# 2000年以前

S. Chippada, C.N. Dawson, M. Martinez, M. Wheeler, Finite element approximations to the System of shallow water equations, Part I: Continuons time a priori error estimates, SIAM J. Numer. Anal. 35 (1995) 692-711.

S. Chippada, C.N. Dawson, M. Martinez, M. Wheeler, Finite element approximations to the System of shallow water equations, Part II: Discrete time a priori error estimates, SIAM J. Numer. Anal. 36 (1996) 226-250.

K.S. Bey, J.T. Oden. 1996. *hp*-version discontinuous Galerkin methods for hyperbolic conservation laws, Comput. Methods Appl. Mech. Engrg. 133: 259-286.

C.E. Baumann, J.T. Oden. 1999. A discontinuous *hp* finite element method for convection-diffusion problems, Comput. Methods Appl. Mech. Engrg. 175: 311-341.

早期的2篇关于hp-自适应间断Galerkin方法求解双曲型方程和对流扩散问题的研究。

C.A. Blain, J. Westerink, R. Luettich. 1998. Grid convergence studies for the prediction of hurricane storm surge, Int. J. Numer. Methods Fluids 26: 1-33.

Blain et al. (1998)就使用ADCIRC模型检验了CG法在风暴潮模拟中的网格收敛性研究。

# 2000年

D. Schwanenberg, J. Kongeter. 2000. A discontinuous Galerkin method for the shallow water equations with source terms, Lect. Notes Comput. Sci. Eng. 11: 419-424.

Schwanenberg and Kongeter(2000)首次将DG法应用于求解带源项浅水方程的论文。

P. Castillo, B. Cockburn, I. Perugia, D. Schotzau. 2000. An a priori error analysis of the local discontinuous Galerkin method for elliptic problems, SIAM J. Numer. Anal. 38: 1676-1706.

# 2001年

V. Aizinger, C. Dawson, B. Cockburn, P. Castillo. 2001. The local discontinuous Galerkin method for contaminant transport, Advances in Water Resources 24: 73-87.

在Cockburn and Shu对DG法的数学分析的基础上，Aizinger,与B. Cockburn, P. Castillo合作(Aizinger et al., 2001)，首次将DG法应用于多孔介质中污染物输移方法的求解。

Bernardo Cockburn and Chi-Wang Shu. 2001. Runge-Kutta Discontinuous Galerkin Methods for Convection-Dominated Problems. Journal of Scientific Computing, 16(3):

Cockburn and Shu (2001)对RKDG法做的全面综述。全文89页！

# 2002年

Vadym Aizinger, Clint Dawson. 2002. A discontinuous Galerkin method for two-dimensional flow and transport in shallow water. Advances in Water Resources 25: 67-84.

早期的，在ADCIRC框架下建立的LDG模型。DG法的优势：（1）使用高阶多项式的FEM，而不是直接计算这些高阶项；（2）传统的Godunov型的FVM无法处理二次导数的扩散项，LDG法可以。应用于Glaveston海湾的潮位预报。

./swem\_2d\_3d

评述：LDG算法的数学原理是抽象的（见第3节的介绍），需要阅读ADCIRC\_DG代码理解其具体实施。

Clint Dawson, Jennifer Proft. 2002. Discontinuous and coupled continuous/discontinuous Galerkin methods for the shallow water equations, Comput. Methods Appl. Mech. Engrg. 191: 4721-4746.

使用DG法和CG法2种联合方法求解浅水方程组，2种方法都是使用DG法求解连续方程，而第1种方法是使CG法求解动量方程，第2种方法是使用特殊的DG法求解连续性方程（即非对称内惩罚Galerkin法—NIP， 近似计算动量）。给出了2种方法的误差评估和数值试验。

使用Euler时间步离散，*P*1型Legendre多项式。

nonsymmetric interior penalty Galerkin (NIPG) method看来是一种早期版本的DG法。NIPG代码可参考github上的代码。

F. X. Giraldo, J. S. Hesthaven, T. Warburton. 2002. Nodal High-Order Discontinuous Galerkin Methods for the Spherical Shallow Water Equations. Journal of Computational Physics, 181: 499-525.

Giraldo et al. (2002)使用节点型DG法求解球坐标上的浅水方程，因为可以解决极地奇点问题(polar singularity)。

评述：节点型DGM常与全球尺度模型结合，算法见第3节介绍。

./nodal dg

# 2003年

Clint Dawson, Jennifer Proft. 2003. Discontinuous/continuous Galerkin methods for coupling the primitive and wave continuity equations of shallow water. Comput. Methods Appl. Mech. Engrg. 192: 5123-5145.

本文包含CG、DG法的数学原理。见2002年的上文。

前言：标准的有限单元法离散SWE，使用相同阶数的空间离散水位和流速，会导致空间上的spurious oscillation。解决该问题的一种方法是：用2阶双曲型“波动连续方程”（WCE）代替1阶的双曲型水位连续方程(Lynch and Gray, 1979)。该方法已经成为很多有限单元模型的基础。GWCE有限单元方法是基于连续的近似空间离散。波动连续方程牺牲了primitive连续方程，因此原始的连续方法在离散形式下不在满足。近年来，已有研究使用不连续的FEM空间离散原始形式的SWE。DG法有几个特点：在数值解中考虑了迎风和稳定性的后处理，可模拟对流占优的流动；另外，DG法是“局部守恒”，即由于流量改变水位的原始连续方程在单元上的弱形式意义上是满足的，该特性对于耦合SWE和输运方程的情况是很重要的，例如污染物输移。DG法已成功用于求解双曲型方程、对流扩散方程和纯扩散方程。DG法的主要缺点是：计算量大。DG法中，数值解的自由度与单元有关，还不是节点值，而在非结构有限单元网格中，单元数远大于节点数。

本文检验一种求解连续方程的耦合DG与CG方法，但这里的耦合方法更难，因为在不同计算域使用不同形式的连续方程。另外，不像Dawson and Proft (2002)，这里我们考虑完全非线性的SWE。在各子区域内，使用连续函数离散动量方程；但是，在耦合界面上允许速度解是不连续的。这种耦合策略具有很多潜在优势。如上所述，DG法的计算量比CG法要大，因此对存在大的水位梯度的区域（对流占优）可使用DG法求解，而其他区域上可使用CG法。还可以在局部守恒是很重要的区域使用DG法，如存在污染物输移的区域，其他区域使用CG法。

D.R. Lynch, W.R. Gray. 1979. A wave equation model for finite element computations, Comput. Fluids 7: 207-228.

耦合CG-DG法还只是初步的数值验证，具体应用于求解波动连续方程，还要实际情况的全面验证。

./early\_dg

O.C. Zienkiewicz, R. Taylor, S.J. Sherwin, J. Peiro. 2003. On discontinuous Galerkin methods, Int. J. Num. Methods Eng. 58: 1119-1148.

C.N. Dawson, M. Martinez-Canales. 2001. A characteristics-Galerkin approximation to a system of shallow water equations, Numer. Math. 86: 239-256.

与Zienkiewicz et al.的CBS FEM求解SWE做个比较。

# 2004年

Clint Dawson, Jennifer Proft. 2004. Coupled discontinuous and continuous Galerkin finite element methods for the depth-integrated shallow water equations. Comput. Methods Appl. Mech. Engrg. 193: 289-318.

是关于CG-DG耦合求解浅水方程的系列论文。本文推导了一个先验的误差评估。

评述：与Dawson and Proft (2003)的内容几乎一致。问题是：如何判断实际应用中大水位梯度？跟踪计算量也会站到相当大的一部分。

Sergio Fagherazzi; Patrick Rasetarinera; M. Youssuff Hussaini; David J. Furbish. 2004. Numerical Solution of the Dam-Break Problem with a Discontinuous Galerkin Method. Journal of Hydraulic Engineering(ASCE), 130(6)

Fagherazzi et al. (2004)在结构网格上实施RKDG法，采用Roe近似黎曼解，应用于模拟溃坝水流。

./case\_applications

D. Schwanenberg, M. Harms. 2004. Discontinuous Galerkin Finite-Element Method for Transcritical Two-Dimensional Shallow Water Flows. Journal of Hydraulic Engineering (ASCE), 130(5)

Schwanenberg and Harms (2004)应用2D RKDG法求解超临界流动，溃坝洪水在干地形上的演进。

./case\_applications

# 2005年

C. Dawson, V. Aizinger, 2005. A discontinuous Galerkin method for three dimensional shallow water equations, J. Scientific Comput. 22-23: 245-267.

Dawson and Aizinger (2005)检验RKDG法用于求解3D浅水方程的精度、L2稳定性和计算效率。

LDG法求解3D浅水方程的稳定性特性研究参考Aizinger and Dawson (2007)。

R.L. Kolar, K.M. Dresback, C.M. Szpilka et al. 2005. A comparison of Continuous and Discontinuous Galerkin Algorithms for Shallow Water Transport. Esturine and Coastal Modelling

Kolar et al. (2005)在ADCIRC模型框架下，对比了CG法与DG法求解浅水方程的数学特性。

./dg\_review

Cheryl Ann Blain, T. Christopher Massey. 2005. Application of a coupled discontinuous-continuous Galerkin finite element shallow water model to coastal ocean dynamics. Ocean Modelling, 10: 283-315.

Blain and Massey(2005)基于GWCE的CG-ADCIRC与DG-ADCIRC模型的实际应用的对比。研究表明：DG\_ADCIRC模型在捕捉小尺度涡动力强对流过程具有优势，这是CG\_ADCIRC模型难以做到的。但DG法的缺点是由于增加DOF导致计算量增大、需要实施坡度限制因子。

./early\_dg

Onno Bokhove. 2005. Flooding and Drying in Discontinuous Galerkin Finite-Element Discretizations of Shallow-Water Equations. Part 1: One Dimension. Journal of Scientific Computing, 22-23, DOI: 10.1007/s10915-004-4136-6

Bokhove (2005)使用1D DG法求解浅水方程，并检验了其中干湿地形处理的数学特性。

./wetting-drying

Amik St.-Cyr, Tephen J. Thomas. High-Order Finite Element Methods for Parallel Atmospheric Modeling. V.S. Sunderam et al. (Eds.): ICCS 2005, LNCS 3514, pp. 256-262, 2005.

Amik St.-Cyr et al. (2005)简要介绍了DG法在大气模式中的应用，可用来捕捉快速移动的重力波和非线性Rossby波，这对于捕捉非线性的大气动力过程是重要的。

./case\_applications

# 2006年

Ethan J. Kubatko, Joannes J. Westerink, Clint Dawson. 2006. *hp* Discontinuous Galerkin methods for advection dominated problems in shallow water flow, Comput. Methods Appl. Mech. Engrg. 196: 437-451.

DG法首次出现于1970s，用于求解SWE约开始于2000年以后，大多用于研究特殊问题，如溃坝流动和水跃，主要采用低阶的p近似。

Kubatko et al. (2006)基于三角网格开发了hp特性的DG模型用于求解浅水方程。介绍了DG法的基本原理、基函数、求积法则（高斯积分）、坡度限制和边界条件施加等。使用包含浅滩算例检验了DG法，并与基于CG法的ADCIRC模型做了对比。研究表明：DG法在捕捉局部涡旋方面较CG法具有优势。数值模拟对比了不同网格密度(h)和不同Dubiner基函数的阶p下的收敛速率，h/p特性发挥了单元的局部高精度。Kubatko et al. (2006)在结论中指出：CG法在使用粗网格时有精度的问题，而高阶DG法不仅精度高，且提高了计算效率，对光滑流动和空间快速变化的流动情况都是如此。另外，p细化方法相对h细化更有计算效率的优势。

./hp-adaptivity

Dawson, C., Westerink, J. J., Feyen, J. C., Pothina, D. 2006. Continuous, discontinuous, and coupled discontinuous-continuous Galerkin finite-element methods for the shallow water equations. Int. J. Numer. Methods Fluids, 52(1): 63-88.

Dawson et al. (2006)系统总结了连续、间断和耦合CG-DG法求解浅水方程的模型及其优缺点。

Ethan J. Kubatko, Joannes J. Westerink, Clint Dawson. 2006. An unstructured grid morphodynamic model with a discontinuous Galerkin method for bed evolution. Ocean Modelling, 15: 71-89.

Kubatko et al. (2006)在DG\_ADCIRC模型基础上建立的泥沙输移和地形演变模型。

./case\_applications

A.K. Patra, C.C. Nichita, A.C. Bauer, E.B. Pitman, M. Bursik, M.F. Sheridan. Parallel adaptive discontinuous Galerkin approximation for thin layer avalanche modeling. Computers & Geosciences 32 (2006) 912-926.

Patra et al. (2006)将DG法应用在薄层的泥石流演变，泥石流的控制方程类似浅水方程，其中使用自适应的Block-structured网格。

./case\_applications

T. Warburton and T. Hagstrom, Taming the CFL number for discontinuous Galerkin methods on structured meshes, J. Comput. Phys. 2006

Warburton and Hagstrom(2006)开展了缓和DG法中的CFL限制条件的研究。

# 2007年

Vadym Aizinger, Clint Dawson. 2007. The local discontinuous Galerkin method for three-dimensional shallow water flow. Comput. Methods Appl. Mech. Engrg. 196: 734-746.

Aizinger and Dawson (2007)建立了使用DG法求解3D浅水方程的模型，模型控制方程为Navier-Stokes方程，基于静水压力假设。平面上采用三角网格，垂向上拉伸形成3D网格。本文分析了模型中的完全非线性（没有简化假设），并考虑了由于自由水面运动引起的网格移动，这是第一次做这样的分析。

Ethan J. Kubatko, Joannes J. Westerink, Clint Dawson. 2007. Semi discrete discontinuous Galerkin methods and stage-exceeding-order, strong-stability-preserving Runge-Kutta time discretizations. Journal of Computational Physics 222: 832-848.

Kubatko et al. (2007)将strong-stability-preserving (SSP) Runge-Kutta时间离散格式与DG空间离散格式结合使用方法，做了论证，即RK法的阶段数s与DG法的空间阶数k之间的相对大小，对计算效率和精度的影响。Kubatko et al. (2007)的研究表明：当s>k的L2稳定性要求比标准RKDG法（s=k）的要低。s>k的RKDG法的计算效率比s=k的要好。

评述：s>k的RKDG法在稳定性和计算效率上均较标准RKDG法有优势。

./CFL&time\_discretization

P.A. Tassi, O. Bokhove, C.A. Vionnet. 2007. Space discontinuous Galerkin method for shallow water flows--kinetic and HLLC flux, and potential vorticity generation. Advances in Water Resources 30: 998-1015.

Tassi et al. (2007)采用HLLC和动力学数值通量格式的DG法，对不连续处局部限制数值振荡。对不规则地形上的bore-vortex相互作用、涡旋预测、收缩渠道中的恒定斜水跃等现象，动力学格式表现处更好的健壮性。

评述：不连续处的数值振荡的抑制，采用新的数值通量格式。

Ethan J. Kubatko, Joannes J. Westerink. 2007. Exact Discontinuous Solutions of Exner’s Bed Evolution Model: Simple Theory for Sediment Bores. Journal of Hydraulic Engineering(ASCE), 133(3)

Exner方程是基于质量守恒描述河床演变的方程。Exner给出的解析解一般称为经典或真解，但这仅当解是连续情况时才成立。Kubatko et al. (2007)使用非线性双曲方程的通用理论，求解Exner方程的不连续解，即可描述河床的不连续变形和传播，也就是sediment bore。

./case\_applications

V.R. Ambati, O. Bokhove. 2007. Space-time discontinuous Galerkin finite element method for shallow water flows. Journal of Computational and Applied Mathematics, 204: 452-462.

Ambati et al. (2007)在局部不连续处，使用HLLC数值通量格式，仅在不连续处采用一个耗散算子（Krivodonov不连续诊断因子），使DG法在时空上具有2阶精度。

./numerical\_flux

Paul-Emile Bernard. Nicolas Chevaugeon. Vincent Legat. Eric Deleersnijder. Jean-François Remacle. 2007. High-order h-adaptive discontinuous Galerkin methods for ocean modelling. Ocean Dynamics, 57: 109-121.

Bernard et al. (2007)将h自适应三角形网格的DG法用于近海岸的水动力模拟。

./hp-adaptivity

L. Pesch, A. Bell, W.E.H. Sollie, V.R. Ambati, O. Bokhove, J.J.W. van der Vegt, 2007. hpGEM-a software framework for discontinuous Galerkin finite element methods, ACM Trans. Math. Software 33 (4): 23.

# 2008年

Ethan J. Kubatko, Clint Dawson, Joannes J. Westerink. 2008. Time step restrictions for Runge-Kutta discontinuous Galerkin methods on triangular grids. Journal of Computational Physics 227: 9697-9710.

Kubatko et al. (2008)使用*p*=*k*-1阶多项式的DG空间离散法的RKDG模型，*k* stage的RK法，给出*k*-1阶精度的RKDG法。Cockburn and Shu (2001)给出了1D RKDG法的近似CFL线性稳定条件：，其中，*c*是波速，是网格间距。该条件仅对*p*=0, *p*=1的情况精确成立。在2种三角形网格（直角和等边三角形），Kubatko et al. (2008)给出了RKDG法的CFL近似条件：，其中*h*为网格参数，类比2D情况。Kubatko et al. (2007)指出s>p对于高阶RKDG法的效率比标准的RKDG法（s=p）好。

G. Kesserwani; R. Ghostine; J. Vazquez; A. Ghenaim; R. Mosé. 2008. Riemann Solvers with Runge-Kutta Discontinuous Galerkin Schemes for the 1D Shallow Water Equations. Journal of Hydraulic Engineering(ASCE), 134(2)

Kesserwani et al. (2008)在DG法中实施了一系列的黎曼求解器，如Roe, Osher, HLL, HLLC, HLLE等，讨论了L1误差、CPU计算耗时、不连续解和源项的影响。

./numerical\_flux

M. Lauter, F. X. Giraldo, D. Handorf, K. Dethloff. 2008. A discontinuous Galerkin method for the shallow water equations in spherical triangular coordinates. Journal of Computational Physics, 227: 10226-10242.

Lauter et al. (2008)基于球面三角形网格建立了RKDG求解（气象）浅水方程。在局部上由球体三角形坐标表示，采用合适的局部坐标映射到三角形上。因此，每个三角形网格单元上，是切向动量的2D表征，仅有2个离散动量方程。采用Rusanov数值通量格式，采用SSP性质的3阶Runge-Kutta时间离散格式。各曲边三角形上的k阶多项式空间由Lagrange基函数表征，因此需要使用高阶求积公式，在单元上和单元的边上求积分。注意：DG法在球坐标上求解浅水方程的数值特性还是有所不同的。

评述：是DG法在大气模拟中的应用。球面三角网格使用AMATOS(Brehens)生成，可用于自适应网格计算。

./spherical\_mesh

S. Rhebergen, O. Bokhove, J.J.W. van der Vegt. 2008. Discontinuous Galerkin finite element methods for hyperbolic nonconservative partial differential equations. Journal of Computational Physics 227: 1887-1922.

标准的DG法不能应用于非守恒形式的双曲型方程（如两相流模型）。因此，对DGFEM公式中的非守恒型product引入弱解理论，这又引起如何定义不连续处左右状态变量间关系的问题。Rhebergen et al. (2008)引入新的数值通量格式来处理非守恒行书的product，并用于2类PDE：带地形源项的SWE（地形导致非守恒product）和沿水深平均的两相流模型（具有更复杂的内在非守恒product）。

数学味太重！！！

./numerical\_flux

P.A. Tassi, S. Rhebergen, C.A. Vionnet, O. Bokhove. 2008. A discontinuous Galerkin finite element model for river bed evolution under shallow flows. Comput. Methods Appl. Mech. Engrg. 197: 2930-2947.

Tassi et al. (2008)基于DG法的水动力、河床演变模型（Exner方程），水动力和地形演变方程都纳入双曲系统，DG法求解2个intertwined的时间步格式（快速的水动力和慢的地形演变分量），其中没有计算悬移质泥沙的对流扩散方程，仅考虑推移质输沙导致的地形演变。

评述：延续了上文Rhebergen, 2008的方法和风格！

./case\_applications

J. C. Dietrich, R. L. Kolar, K. M. Dresback. 2008. Mass Residuals as a Criterion for Mesh Refinement in Continuous Galerkin Shallow Water Models. Journal of Hydraulic Engineering, 134(5)

CG法一直受到全局和局部质量守恒问题的困扰，如ADCIRC模型，降低质量守恒误差只能依靠细化网格的方法。ADCIRC模型计算GWC方程，与基于连续方程的浅水方程求解不一样，需要细心选择数值参数G。GWC方程的离散等价于Telemac模型中使用quasi-bubble格式离散连续方程(Atkinson et al., 2004)。总之，CG法都没有实施局部的质量守恒，质量计算残差是网格细化的主要准则。

评述：CG法模型的计算精度严重依赖于网格密度。

./dg\_review

Clint Dawson. 2008. A continuous/discontinuous Galerkin framework for modeling coupled subsurface and surface water flow. Comput Geosci, 12: 451-472.

Dawson(2008)应用CG法法求解地下水方程（Richards方程），DG法耦合地表水与地下水流动模型。

./case\_applications

Chen Zhiyun, Holger Steeb, Stefan Diebels. 2008. A space-time discontinuous Galerkin method applied to single-phase flow in porous media. Comput Geosci, 12: 525-539.

Chen et al. (2009)应用DG法求解多孔介质中的单相流。

./case\_applications

# 2009年

Shintaro Bunya, Ethan J. Kubatko, Joannes J. Westerink, Clint Dawson. 2009. A wetting and drying treatment for the Runge-Kutta discontinuous Galerkin solution to the shallow water equations, Comput. Methods Appl. Mech. Engrg. 198: 1548-1562.

Bunya et al. (2009)将RKDG模型求解浅水方程时的干湿地形处理，即数值通量计算后处理，保证水深为正值。

Kubatko EJ, Bunya S, Dawson C, Westerink JJ, Mirabito C. 2009. A performance comparison of continuous and discontinuous finite element shallow water models. J Sci Comput, 40(1-3): 315-39.

Kubatko et al.(2009)对比了CG法与DG法求解浅水方程的计算效率。

Ethan J. Kubatko, Shintaro Bunya, Clint Dawson, Joannes J. Westerink. 2009. Dynamic p-adaptive Runge-Kutta discontinuous Galerkin methods for the shallow water equations. Comput. Methods Appl. Mech. Engrg. 198: 1766-1774

Kubatko et al. (2009)详细论证了p-自适应的RKDG模型，结合SSP RK时间离散，p-自适应可显著提交计算效率（高阶精度抵消了CPU耗时），应结合h-自适应和实际应用。

./hp-adaptivity

Sebastian Roger , Benjamin J. Dewals, Sébastien Erpicum, Dirk Schwanenberg, Holger Schüttrumpf, Jürgen Köngeter, Michel Pirotton. 2009. Experimental and numerical investigations of dike-break induced flows, Journal of Hydraulic Research (IAHR), 47(3): 349-359.

Roger et al. (2009)针对一个溃堤试验，采用RKDG法和FVS的FVM模型做了模拟。

./case\_applications

# 2010年

Rabih Ghostine, Emmanuel Mignot, Maher Abdallah, Fabrice Lawniczak, José Vazquez, Robert Mosé, Caroline Grégoire. 2010. Discontinuous Galerkin Finite-Element Method for Simulation of Flood in Crossroads. Journal of Hydraulic Engineering (ASCE), 136(8)

Ghostine et al. (2010)应用2D RKDG模拟了交叉路口洪水的演进。Ghostine et al. (2010)对Ghostine et al. (2010)的研究做了讨论。

Rabih Ghostine, Emmanuel Mignot, Maher Abdallah, Fabrice Lawniczak, José Vazquez, Robert Mosé, and Caroline Grégoire. 2010. Discussion of “Discontinuous Galerkin Finite-Element Method for Simulation of Flood in Crossroads” 136(8): 474–482. DOI: 10.1061/(ASCE)HY.1943-7900.0000209

./case\_applications

G. Kesserwani, Liang Q.H. 2010. Well-balanced RKDG2 solutions to the shallow water equations over irregular domains with wetting and drying. Computers & Fluids, 39: 2040-2050.

Kesserwani and Liang (2010)建立了1D 2阶RKDG格式求解具有复杂地形的浅水方程。可处理干湿地形，将MUSCL格式实施到RKDG2求解器。摩阻源项采用分裂隐格式离散，实施物理停止条件来保证稳定性。最后，采用考虑和不考虑摩阻效应的恒定和非恒定算例验证了建立的模型。

./swem\_2d\_3d

G. Kesserwani, Liang Q.H. 2010. A discontinuous Galerkin algorithm for the two-dimensional shallow water equations, Comput. Methods Appl. Mech. Engrg. 199: 3356-3368.

Kesserwani and Liang (2010)应用RKDG2法求解具有干湿前锋的浅水方程。实施了FV的坡度限制因子。单元间的通量采用HLLC近似黎曼解。

评述：Kesserwani较好地将DG法用于具有干湿前锋的浅水方程求解，很好地将FV与DGFEM结合了。

./numerical\_flux

Xing Yulong, Zhang Xiangxiong, Shu Chi-Wang. 2010. Positivity-preserving high order well-balanced discontinuous Galerkin methods for the shallow water equations. Advances in Water Resources 33: 1476-1493.

Xing et al. (2010)深入分析了DG法求解具有非平底的浅水方程时，如何保持静水通量平衡和保证状态变量正值方面的特性。

评述：重点阅读！

./numerical\_flux

# 2011年

C. Dawson, E.J. Kubatko, J.J. Westerink, C. Trahan, C. Mirabito, C. Michoski, N. Panda. 2011. Discontinuous Galerkin methods for modeling hurricane storm surge, Adv. Water Resour. 34: 1165-1176.

Dawson et al. (2011)采用DG法与CG-ADCIRC模型模拟风暴潮下的复杂水网内的增水。由于使用的网格是针对ADCIRC模型的（断面上有2个单元），即在上端航道内的断面上有2个网格（网格密度较大），不适合于DG模型（不是基于节点的，而是基于单元和边的计算，断面上有1个单元即可）。因此，DG模型模拟的水位增幅偏低。（问题：网格太密了，反而对基于单元的DG模型不适合？？？）

评述：是SWEM2D模型的重要原理性论文，具备捕捉干湿前锋的能力和质量守恒，成功应用于复杂河网的水动力计算，有效抑制了数值耗散引起的水位（数值上的）衰减问题。

./case\_applications

C. Mirabito, C. Dawson, E.J. Kubatko, J.J. Westerink, S. Bunya. 2011. Implementation of a discontinuous Galerkin morphological model on two-dimensional unstructured meshes. Comput. Methods Appl. Mech. Engrg. 200: 189-207.

Mirabito et al. (2011)基于Rhebergen et al. (2008)修正了DG法，并在DG\_ADCIRC模型中实施了泥沙输移和地形演变模拟。

./case\_applications

Jagadeesh Anmala, Rabi H. Mohtar. 2011. Fourier stability analysis of two-dimensional finite element schemes for shallow water equations, International Journal of Computational Fluid Dynamics, 25: 2, 75-94.

Anmala et al. (2011)对显格式、半隐格式和隐格式的FEM求解浅水方程做了Fourier稳定性分析。

./dg\_review

Ramachandran D. Nair, Michael N. Levy, and Peter H. Lauritzen. 2011. Emerging Numerical Methods for Atmospheric Modeling. P.H. Lauritzen et al. (eds.), Numerical Techniques for Global Atmospheric Models, Lecture Notes in Computational Science and Engineering 80, DOI 10.1007/978-3-642-11640-7\_9.

Nair et al.(2011)综述了大气模拟中新出现的数值方法—DG法。

./dg\_review

# 2012年

Caleffi V, Valiani A. A well-balanced third-order-accurate RKDG scheme for SWE on curved boundary domains. Adv Water Resour 2012; 46(0): 31-45.

Caleffi and Valiani (2012)在曲线边界区域上实施了3阶精度的RKDG法求解浅水方程。

Corey J. Trahan, Clint Dawson. Local time-stepping in Runge-Kutta discontinuous Galerkin finite element methods applied to the shallow-water equations. Comput. Methods Appl. Mech. Engrg. 217-220 (2012) 139-152.

Trahan et al. (2012)实施了DG局部时间步长法。

Georges Kesserwani, Liang Qiuhua. Dynamically adaptive grid based discontinuous Galerkin shallow water model. Advances in Water Resources 37 (2012) 23-39.

Kesserwani and Liang (2012)实施了h-自适应的RKDG2模型，基于均匀四边形网格，是自适应结构化网格。

Georges Kesserwani, Qiuhua Liang. 2012. Influence of Total-Variation-Diminishing Slope Limiting on Local Discontinuous Galerkin Solutions of the Shallow Water Equations. Journal of Hydraulic Engineering (ASCE), 138(2)

Kesserwani and Liang (2012)指出FV坡度限制对DG法的数值解有一些副作用，如精度下降、增加计算耗时。本文对局部限制、全局限制和无坡度限制的线性前锋跟踪等3种模式对计算精度的影响做了详细研究。

./slope\_limiter

Georges Kesserwani, Qiuhua Liang. Locally Limited and Fully Conserved RKDG2 Shallow Water Solutions with Wetting and Drying. J Sci Comput (2012) 50:120–144

Kesserwani and Liang (2012)在RKDG2模型中，实现干湿地形处理的局部限制和完全守恒的浅水方程求解。

./wetting-drying

T. Toulorge, W. Desmet. CFL Conditions for Runge-Kutta discontinuous Galerkin methods on triangular grids. Journal of Computational Physics 230 (2011) 4657-4678.

在三角形网格上，Toulorge and Desmet(2011)考察RKDG法的线性稳定性限制条件，局部稳定性标准导致过于严格的限制。

W. Lai, A. A. Khan. 2012. Discontinuous Galerkin Method for 1D Shallow Water Flows in Natural Rivers, Engineering Applications of Computational Fluid Mechanics, 6:1, 74-86.

Lai and Khan (2012)将RKDG 1D用于求解天然河道。

./rkdg\_1d

Lai Wencong, KHAN Abdul A. Modeling dam-break flood over natural rivers using discontinuous galerkin method. Journal of Hydrodynamics, 2012,24(4):467-478

Lai and Khan (2012)采用HLLC数值通量的DG法模拟河道洪水的实际案例。

W. Lai, A. A. Khan. 2012. Discontinuous Galerkin Method for 1D Shallow Water Flow in Nonrectangular and Nonprismatic Channels. Journal of Hydraulic Engineering, 138(3)

Lai and Khan (2012)将1D RKDG法用于非矩形和非棱柱断面河道，研究表明：Roe和HLLC具有相似的精度，但对于非矩形的天然河道，使用坡度限制器可得到更精确的数值解。

./slope\_limiter

# 2013年

Clint Dawson, Corey Jason Trahan, Ethan J. Kubatko, Joannes J. Westerink. 2013. A parallel local timestepping Runge–Kutta discontinuous Galerkin method with applications to coastal ocean modeling. Comput. Methods Appl. Mech. Engrg. 259: 154-165.

Dawson et al. (2013)应用局部时间步长的DG法应用于近海岸区域水动力模拟。

./local timestep

Nappi A. 2013. Development and Application of a Discontinuous Galerkin-Based Wave Prediction Model. Master’s thesis. The Ohio State University.

Nappi(2013)基于DG法的风生波浪模型，计算效率比SWAN要高很多。

./case\_applications

Xing Yulong, Zhang Xiangxiong. 2013. Positivity-Preserving Well-Balanced Discontinuous Galerkin Methods for the Shallow Water Equations on Unstructured Triangular Meshes. J Sci Comput, 57: 19-41.

在“Xing et al. Adv. Water Resourc. 33: 1476–1493, 2010”论文的基础上，Xing and Zhang (2013)研究静水平衡在三角形非结构网格上的特性，构建了正值保证的限制因子，证明该格式可保证计算水深为正值、通量平衡以及高精度。

./numerical\_flux

Vadym Aizinger, Jennifer Proft, Clint Dawson, Dharhas Pothina, Solomon Negusse. 2013. A three-dimensional discontinuous Galerkin model applied to the baroclinic simulation of Corpus Christi Bay. Ocean Dynamics, 63: 89-113.

Aizinger et al. (2013)开发了MPI并行化的University of Texas Bays and Estuaries 3D (UTBEST3D)，评估了其并行计算效率。目前已知的仅有UTBEST3D和SLIM这2个基于RKDG法的3D海洋动力学模型。

./rkdg\_2d\_3d

LAI Wencong, KHAN Abdul A. Time stepping in discontinuous Galerkin method. Journal of Hydrodynamics, 2013, 25(3): 321-329.

Lai and Khan (2013)考察了时间离散格式在DG法中对稳定性和计算效率的影响。使用不同的坡度限制因子考虑额Euler格式和RK2格式。RK2格式比Euler格式的数值扩散性更强，RK2格式允许使用更大的时间步长。Euler格式的计算效率和精度比RK2更好。

Cass T. Miller, Clint N. Dawson, Matthew W. Farthing, Thomas Y. Hou, Jingfang Huang, Christopher E. Kees, C.T. Kelley, Hans Petter Langtangen. Numerical simulation of water resources problems: Models, methods, and trends. Advances in Water Resources 51 (2013) 405-437.

Miller et al.(2013)全面综述了计算水力学中用到的一些数值方法，包括FVM, RKDG法等。

./dg\_review

Andreas Meister, Sigrun Ortleb. 2013. The DG Scheme on Triangular Grids with Adaptive Modal and Variational Filtering Routines Applied to Shallow water Flows. R. Ansorge et al. (Eds.): Recent Developments in the Numerics, NNFM 120, pp. 253-266.

Meister and Ortleb (2013)应用基于衰减策略的谱粘度，增加DG法中的数值耗散，该策略是由直接用于格式中的系数的有效模式过滤组成，实现通量平衡。

./numerical\_flux

Valerio Caleffi, Alessandro Valiani. 2013. A 2D local discontinuous Galerkin method for contaminant transport in channel bends, Computers & Fluids 88: 629-642.

Caleffi and Valiani (2013)基于3阶LDG法求解浅水方程和污染物输移方程，并应用于弯道的模拟。LDG法的数学原理见Begnudelli et al.(2010)，数值积分使用Calffi and Valiani (2012)推荐的格式。

# 2014年

Lee Haegyun. Application of Runge-Kutta Discontinuous Galerkin Finite Element Method to Shallow Water Flow. KSCE Journal of Civil Engineering (2014) 18(5):1554-1562.

Lee Haegyun (2014)应用RKDG法求解浅水方程，对RKDG法做了简洁的介绍，可以作为初步理解RKDG法的开始。

./early\_dg

Jessica Meixner, J. Casey Dietrich, Clint Dawson, et al. A Discontinuous Galerkin Coupled Wave Propagation/Circulation Model. J Sci Comput (2014) 59:334–370

Meixner et al. (2014)将DG法用于波浪模拟，与DGSWEM松散耦合，并与DGSWEM与SWAN紧密耦合计算结果，对比了DGWAVE与SWAN的计算效率。

./case\_applications

Kubatko E.J., Yeager B.A., Ketcheson D.I., 2014. Optimal strong-stability-preserving Runge Kutta time discretizations for discontinuous Galerkin methods. J. Sci. Comput. 60: 313-344.

Kubatko et al. (2014)给出了优化的SSP RK时间离散格式。

D. Wirasaet, E.J. Kubatko, C.E. Michoski, S. Tanaka, J.J. Westerink, C. Dawson, 2014. Discontinuous Galerkin methods with nodal and hybrid modal/nodal triangular, quadrilateral, and polygonal elements for nonlinear shallow water flow, Comput. Methods Appl. Mech. Eng. 270: 113-149.

Wirasaet et al. (2014)开发了基于三角形、四边形和多边形单元的节点型、混合模式/节点的DG模型来求解非线性的SWE。

./nodal\_dg

Xing Yulong. 2014. Exactly well-balanced discontinuous Galerkin methods for the shallow water equations with moving water equilibrium. Journal of Computational Physics, 257: 536-553.

带源项的双曲律方程要得到恒定态数值解要求通量与源项间达到平衡(well-balanced)。Xing (2014)建立了well-balanced的间断Galerkin法求解浅水方程，不仅可达到静水平衡，还可以实现更一般的动水平衡，关键点是一种特殊的源项近似、基于通用的静水压力重构得到近似数值通量。通过数值试验验证了光滑解和不连续解的well-balanced特性和较好的精度。

评述：详细的数学分析，对理解通量平衡的DG法的数学原理很有价值。

./numerical\_flux

A. Duran, F. Marche. 2014. Recent advances on the discontinuous Galerkin method for shallow water equations with topography source terms. Computers & Fluids, 101: 88-104.

Duran and Marche (2014)系统总结和综述了DG法求解带源项的浅水方程的研究进展。

评述：数学味太重！

Pablo Ortiz. 2014. Shallow water flows over flooding areas by a flux-corrected finite element method, Journal of Hydraulic Research, 52(2): 241-252.

Ortiz (2014)对连续FEM引入flux corrected transport (FCT)概念(Ortiz, 2009)，得到2阶精度的FEM，并用于溃坝洪水的模拟。

./numerical\_flux

Ortiz P. 2009. A positive definite continuous FEM model for advection. Adv. Water Res. 32: 1359-1371.

Ortiz, P., Zienkiewicz, O.C., Szmelter, J. 2006. Hydrodynamics and transport in estuaries and rivers by the CBS finite element method. Int. J. Numer. Meth. Eng. 66, 1569-1586.

Ortiz的FEM模型是建立的CG法及Zienkiewicz的CBS FEM。

Georges Kesserwani, Alireza Shamkhalchian, Mahboobeh Jomeh Zadeh. Fully Coupled Discontinuous Galerkin Modeling of Dam-Break Flows over Movable Bed with Sediment Transport. Journal of Hydraulic Engineering(ASCE), 2014

Kesserwani et al. (2014)建立了1D RKDG,用于模拟溃坝流动下的动床演变和悬移质泥沙输移，考虑了泥沙输移和河床演变对水流的作用。

./case\_applications

Stefan Vater and Jörn Behrens. Well-Balanced Inundation Modeling for Shallow-Water Flows with Discontinuous Galerkin Schemes. 2014. J. Fuhrmann et al. (eds.), Finite Volumes for Complex Applications VII - Elliptic, 965 Parabolic and Hyperbolic Problems, Springer Proceedings in Mathematics & Statistics 78, DOI: 10.1007/978-3-319-05591-6\_98

Stefan Vater and Jörn Behrens(2014)使用DG法建立了通量平衡的洪水淹没模型，模拟近海地区的海啸或风暴潮淹没中，指出DG法在保持质量守恒、通量平衡和干湿地形变化是具有挑战性的。本文简要介绍了RKDG法在这方面的实际应用和优势。

./wetting-drying

Clint Dawson. 2014. A local timestepping runge–kutta discontinuous galerkin method for hurricane storm surge modeling. X. Feng et al. (eds.), Recent Developments in Discontinuous Galerkin Finite Element Methods for Partial Differential Equations, The IMA Volumes in Mathematics and its Applications 157, DOI 10.1007/978-3-319-01818-8-5

Dawson(2014) 简要介绍了局部时间步长法在RKDG法模拟风暴潮中的应用。

./local\_timestep

Shu Chi-Wang. 2014. Discontinuous galerkin method for time-dependent problems: survey and recent developments. X. Feng et al. (eds.), Recent Developments in Discontinuous Galerkin Finite Element Methods for Partial Differential Equations, The IMA Volumes in Mathematics and its Applications 157, DOI 10.1007/978-3-319-01818-8-2

Shu (2014)综述了DG法求解非恒定问题（包括对流占优和对流扩散问题），同时综述了近期DG法求解各种不同类型方程的研究进展。

./dg\_review

# 2015年

Prapti Neupane, Clint Dawson. 2015. A discontinuous Galerkin method for modeling flow in networks of channels. Advances in Water Resources, 79: 61-79.

Neupane and Dawson (2015)建立了RKDG法用于求解河网（1D与2D的耦合模拟）的模型。

在github有代码学习。

D. Wirasaet, S.R.Brus, C.E. Michoski, E.J. Kubatko, J.J. Westerink, C. Dawson. 2015. Artificial boundary layers in discontinuous Galerkin solutions to shallow water equations in channels. Journal of Computational Physics 299: 597-612

DG法用于明渠流动模拟时，采用曲边固体边界时，No-normal水流边界条件直接施加，会导致低精度的结果。常用方法是人为增加一个边界层（1个网格厚度）的流速场，使上游方向的水位计算偏大，Wirasaet et al. (2015)给出了解决方法。

./curved element

Stefan Vater, Nicole Beisiegel, Jörn Behrens. 2015. A limiter-based well-balanced discontinuous Galerkin method for shallow-water flows with wetting and drying: One-dimensional case. Advances inWater Resources 85: 1-13.

Vater et al. (2015)建立了模拟洪水淹没的新的1D DG算法，该方法是基于流速限制各单元中的动量分配，可防止干湿地形过程中不稳定问题发生。对水深的限制保证水深的正值，从而保证局部质量守恒。干湿界面处的单元通量修正引出通量平衡方法，保证lake at rest。DG格式使用Lagrange基函数的节点形式公式。因此，该模型很适合用于海啸和风暴潮情况下的干湿地形变化显著的近海区域。

评述：坡度限制因子。

Daniel Caviedes-Voullieme, Georges Kesserwani. 2015. Benchmarking a multi resolution discontinuous Galerkin shallow water model: Implications for computational hydraulics. Advances inWater Resources, 86: 14-31.

Daniel Caviedes-Voullieme and Kesserwani. (2015)使用一种新的网格自适应技术—MultiWavelets (MW) RKDG法。

./hp-adaptivity

Safarzadeh Maleki Farzam, Khan Abdul A. 2015. Effect of channel shape on selection of time marching scheme in the discon- tinuous Galerkin method for 1-D open channel flow. Journal of Hydrodynamics, 27(3): 413-426.

明渠形状对RKDG法的时间离散方法也有影响？

Farzam and Khan(2015)使用3种时间离散方法（Euler向前、2阶Adams-Bashforth，多步RK法）比较了矩形、梯形、三角形、抛物型断面河道的计算精度。表明：AB时间推进格式具有最佳精度和计算效率。

./CFL&time\_discretization

Robert L. Higdon. Multiple time scales and pressure forcing in discontinuous Galerkin approximations to layered ocean models. Journal of Computational Physics 295 (2015) 230-260.

Higdo (2015)分析了用于地转流调整的多层流体的3D DG FEM模型中，多个时间尺度和压力驱动的影响，表明：DG法可以获得比标准有限差分法好的结果。

./multi-layers

Florian Frank, Balthasar Reuter, Vadym Aizinger, Peter Knabner. 2015. FESTUNG: A MATLAB/GNU Octave toolbox for the discontinuous Galerkin method, Part I: Diffusion operator. Computers and Mathematics with Applications 70: 11-46.

Frank et al. (2015)采用GNU OCTAVE/MATLAB语言编写的DG模型，通过4篇系列论文，系统阐述了DG法的原理，MATLAB语言编程加快了DG法的教学和科研进程。本模型是理解DG法及其编程实现的范例。

评述：DGFEM的教学软件，可见德国人在研发、教学和工业化方面的优秀品质。

# 2016年

Colton J. Conroy, Ethan J. Kubatko. *hp* discontinuous Galerkin methods for the vertical extent of the water column in coastal settings part I: Barotropic forcing. Journal of Computational Physics 305 (2016) 1147-1171.

Conroy and Kubatko (2016)建立了3D hp自适应的DG海洋模型，用于分析近海岸3D水体环境下的斜压力作用。

D.T. Steinmoeller, M. Stastna, K.G. Lamb. 2016. Discontinuous Galerkin methods for dispersive shallow water models in closed basins: Spurious eddies and their removal using curved boundary methods. Ocean Modelling, 107: 112-124.

DG法求解复杂区域上的近似双曲系统（即扩散修改的浅水方程），在尖锐障碍物附近会产生spurious eddies。Steinmoeller et al. (2016)通过增加人工耗散（即涡粘度系数）来消除这种振荡涡旋，研究表明：中等阶数的DG法可扩展到曲边三角形单元，积分公式可使用高阶的quadrature和cubature准则，可消除spurious eddies。最后，用理想复杂区域和现实区域做了检验。

Xiao Wen, Zhen Gao, Wai Sun Don, Yulong Xing, Peng Li. Application of positivity-preserving well-balanced discontinuous Galerkin method in computational hydrology. Computers and Fluids 139 (2016) 112–119.

Xiao et al. (2016)将well-balanced RKDG (Xing et al., 2013)应用于斜水跃和tidal bore问题的模拟。

./case\_applications

Nouh Izem, Mohammed Seaid, Mohamed Wakrim. 2016. A discontinuous Galerkin method for two-layer shallow water equations, Mathematics and Computers in Simulation 120: 12-23.

Izem et al. (2016)基于DG法建立了双层浅水方程数学模型（垂向水体密度不同的情况），当表层与底层的水体密度相同时，将退化为单层模型。

Nouh Izem, Mohammed Seaid, Imad Elmahi, Mohamed Wakrim. 2016. Discontinuous Galerkin method for two-dimensional bilayer shallow water equations. J Eng Math, 96: 1–21.

评述：比较下上面2篇论文的差异？

./multi-layers

Farzam Safarzadeh Maleki, Abdul A. Khan. 2016. 1-D coupled non-equilibrium sediment transport modeling for unsteady flows in the discontinuous Galerkin framework. Journal of Hydrodynamics, 28(4): 534-543.

Maleki and Khan(2016)建立了1D RKDG法求解非恒定流和泥沙输移。

Farzam Safarzadeh Maleki, Abdul A. Khan. 2016. A novel Local Time Stepping algorithm for shallow water flow simulation in the discontinuous Galerkin framework. Applied Mathematical Modelling, 40: 70-84.

Maleki and Khan(2016)建立了DG框架下的新的局部时间步长法。与Dawson et al. (2013)做个比较。

C. Michoski, C. Dawson, E. J. Kubatko, D. Wirasaet, S. Brus, J. J. Westerink. 2016. A Comparison of Artificial Viscosity, Limiters, and Filters, for High Order Discontinuous Galerkin Solutions in Nonlinear Settings. J Sci Comput, 66: 406-434.

Michoski et al. (2016)系统总结了DG法求解对流扩散反应方程中，实施的坡度限制因子方法、模式过滤、模式系数的人工扩散系数。

Zhao Zhangyi, Zhang Qinghe, Zhao Hongbo,Yang Hua. 2016. A three-dimensional model for suspended sediment transport based on the compact discontinuous Galerkin method. International Journal of Sediment Research, 31: 36-43.

Zhao et al.(2016)应用紧致型DG法进行了3D的悬移质泥沙输移模拟。

Lee Haegyun, Lee Namjoo. 2016. Wet-Dry Moving Boundary Treatment for Runge-Kutta Discontinuous Galerkin Shallow Water Equation Model. KSCE Journal of Civil Engineering, 20(2): 978-989.

Lee and Lee (2016)介绍了DG法求解浅水方程中的干湿变化边界的处理。

Simone Marras, James F. Kelly, Margarida Moragues. 2016. A Review of Element-Based Galerkin Methods for Numerical Weather Prediction: Finite Elements, Spectral Elements, and Discontinuous Galerkin. Arch Computat Methods Eng, 23:673-722.

Marras et al. (2016)系统综述了在大气模式开发中用到的FEM，谱元法和间断Galerkin法。

./dg\_review

Maya Briani, Benedetto Piccoli, Jing-Mei Qiu. 2016. Notes on RKDG Methods for Shallow-Water Equations in Canal Networks. J Sci Comput, 68:1101-1123.

Briani et al. (2016)系统分析了1D RKDG法求解河网水动力的数学特性，显示了相比1阶格式的优势。

./rkdg\_1d

Shu Chi-Wang. 2016. Discontinuous Galerkin Methods for Time-Dependent Convection Dominated Problems: Basics, Recent Developments and Comparison with Other Methods. G.R. Barrenechea et al. (eds.), Building Bridges: Connections and Challenges in Modern Approaches to Numerical Partial Differential Equations, Lecture Notes in Computational Science and Engineering 114, DOI 10.1007/978-3-319-41640-3\_12

Shu Chi-Wang (2016)综述了DG法求解非恒定对流占优流动问题的基本原理和近期发展，并将DG法与其他数值方法做了比较。

./dg\_review

# 2017年

Li Longxiang, Zhang Qinghe. 2017. A new vertex-based limiting approach for nodal discontinuous Galerkin methods on arbitrary unstructured meshes. Computers and Fluids, 159: 316-326.

节点型DG法中的限制器，可用于三角形和四边形单元的非结构网格。

Dustin W. West, Ethan J. Kubatko, Colton J. Conroy, Mariah Yaufman, Dylan Wood. 2017. A multidimensional discontinuous Galerkin modeling framework for overland flow and channel routing. Advances in Water Resources 102: 142-160.

West et al. (2017)建立了RKDG法用于求解坡面流动和河网，与Prapti Neupane (2015)的研究很类似。

Niklas Wintermeyer, Andrew R. Winters, Gregor J. Gassner, David A. Kopriva, 2017. An entropy stable nodal discontinuous Galerkin method for the two dimensional shallow water equations on unstructured curvilinear meshes with discontinuous bathymetry, J. Comput. Phys. 340: 200-242.

Wintermeyer et al. (2017)建立了一种熵稳定的高阶节点间断Galerkin谱单元，近似求解非线性2D浅水方程的DGSEM方法，该算法是在结构网格上实施的。

评述：DGSEM的大量数学分析。

# 2018年

Colton J. Conroy, Ethan J. Kubatko, Angela Nappi, Rachel Sebian, Dustin West, Kyle T. Mandli. 2018. hp discontinuous Galerkin methods for parametric, wind-driven water wave models. Advances in Water Resources 119: 70-83.

Conroy et al. (2018)基于DG法建立了基于动量平衡方程的2参数风浪模型，在大湖风浪环境下，计算得到的波普误差与SWAN模型（基于能量平衡方程）的结果对比，误差相当，但基于DG法的2参数风浪模型的计算效率显著提高（30s vs 3h）。Conroy et al. (2018)建立的2参数DG波浪模型仅适用于大湖(Nappi, 2013)，应用于近海岸的风暴潮情况时，需要重新设计数学方程的项(Conroy et al., 2018)，例如将大气紊流和风生浪紧密耦合到涌浪(swell)上去。

代码见DGwave

./case\_applications

C.J. Conroy, K.T. Mandli, and E.J. Kubatko, Moment field equations for high-frequency gravity waves, To be submitted, pp. 1–35, 2018a.

Conroy C.J. , Mandli K.T. , Kubatko E.J. , 2018b. A self-affine multiplicative interpolation method for turbulent flow fields. To be submitted to. J. Fluid Mech. 1–25 .

Li Gang, Song Lina, Gao Jinmei. 2018. High order well-balanced discontinuous Galerkin methods based on hydrostatic reconstruction for shallow water equations. Journal of Computational and Applied Mathematics 340: 546-560.

Li et al. (2018)建立了1D高阶DG模型，DG法中的数值通量采用静水压力重构，联合使用一个新的源项近似方法和分解算法。基于严格的理论分析和数值试验，证明该格式具有稳定性、精度，且能捕捉小扰动，获得光滑解的真实高阶精度。

评述：数学味太重！

Georges Kesserwani, Janice Lynn Ayog, Domenico Bau. 2018. Discontinuous Galerkin formulation for 2D hydrodynamic modelling: Trade-offs between theoretical complexity and practical convenience. Comput. Methods Appl. Mech. Engrg. 342: 710-741.

Kesserwani et al. (2018)在四边形单元网格上建立RKDG2模型，采用所谓的“slope-decoupled"方法对标准的RKDG法做了简化，降低了DG法的复杂度而便于实施，表明简化的RKDG模型具有健壮性和精确性，计算效率更高。

Boris Bonev, Jan S. Hesthaven, Francis X. Giraldo, Michal A. Kopera. 2018. Discontinuous Galerkin scheme for the spherical shallow water equations with applications to tsunami modeling and prediction. Journal of Computational Physics 362: 425-448.

Bonev et al.(2018) 的DG法求解球坐标上的浅水方程，可处理干湿变化和non-conforming mesh。干湿处理方法可适用于任意阶的多项式，没有引入人工粘度、人工孔隙率等。

Mohammad Kazem Sharifian, Georges Kesserwani, Yousef Hassanzadeh. A discontinuous Galerkin approach for conservative modeling of fully nonlinear and weakly dispersive wave transformations. Ocean Modelling 125 (2018) 61–79

Sharifian et al. (2018)建立了2D RKDG 求解Green-Naghdi方程（弱色散浅水方程），对模拟波浪爬高与船舶行进中波浪阻力有应用价值。

./case\_applications

Qian Shouguo, Li Gang, Fengjing Shao, Yulong Xing. Positivity-preserving well-balanced discontinuous Galerkin methods for the shallow water flows in open channels. Advances in Water Resources 115 (2018) 172–184

Qian et al. (2018)将well-balanced RKDG (Xing et al., 2013)应用于不规则边界和不规则地形的明渠流动模拟。

.\case\_applications

Vadym Aizinger, Andreas Rupp, Jochen Schutz, Peter Knabner. Analysis of a mixed discontinuous Galerkin method for instationary Darcy flow. Comput Geosci (2018) 22:179-194.

Aizinger et al. (2018)论述了将Mixed DG FEM，Compact DG，hydridized DG用于求解非静止的Darcy流动的原理。全面了解DG法，可参考此文。

./dg\_review

# 2019年

S.R. Brus, D. Wirasaet, E.J. Kubatko, J.J. Westerink, C. Dawson. 2019. High-order discontinuous Galerkin methods for coastal hydrodynamics applications, Comput. Methods Appl. Mech. Engrg. 355: 860-899. （SWEM2D\_Brus模型原理）

针对低阶DG法用于模拟长波的问题，Brus et al. (2019)提出高阶的DG法，并应用于Galveston湾。高阶格式与相对稀疏网格，可在保证求解精度下提高计算效率。关键是高阶精度网格的生成技术（GMSH），可通过等参单元和超参单元实现弯曲边界，并使用等阶多项式描述地形。本文针对Dawson et al. (2010)中提到的低阶DG用于模拟近海岸流体的问题，Brus et al. (2019)提出了高阶DG法的解决方案。

Michael Herty, Nouh Izem, Mohammed Seaid. 2019. Fast and accurate simulations of shallow water equations in large networks. Computers and Mathematics with Applications, 78: 2107-2126.

Herty et al. (2019)建立的DG法用于1D河网水动力计算，并与传统2D求解浅水方程的模拟做了对比。在同等精度下，1D河网模型具有更高的计算效率。

评述：与Neupane and Dawson (2015)的1D RKDG研究做个比较。

Stefan Vater, Nicole Beisiegel, Jorn Behrens. A limiter-based well-balanced discontinuous Galerkin method for shallow-water flows with wetting and drying: Triangular grids. Int J Numer Meth Fluids. 2019, 91: 395-418.

Stefan Vater et al. (2019)在同形三角形单元网格上，使用2阶RKDG法求解非线性浅水方程。Stefan Vater et al. (2019)使用坡度限制因子，使模型具备了干湿前锋的捕捉能力。

Du Huijing, Yingjie Liu, Yuan Liu, Zhiliang Xu. 2019. Well-Balanced Discontinuous Galerkin Method for Shallow Water Equations with Constant Subtraction Techniques on Unstructured Meshes. Journal of Scientific Computing, 81: 2115-2131

Du et al. (2019)建立通量平衡的RKDG法求解浅水方程，在非结构网格上使用Constant Subtraction Techniques实现通量平衡。

评述：数学分析类的论文！

./numerical\_flux

Ran Guoquan, Zhang Qinghe, Li Longxiang. A discontinuous galerkin method for two-dimensional depth integrated non-hydrostatic shallow water model. Proceedings of the 10th International Conference on Asian and Pacific Coasts (APAC 2019) Hanoi, Vietnam, September 25-28, 2019.

Ran et al. (2019)使用DG法求解考虑非静水压力的平面2D浅水方程，可有效模拟有弱扩散的水波。浅水方程使用节点型的无积分NDG法离散，时间项采用4阶RK法离散。

./nodla\_dg

Balthasar Reuter, Andreas Rupp, Vadym Aizinger, Peter Knabner. Discontinuous Galerkin method for coupling hydrostatic free surface flows to saturated subsurface systems. Computers and Mathematics with Applications 77 (2019) 2291-2309

Reuter et al. (2019)应用LDG法求解地下水模型（Darcy定律）。

./case\_applications

# 2020年

Dylan Wood, Ethan J. Kubatko, Mehrzad Rahimi, Abdollah Shafieezadeh, Colton J. Conroy. 2020. Implementation and evaluation of coupled discontinuous Galerkin methods for simulating overtopping of flood defenses by storm waves, Advances in Water Resources 136: 103501

Wood et al. (2020)在DGSWE模型中的考虑内部障碍物处理算法，用于模拟了风暴潮情况下，河堤的漫顶水流现象。

Robert L. Higdon. 2020. Discontinuous Galerkin methods for multi-layer ocean modeling: Viscosity and thin layers. Journal of Computational Physics, 401: 109018

Higdon (2020)将DG法应用于多层的流体模拟时还是有很多需要关注的问题，例如等密度流体中，粘性和分层厚度对模拟结果的影响。

./multi-layers

Daniel Caviedes-Voullième, Nils Gerhard, Aleksey Sikstel, Siegfried Müller. 2020. Multiwavelet-based mesh adaptivity with Discontinuous Galerkin schemes: Exploring 2D shallow water problems. Advances in Water Resources, 138: 103559

Daniel Caviedes-Voullième et al. (2020)基于Multi-wavelet的网格自适应。

./hp-adaptivity

# 2021年

Colton J. Conroy, Einat Lev. 2021. A discontinuous Galerkin finite-element model for fast channelized lava flows v1.0. Geosci. Model Dev., 14, 3553-3575.

Conroy and Lev (2021)应用RKDG法模拟河道中的快速岩浆流动(lava flow)，同样表现出较好的数值性能。

Matlab语言编程。可作为学习用代码。

./case\_applications

Matteo Giacomini, Ruben Sevilla，Antonio Huerta. HDGlab: An Open‑Source Implementation of the Hybridisable Discontinuous Galerkin Method in MATLAB. Archives of Computational Methods in Engineering (2021) 28:1941-1986.

Matlab语言编程的Hybridisable Discontinuous Galerkin的模型。特点：（1）实施了9阶的高阶多项式形函数；（2）支持2D和3D的等参曲边单元；（3）支持非均匀阶的多项式近似，灵活支持阶自适应(p-)策略。与高阶网格生成器GMSH有接口，可解决实际问题。

./semi-implicit&hybridized dg

# 2022年

Jordi Vila-Pérez, R. Loek Van Heyningen, Ngoc-Cuong Nguyen, Jaume Peraire. Exasim: Generating Discontinuous Galerkin Codes for Numerical Solutions of Partial Differential Equations on Graphics Processors. arXiv:2205.07824v1 [cs.MS] 16 May 2022

开源的DG法的软件Exasim，有高级语言（Julia, Matlab, Python）的接口，可运行在CPU与GPU集群上。

Exasim采用时间隐格式的无矩阵方法，与ExaDG, deal.II和MFEM等软件的原理类似。



Exasim的源码架构示意图

D. Arndt, N. Fehn, G. Kanschat, K. Kormann, M. Kronbichler, P. Munch, W. A.Wall, J.Witte, ExaDG: High-order discontinuous Galerkin for the exascale, in: H.-J. Bungartz, S. Reiz, B. Uekermann, P. Neumann, W. E. Nagel(Eds.), Software for Exascale Computing - SPPEXA 2016-2019, Springer International Publishing, Cham, 2020, pp. 189-224.

D. Arndt, W. Bangerth, D. Davydov, T. Heister, L. Heltai, M. Kronbichler, M. Maier, J.-P. Pelteret, B. Turcksin, D. Wells, The deal.II finite element library: Design, features, and insights, Computers & Mathematics with Applications 81 (2021) 407-422.

R. Anderson, J. Andrej, A. Barker, J. Bramwell, J.-S. Camier, J. Cerveny, V. Dobrev, Y. Dudouit, A. Fisher, T. Kolev,W. Pazner, M. Stowell, V. Tomov, I. Akkerman, J. Dahm, D. Medina, S. Zampini, MFEM: A modular finite element methods library, Computers & Mathematics with Applications 81 (2021) 42-74.

Eric J. Ching, Brett Bornhoft, Ali Lasemi, Matthias Ihme. Quail: A lightweight open-source discontinuous Galerkin code in Python for teaching and prototyping. SoftwareX 17 (2022) 100982

Python语言编写的DG法的教学代码QUAIL，可作为入门级学习用代码，也可用来验证和实施新的数值算法。

QUAIL在1D和2D非结构网格上求解1阶或2阶PDE。实施了多种时间积分格式、积分准则、基函数类型、方程组和其他特性。example代码包括一个2D涡和一个带重力加速度g源项的Riemann问题求解。

# 2023年

Shallow-Flow Velocity Predictions Using Discontinuous Galerkin Solutions. Journal of Hydraulic Engineering, Volume 149, Issue 5

<https://doi.org/10.1061/JHEND8.HYENG-13244>

# DG与FVM等算法的比较

## WENO与DG

Kesserwani, G., Y. Wang 2014. Discontinuous Galerkin flood model formulation: Luxury or necessity? Water Resour. Res., 50, 6522-6541.

DG法应用于浅水方程求解的必要性？

DG法的复杂性（算法和编码）得到了回报：当地形数据有限或分辨率很低，在粗网格上，DG法的求解精度较FVM高，流速预测精度得到改善。

比较FVM与DG法的研究还有Zhou and Shu (2001)

Zhou, T., Y. Li, and C. W. Shu (2001), Numerical comparison of WENO finite volume and Runge-Kutta discontinuous Galerkin methods, J. Sci. Comput., 16(2), 145-171.

本文中，Kesserwani and Wang (2014)应用几个算例（包括Toce试验等），在不同分辨率网格上，比较了二阶RKDG模型与MUSCL格式的FVM模型，模拟水深和流速的差异。研究表明：细网格上，两者的计算差异不大。但在粗网格上，RKDG2模型的优势很明显。

本文还较精炼地说明了两种算法的实施步骤。

Shu Chi-Wang. 2003. High-order Finite Difference and Finite Volume WENO Schemes and Discontinuous Galerkin Methods for CFD. International Journal of Computational Fluid Dynamics, 17(2): 107-118.

Shu (2003)综述了3种在CFD领域的高阶数值方法：WENO FDM，WENO FVM和DG法。本文主要从实践角度阐述3种方法的相对优势、算法的具体实施和不同情况的适用性等。DG法的理论特性综述可参考Cockburn and Shu (2001)的文章。

./dg\_review

Xing Yulong, Shu Chi-Wang. 2006. High order well-balanced finite volume WENO schemes and discontinuous Galerkin methods for a class of hyperbolic systems with source terms. Journal of Computational Physics, 214: 567-598.

设计了well-balanced的WENO FVM和DG FEM，在1D和2D情况下论证了2种算法求解带源项的浅水方程的数学特性和高阶精度，主要是稳定态下平衡律的守恒性、带不连续处的一般解的非振荡特性和光滑区的高阶精度。

数学味太重！

./dg\_reivew

## GWCE与quasi-bubble

Atkinson, J. H., Westerink, J. J., Hervouet, J. M. 2004. Similarities between the quasi-bubble and the generalized wave continuity equation solutions to the shallow water equations. Int. J. Numer. Methods Fluids, 45, 689-714.

GWCE方法与quasi-bubble格式方法的相似性分析。

# DG模型的并行化

## OpenMP/MPI并行

Balthasar Reuter, Vadym Aizinger, Harald Kostler. 2015. A multi-platform scaling study for an OpenMP parallelization of a discontinuous Galerkin ocean model. Computers & Fluids 117: 325-335.

Reuter et al. (2015)使用OpenMP并行化了近海区域3D海洋UTBEST3D模型，一个基于DG法的动力学模型，基于非结构网格，目的是利用Intel MIC处理器（协处理器），是第一个针对MIC处理器开发的并行化DG法海洋动力学模型。

S. R. Brus, Wirasaet, J.J. Westerink,·C. Dawson. 2017. Performance and Scalability Improvements for Discontinuous Galerkin Solutions to Conservation Laws on Unstructured Grids. J Sci Comput, 70: 210–242.

Brus et al. (2017)对DGSWEM模型的HPC开发做了深入分析，特别是循环矢量化。

高阶DGSWEM模型的代码结构，基于数据循环矢量化、openMP和MPI并行化DG算法。

Lai Wencong, Abdul A. Khan. A Parallel Two-Dimensional Discontinuous Galerkin Method for Shallow-Water Flows Using High-Resolution Unstructured Meshes. Journal of Computing in Civil Engineering (ASCE), DOI: 10.1061/(ASCE) CP.1943-5487.0000647.

MPI并行化RKDG 2D。

./parallelized

Andrea Crivellini, Matteo Franciolini, Alessandro Colombo, Francesco Bassi. OpenMP Parallelization Strategies for a Discontinuous Galerkin Solver. International Journal of Parallel Programming (2019) 47: 838-873.

采用OpenMP 4.0并行化了DG法模型。

## GPU并行

GPU并行的方法可分为以下几类：

（1）CUDA C/C++编程（最基础的）；

（2）Domain Specific language;

（3）Automated code generation

（4）OpenACC, OpenCL等（与编译器有关，指令集）；

（5）基于第三方库(PETSc)的二次开发。

CUDA C/C++编程

D. Schwanenberg, S. Horsten, S. Roger. 2010. Discontinuous Galerkin shallow water slolver on CUDA architetures. 9th International Conference on Hydroinformatics HIC, 2010, Tianjin, China

Schwanenberg et al. (2010)使用CUDA并行化了RKDG模型，用于求解浅水方程。

Martin Fuhry, Lilia Krivodonova. 2013. Discontinuous Galerkin Methods on Graphics Processing Units for Nonlinear Hyperbolic Conservation Laws. 硕士论文

Fuhry and Krivodonova (2013)使用CUDA并行化DG模型，求解3D欧拉方程，达到50倍加速。

Dawei Mu, Po Chen, Liqiang Wang. Accelerating the discontinuous Galerkin method for seismic wave propagation simulations using the graphic processing unit (GPU)--single-GPU implementation. Computers & Geosciences 51 (2013) 282–292

实施了SeiSol模型的GPU并行化(CUDA)。

OpenACC指令集

Michael DuChene, Anna Maria Spagnuolo, Ethan Kubatko, et al. 2011. A Framework for Running the ADCIRC Discontinuous Galerkin Storm Surge Model on a GPU, Procedia Computer Science 4: 2017-2026.

DuChene et al. (2011)对DG法的ADCIRC模型进行异构并行，首先分析了下列并行化方式的优缺点和可行性：

（1）PGI的openACC并行：ADCIRC有很多子程序，子程序间涉及大量的CPU/GPU数据转移，降低并行效率。

（2）F2C-ACC，不支持FORTRAN语言，也不支持子程序间的共享数据；

（3）PGI CUDA-FORTRAN，开发成本过高。

（4）Corrigan et al.的半自动转换方法，没有python代码，无法实现。

因此，DuChene et al. (2011)重写了部分ADCIRC模型的子程序，将FORTRAN重写为C++语言，调用CUDA库，用nvcc优化。

看来：如果能用openACC解决了子程序间的数据共享，是一个好办法。

Xia Yidong, Luo Hong, Luo Lixiang, Jack Edwards, Lou Jialin. OpenACC-based GPU Acceleration of a 3-D Unstructured Discontinuous Galerkin Method. AIAA 年代？？

Xia et al.(??)使用openACC技术，实现DG法的3D可压缩NS方程求解模型的异构并行化，Runge-Kutta显格式时间层迭代中，一次性实现CPU/GPU间的数据转移，提高了异构并行计算效率，达到24倍和1.6倍加速比（分别与单核和16进程的MPI并行计算时间比较）。

自动代码生成

基于Python语言快速实现DG法的GPU并行化：

Andreas Klöckner, Tim Warburton, Jeff Bridge, Jan S. Hesthaven, 2009. Nodal discontinuous Galerkin methods on graphics processors, J. Comput. Phys. 228(21): 7863-7882.

Kristian B. Ølgaard, Anders Logg, Garth N. Wells. Automated code generation for discontinuous galerkin methods. arXiv:1104.0628v1 [cs.MS] 4 Apr 2011

在FEniCS框架下实施的DG法。

Andreas Klöckner, Timothy Warburton, Jan S. Hesthaven, Solving wave equations on unstructured geometries, in: GPU Computing Gems, vol.2, 2012, p.225.

Andreas Klöckner, Timothy Warburton, Jan S. Hesthaven. 2013. High-Order Discontinuous Galerkin Methods by GPU Metaprogramming. D. A. Yuen et al. (eds.), GPU Solutions to Multi-scale Problems in Science and Engineering, 353 Lecture Notes in Earth System Sciences, DOI: 10.1007/978-3-642-16405-7\_23.

基于OCCA库的DG法(Timothy Warburton研究组)

**下面Timothy Warburton研究组在GPU加速DG算法方面的研究应关注。**

David S. Medina, Amik St-Cyr, Timothy Warburton, OCCA: a unified approach to multi-threading languages, arXiv preprint, arXiv:1403 .0968, 2014.

OCCA库的介绍。

David S. Medina, Amik St-Cyr, Timothy Warburton, High-order finite-differences on multi-threaded architectures using OCCA, arXiv:1410.1387, 2014.

高阶有限差分法的GPU并行。

A. Modave, A. St-Cyr, T. Warburton, W.A. Mulder, Accelerated discontinuous Galerkin time-domain simulations for seismic wave propagation, in: 77th EAGE Conference and Exhibition 2015.

Rajesh Gandham, D.S. Medina, Timothy Warburton, GPU Accelerated discontinuous Galerkin methods for shallow water equations, Commun. Comput. Phys. (2015).18:37-64

./parallelized/pasidg\_papers

A. Modave, A. St-Cyr, T. Warburton. 2016. GPU performance analysis of a nodal discontinuous Galerkin method for acoustic and elastic models. Computers &Geosciences, 91: 64-76.

Modave et al. (2016)基于OCCA框架的GPU并行化高阶DG法。

J.-F. Remacle, R. Gandham, T. Warburton. GPU accelerated spectral finite elements on all-hex meshes. Journal of Computational Physics 324 (2016) 246–257

GPU加速的谱单元法的有限单元模型，在六面体网格上求解椭圆型方程。

A. Karakus, T. Warburton, M.H. Aksel & C. Sert (2016) A GPU-accelerated adaptive discontinuous Galerkin method for level set equation, International Journal of Computational Fluid Dynamics, 30:1, 56-68.

GPU加速的DG法求解Level Set方程。

Kasia Swirydowicz, Noel Chalmers, Ali Karakus, Timothy Warburton, Acceleration of tensor-product operations for high-order finite element methods. arXiv preprint, arXiv:1711.00903, 2017.

Niklas Wintermeyer, Andrew R. Winters, Gregor J. Gassner, Timothy Warburton. 2018. An entropy stable discontinuous Galerkin method for the shallow water equations on curvilinear meshes with wet/dry fronts accelerated by GPUs. Journal of Computational Physics 375: 447-480.

拓展了熵稳定的高阶节点间断Galerkin谱单元近似求解非线性2D浅水方程的方法(Wintermeyer et al., 2017)，具备激波捕捉和正值保证能力来处理干湿地形变化问题。该数值格式是well-balanced的、可用于非结构曲边四边形单元网格。为捕捉激波，引入人工粘度，证明数值格式仍然是熵稳定的。

A. Karakus, N. Chalmers, K. Swirydowicz, T. Warburton. 2019. A GPU accelerated discontinuous Galerkin incompressible flow solver. Journal of Computational Physics 390: 380-404.

半隐格式DG法求解非恒定不可压缩NS方程。显格式处理非线性项。压力方程使用GPU加速的多网格预处理的CG法求解器。

Andrew C. Kirby, Dimitri J. Mavriplis. GPU-Accelerated Discontinuous Galerkin Methods: 30x Speedup on 345 Billion Unknowns. arXiv: 2006.15698v3

基于OCCA库实施了DG法的GPU加速，用于求解可压缩Euler方程，在笛卡尔网格上实施离散。相比CPU（MPI并行），GPU加速30X。另外，还比较了CUDA-aware MPI与non-GPUDirect的效率，CUDA-aware MPI提高效率24%。

Wu Xinhui, Ethan J. Kubatko, Jesse Chan. 2021. High-order entropy stable discontinuous Galerkin methods for the shallow water equations: Curved triangular meshes and GPU acceleration. Computers and Mathematics with Applications 82: 179-199.

高阶熵稳定的间断Galerkin（ESDG）方法，在曲边三角形网格上求解2D浅水方程。在四边形单元网格上，实施了GPU并行的高阶DG法。

## 基于第三方库(HPX, Nektar++, DUNE…)

例如，基于HPX库开发的：

Zachary D. Byerly, Hartmut Kaiser, Steven Brus, Andreas Schaefer. A Non-intrusive Technique for Interfacing Legacy Fortran Codes with Modern C++ Runtime Systems.

介绍将SWEM模型（FORTRAN）移植到HPX库，利用HPC技术。

Maximilian Bremer, Kazbek Kazhyken, Hartmut Kaiser, Craig Michoski, Clint Dawson. Performance Comparison of HPX Versus Traditional Parallelization Strategies for the Discontinuous Galerkin Method. Journal of Scientific Computing (2019) 80:878-902.

基于HPX实施的SWEM模型的并行化技术细节，并与传统的并行化方法的计算效率做了比较。

基于Nektar++库开发的：

Claes Eskilsson, Yaakoub El-Khamra, David Rideout, Gabrielle Allen, Q. Jim Chen, Mayank Tyagi. A Parallel High-Order Discontinuous Galerkin Shallow Water Model. G. Allen et al. (Eds.): ICCS 2009, Part I, LNCS 5544, pp. 63-2, 2009.

基于Nektar++库开发的用于求解浅水方程的DG法模型。

基于DUNE库开发的：

Andreas Dedner, Dietmar Kröner, Nina Shokina. 2011. Adaptive Modelling of Two-Dimensional ShallowWater Flows with Wetting and Drying. E. Krause et al. (Eds.): Computational Sci., & High Performance Computing IV, NNFM 115, pp. 1–15.

在DUNE(the **D**istributed and **U**nified **N**umerics **E**nvironment)框架中实施DG法求解浅水方程，耦合地表水与地下水的水文模型（实际上还是水动力学模型）。