CS 61A
Summer 2022

Trees, Linked Lists

Discussion 7: July 19, 2022

Trees

In computer science, **trees** are recursive data structures that are widely used in various settings and can be implemented in many ways.

Generally in computer science, you may see trees drawn "upside-down." We say the **root** is the node where the tree begins to branch out at the top, and the **leaves** are the nodes where the tree ends at the bottom.

Some terminology regarding trees:

- **Parent Node**: A node that has at least one branch.
- **Child Node**: A node that has a parent. A child node can only have one parent.
- **Root**: The top node of the tree.
- **Label**: The value at a node.
- **Leaf**: A node that has no branches.
- **Branch**: A subtree of the root. Trees have branches, which are trees themselves: this is why trees are *recursive* data structures.
- **Depth**: How far away a node is from the root. We define this as the number of edges between the root to the node. As there are no edges between the root and itself, the root has depth 0.
- **Height**: The depth of the lowest (furthest from the root) leaf.

In computer science, there are many different types of trees, used for different purposes. Some vary in the number of branches each node has; others vary in the structure of the tree.

A tree has a root value and a list of branches, where each branch is itself a tree.

- The `Tree` constructor takes in a value `label` for the root, and an optional list of branches `branches`. If `branches` isn't given, the constructor uses the empty list `[]` as the default.
- To get the label of a tree `t`, we access the instance attributes `t.label`.
- Accessing the instance attribute `t.branches` will give us a **list of branches**. Treating the return value of `t.branches` as a list is then part of how we define trees.

Q1: Height

Write a function that returns the height of a tree. Recall that the height of a tree is the length of the longest path from the root to a leaf.

```
def height(t):
    """Return the height of a tree.

    >>> t = Tree(3, [Tree(5, [Tree(1)]), Tree(2)])
    >>> height(t)
    2
    >>> t = Tree(3, [Tree(1), Tree(2, [Tree(5, [Tree(6)]), Tree(1)])
    ])
    >>> height(t)
    3
    """
    "*** YOUR CODE HERE ***"









# You can use more space on the back if you want
```

Q2: Maximum Path Sum

Write a function that takes in a tree and returns the maximum sum of the values along any path in the tree. Recall that a path is from the tree's root to any leaf.

```
def max_path_sum(t):
    """Return the maximum path sum of the tree.

    >>> t = Tree(1, [Tree(5, [Tree(1), Tree(3)]), Tree(10)])
    >>> max_path_sum(t)
    11
    """
    "*** YOUR CODE HERE ***"









# You can use more space on the back if you want
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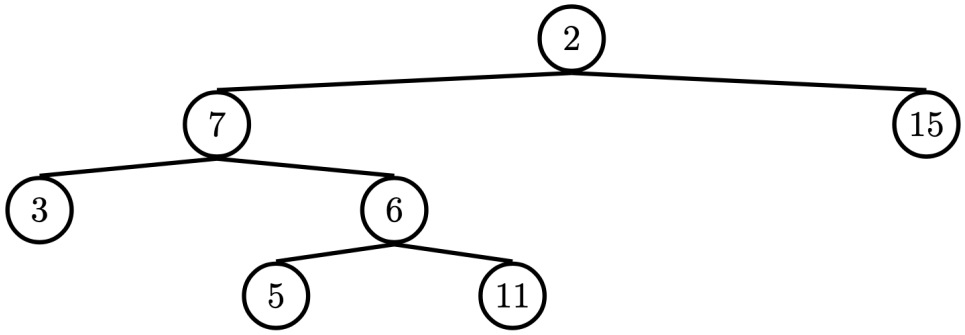
Q3: Find Path

Write a function that takes in a tree and a value `x` and returns a list containing the nodes along the path required to get from the root of the tree to a node containing `x`.

If `x` is not present in the tree, return `None`. Assume that the entries of the tree are unique.

For the following tree, `find_path(t, 5)` should return `[2, 7, 6, 5]`



Example Tree

```
def find_path(t, x):
    """
    >>> t = Tree(2, [Tree(7, [Tree(3), Tree(6, [Tree(5), Tree(11)])
    ]), Tree(15)])
    >>> find_path(t, 5)
    [2, 7, 6, 5]
    >>> find_path(t, 10)  # returns None
    """
    if ______________________________:
        return ______________________________
    ______________________________:
        path = ______________________
        if ______________________________:
            return ______________________________
```

Q4: Prune Small

Complete the function `prune_small` that takes in a `Tree t` and a number `n` and prunes `t` mutatively. If `t` or any of its branches has more than `n` branches, the `n` branches with the smallest labels should be kept and any other branches should be *pruned*, or removed, from the tree.

```
def prune_small(t, n):
    """Prune the tree mutatively, keeping only the n branches
    of each node with the smallest label.

    >>> t1 = Tree(6)
    >>> prune_small(t1, 2)
    >>> t1
    Tree(6)
    >>> t2 = Tree(6, [Tree(3), Tree(4)])
    >>> prune_small(t2, 1)
    >>> t2
    Tree(6, [Tree(3)])
    >>> t3 = Tree(6, [Tree(1), Tree(3, [Tree(1), Tree(2), Tree(3)]),
     Tree(5, [Tree(3), Tree(4)])])
    >>> prune_small(t3, 2)
    >>> t3
    Tree(6, [Tree(1), Tree(3, [Tree(1), Tree(2)])])
    """
    while ____________________________:
        largest = max(_________________, key=____________________)
        _________________________
    for __ in _____________:
        ___________________
```

Linked Lists

There are many different implementations of sequences in Python. Today, we'll explore the linked list implementation.

A linked list is either an empty linked list, or a Link object containing a `first` value and the `rest` of the linked list.

To check if a linked list is an empty linked list, compare it against the class attribute `Link.empty`:

```
if link is Link.empty:
    print('This linked list is empty!')
else:
    print('This linked list is not empty!')
```

You can find an implementation of the `Link` class below:

```
class Link:
    """A linked list."""
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest_repr = ', ' + repr(self.rest)
        else:
            rest_repr = ''
        return 'Link(' + repr(self.first) + rest_repr + ')'

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + '>'
```

Q5: WWPD: Linked Lists

What would Python display?

> Note: If you get stuck, try drawing out the box-and-pointer diagram for the linked list or running examples in 61A Code.

```
>>> link = Link(1, Link(2, Link(3)))
>>> link.first
```

```
>>> link.rest.first
```

```
>>> link.rest.rest.rest is Link.empty
```

```
>>> link.rest = link.rest.rest
>>> link.rest.first
```

```
>>> link = Link(1)
>>> link.rest = link
>>> link.rest.rest.rest.rest.first
```

```
>>> link = Link(2, Link(3, Link(4)))
>>> link2 = Link(1, link)
>>> link2.first
```

```
>>> link2.rest.first
```

Q6: Remove All

Implement a function `remove_all` that takes a `Link`, and a `value`, and remove any linked list node containing that value. You can assume the list already has at least one node containing `value` and the first element is never removed. Notice that you are not returning anything, so you should mutate the list.

Note: Can you create a recursive and iterative solution for `remove_all`?

```
def remove_all(link, value):
    """Remove all the nodes containing value in link. Assume that
    the
    first element is never removed.

    >>> l1 = Link(0, Link(2, Link(2, Link(3, Link(1, Link(2, Link(3)
    ))))))
    >>> print(l1)
    <0 2 2 3 1 2 3>
    >>> remove_all(l1, 2)
    >>> print(l1)
    <0 3 1 3>
    >>> remove_all(l1, 3)
    >>> print(l1)
    <0 1>
    >>> remove_all(l1, 3)
    >>> print(l1)
    <0 1>
    """
    "*** YOUR CODE HERE ***"









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Q7: Slice

Implement a function `slice_link` that slices a given `link`. `slice_link` should return a new `Link` containing the elements of `link` starting at index `start` and ending one element before index `end`, just like slicing a normal Python list.

```
def slice_link(link, start, end):
    """Slices a Link from start to end (as with a normal Python list
    ).

    >>> link = Link(3, Link(1, Link(4, Link(1, Link(5, Link(9))))))
    >>> new = slice_link(link, 1, 4)
    >>> print(new)
    <1 4 1>
    """
    "*** YOUR CODE HERE ***"

# You can use more space on the back if you want
```