \mu_i - \mu_r$

$\delta V_i = \frac{1}{2}(1-d_i)\sigma_i^2 - \frac{1}{2}(1-d_r)\sigma_r^2$

$\langle E_j \rangle = \langle \ln g_{j,t} \rangle$

$\mu_j = \langle \ln g_{j,t} \rangle - \ln(1-s_j)$

$\delta\mu_i = \mu_i - \mu_r$ (OR) $\mu_i - \bar{\mu}_r$ // where $\bar{\mu}_r$ is average over resident spp

$var(E_j) = V(\ln(g_{j,t}))$

$\delta V_i = \frac{1}{2} s_i V(\ln g_{i,t}) - \frac{1}{2} S_r V(\ln g_{r,t})$ (OR) $\frac{1}{2} s_i V(\ln g_{i,t}) - \frac{1}{2}\overline{S_r V(\ln g_{r,t})}$ avg over all residents

$g_{j,t} = G_{max} e^{-h(\tau_p - \tau_i)^2}$

$\tau_p \sim B(p, q)$

$\ln g_{j,t} = \ln G_{max} - h(\tau_p(t) - \tau_i)^2$

$(\tau_p(t) - \tau_i)(\tau_p(t) - \tau_i)$

$= \tau_p(t)^2 - 2\tau_p(t)\tau_i + \tau_i^2$

$\langle \ln g_{j,t} \rangle = \ln G_{max} - h\langle (\tau_p(t) - \tau_i)^2 \rangle$

$= \ln G_{max} - h(\langle \tau_p(t)^2 \rangle - 2\tau_i \langle \tau_p(t) \rangle + \tau_i^2)$

$= \ln G_{max} - h(var(\tau_p(t) + \langle \tau_p \rangle^2 - 2\tau_i \langle \tau_p \rangle + \tau_i^2)$

$= \ln G_{max} - h\left[\sigma_{\tau_p}^2 + \mu_{\tau_p}^2 - 2\tau_i \mu_{\tau_p} + \tau_i^2\right]$

$var\, \tau_p = E(\tau_p^2) - (E(\tau_p))^2$

$var\, \tau_p + E(\tau_p)^2$

$E(\tau_p) = \frac{p}{p+q}$

$V(\tau_p) = \frac{pq}{(p+q)^2(p+q+1)}$

→ fixed after p. 5 ←

$$\frac{\Delta I_i}{d_i} = \overline{(1-d_s)\chi_s^{(-i)}}^{(s\neq i)} - (1-d_i)\chi_i^{(-i)}$$

$$\chi_j^{(-i)} = \mathrm{Cov}(E_j, C^{(-i)})$$ where $C^{(-i)}$ is competition when sp i is invader (i.e. competition from everyone except sp i)

$$\chi_s^{(-i)} = \mathrm{Cov}\left(\ln g_{r,t}, \ln\left(\frac{\lambda_r}{1+\alpha_{rr} g_{r,t} N_{r,t}} - s_r\right)\right)$$ for 2 spp

$$= \mathrm{Cov}\left(\ln g_{r,t}, \ln\left(\frac{\lambda_r}{1+\sum_{l\neq i}\alpha_{rl} g_{r,t} N_{r,t}} - s_r\right)\right)$$

r is all resident species

$$\chi_i^{(-i)} = \mathrm{cov}\left(\ln g_{i,t}, \ln\left(\frac{\lambda_i}{1+\sum_{l=r}\alpha_{il} g_{l,t} N_{l,t}} - s_i\right)\right)$$

$$\frac{\Delta N_i}{d_i} = \frac{1}{2}V(C^{\{-i\}})(d_i - \bar{d}_s^{\{s\neq i\}})$$

$$V(C^{\{-i\}}) = \mathrm{var}\left(\ln \frac{\lambda_j}{1+\sum_l \alpha_{il} g_{l,t} N_{l,t}} - s_j\right)$$

$$\bar{d}_s^{\{s\neq i\}} = \frac{1}{n-1}\sum_{l=i}(1-s_l)$$

Potential problem: with current definition d varies with time

$$\frac{N_{t+1}}{N_t} = (1-d_j)N_j(t) + R_j(t)N_j(t)$$

$$= \underbrace{s_j}_{\text{surv}}\underbrace{(1-g_j(t))}_{\text{did not germ}} N_j(t) + g_j(t)F_j(t)N_j(t)$$

$$= s_j N_j(t) + g_j(t)(F_j(t)-s_j)N_j(t)$$

$$+ g_j(t)F_j(t)N_j(t) - g_j(t)s_jN_j(t)$$

If we stick to a pre-season count, then

$$\frac{N_{j,t+1}}{N_{jt}} = (1-d_j)N_{jt} - R_{j,t}N_{jt}$$

$$= s_jN_{j,t} + g_{j,t}(F_{j,t}-s_j)N_{j,t}$$

$$\ln R_{j,t} = \ln(g_{j,t}(F_{j,t}-s_j))$$

$$= \ln\left(g_{j,t}\left(\frac{\lambda_j}{1+\sum_l \alpha_{jl} g_{l,t}N_{l,t}} - s_j\right)\right)$$

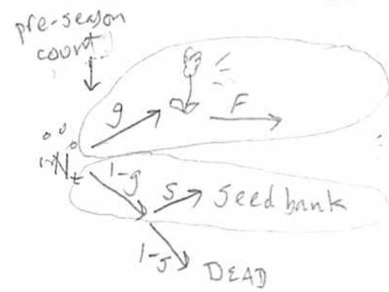
$$= \underbrace{\ln g_{j,t}}_{E_{j,t}} + \underbrace{\ln\left(\frac{\lambda_j}{1+\sum_l\alpha_{jl}g_{l,t}N_{l,t}} - s_j\right)}_{-C_{j,t}}$$

$s(1-g) + gF$ germ

$s_j - s_jg_{j,t} + g_{j,t}F_{j,t}$

$s_j - g_{j,t}(F_{j,t} - s_j)$

how to conceptualize the $-gs$ term?

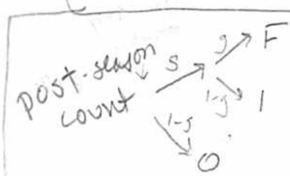


$NgF + N(1-g)s*1 + N(1-g)(1-s)*0$

$gFN + s(1-g)N$

$sN - sgN + gFN.$

$sN + g(F-s)N$



$sgFN_t + s(1-g)N_t*1 + \cancel{(1-s)}N_t*0$

$sgFN_t + sN_t - sgN_t$

$sN_t + sg(F-1)N_t.$