# DG法文献分析

下载截止2020年3月的几种数据库中的RKDG法文献。下面按照年份排序，整理阅读相关内容。

阅读后的文献整理进入RKDG\_literature文件夹(2021/6/25)。

文献阅读主要针对RKDG法(Shu Chi-Wang)用于求解浅水方程（SWE）的论文，侧重实际应用和算法实现，忽略了DG法数学分析和其他PDE方程求解（如Maxwell, Kdv方程）的文献。忽略了数学味太重及抽象的数学分析类型的论文。

从文献中了解的几个用于SWE求解的DG模型：

DGSWEM2D：早期依赖于ADCIRC建立，后期脱离了ADCIRC模型

DGSWEM3D: V. Aizinger and C. Dawson (2007)

UTBEST3D：Vadym Aizinger，Reuter et al. (2015)

SLIM3D：比利时鲁文大学研发的开源C++代码程序。

从文献中，还可以看到一些作者是属于一个研究组，使用相同的模型，总结如下：

美国Clint Dawson研究组（DGSWEM模型）：Vadym Aizinger, Shintaro Bunya, Ethan J. Kubatko, Joannes J. Westerink, Clint Dawson, S.R. Brus, D. Wirasaet, Colton J. Conroy

一波意大利人：Valerio Caleffi, Alessandro Valiani, Lorenzo Minatti, Pina Nicoletta De Cicco , Luca Solari

一波德国人（Dumbser研究组，与意大利人有合作，主要是DG-ADER算法）：Michael Dumbser, Maurizio Tavelli, Balthasar Reuter, Vadym Aizinger, Harald Kostler

荷兰人：P.A. Tassi, S. Rhebergen, C.A. Vionnet, O. Bokhove

G. Kesserwani极大地推进了DG法在浅水方程求解方面的应用。

J. S. Hesthaven和T. Warburton在节点间断Galerkin法和GPU并行化方面的工作引人注目。

Cockburn and Shu在DG法的开拓性研究需要深入研究。

## 1、文献统计

DG法用于SWE求解研究的主要作者发表年份及主题分析

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| --- | --- | --- |
| 主要作者 | 发表年份 | 简要说明 |
| Schwanenberg et al. | 2000, 2004 | 最早将DG法用于SWE求解的 |
| V. Aizinger et al. | 2001, 2002, 2007, 2013 | LDG法求解污染物输移、UTBEST3D |
| Clint Dawson | 2002, 2003, 2004, 2005, 2006, 2008, 2012, 2013, 2014, 2015 | 最早在ADCIRC框架下实施DG法、耦合CG-DG、3D DG模型、LTS、波浪DG模型、1D-2D耦合DG模型 |
| Kolar | 2005, 2008 | 比较CG与DG、网格细化准则 |
| Colton J. Conroy | 2016, 2018, 2020 | 3D DG，波浪DG模型，堤坝漫顶 |
| E. J. Kubatko | 2006a, 2006b, 2007, 2008, 2009, 2011, 2014, 2015, 2016, 2017, 2018, 2020 | hp自适应性DG、泥沙模型、时间离散、Exner方程的间断解、DG-SWEM2D、边界条件施加、3D DG |
| Xing Yulong | 2006, 2010, 2013, 2014 | well-balanced、Positivity-preserving |
| Kesserwani, Liang | 2008, 2010, 2012, 2014, 2015, 2018 | 1D、2D DG, 数值通量、泥沙输移 |
| Bunya, Mirabito | 2009, 2011 | 日本人，与Dawson合作 |
| Lai Wencong, Khan | 2012, 2013, 2015, 2016 | 一些实际应用(1D 2D ), 坡度限制 |
| Haegyun Lee | 2014, 2016 | 韩国人，一些DG法的初步应用 |
| Stefan Vater | 2015, 2019 | 德国人，限制因子 |
| S. Brus | 2017ab, 2019 | 高阶DG、HPC |
| Li Longxiang | 2017 | 中国人，天津大学，节点DG |

该表是统计了一些关于DG法求解SWE多产的作者，还有一些零星作者的研究未列入上表。

DG法综述性的论文有：Cockburn and Shu (2001): 89页、Shu (2003, 2014)、Xing and Shu (2006)、Miller et al. (2013)、Duran et al. (2014)、Kesserwani and Wang (2014)、Marras et al. (2016)和Ramachandran (2011)：DG法用于大气模式、Aizinger et al. (2018)。

## 2、早期DG模型、CG-DG耦合/对比及综述

DG法首次出现于1970s，用于求解SWE约开始于2000年以后，大多用于研究特殊问题，如溃坝流动和水跃，主要采用低阶的p近似。

20世纪80-90年代，Cockburn and Shu对间断Galerkin法的数学特性进行了大量研究和综述(Cockburn and Shu, 2001; Zienkiewicz et al., 2003)，可参考学习。下面主要讲述DG法在求解浅水方程（SWE）方面的应用研究。

Schwanenberg and Kongeter(2000)首次将DG法应用于求解带源项浅水方程的论文。

早期的，在ADCIRC框架下建立的LDG模型。DG法的优势：（1）使用高阶多项式的FEM，而不是直接计算这些高阶项；（2）传统的Godunov型的FVM无法处理二次导数的扩散项，LDG法可以。应用于Glaveston海湾的潮位预报。

CG法与DG法各有优缺点，两者在早期通过耦合方式，联合求解线性的浅水方程、波动连续方程和完全非线性的浅水方程(Dawson and Proft, 2002, 2003, 2004)。

Kolar et al. (2005)在ADCIRC模型框架下，对比了CG法与DG法求解浅水方程的数学特性。

Blain and Massey(2005)基于GWCE的CG-ADCIRC与DG-ADCIRC模型的实际应用的对比。研究表明：DG\_ADCIRC模型在捕捉小尺度涡动力强对流过程具有优势，这是CG\_ADCIRC模型难以做到的。但DG法的缺点是由于增加DOF导致计算量增大、需要实施坡度限制因子。

Dawson et al. (2006)系统总结了连续、间断和耦合CG-DG法求解浅水方程的模型。

Dawson(2008)应用CG法法求解地下水方程（Richards方程），DG法耦合地表水与地下水流动模型。

实际上，网格密度对CG法和DG法的收敛性影响很大，DG法可以在相对稀疏网格情况下，得到收敛的数值解，这是DG法的一大优势。Blain et al. (1998)就使用ADCIRC模型检验了CG法在风暴潮模拟中的网格收敛性研究。CG法一直受到全局和局部质量守恒问题的困扰，如ADCIRC模型，降低质量守恒误差只能依靠细化网格的方法。ADCIRC模型计算GWC方程，与基于连续方程的浅水方程求解不一样，需要细心选择数值参数G。GWC方程的离散等价于Telemac模型中使用quasi-bubble格式离散连续方程(Atkinson et al., 2004)。总之，CG法都没有实施局部的质量守恒，质量计算残差是网格细化的主要准则。

Kubatko et al.(2009)对比了CG法与DG法求解浅水方程的计算效率。

Anmala et al. (2011)对显格式、半隐格式和隐格式的FEM求解浅水方程做了Fourier稳定性分析。

Miller et al.(2013)全面综述了计算水力学中用到的一些数值方法，包括FVM, RKDG法等。

Lee Haegyun (2014)应用RKDG法求解浅水方程，对RKDG法做了简洁的介绍，可以作为初步理解RKDG法的开始。

Duran and Marche (2014)系统总结和综述了DG法求解带源项的浅水方程的研究进展。

Shu (2014)综述了DG法求解非恒定问题（包括对流占优和对流扩散问题），同时综述了近期DG法求解各种不同类型方程的研究进展。

Shu Chi-Wang (2016)综述了DG法求解非恒定对流占优流动问题的基本原理和近期发展，并将DG法与其他数值方法做了比较。

Kesserwani et al. (2018)在四边形单元网格上建立RKDG2模型，采用所谓的“slope-decoupled"方法对标准的RKDG法做了简化，降低了DG法的复杂度而便于实施，表明简化的RKDG模型具有健壮性和精确性，计算效率更高。

Aizinger et al. (2018)论述了将Mixed DG FEM，Compact DG，hydridized DG用于求解非静止的Darcy流动的原理。全面了解DG法，可参考此文。

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Vadym Aizinger, Andreas Rupp, Jochen Schutz, Peter Knabner. Analysis of a mixed discontinuous Galerkin method for instationary Darcy flow. Comput Geosci (2018) 22:179-194.

## 3、RKDG法模拟洪水

Fagherazzi et al. (2004)在结构网格上实施RKDG法，采用Roe近似黎曼解，应用于模拟溃坝水流。

Schwanenberg and Harms (2004)应用2D RKDG法求解超临界流动，溃坝洪水在干地形上的演进。

Roger et al. (2009)针对一个溃堤试验，采用RKDG法和FVS的FVM模型做了模拟。

Ghostine et al. (2010)应用2D RKDG模拟了交叉路口洪水的演进。Ghostine et al. (2010)对Ghostine et al. (2010)的研究做了讨论。

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## 4、干湿地形处理

Bokhove (2005)使用1D DG法求解浅水方程，并检验了其中干湿地形处理的数学特性。

Bunya et al. (2009)将RKDG模型求解浅水方程时的干湿地形处理，即数值通量计算后处理，保证水深为正值。

Kesserwani and Liang (2010)建立了1D 2阶RKDG格式求解具有复杂地形的浅水方程。可处理干湿地形，将MUSCL格式实施到RKDG2求解器。摩阻源项采用分裂隐格式离散，实施物理停止条件来保证稳定性。最后，采用考虑和不考虑摩阻效应的恒定和非恒定算例验证了建立的模型。

Kesserwani and Liang (2010)应用RKDG2法求解具有干湿前锋的浅水方程。实施了FV的坡度限制因子。单元间的通量采用HLLC近似黎曼解。

Kesserwani and Liang (2012)在RKDG2模型中，实现干湿地形处理的局部限制和完全守恒的浅水方程求解。

Lee and Lee (2016)介绍了DG法求解浅水方程中的干湿变化边界的处理。

Bonev et al.(2018) 的DG法求解球坐标上的浅水方程，可处理干湿变化和non-conforming mesh。干湿处理方法可适用于任意阶的多项式，没有引入人工粘度、人工孔隙率等。

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Boris Bonev, Jan S. Hesthaven, Francis X. Giraldo, Michal A. Kopera. 2018. Discontinuous Galerkin scheme for the spherical shallow water equations with applications to tsunami modeling and prediction. Journal of Computational Physics 362: 425-448.

## 5、数值通量格式与通量平衡

Tassi et al. (2007)采用HLLC和动力学数值通量格式的DG法，对不连续处局部限制数值振荡。对不规则地形上的bore-vortex相互作用、涡旋预测、收缩渠道中的恒定斜水跃等现象，动力学格式表现处更好的健壮性。

Ambati et al. (2007)在局部不连续处，使用HLLC数值通量格式，仅在不连续处采用一个耗散算子（Krivodonov不连续诊断因子），使DG法在时空上具有2阶精度。

Kesserwani et al. (2008)在DG法中实施了一系列的黎曼求解器，如Roe, Osher, HLL, HLLC, HLLE等，讨论了L1误差、CPU计算耗时、不连续解和源项的影响。

Xing et al. (2010)深入分析了DG法求解具有非平底的浅水方程时，如何保持静水通量平衡和保证状态变量正值方面的特性。

Lai and Khan (2012)采用HLLC数值通量的DG法模拟河道洪水的实际案例。

在“Xing et al. Adv. Water Resourc. 33: 1476–1493, 2010”论文的基础上，Xing and Zhang (2013)研究静水平衡在三角形非结构网格上的特性，构建了正值保证的限制因子，证明该格式可保证计算水深为正值、通量平衡以及高精度。

Meister and Ortleb (2013)应用基于衰减策略的谱粘度，增加DG法中的数值耗散，该策略是由直接用于格式中的系数的有效模式过滤组成，实现通量平衡。

带源项的双曲律方程要得到恒定态数值解要求通量与源项间达到平衡(well-balanced)。Xing (2014)建立了well-balanced的间断Galerkin法求解浅水方程，不仅可达到静水平衡，还可以实现更一般的动水平衡，关键点是一种特殊的源项近似、基于通用的静水压力重构得到近似数值通量。通过数值试验验证了光滑解和不连续解的well-balanced特性和较好的精度。

Stefan Vater and Jörn Behrens(2014)使用DG法建立了通量平衡的洪水淹没模型，模拟近海地区的海啸或风暴潮淹没中，指出DG法在保持质量守恒、通量平衡和干湿地形变化是具有挑战性的。

Vater et al. (2015)建立了模拟洪水淹没的新的1D DG算法，该方法是基于流速限制各单元中的动量分配，可防止干湿地形过程中不稳定问题发生。对水深的限制保证水深的正值，从而保证局部质量守恒。干湿界面处的单元通量修正引出通量平衡方法，保证lake at rest。DG格式使用Lagrange基函数的节点形式公式。因此，该模型很适合用于海啸和风暴潮情况下的干湿地形变化显著的近海区域。

Xiao et al. (2016)将well-balanced RKDG (Xing et al., 2013)应用于斜水跃和tidal bore问题的模拟。

Qian et al. (2018)将well-balanced RKDG (Xing et al., 2013)应用于不规则边界和不规则地形的明渠流动模拟。

Du et al. (2019)建立通量平衡的RKDG法求解浅水方程，在非结构网格上使用Constant Subtraction Techniques实现通量平衡。

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Du Huijing, Yingjie Liu, Yuan Liu, Zhiliang Xu. 2019. Well-Balanced Discontinuous Galerkin Method for Shallow Water Equations with Constant Subtraction Techniques on Unstructured Meshes. Journal of Scientific Computing, 81: 2115-2131

## 6、DG法求解3D（多分层）浅水方程

Dawson and Aizinger (2005)检验RKDG法用于求解3D浅水方程的精度、L2稳定性和计算效率。

Aizinger and Dawson (2007)建立了使用DG法求解3D浅水方程的模型，模型控制方程为Navier-Stokes方程，基于静水压力假设。平面上采用三角网格，垂向上拉伸形成3D网格。本文分析了模型中的完全非线性（没有简化假设），并考虑了由于自由水面运动引起的网格移动，这是第一次做这样的分析。

Aizinger et al. (2013)开发了MPI并行化的University of Texas Bays and Estuaries 3D (UTBEST3D)，评估了其并行计算效率。目前已知的仅有UTBEST3D和SLIM这2个基于RKDG法的3D海洋动力学模型。

Higdo (2015)分析了用于地转流调整的多层流体的3D DG FEM模型中，多个时间尺度和压力驱动的影响，表明：DG法可以获得比标准有限差分法好的结果。

Conroy and Kubatko (2016)建立了3D hp自适应的DG海洋模型，用于分析近海岸3D水体环境下的斜压力作用。

Izem et al. (2016a,b)基于DG法建立了双层浅水方程数学模型（垂向水体密度不同的情况），当表层与底层的水体密度相同时，将退化为单层模型。

Higdon (2020)将DG法应用于多层的流体模拟时还是有很多需要关注的问题，例如等密度流体中，粘性和分层厚度对模拟结果的影响。

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## 7、DG法的CFL条件和时间离散

CFL条件和时间步长

Warburton and Hagstrom(2006)开展了缓和DG法中的CFL限制条件的研究。

Trahan et al. (2012)实施了DG局部时间步长法。

在三角形网格上，Toulorge and Desmet(2011)考察RKDG法的线性稳定性限制条件，局部稳定性标准导致过于严格的限制。

Dawson et al. (2013)应用局部时间步长的DG法应用于近海岸区域水动力模拟。

Dawson(2014) 简要介绍了局部时间步长法在RKDG法模拟风暴潮中的应用。

Lai and Khan (2013)考察了时间离散格式在DG法中对稳定性和计算效率的影响。使用不同的坡度限制因子考虑额Euler格式和RK2格式。RK2格式比Euler格式的数值扩散性更强，RK2格式允许使用更大的时间步长。Euler格式的计算效率和精度比RK2更好。

Maleki and Khan(2016)建立了DG框架下的新的局部时间步长法。与Dawson et al. (2013)做个比较。

时间离散

Kubatko et al. (2007)将strong-stability-preserving (SSP) Runge-Kutta时间离散格式与DG空间离散格式结合使用方法，做了论证，即RK法的阶段数s与DG法的空间阶数k之间的相对大小，对计算效率和精度的影响。Kubatko et al. (2007)的研究表明：当s>k的L2稳定性要求比标准RKDG法（s=k）的要低。s>k的RKDG法的计算效率比s=k的要好。

Kubatko et al. (2008)使用*p*=*k*-1阶多项式的DG空间离散法的RKDG模型，*k* stage的RK法，给出*k*-1阶精度的RKDG法。Cockburn and Shu (2001)给出了1D RKDG法的近似CFL线性稳定条件：，其中，*c*是波速，是网格间距。该条件仅对*p*=0, *p*=1的情况精确成立。在2种三角形网格（直角和等边三角形），Kubatko et al. (2008)给出了RKDG法的CFL近似条件：，其中*h*为网格参数，类比2D情况。

Kubatko et al. (2014)给出了优化的SSP RK时间离散格式。

Farzam and Khan(2015)使用3种时间离散方法（Euler向前、2阶Adams-Bashforth，多步RK法）比较了矩形、梯形、三角形、抛物型断面河道的计算精度。表明：AB时间推进格式具有最佳精度和计算效率。

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## 8、坡度限制

Kesserwani and Liang (2012)指出FV坡度限制对DG法的数值解有一些副作用，如精度下降、增加计算耗时。本文对局部限制、全局限制和无坡度限制的线性前锋跟踪等3种模式对计算精度的影响做了详细研究。

Michoski et al. (2016)系统总结了DG法求解对流扩散反应方程中，实施的坡度限制因子方法、模式过滤、模式系数的人工扩散系数。

Stefan Vater et al. (2019)在同形三角形单元网格上，使用2阶RKDG法求解非线性浅水方程。Stefan Vater et al. (2019)使用坡度限制因子，使模型具备了干湿前锋的捕捉能力。

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## 9、hp-自适应的DG法开发及应用

Kubatko et al. (2006)基于三角网格开发了hp特性的DG模型用于求解浅水方程。介绍了DG法的基本原理、基函数、求积法则（高斯积分）、坡度限制和边界条件施加等。使用包含浅滩算例检验了DG法，并与基于CG法的ADCIRC模型做了对比。研究表明：DG法在捕捉局部涡旋方面较CG法具有优势。数值模拟对比了不同网格密度(h)和不同Dubiner基函数的阶p下的收敛速率，h/p特性发挥了单元的局部高精度。Kubatko et al. (2006)在结论中指出：CG法在使用粗网格时有精度的问题，而高阶DG法不仅精度高，且提高了计算效率，对光滑流动和空间快速变化的流动情况都是如此。另外，p细化方法相对h细化更有计算效率的优势。

Bernard et al. (2007)将h自适应三角形网格的DG法用于近海岸的水动力模拟。

Kubatko et al. (2009)详细论证了p-自适应的RKDG模型，结合SSP RK时间离散，p-自适应可显著提交计算效率（高阶精度抵消了CPU耗时），应结合h-自适应和实际应用。

Kesserwani and Liang (2012)实施了h-自适应的RKDG2模型，基于均匀四边形网格，是自适应结构化网格。

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## 10、DG法的应用（扩散、地形演变、地下水、波浪）

LDG法求解物质输移（对流扩散方程中的二阶扩散项）

在Cockburn and Shu对DG法的数学分析的基础上，Aizinger,与B. Cockburn, P. Castillo合作(Aizinger et al., 2001)，首次将LDG法应用于多孔介质中污染物输移方法的求解。

Caleffi and Valiani (2013)基于3阶LDG法求解浅水方程和污染物输移方程，并应用于弯道的模拟。LDG法的数学原理见Begnudelli et al.(2010)，数值积分使用Calffi and Valiani (2012)推荐的格式。

泥沙输移和地形演变

Kubatko et al. (2006)在DG\_ADCIRC模型基础上建立的泥沙输移和地形演变模型。

Patra et al. (2006)将DG法应用在薄层的泥石流演变，泥石流的控制方程类似浅水方程，其中使用自适应的Block-structured网格。

Exner方程是基于质量守恒描述河床演变的方程。Exner给出的解析解一般称为经典或真解，但这仅当解是连续情况时才成立。Kubatko et al. (2007)使用非线性双曲方程的通用理论，求解Exner方程的不连续解，即可描述河床的不连续变形和传播，也就是sediment bore。

Tassi et al. (2008)基于DG法的水动力、河床演变模型（Exner方程），水动力和地形演变方程都纳入双曲系统，DG法求解2个intertwined的时间步格式（快速的水动力和慢的地形演变分量），其中没有计算悬移质泥沙的对流扩散方程，仅考虑推移质输沙导致的地形演变。

Mirabito et al. (2011)基于Rhebergen et al. (2008)修正了DG法，并在DG\_ADCIRC模型中实施了泥沙输移和地形演变模拟。

Kesserwani et al. (2014)建立了1D RKDG,用于模拟溃坝流动下的动床演变和悬移质泥沙输移，考虑了泥沙输移和河床演变对水流的作用。

Maleki and Khan(2016)建立了1D RKDG法求解非恒定流和泥沙输移。

Zhao et al.(2016)应用紧致型DG法进行了3D的悬移质泥沙输移模拟。

地表/地下水

Dawson(2008)应用CG法法求解地下水方程（Richards方程），DG法耦合地表水与地下水流动模型。

Chen et al. (2009)应用DG法求解多孔介质中的单相流。

Reuter et al. (2019)应用LDG法求解地下水模型（Darcy定律）。

风暴潮模拟

Dawson et al. (2011)采用DG法与CG-ADCIRC模型模拟风暴潮下的复杂水网内的增水。由于使用的网格是针对ADCIRC模型的（断面上有2个单元），即在上端航道内的断面上有2个网格（网格密度较大），不适合于DG模型（不是基于节点的，而是基于单元和边的计算，断面上有1个单元即可）。因此，DG模型模拟的水位增幅偏低。

Wood et al. (2020)在DGSWE模型中的考虑内部障碍物处理算法，用于模拟了风暴潮情况下，河堤的漫顶水流现象。

河网（1D）

Lai and Khan (2012)将RKDG 1D用于求解天然河道。

Lai and Khan (2012)将1D RKDG法用于非矩形和非棱柱断面河道，研究表明：Roe和HLLC具有相似的精度，但对于非矩形的天然河道，使用坡度限制器可得到更精确的数值解。

Neupane and Dawson (2015)建立了RKDG法用于求解河网（1D与2D的耦合模拟）的模型。

Briani et al. (2016)系统分析了1D RKDG法求解河网水动力的数学特性，显示了相比1阶格式的优势。

West et al. (2017)建立了RKDG法用于求解坡面流动和河网，与Prapti Neupane (2015)的研究很类似。

Herty et al. (2019)建立的DG法用于1D河网水动力计算，并与传统2D求解浅水方程的模拟做了对比。在同等精度下，1D河网模型具有更高的计算效率。

波浪模拟

Nappi (2013)基于DG法的风生波浪模型，计算效率比SWAN要高很多。

Meixner et al. (2014)将DG法用于波浪模拟，与DGSWEM松散耦合，并与DGSWEM与SWAN紧密耦合计算结果，对比了DGWAVE与SWAN的计算效率。

Conroy et al. (2018)基于DG法建立了基于动量平衡方程的2参数风浪模型，在大湖风浪环境下，计算得到的波普误差与SWAN模型（基于能量平衡方程）的结果对比，误差相当，但基于DG法的2参数风浪模型的计算效率显著提高（30s vs 3h）。Conroy et al. (2018)建立的2参数DG波浪模型仅适用于大湖(Nappi, 2013)，应用于近海岸的风暴潮情况时，需要重新设计数学方程的项(Conroy et al., 2018)，例如将大气紊流和风生浪紧密耦合到涌浪(swell)上去。

求解其他方程(Grenn-Naghdi, Maxwell)

Sharifian et al. (2018)建立了2D RKDG 求解Green-Naghdi方程（弱色散浅水方程），对模拟波浪爬高与船舶行进中波浪阻力有应用价值。

Conroy and Lev (2021)应用RKDG法模拟河道中的快速岩浆流动(lava flow)，同样表现出较好的数值性能。

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## 11、高阶DK法求解SWE

Caleffi and Valiani (2012)在曲线边界区域上实施了3阶精度的RKDG法求解浅水方程。

DG法用于明渠流动模拟时，采用曲边固体边界时，No-normal水流边界条件直接施加，会导致低精度的结果。常用方法是人为增加一个边界层（1个网格厚度）的流速场，使上游方向的水位计算偏大，Wirasaet et al. (2015)给出了解决方法。

DG法求解复杂区域上的近似双曲系统（即扩散修改的浅水方程），在尖锐障碍物附近会产生spurious eddies。Steinmoeller et al. (2016)通过增加人工耗散（即涡粘度系数）来消除这种振荡涡旋，研究表明：中等阶数的DG法可扩展到曲边三角形单元，积分公式可使用高阶的quadrature和cubature准则，可消除spurious eddies。最后，用理想复杂区域和现实区域做了检验。

Wintermeyer et al. (2017)建立了一种熵稳定的高阶节点间断Galerkin谱单元，近似求解非线性2D浅水方程的DGSEM方法，该算法是在结构网格上实施的。

Li et al. (2018)建立了1D高阶DG模型，DG法中的数值通量采用静水压力重构，联合使用一个新的源项近似方法和分解算法。基于严格的理论分析和数值试验，证明该格式具有稳定性、精度，且能捕捉小扰动，获得光滑解的真实高阶精度。

针对低阶DG法用于模拟长波的问题，Brus et al. (2019)提出高阶的DG法，并应用于Galveston湾。高阶格式与相对稀疏网格，可在保证求解精度下提高计算效率。关键是高阶精度网格的生成技术（GMSH），可通过等参单元和超参单元实现弯曲边界，并使用等阶多项式描述地形。本文针对Dawson et al. (2010)中提到的低阶DG用于模拟近海岸流体的问题，Brus et al. (2019)提出了高阶DG法的解决方案。

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## 12、节点型DG法

Giraldo et al. (2002)使用节点型DG法求解球坐标上的浅水方程，因为可以解决极地奇点问题(polar singularity)。

Wirasaet et al. (2014)开发了基于三角形、四边形和多边形单元的节点型、混合模式/节点的DG模型来求解非线性的SWE。

Li Longxiang and Zhang Qinghe (2017)建立了节点型DG法中的限制器，可使用三角形和四边形单元的非结构网格，进行海洋2D模拟。

Ran et al. (2019)使用DG法求解考虑非静水压力的平面2D浅水方程，可有效模拟有弱扩散的水波。浅水方程使用节点型的无积分NDG法离散，时间项采用4阶RK法离散。

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## 13、DG法的程序库开发

DG法是FEM中具有一定模式的模型，可基于面向对象（C++, Python）开发程序库，可加速特殊应用领域的DG模型开发效率。例如：

hpGEM

Firedrake

Nektar++

DUNE

Frank et al. (2015)采用GNU OCTAVE/MATLAB语言编写的DG模型，通过4篇系列论文，系统阐述了DG法的原理，MATLAB语言编程加快了DG法的教学和科研进程。本模型是理解DG法及其编程实现的范例。

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## 14、DG法在大气模式中的应用

Amik St.-Cyr et al. (2005)简要介绍了DG法在大气模式中的应用，可用来捕捉快速移动的重力波和非线性Rossby波，这对于捕捉非线性的大气动力过程是重要的。

Lauter et al. (2008)基于球面三角形网格建立了RKDG求解（气象）浅水方程。在局部上由球体三角形坐标表示，采用合适的局部坐标映射到三角形上。因此，每个三角形网格单元上，是切向动量的2D表征，仅有2个离散动量方程。采用Rusanov数值通量格式，采用SSP性质的3阶Runge-Kutta时间离散格式。各曲边三角形上的k阶多项式空间由Lagrange基函数表征，因此需要使用高阶求积公式，在单元上和单元的边上求积分。注意：DG法在球坐标上求解浅水方程的数值特性还是有所不同的。球面三角网格使用AMATOS(Brehens)生成，可用于自适应网格计算。

Nair et al.(2011)综述了大气模拟中新出现的数值方法—DG法。

Marras et al. (2016)系统综述了在大气模式开发中用到的FEM，谱元法和间断Galerkin法。

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## 15、CBS FEM (Zienkiewicz)

Ortiz的FEM模型是建立的CG法及Zienkiewicz的CBS FEM。

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