Homework 1

Deadline: 28 September 2025, 23:30.
All solutions must be in a single PDF file and uploaded to the LMS portal.

1. (0.25 point) Prove that the sum of two functions of bounded first-order variation also has bounded first-order variation.
2. (0.75 point) Prove that the product of two functions of bounded first-order variation also has bounded first-order variation.
3. (1 point) Is it true that the quadratic variation of function

$$f(x) = \begin{cases} x \sin(\frac{1}{x}), & x \in (0, 1], \\ 0, & x = 0. \end{cases}$$

   is bounded?
4. (2 points) Verify that the explicit call option price formula

$$C_t = S_t N(d_1) - K e^{-r(T-t)} N(d_2),$$
$$d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

   satisfies the Black–Scholes–Merton equation

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0.$$