Homework 2

Deadline: 5 October 2025, 23:30.

All solutions must be in a single PDF file and uploaded to the LMS portal.

1. (1 point) Find the variance of the sample third-order variation

$$\sum_{i=1}^{n} |W_{t_i} - W_{t_{i-1}}|^3$$

   of Brownian motion on $[0, T]$.

2. (1 point) The Ornstein–Uhlenbeck process is defined by

$$dX_t = -\kappa X_t dt + \sigma dW_t.$$

   Find the quadratic variation of $\sin(X_t)$.

3. (a) (0.5 point) Show that the process defined as

$$(T - t)W_t$$

   is differentiable at point T almost surely.

   (b) (0.5 point) Are there other points at which the process is differentiable almost surely?

$(T - t)W_t$