**FINM4033: Financial Modelling Assignment 2**

**Xinyi Li（李心伊）**

**1930018036**

**Note: For the version of Excel, VBA function may not show on your computer, so I offered the code of VBA in the attachment and snapshot in the PDF file.**

*Please select a stock or stock index from WIND database or yahoo finance. Suppose that the risk-free rate is $r_f = 2\%$. You are required to use the following four methods to price a hypothetical at-the-money European option (S = X) that expires in 6 months:*

*1) Single Period Binomial Tree*

*2) 20-Period Binomial Tree*

*3) Black-Scholes Model*

*4) Monte Carlo Simulation with 100 paths.*

1. *Please briefly explain the methodology of each aforementioned methods and compare the results from the four models and summarize the results.*

| | SPBT | MPBT | BS | MC |
|---|---|---|---|---|
| Call | 0.197028784 | 0.2024 | 0.190206187 | 0.2184367 |
| Put | 0.141855024 | 0.146715728 | 0.162413998 | 0.1679356 |

The stock I chose is 600015.SS Hua Xia Bank.

I computed the sigma using the historical return, not the implied volatility. Moreover, the annualized mean return $\mu$ was also calculated at the same time. Then I used Up=exp (m*Dt + s*sqrt(Dt)), Down = exp(m*Dt - s*sqrt(Dt)) to get the U and D. The U, D, $\mu$,sigma would be applied to the formulas of other methodologies.

1. **Single Period Binomial Tree:**

In this methodology, I used discrete compounding so the risk-free rate of half year is $(1+0.02)^{0.5}-1\approx 0.01$.

State price and risk-neutral probability are used in the replication of stock and bond.

State price: There is actually a simpler (and more general) way to solve this problem: Viewed from today, there are only two possibilities for next period: Either the stock price goes up or it goes down. Think about the market determining a price $q_U$ for \$1 in the "up" state of the world and a price $q_D$ for \$1 in the "down" state of the world.

$$C = q_U \max(S*U - X, 0) + q_D \max(S*D - X, 0)$$
$$P = q_U \max(X - S*U, 0) + q_D \max(X - S*D, 0)$$

or, if priced by put-call parity,

$$P = C + PV(X) - S$$

Then both the bond and the stock have to be priced using these state prices. The put-call parity also holds.

Risk-neutral probability: multiplying the state prices by 1 plus the interest rate R gives the risk neutral prices: $\pi_U = q_U R$, $\pi_D = q_D R$. The risk-neutral prices look like a probability distribution of the states, since they sum to 1:

$$\pi_U + \pi_D = q_U R + q_D R = \frac{R - D}{R(U - D)} R + \frac{U - R}{R(U - D)} R = 1$$

Suppose an asset has statedependent payoffs XU in the "up" state and XD in the "down" state of a two-date model. Then the date-0 price of the asset using the state prices is $q_U X_U + q_D X_D$, and the date-0 price of the asset using the risk-neutral prices is the discounted expected asset payoff, where the expectation is computed using the risk-neutral prices as if they are the actual state probabilities:

$$\frac{\pi_U X_U + \pi_D X_D}{R} = \frac{\text{"Expected" asset payoff using risk-neutral prices}}{1 + r}$$
$$= q_U X_U + q_D X_D$$

2. **20-Period Binomial Tree:**

In this methodology, I used continuous compounding and the risk-free rate is exp (0.02*the length of one division).

The binomial model can easily be extended to more than one period. The binomial model is intuitive and easy to implement. This model of multiple periods binomial tree can be put into a spread sheet and programmed in VBA.

3. **Black Scholes:**

In this methodology, I used Excel and R to compute.

The Black Scholes model uses the following formula to price European calls on a stock:

$$C = SN(d_1) - Xe^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Here C denotes the price of a call, S is the price of the underlying stock, X is the exercise price of the call, T is the call's time to exercise, r is the interest rate, and σ is the standard deviation of the logarithm of the stock's return. N() denotes a value of the standard normal distribution. It is assumed that the stock will pay no dividends before date T.By the put-call parity theorem, a put with the same exercise date T and exercise price X written on the same stock will have price $P = C - S + Xe^{-rT}$. Substituting for C in this equation and doing some algebra gives the Black-Scholes European-put-pricing formula: $P=Xe^{-rT} N(-d_2)-SN(-d_1)$.

The stock price S, the option exercise price X, the option's time to maturity T, the interest rate r, and the standard deviation of the returns of the stock underlying the option are the five variables that make up the Black-Scholes formula (sigma). The fifth parameter sigma, is challenging but the other four are easy to understand. There are two typical approaches to computing: There are two typical approaches to computing sigma: stock's historical returns or implied volatility. What I used is stock's historical returns on the sheet "stock".

4. **Monte Carlo:**

In this methodology, I used the R and MATLAB to calculate.

Monte Carlo pricing of options depends on a simulation of the price path of the underlying asset. Monte Carlo methods are experimental techniques for determining the numerical value of a function or a procedure. In general these methods are to be avoided when there is another, closed-form, way of determining the value. In cases where no such method exists, however, you can use Monte Carlo to approximate the value.

It is easy to extend the risk-neutral pricing scheme to multiperiod frame works. Consider a multiperiod binomial setting where U and D do not change over time, and indicate the date-n state payoffs by $Payoff_{n,j}$, j = 0, . . . , n. The notation $Payoff_{n,j}$ indicates the date-n payoff of the asset in a state where there are j up moves on the binomial tree; for a call in a binomial framework, $\text{Payoff}_{n,j} = \max(S^*U^j D^{n-j} - X,0)$

Then the value of this asset is given by

$$\text{Asset value today} = \sum_{j=0}^{n} \binom{n}{j} q_U^j q_D^{n-j} \text{Payoff}_{n,j} = \underbrace{\frac{1}{R^n} \sum_{j=0}^{n} \binom{n}{j} \pi_U^j \pi_D^{n-j} \text{Payoff}_{n,j}}_{\substack{\uparrow \\ \text{The risk-neutral expected} \\ \text{discounted value}}}$$

This particular notation assumes that the tree is recombining. It assumes, in other words, that the date-n payoffs are path independent—the option payoff is a function only of the terminal stock price and does not depend on the path by which this price is reached.

Monte Carlo Plain-Vanilla Call Pricing(used in the Q3):

Our basic setup is as follows: We price a European call on a stock whose current price is $S_0$. The option's exercise price is X, and the time to maturity of the option is T. We assume that the stock price is lognormally distributed with mean μ and standard deviation σ. To price the call using Monte Carlo, we divide the unit time interval into n divisions. Therefore, $\Delta t = 1/n$. For each Δt, we define $Up_{\Delta t} = \exp[\mu\Delta t + \sigma\sqrt{\Delta t}]$ and $Down_{\Delta t} = \exp[\mu\Delta t - \sigma\sqrt{\Delta t}]$. The interest rate on the interval Δt is $R_{\Delta t} = \exp[r\Delta t]$.

Therefore, the state prices and risk-neutral probabilities are given by

$$q_u = \frac{R_{\Delta t} - \text{Down}_{\Delta t}}{R_{\Delta t}(\text{Up}_{\Delta t} - \text{Down}_{\Delta t})}, \quad q_d = \frac{\text{Up}_{\Delta t} - R_{\Delta t}}{R_{\Delta t}(\text{Up}_{\Delta t} - \text{Down}_{\Delta t})}$$

$$\pi_u = \frac{R_{\Delta t} - \text{Down}_{\Delta t}}{\text{Up}_{\Delta t} - \text{Down}_{\Delta t}}, \qquad \pi_d = \frac{\text{Up}_{\Delta t} - R_{\Delta t}}{\text{Up}_{\Delta t} - \text{Down}_{\Delta t}} = 1 - \pi_u$$

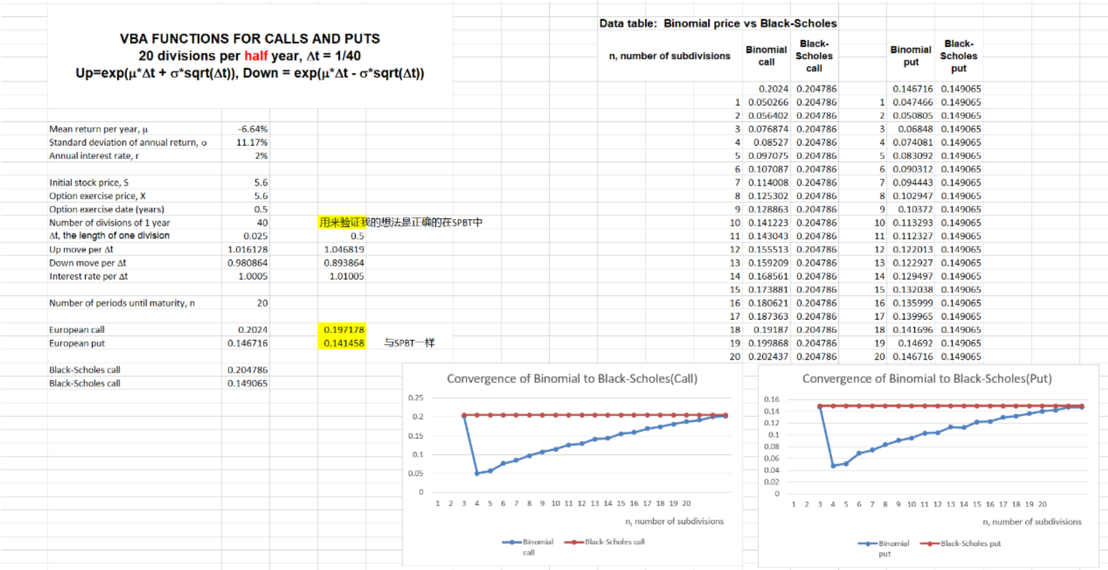
Since the time to maturity of the option is T, the price path to T requires $m = T/\Delta t$ periods. A price path of length m is created by determining the Up or Down move of the stock as a function of a random number between 0 and 1 and the risk-neutral probability $\pi_d$.

The principle is as follows:

Price paths are generated by using the risk-neutral probabilities. In the program Vanillacall, for example, the price of the stock moves Up if the random-number generator is greater than $\pi_d$ and moves Down if the random-number generator is less than or equal to $\pi_d$. Effectively, therefore, the risk-neutral probabilities $\{\pi_U = 1 - \pi_d, \pi_d\}$ of each price path are incorporated into the price path itself. The value of the option using Monte Carlo is determined by the discounted value of the simple average of all results over the price paths generated.

2.

*In the binomial tree case, please study the case when the period extends from 1 to 20. Also compare the result with a Black-Scholes (BS) solution. (20 marks)*



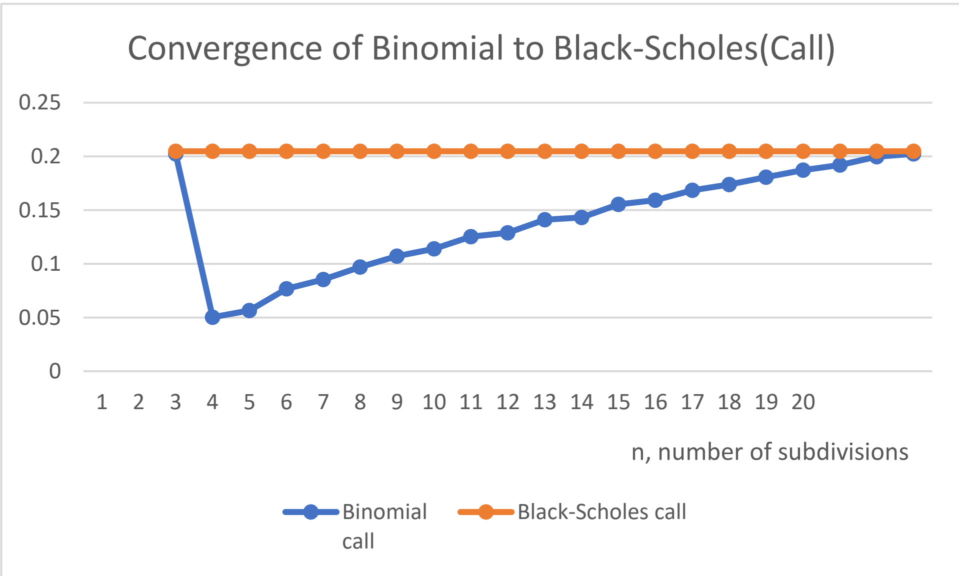
**VBA FUNCTIONS FOR CALLS AND PUTS**
**20 divisions per half year, Δt = 1/40**
**Up=exp(μ*Δt + σ*sqrt(Δt)), Down = exp(μ*Δt - σ*sqrt(Δt))**

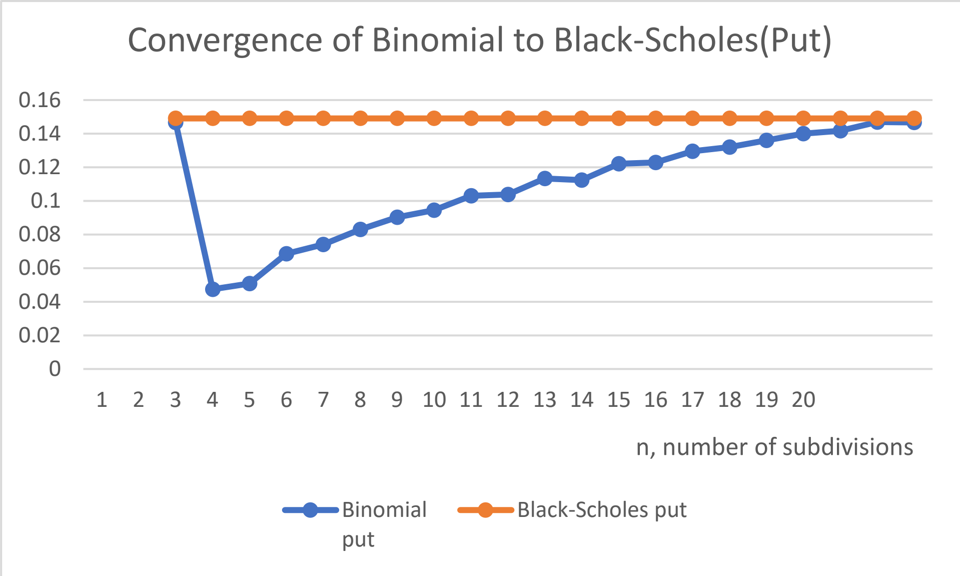
| | | | |
|---|---|---|---|
| Mean return per year, μ | -6.64% | | |
| Standard deviation of annual return, σ | 11.17% | | |
| Annual interest rate, r | 2% | | |
| | | | |
| Initial stock price, S | 5.6 | | |
| Option exercise price, X | 5.6 | | |
| Option exercise date (years) | 0.5 | | |
| Number of divisions of 1 year | 40 | 用来验证我的想法是正确的在SPBT中 | |
| Δt, the length of one division | 0.025 | 0.5 | |
| Up move per Δt | 1.016128 | 1.046819 | |
| Down move per Δt | 0.980864 | 0.893864 | |
| Interest rate per Δt | 1.0005 | 1.01005 | |
| | | | |
| Number of periods until maturity, n | 20 | | |
| | | | |
| European call | 0.2024 | 0.197178 | |
| European put | 0.146716 | 0.141458 | 与SPBT一样 |
| | | | |
| Black-Scholes call | 0.204786 | | |
| Black-Scholes call | 0.149065 | | |

**Data table: Binomial price vs Black-Scholes**

| n, number of subdivisions | Binomial call | Black-Scholes call | | Binomial put | Black-Scholes put |
|---|---|---|---|---|---|
| | 0.2024 | 0.204786 | | 0.146716 | 0.149065 |
| 1 | 0.050266 | 0.204786 | 1 | 0.047466 | 0.149065 |
| 2 | 0.056402 | 0.204786 | 2 | 0.050805 | 0.149065 |
| 3 | 0.076874 | 0.204786 | 3 | 0.06848 | 0.149065 |
| 4 | 0.08527 | 0.204786 | 4 | 0.074081 | 0.149065 |
| 5 | 0.097075 | 0.204786 | 5 | 0.083092 | 0.149065 |
| 6 | 0.107087 | 0.204786 | 6 | 0.090312 | 0.149065 |
| 7 | 0.114008 | 0.204786 | 7 | 0.094443 | 0.149065 |
| 8 | 0.125302 | 0.204786 | 8 | 0.102947 | 0.149065 |
| 9 | 0.128863 | 0.204786 | 9 | 0.10372 | 0.149065 |
| 10 | 0.141223 | 0.204786 | 10 | 0.113293 | 0.149065 |
| 11 | 0.143043 | 0.204786 | 11 | 0.112327 | 0.149065 |
| 12 | 0.155513 | 0.204786 | 12 | 0.122013 | 0.149065 |
| 13 | 0.159209 | 0.204786 | 13 | 0.122927 | 0.149065 |
| 14 | 0.168561 | 0.204786 | 14 | 0.129497 | 0.149065 |
| 15 | 0.173881 | 0.204786 | 15 | 0.132038 | 0.149065 |
| 16 | 0.180621 | 0.204786 | 16 | 0.135999 | 0.149065 |
| 17 | 0.187363 | 0.204786 | 17 | 0.139965 | 0.149065 |
| 18 | 0.19187 | 0.204786 | 18 | 0.141696 | 0.149065 |
| 19 | 0.199868 | 0.204786 | 19 | 0.14692 | 0.149065 |
| 20 | 0.202437 | 0.204786 | 20 | 0.146716 | 0.149065 |








Assuming understanding of lognormality and the Black-Scholes option-pricing formula, it is a finite approximation. As $\Delta t \to 0$ , the resulting distribution of the stock returns approaches the lognormal distribution.

The binomial model offers a good approximation to the Black-Scholes. As the n, number of subdivisions of T gets larger, this approximation gets better, although the convergence to the Black-Scholes price is not smooth. From the two graphs above, it is obvious that the when n=20, there is a intersection between two lines while there is a significant gap at the beginning.

3.

*In the Monte Carlo Simulation, please also study the case when number of path increases from 10 to 100, as compared to the BS price. (20 marks)*

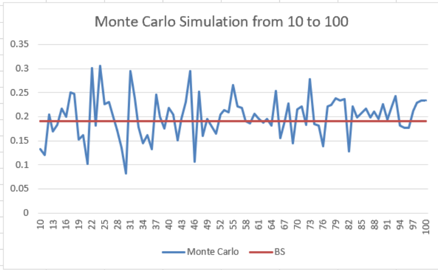
In this question, I used R with the number of paths increased by 10 (Please refer to the HTML) and Excel with the number of paths increased by 1 to compare.

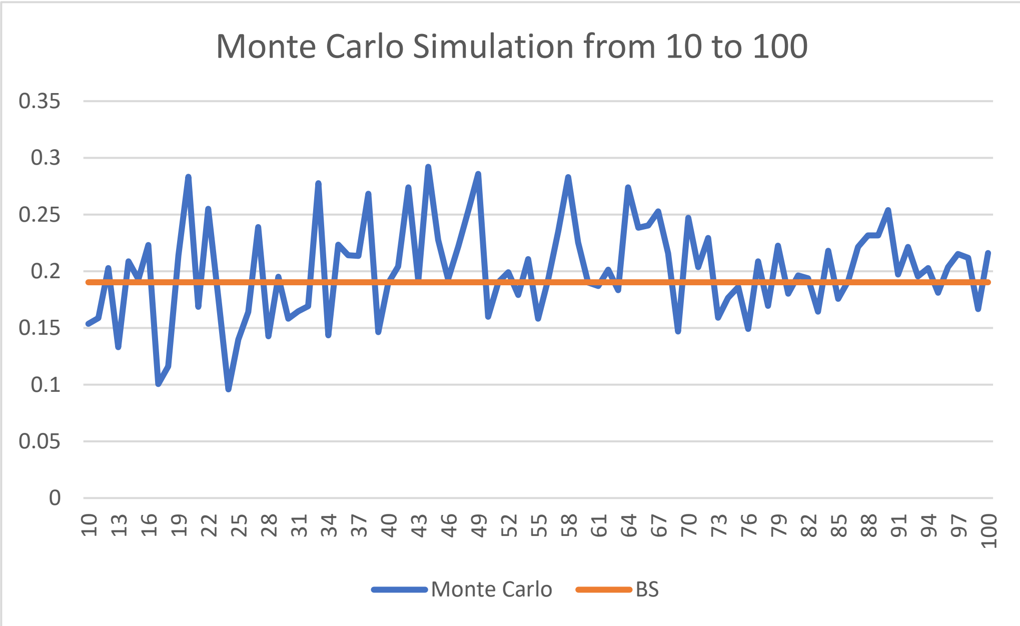
Monte Carlo methods—simulations of option pricing by tracing out many paths of the stock price—are at best a “second best” method of pricing. But in cases where no analytical formulas are available, Monte Carlo is easy to program in VBA and easy to see in Excel.

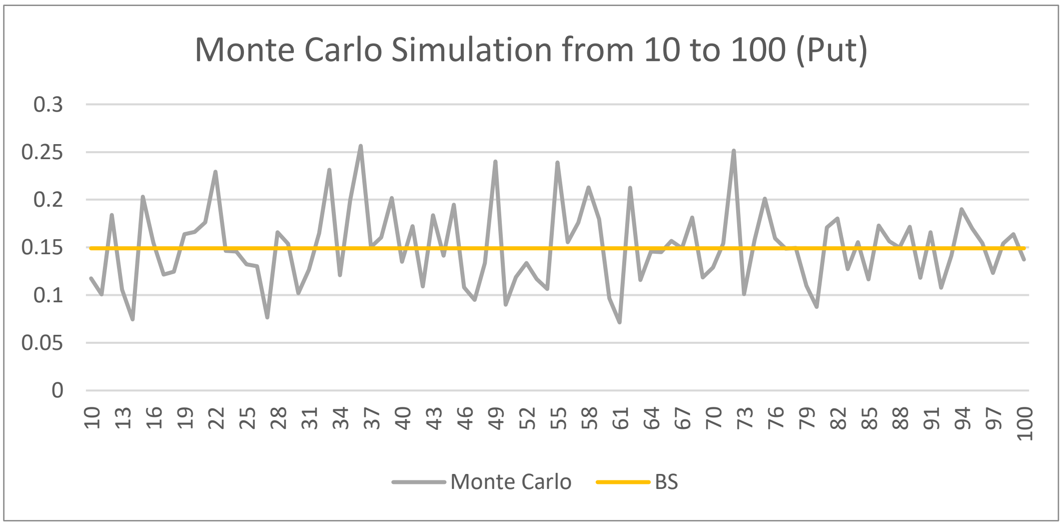
Since plain-vanilla options are accurately priced using the Black-Scholes formula, this exercise allows us to check our pricing method against a known result and also allows us to develop the proper intuitions about the Monte Carlo pricing of more complicated options.

| MONTE CARLO PRICING OF PLAIN VANILLA CALLS | | |
|---|---|---|
| $S_0$ , current | 5.6 | |
| X, exercise | 5.6 | |
| r, interest ra | 2% | |
| T, time | 0.5 | |
| μ, mean sto | -6.64% | |
| σ, sigma--s | 11.17% | |
| | | |
| n, divisions | 20 | |
| Runs | 100 | |
| | | |
| VanillaCall | 0.1798 | <-- =vanillacall(B2,B3,B6,B7,B4,B5,B9,B10) |
| VanillaPut | 0.0700 | |
| BS call | 0.2048 | <-- =BSCall(B2,B3,B5,B4,B7) |
| BS put | 0.149065 | 0.0950 |

| | | |
|---|---|---|
| | 0.1798 | |
| 10 | 0.133392 | 0.190276 |
| 11 | 0.120521 | 0.190276 |
| 12 | 0.203931 | 0.190276 |
| 13 | 0.169755 | 0.190276 |
| 14 | 0.183879 | 0.190276 |
| 15 | 0.217343 | 0.190276 |
| 16 | 0.19981 | 0.190276 |
| 17 | 0.249973 | 0.190276 |
| 18 | 0.247701 | 0.190276 |
| 19 | 0.152632 | 0.190276 |
| 20 | 0.162189 | 0.190276 |
| 21 | 0.102782 | 0.190276 |
| 22 | 0.30119 | 0.190276 |
| 23 | 0.181983 | 0.190276 |
| 24 | 0.305315 | 0.190276 |
| 25 | 0.225471 | 0.190276 |
| 26 | 0.23059 | 0.190276 |



It is apparent that when the number of paths increases, the result of Monte Carlo is closer to the result of Black Scholes, which is like an evaluation metric to check the accuracy. Therefore, when the number of paths is larger, the result will be more precise.

4. Please also discuss the pros and cons of each method. (20 marks)

| | Pros | Cons |
|---|---|---|
| Binomial Model | • The binomial model also offers more flexibility because the user can alter the inputs at each step in the process to account for differences in the ability to exercise a particular option that shows non-standard features.<br>• The binomial trees are powerful, intuitive methods to value both American and European option.<br>• | • Binomial models are complex to construct and depending on the number of steps used in the model, can be incredibly unwieldy in terms of size of the spreadsheet and computing power needed to run.<br>• It lacks of efficient in a situation where effects of cash dividends should be analyzed.<br>• The binomial tree models are inefficient in valuing American options compared with European option. And it is less efficient and accurate than finite difference methods for multiple options valuation. This is because it has a conditional starting point. |
| Black Scholes | Timesaving and accurate. The determination of the value or worth of an asset or liability just | It is a black box calculator and it doesn't offer the flexibility required to value options |

| | | |
|---|---|---|
| | depends on one formula. It relies on fixed inputs (current stock price, strike price, time until expiration, volatility, risk free rates, and dividend yield). You just need to find the right inputs and use a good online calculator to make it correct. | with non-standard features, such as a price reset feature or a mandatory exercise requirement. |
| Monte Carlo | • It can generate different graphs depending on the outcome of each simulation. Graphical representation is easy to understand and explain the scenarios.<br>• Monte Carlo enables sensitivity analysis. It is easier to determine the inputs that had the biggest effect on bottom-line results.<br>• Monte Carlo simulation is simple, flexible. It can be easily modified to adapt different processes which involved governing stock returns. For example, in general a path-dependent option | • It is wasteful to calculate many times and difficult to control situations when there are early exercise opportunities.<br>• It is time-consuming to calculate the average of all the calculated results of abundant paths.<br>• There is no fixed results after running program. |

| | | | |
|---|---|---|---|
| | | does not have an analytic price solution like Asian Option and barrier option, and Monte Carlo provides us with a handy numerical tool for pricing. | |