Анализ модельного риска
дз3
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1

Structured deposit means that the funding rate (r_f) is bigger than the model rate (r_m), so:

a) The guaranteed S_0 at T is a zero-coupon: model PV is $S_0 e^{-r_m T}$, funded PV is $S_0 e^{-r_f T}$. Since $r_f > r_m$, we get that $e^{-r_f T} < e^{-r_m T}$, so the funding adjustment $S_0(e^{-r_m T} - e^{-r_f T}) > 0$ (benefit), but the call we must finance at r_f is more expensive, than under the model, giving a negative adjustment. So the total is not always positive $\implies$ the answer to the question is **no**

b) The same reason. The bond price is positive, only the option price is negative. The sum can have either sign depending on their relative sizes $\implies$ the answer to the question is **no**

2

Work in discounted units $\hat{S}_t = e^{-rt} S_t$. The discounted call payoff is $(\hat{S}_T - \hat{K})^+$, with $\hat{K} = Ke^{-rT}$
For continuous paths that cross the level only finitely many times, Tanaka's formula gives $(\hat{S}_T - \hat{K})^+ = (\hat{S}_0 - \hat{K})^+ + \int_0^T \mathbb{1}_{\{\hat{S}_t > \hat{K}\}} d\hat{S}_t$
So hold one discounted share whenever $\hat{S}_t > \hat{K}$ and 0 otherwise, starting with capital $(\hat{S}_0 - \hat{K})^+$, keep the rest in the bank account (self-financing), so returning to original units this is:
initial capital $V_0 = (S_0 - Ke^{-rT})^+$, stock position $\Delta t = \mathbb{1}_{\{S_t > Ke^{-r(T-t)}\}}$ own 1 share above the moving boundary $Ke^{-r(T-t)}$, none below. Cash $= V_t - \Delta_t S_t$ earning rate r
This produces $V_T = (S_T - K)^+$ for any continuous path with finitely many crossings.

3

Let S_t be a Brownian motion with volatility σ (zero rates)
For a european call with payoff $h(S_T) = (S_T - K)^+$, the naive hedge takes a constant delta $\Delta = h'(S_0) = \mathbb{1}_{\{S_0 > K\}}$ and never rebalances
Let's write a 2order Taylor expansion of the payoff around S_0 :
$h(S_T) \approx h(S_0) + h'(S_0)(S_T - S_0) + \frac{1}{2} h''(S_0)(S_T - S_0)^2$
With constant-delta hedge, the terminal hedging error is $\varepsilon_T = h(S_T) - \Delta S_T + \Delta S_0 \approx \frac{1}{2} h''(S_0)(S_T - S_0)^2$
For a call, the curvature h'' is concentrated at the strike K
So only paths near the strike contribute, so we can replace $h''(S_0)$ by the mass of the delta spike, which for a one-period approximation is $\Gamma_{ATM} \approx \dfrac{1}{\sigma\sqrt{2\pi T}}$
Since $S_T - S_0 = \sigma W_T$, with $W_T \sim N(0, T)$
$std(\varepsilon_T) \approx \frac{1}{2}\Gamma_{ATM} std[(\sigma W_T)^2] = \frac{1}{2}\Gamma_{ATM}\sqrt{Var(W_T^2)} = \frac{1}{2}\Gamma_{ATM}\sigma^2\sqrt{2}T$
Plugging $\Gamma_{ATM} \approx \dfrac{1}{\sigma\sqrt{2\pi T}}$ gives $std(\varepsilon_T) \approx \dfrac{\sigma^2\sqrt{2}T}{2\sigma\sqrt{2\pi T}} = \dfrac{\sigma\sqrt{T}}{2\sqrt{\pi}}$.
With zero rates, a zero-strike option that pays S_T has premium S_0, so the ratio of the standard deviation of the hedging error to the premium of a zero-strike option $= \dfrac{\sigma\sqrt{T}}{2S_0\sqrt{\pi}}$