# AMR研究文献分类及阅读记录

## AMR简介

CFD使用的AMR技术可分为3类(Tang Tao)：

* h-方法：基于先验的的误差评估或误差指标，实施自动化的空间网格的细化或粗化。该方法包含2个独立的部分：求解算法和网格选择算法。
* p-方法：自适应提高多项式的阶数。通常用于间断Galerkin有限单元模型中。
* r-方法：也称之为移动网格方法(moving mesh method, MMM)，该方法重新布置网格的节点，但不改变节点的固定数目，使网格在数值解快速变化的区域加密。该方法常用于流固耦合问题（FSI）中界面追踪，如船舶工程。

自适应网格加密技术（AMR）一般采用各向异性自适应(anisotropic adaptivity, C. Pain, 2001)、p加密和h加密。

实施AMR的软件众多，例如PARAMESH, Chombo, deal.II, AMRClaw。

H-自适应又可分为3类(Andrew and Lilia, 2019)：

* patch-based网格细化
* block-based网格细化
* cell-based网格细化

patch-based网格细化，或称为component grids，就是逐步细化笛卡尔网格，直到获得期望的精度，如图1(b)，子网格与相互之间通信，使用局部时间步长在时间层推进计算。

block-based网格细化，预定义一些网格单元，分组为blocks。在单元的blocks上执行网格细化和粗化操作，而不是单个的网格单元（图1(c)）。仅需要给出block间的连接关系。block-based AMR类似于气象模型中常用的嵌套网格(nested grid)，便于并行化。如果一个网格节点/单元需要细化，整个block的分辨率都要提高（网格细化）。

cell-based网格细化，单元相互之间是独立细化的，相比block-based网格细化，cell-based细化要求更多的连接关系数据（图1(d)）。通常粗化和细化单元的父-子关系组织为quadtree或octree数据结构（分别对于2D和3D情况）。该方法的优势是：对于预设的误差水平，需要较少的单元数，这很适合于非结构网格。三角形非结构网格上的h自适应网格见图2.



图1 规则网格上的patch-, block-, cell-based网格细化策略



图2 三角形非结构网格上实施的cell-based网格细化

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## AMR相关软件分类

目前已有很多降低AMR数据管理难度的程序，如Behrens et al., 2005; Burstedde et al., 2011; Adams et al., 2015。

下面根据AMR分类对程序库进行整理。

Patch-based AMR

最早地，Berger and Oliger (1984)建立的分层级结构化网格方法（称之为BOSAMR方法），就是patch-based AMR，这种AMR对需要网格细化的patch很灵活，如图3.

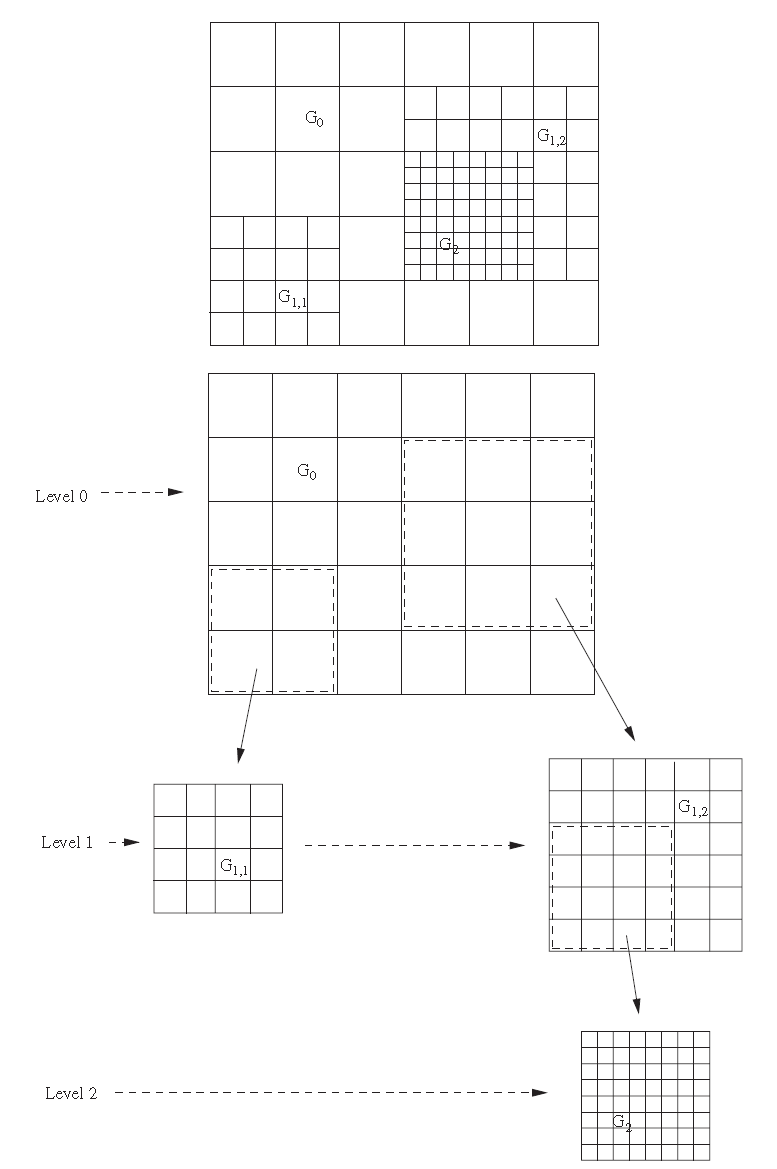


图3 Patch-based AMR分级网格示意图(From AGRIF)

* GeoClaw(Berger et al., 2011)，是基于AMRClaw程序的可模拟地球流体力学现象的FORTRAN程序，有源码。AMR的GeoClaw已经基于CUDA实现了异构并行(Qin et al., 2019)。Mandli and Dawson (2014)使用AMR的GeoClaw模拟了风暴潮淹没过程，并与静态网格的ADCIRC模拟结果做了对比。
* SAMRAI (Wissink et al., 2001)：Patch-based Structured AMR，面向对象C++编程，structured adaptive mesh refinement (SAMR)，Lawrence Livermore National Laboratory开发。
* AGRIF ()，tree-based, FORTRAN 90编程，已用于ROMS模型。法国人开发

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Block-based AMR

* ENZO，主要用于天文领域的模拟。Brummel-Smith et al., (2019)
* Chombo，并行化的AMR库，C++语言，基于此建立了非静水压力的斜压海洋模型Somar (Santilli, 2015)。Lawrence Berkeley National Laboratory开发。
* SOMAR (Santilli, 2015)，基于AMR求解NS方程，很好地求解Lock Exchange问题。

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Cell-based (Tree-based) AMR

Cell-based AMR，也称之为tree-based AMR（通常在结构网格的情况下）。Cell-based AMR可拓展至一般性的非结构网格。这是最灵活的一种AMR，无需细化不必要的单元，在网格的单元数方面更有效。



图4 Tree-based离散和对应的树状数据结构示意图（From PARAMESH）

基于四叉树/八叉树AMR技术的程序，如：

* Gerris (Popinet, 2003; 2006)
* QTAdaptive，串行的开源FORTRAN程序，在洪水模拟的应用见Liang et al. (2009)。
* PARAMESH (Olsona, 2005), FORTRAN 90，cell-based AMR，开源代码
* p4est (Burstedde et al., 2011), Forest of trees，C++编程。已有ForestCLAW和FORTRAN/Python wrapper程序

基于SFC的非结构网格AMR库：

* amotos，FORTRAN 90编程，串行，间断Galerkin算法的风暴潮/洪水淹没模拟。
* sam(oa)2，FORTRAN 90编程，OpenMP/MPI并行化
* Sierip\_Project, 完美融合AMR, OpenGL, HPC, DG等于一身的C++模型，可模拟实际地形下的海啸淹没过程。
* PRAGMATIC, C++语言编程，是一个并行化的非结构网格AMR程序库，主要是嵌入到Firedrake项目。也可自己开发相应的AMR模型。支持C/FORTRAN编程。

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## 并行化的h自适应网格加密程序

* PLTMG (Bank, 1994)，一个MPI并行化的AMR库，求解椭圆型方程，FORTRAN语言，有源码。
* PTETRAD (SELWOOD, 1999), 是对TETRAD(SPEARES, 1997)的并行化，四面体h自适应网格加密程序，C语言，未见源码。

通常，AMR执行后，需要动态平衡荷载，一般可使用Zoltan, ParMETIS, SCOTCH等区域分解程序库。此类的程序库有：

* RefficientLib (Joan Baiges and Camilo Bayona, 2017)，未开源的FORTRAN程序。
* PYRAMID (Norton et al., 2002)，加州理工JPL开发的AMR，FORTRAN程序，开源，但找不到下载源了。
* Fluidity，基于ani2d和Zoltan的复杂AMR的海洋动力学模型，Pain (UK)课题组开发，开源的FORTRAN程序。
* sierpi\_Project, 德国人Martin Schreiber开发的，交互式可视化，OpenGL，并行化AMR的海啸模拟程序，开源代码。
* omega\_h
* girdap-master，基于贴体结构网格的并行AMR，求解不可压缩NS方程。
* amotos
* sam(oa)2
* PRAGMATIC

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## CUDA AMR

大量的AMR库是基于CPU，较少用在GPU上，原因是AMR的数据结。一些基于CPU的AMR库，如Enzo, Cactus, SAMRAI都拓展至GPU，但主要都是测试目的。

* Wang et al (2010)首次将AMR求解器迁移到GPU，是基于ENZO水动力模型，采用block-structured AMR。Experimental
* CLAMR(Nicholaeff et al., 2012)也测试了使用MPI与OpenCL的并行模式加速求解浅水方程。Cell-based AMR Miniapp, LANL开发。OpenGL，目前仅能细化，没有粗化过程。边界条件也有待完善。
* Sætra et al., 2014首次采用Block-based (or tile-based in 2D case) AMR的CUDA加速洪水模型，采用2阶精度的Kurganov-Petrova格式。本文也介绍了几种基于GPU加速的AMR模型。C++编程。挪威人。
* gamer2 (Schive et al., 2010)，Patch-based AMR，主要用于天文模拟，但也包含水动力计算的模块（试运行，尚不具备浅水方程求解的功能）。
* GeoClaw(Berger et al., 2011)，基于patch-based AMR的CUDA FORTRAN程序(Qin et al., 2019)，是可以实际应用的模型。LeVeque
* 采用BUQ (Block Uniform Quadtree) AMR的CUDA加速(Vacondio et al., 2014)。意大利人。
* Daino (Mohamed Wahib et al., 2016)，Octree-based AMR, 日本人开发的GPU加速的AMR求解器，C++，其中有求解Hydro的模块（目前还不是浅水方程，是Euler方程的shock问题）。日本人。

总结：CUDA加速的AMR技术，涉及较复杂的适于GPU架构的数据结构。目前，已有很多程序进入这个领域，例如gamer2，CLAMR，Daino等，但大多都是测试一些基本的水动力现象，不能实际求解SWE。目前能较好地求解SWE的程序是GeoClaw-AMR, BUQ-AMR, Daino(实施SWE求解的最佳库，关注一下)。

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## Firedrake的自适应模块

基于Firedrake的Python程序的自适应技术。

求解近海岸动力学模型Thetis，构建于Firedrake之上，可使用PETSc代码自动生成的Python语言求解FEM问题。

目前仅支持：浅水方程，标量输移（保守或非保守标量），泥沙输移和Exner方程。

各向异性网格自适应基于Riemann Metric Field，使用PRAgMatic程序。

This is research of the Applied Modelling and Computation Group ([AMCG](http://www.imperial.ac.uk/earth-science/research/research-groups/amcg/)) at Imperial College London.

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