

*Performance  
Evaluation*

*Trading  
Strategy*

# Refining Cox-Ingersoll-Ross Dynamics:

Integration of Jump Risks for Advanced Treasury Yield Curve Forecasting

*Simulation  
&  
Calibration*

*Conclusion*

Group 15

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*Building  
Models*

*Intro*

***Thank You!***

# ***Project Overview***

- Built upon traditional CIR model by including jump risks in its framework
- Assessed and fine-tuned the enhanced model and original model
- Prediction effect of CIR and ECIR on zero coupon bond price measured and compared
- Trading strategy built based on yield curve predictions

***Data***

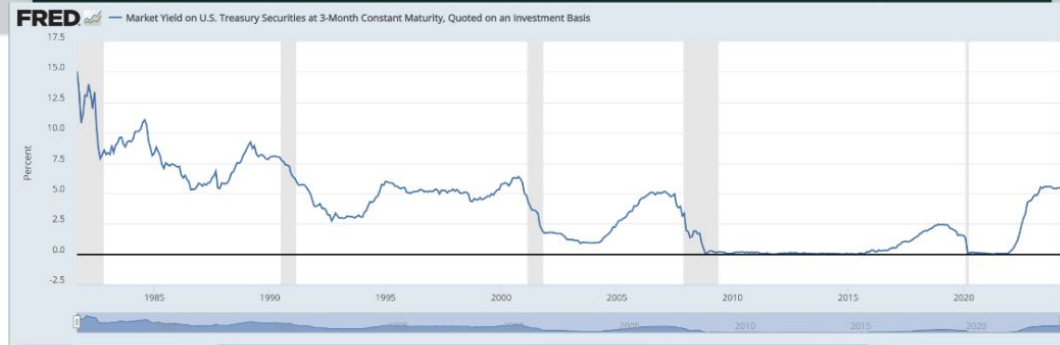
# ***US Treasury Zero-Coupon Yield Curve***

- Downloaded daily yield curve data for different maturities from FRED (Federal Reserve Economic Data)
- Completed yield curve for discrete years via **Cubic Spline interpolation**

***Additional  
Yield  
Measures***

## ***Additional Yield Measures***

- 3-mon Yield to Maturity (YTM) for Treasury Bond Yields from FRED
- Model parameters fitting using the Maximum Likelihood Estimation (MLE) approach (discussed later)



# *Short Rate Models*

- Directly model the future instantaneous rate
- a structured framework that facilitates the incorporation of new mathematical features

*CIR and  
ECIR*

# *The CIR and Extended CIR Model*

CIR  $dr(t) = \kappa(t)[\mu_r(t) - r(t)]dt + \sigma(t)\sqrt{r(t)}dW(t)$

## **Extended CIR**

The extension of the standard CIR model allows us to introduce a jump component.  $-r(t-) < J < r(t-)$ . This ensures the rate  $r(t) = r(t-) + J$  remains non-negative.

$$dr(t) = \kappa(t)[\mu_r(t) - r(t)]dt + \sigma(t)\sqrt{r(t)}dW(t) + JdP$$

$$J = N_{\text{trunc}}(\mu, \gamma, -r(t-), r(t-)), P = \text{Poisson}(dt * h)$$



# ***Simulation & Calibration***

Approach 1: Yield Curve Fitting

Approach 2: Maximum Likelihood Estimation

***Yield  
Curve***

***Maximum  
Likelihood  
Estimation***

# *Yield Curve Fitting Approach*

- Market data: Treasury Bonds' yield curve across 30Y maturities snapshot on 2024-03-15
- Assuming there are 252 trading days per year,  $dt = 1/252$

## Calibrated Model Parameters

	Calibration Date	$\kappa$	$\mu_r$	$\sigma$	$\mu$	$\gamma$
CIR	3.15.2024	1.2310	0.0459	0.1410	-	-
ECIR	3.15.2024	3.071	0.007668	0.3419	0.08226	0.04355



# *Maximum Likelihood Estimation Approach*

- Market data: 3 month treasury bill YTM for the past 6 months from 2023-12-15 to 2024-03-15
- Assuming there are 252 trading days per year,  $dt = 1/252$

## Calibrated Model Parameters

	Calibration Date	$\kappa$	$\mu_r$	$\sigma$	$\mu$	$\gamma$
CIR	3.15.2024	0.0565	0.0996	0.0124	-	-
ECIR	3.15.2024	4.989	0.05481	0.01116	0.00000	0.05628

# ***Performance Evaluation***



***Yield Rate***

***Standard  
Deviations***

***Absolute  
Errors***

***Mean  
Absolute  
Errors***

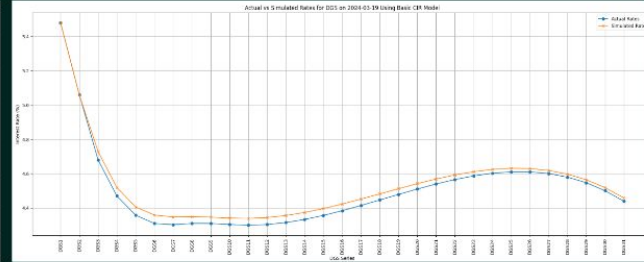
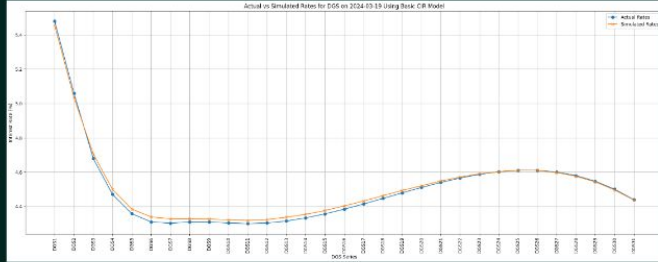
***Bond Price***

# Yield Rate

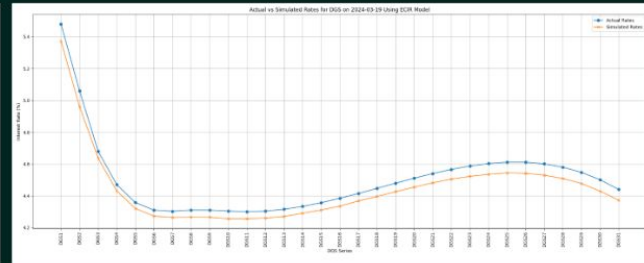
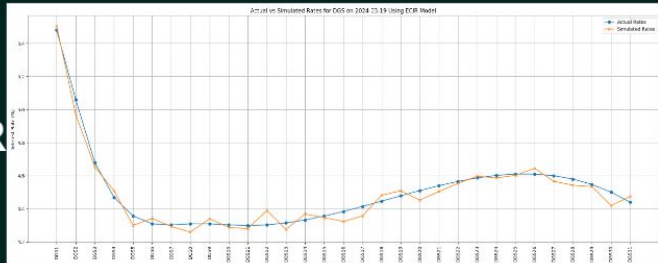
## Yield Curve

## MLE

CIR



ECIR

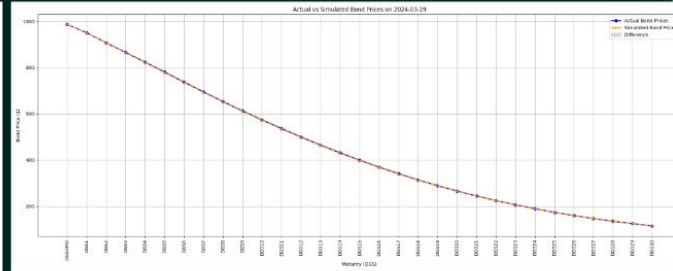
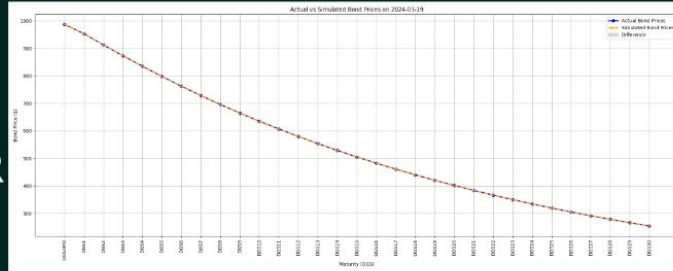


# Bond Price

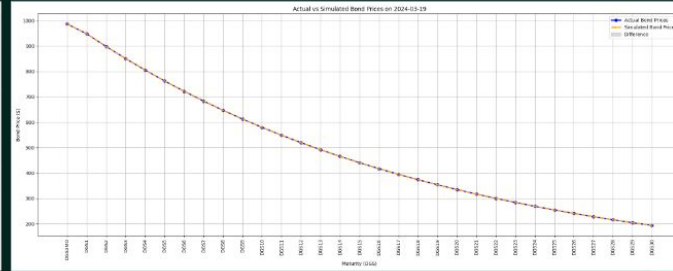
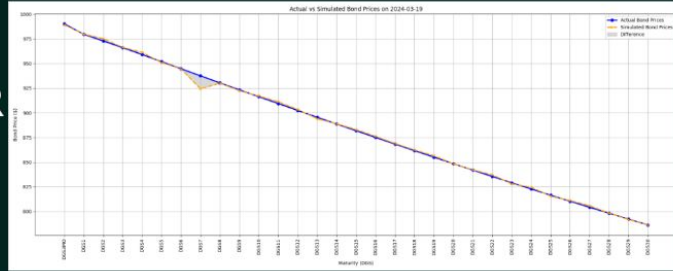
## Yield Curve

## MLE

CIR



ECIR



# Standard Deviations

## Yield Curve

## MLE

Yield  
Rate

	1Y	3Y	5Y	7Y	10Y	20Y	30Y
CIR	0.000898	0.000786	0.000663	0.000515	0.000455	0.000356	0.000286
ECIR	0.147668	0.372337	0.290388	0.175888	0.145358	0.261232	0.268256

	1Y	2Y	3Y	5Y	7Y	10Y	20Y	30Y
CIR	0.000622	0.000500	0.000671	0.000616	0.000625	0.000667	0.000466	0.000595
ECIR	0.000484	0.000361	0.000322	0.000192	0.000219	0.000182	0.000126	0.000098

Bond  
Price

	1Y	2Y	3Y	5Y	7Y	10Y	20Y	30Y
CIR	0.006866	0.008105	0.008279	0.007868	0.006959	0.005713	0.003953	0.002919
ECIR	0.062509	0.237724	0.237295	0.225276	0.229205	0.310449	0.276522	0.209865

	1Y	3Y	5Y	7Y	10Y	20Y	30Y
CIR	0.014829	0.022496	0.021839	0.019051	0.016152	0.006897	0.002681
ECIR	0.002087	0.001950	0.001772	0.001682	0.001438	0.001036	0.000657

# Absolute Errors

## Yield Curve

## MLE

Yield  
Rate

DGS Series	CIR Absolute Error	ECIR Absolute Error
DGS1	0.024757	0.089364
DGS2	0.026022	0.026638
DGS3	0.029019	0.039698
DGS5	0.029237	0.032193
DGS7	0.017509	0.049650
DGS10	0.018796	0.019382
DGS20	0.008551	0.034772
DGS30	0.002222	0.034963

DGS Series	CIR Absolute Error	ECIR Absolute Error
DGS1	0.001072	0.098887
DGS3	0.049052	0.039649
DGS5	0.048890	0.037481
DGS7	0.039070	0.043922
DGS10	0.038985	0.044721
DGS20	0.028976	0.051889
DGS30	0.018926	0.067676

Bond  
Price

DGS Series	CIR Absolute Error	ECIR Absolute Error
DGS1	0.061668	0.100842
DGS2	0.223623	2.387464
DGS3	0.276758	0.353862
DGS5	0.251260	0.892857
DGS7	0.196316	13.170066
DGS10	0.096901	0.674075
DGS30	0.034563	0.132302

DGS Series	CIR Absolute Error	ECIR Absolute Error
DGS1	0.017588	0.012356
DGS3	1.174431	0.101483
DGS5	1.708049	0.109931
DGS7	1.682633	0.076709
DGS10	1.770614	0.092740
DGS20	0.973306	0.015818
DGS30	0.330961	0.015735



# *Mean Absolute Error*

## Yield Curve

	Yield Rate Estimates	Bond Prices MAE
CIR	0.494251	0.38846
ECIR	0.494251	0.611413

## MLE

	Yield Rate Estimates	Bond Prices MAE
CIR	0.440558	0.619146
ECIR	0.440558	0.441298

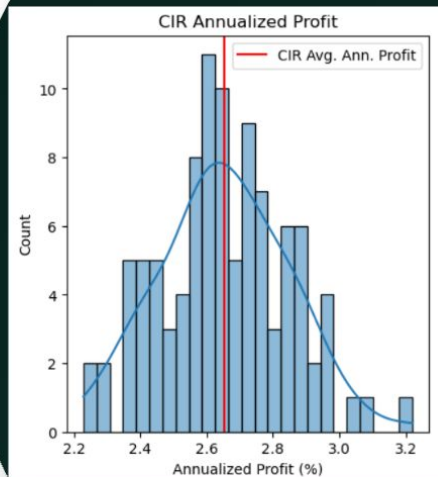
# ***Trading Strategy***

- Predict yield curve and bond prices
- Determine if actual prices are over- or under-valued
- Invest in top 5 smallest prediction errors
  - Short if over-valued
  - Long if under-valued
- Close position the following day
- Repeat, re-investing all capital including any profit

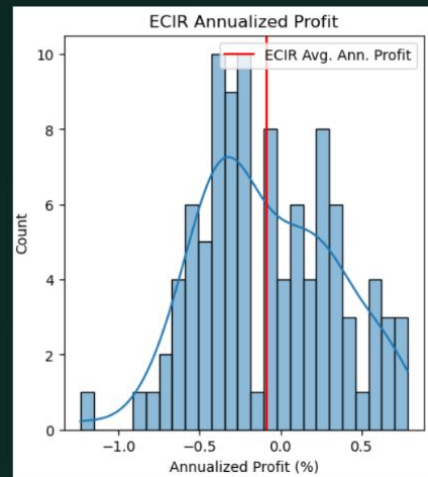
***Overall  
Performance***

***Individual  
Performance***

## ***Total Annualized Returns***



Average: 2.65%  
Sharpe: 1.624



Average: -0.08%  
Sharpe: -0.037

# Individual Annualized Returns

	DGS3MO	DGS1	DGS2	DGS3	DGS4	DGS5	DGS6	DGS7	DGS8	DGS9	DGS10	DGS11	DGS12	DGS13	DGS14	DGS15
CIR	0.0035	0.1785	-0.0603	-0.1694	0.0729	0.0000	0.0000	0.0221	-0.0120	-0.0009	0.0000	0.0000	0.0000	-0.0210	-0.0415	-0.0572
ECIR	0.0069	0.0001	-0.0080	0.0141	0.0124	-0.0065	-0.0023	0.0170	0.0010	-0.0039	0.0022	-0.0069	-0.0016	-0.0085	0.0163	-0.0162
	DGS16	DGS17	DGS18	DGS19	DGS20	DGS21	DGS22	DGS23	DGS24	DGS25	DGS26	DGS27	DGS28	DGS29	DGS30	
CIR	-0.0295	-0.0007	-0.0017	-0.0021	0.0513	-0.0485	0.0232	0.0722	0.2766	0.4636	0.6129	0.5716	0.4141	0.2274	0.1076	
ECIR	-0.0066	0.0065	-0.0043	-0.0114	-0.0164	0.0024	-0.0256	-0.0007	-0.0090	0.0016	-0.0107	0.0059	-0.0117	-0.0113	-0.0071	

# *Conclusion*

## **Yield Curve Fitting Approach...**

- ...has volatility in predictions
- ...is not guaranteed to converge
- ...is relevant for active trading

## **MLE Method...**

- ...demonstrates lower variability in predictions
- ...delivers more consistent and accurate results across all maturities
- ...less likely to be overfit

## **Trading Strategy...**

- ...generates higher returns under the CIR model during a period with no jumps

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*Intro*



Prezi