1 Model

For our example, we adapt the model from Wolkovich & Donahue 2020, which is itself an adaptation of a common model of community dynamics, often conceptualized for annual plants with a seedbank (as we do here). A more detailed description of the model can be found in Wolkovich & Donahue 2020.
In this model, a community of n species, the seedbank (N) of species i at time $t+1$ (where t represented interannual time) is determined by the survival (s) of ungerminated seeds from the previous season $(1-g_i(t))$ plus new biomass (B_i) produced during the following growing season (δ), which is converted to seeds at rate ϕ:

$$N_i(t+1) = sN_i(t)(1-g_i(t)) + \phi B_i(t+\delta) \tag{1}$$

Basic R^* competition controls new biomass each season, depending on species i's resource uptake ($f(R)$ converted into biomass at rate c_i) minus maintenance costs (m), with resource uptake controlled by species-level parameters that define how the uptake increases as R increases (a), the inverse of the maximum uptake (u), and the shape of the uptake (θ):

$$\frac{\mathrm{d}B_i}{\mathrm{d}t} = (c_i f(R) - m)B_i + G_i(t) \tag{2}$$

$$f(R) = \frac{aR^\theta}{1+auR^\theta} \tag{3}$$

The resource (R) itself declines across a growing season due to uptake by all species and abiotic loss (ϵ):

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -\sum_{i=1}^{n} f_i(R)B_i - \epsilon R \tag{4}$$

Our model departs from many similar community models but explicitly modeling germination over the season ($G_i(t)$) by modeling more of the physiology of germination—specifically that species may delay germination beyond the start of the season (according to the resource) depending on winter conditions, including what we call 'chilling.' Germination thus depends both on the traits of the species and on the environment each year. To achieve this, we modeled $G_i(t)$ germination as a time dependent forcing function that adds daily biomass to the ODE according to a Hill equation (see IMPT note in comments):

$$G_i(t) = N_0 s_i b_i g_i(t) \tag{5}$$

$$g_i(t) = g_{max,i}\left(\frac{\text{within-season-time}^{nH}}{\tau_{g50}^{nH} + \text{within-season-time}^{nH}}\right) \tag{6}$$

where b_i is the biomass per seed and where τ_{g50} is a Poisson random variable for the days of delay due to chilling, ξ, according to:

$$\tau_{g50} \sim \text{Poisson}(\tau_{delay}) \tag{7}$$

$$\tau_{delay} = \tau_{max} e^{-\tau_\xi \xi} \tag{8}$$

ADD: While explaining the model, will need to reference (and include in figures), the plots that are made in `Prieff_Species.R` to visualize.

Simulations:
Given an overview of simulations performed, be sure to mention: two-species communities, species vary in *what* and explain a little what that does (for example: We varied species' resource use efficiency (*via* c_i), yielding species with different R^* (a metric of resource competition where a lower R^* means a species can draw the resource down to a lower level and is thus considered the superior competitor)) and then state something like "All other parameters were identical between species (Table **??**)." Also need: starting conditions (for example: both species were initialized with a census size of $N(0) = 100$ per unit area, and the temporally varying parameters $R_0(t)$ and $\tau_p(t)$ were generated) and then some re-wording of this:

> Within-year R^* competition dynamics were solved using an ode solver (`ode` in the R package `deSolve`) and ended when the resource was drawn down to $min(R^*)$, i.e. the R^* value of the better resource competitor. The end-of-season biomass of each species was converted to seeds, and the populations were censused. At each census, a minimum cutoff was applied to define extinction from the model. Note that 'coexistence' in this model is defined by joint persistence through time and not by low density growth rate.

Table S1: Parameter values, definitions, and units.

Parameter	Value(s)	Definition	Unit
N_i	initial conditions $N_i(0) = 100; min(N_i(t)) = 10^{-4}$	census of seedbank of species i	seeds per unit area
s	0.8	survival of species i	unitless
B_i	see Eqn 2	biomass of species i	biomass
R_0	$\sim\ logN(\mu,\sigma)mu = log(2), \sigma = 0.2$	annually varying initial value of resource at the beginning of the growing season	resource
ξ_0	$\sim$??	chilling	chilling
c_i	$\sim$Unif(8,20)	conversion efficiency of R to biomass of species i	$\frac{\text{biomass}}{\text{resource}}$
m	0.05	maintenance costs during growth season i	days^{-1}
a	20	uptake increase as R increases for species i	days^{-1}
u	1	inverse of maximum uptake for species i	$\frac{\text{(days)(biomass)}}{\text{resource}}$
θ	1	shape of uptake for species i	unitless
ϕ	0.05	conversion of end-of-season biomass to seeds	biomass^{-1}, but conceptually $\frac{\text{seeds}}{\text{(biomass)(seeds)}}$
ϵ	1	abiotic loss of R	days^{-1}
g_{max}	0.5	maximum germination rate of species	unitless
h	100	controls the the rate at which germination declines as τ_p deviates from optimum for species i	days^{-2}
g_i	see Eqn 6	germination rate	unitless
τ_g	$\sim \beta(?,?)$	days of delay (given on weeks of chilling)	days
$\tau_{g_{50}}$	$\sim$ Pois(τ_g)	??	days
ξ_τ	$\sim$?,?	rate of delay (given weeks of chilling) i	days/week
b_i	1	biomass of a seedling	$\frac{\text{biomass}}{\text{seeds}}$
$f(R)$	see Eqn 3	resource uptake rate for species i	$\frac{\text{resource}}{\text{(days)(biomass)}}$
t	1	annual timestep	years
$0 \to \delta$	determined by rate of resource depletion	time during the growing season	days