A NEW MODEL FOR BERTH ALLOCATION PROBLEM IN MARITIME LOGISTICS

FINAL PROJECT REPORT

submitted to

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

By

LEO JUSTINE(MAC17ME078)

in partial fulfillment for the award of the degree

of

BACHELOR OF TECHNOLOGY

in

MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

MAR ATHANASIUS COLLEGE OF ENGINEERING

KOTHAMANGALAM 686 666, KERALA,INDIA

JUNE 202

ACKNOWLEDGEMENT

This project at first was challenging for us and now I have successfully reached the end of our project. At this stage, we would like to gratefully acknowledge that we would not have been able to complete this project without the invaluable support of our benefactors, especially the Mechanical Department faculty.

First of all, I thank God Almighty for his blessings as it is only through his grace that we were able to complete our project successfully.

I take this opportunity to extend our sincere thanks to our project guide, Prof.BobinCherian Jos, Associate Professor, Department of Mechanical Engineering for his constant support and immense contribution to the success of our project.

I also extend our sincere thanks to Project Coordinator, Dr. Biju B, Professor,Mechanical Engineering Department and our Faculty Advisor Dr. Deepak Eldho Babu, Assistant Professor and all other members of the Department of Mechanical Engineering for sharing their valuable comments during the preparation of the project.

I am also grateful to Dr. Binu Markose, Head Of the Mechanical Engineering Department, and Dr. Jeoju M Issac, former Head of the Mechanical Engineering Department, for the valuable guidance as well as timely advice which helped us a lot during the preparation of the project

I am deeply indebted to Dr. Mathew K, Principal, Mar Athanasius College of Engineering for his encouragement and support.

I whole-heartedly thank all our classmates, for their valuable suggestions and for the spirit of healthy competition that existed between us

ABSTRACT

KEYWORDS: *Dynamic Berth Allocation Problem (DBAP), Mixed-Integer Linear Programming (MILP), Maritime logistics,tide.*

The Dynamic Berth Allocation Problem (DBAP) represents one of the significant port optimization problems associated with maritime logistics. Its main objective is to allocate berth space efficiently for vessels. This study focuses on the problem of defining berth space to vessels coming dynamically in a public berth system. Proper assignment of berths to incoming ships for their cargo handlings plays a vital role in minimizing the delay time. The main objective of our study is to assign berths to the incoming ships in such a way that it can minimize the overall delay time the ships spend in a tide affected port . To obtain an optimal solution with considerably small computational effort, we developed a novel Mixed-Integer Linear Programming Model (MILP), which is adaptable to real-world applications.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABBREVIATIONS

BAP	:	Berth Allocation Problem
SBAP	:	Static Berth Allocation Problem
DBAP	:	Dynamic Berth Allocation Problem
MILP	:	Programming
FCFS	:	First Come-First Serve
MUT	:	Multi-User Container Terminal
MCBAP	:	Minimum Cost Berth Allocation Problem
BAPC	:	Continuous Berth Allocation Problem

NOTATIONS

- *Sets*
- $I = \{1, 2\ldots, K\}$ Berths' set
- $J = \{1, 2\ldots, N\}$ Vessels' set
- *Parameters*
- *Aj* Vessel *j* estimated arrival time
- *Hj* Vessel *j* estimated loading and unloading time
- *lj* The length of vessel *j* including vessel safe distance for mooring
- *dj* Draft of vessel *j*
- *Rj* Scheduled departure time of vessel *j*
- *Li* The length of berth *i*

Di The depth of berth *i*
αjt Binary matrix, 1 if vessel *j* at time *t* can go through the access channel, otherwise 0.

M A very large number
v_i The time when berth *i* becomes
empty at the beginning of the
time horizon
Yi Coordinates of the middle of berth *i*

Decision variables
St*j* Mooring time of vessel *j*
Ft*j* Departure time of vessel *j*
xij Binary variable, 1 if vessel *j* moors at berth *i*, otherwise 0
fj 1 if vessel *j* is the first vessel that moors at its allocated berth, otherwise 0
ej 1 if vessel *j* is the last vessel that moors at its allocated berth, otherwise 0
yab If *xia* = *xib* and vessel *b* is serviced immediately after vessel *a* 1, otherwise 0
wjt Binary variable, 1 if vessel *j* moors at time *t* it is set, otherwise 0
yj Relative mooring position of
qjt Binary variable, 1 if vessel *j* departs the port at time *t* it is set, otherwise 0

CHAPTER 1

INTRODUCTION

Throughout recorded history, the sea has been widely used for freight transport. The introduction of aviation resulted in the decline of the importance of sea travel for passengers. However, it is still popular for pleasure cruises and short trips. The low cost of transport by ships compared to airplanes make it still suitable for large cargo transport across the globe. Ships can carry a huge load of cargo compared to airplanes making it the more efficient way economically. 90% of the world's trade is carried by sea. About two-thirds of goods transported worldwide are accounted for by developing economies. By 2017, the International trade through sea gained a volume spike of about 4.0 percent (Hoffman et al., 2018). It was the quickest surge that was recorded in the past 5 years. It is predicted to increase in the future years. This huge volume of transport can result in heavy container traffic. To reduce this container traffic, optimizing port operations are given priority by effective use of resources.

On the other hand, there are container terminals, operated as Common User Terminals or also known as Multi User Container Terminals, which are mostly state- owned. Most of these terminals use the "First Come First Served" criterion for their day-to-day operations. These terminals can be defined to be with long berths, which are capable of serving more than one ship at a time, depending on the size of the terminal facilities. Moreover, ships would be allocated to specific berths, taking into account the arrival times of the vessels, so that a ship may not always be assigned to a specific berth every time it calls.

The operations of a container terminal include seaside operations, yard operations, and land-side operations. One important issue in seaside operations is the assignment of berthing position to a defined set of ships that must be served within a defined planning horizon. This is esteemed as a key process for container terminals. Due to operational correlation, container terminal operators and ocean carrier share the common objective in minimizing the time ships spend in port. The berth allocation problem (BAP) thus arises in dealing with terminal assignments of berths to ships, and one of its major aims is to minimize the total delay time, that is, actual departure time minus scheduled departure time. Various types of aims/objectives pursued by the BAP exist and can be referred to as in the excellent reviews of the literature for the corresponding objective functions.

Figure 1.1: Operations of a container terminal

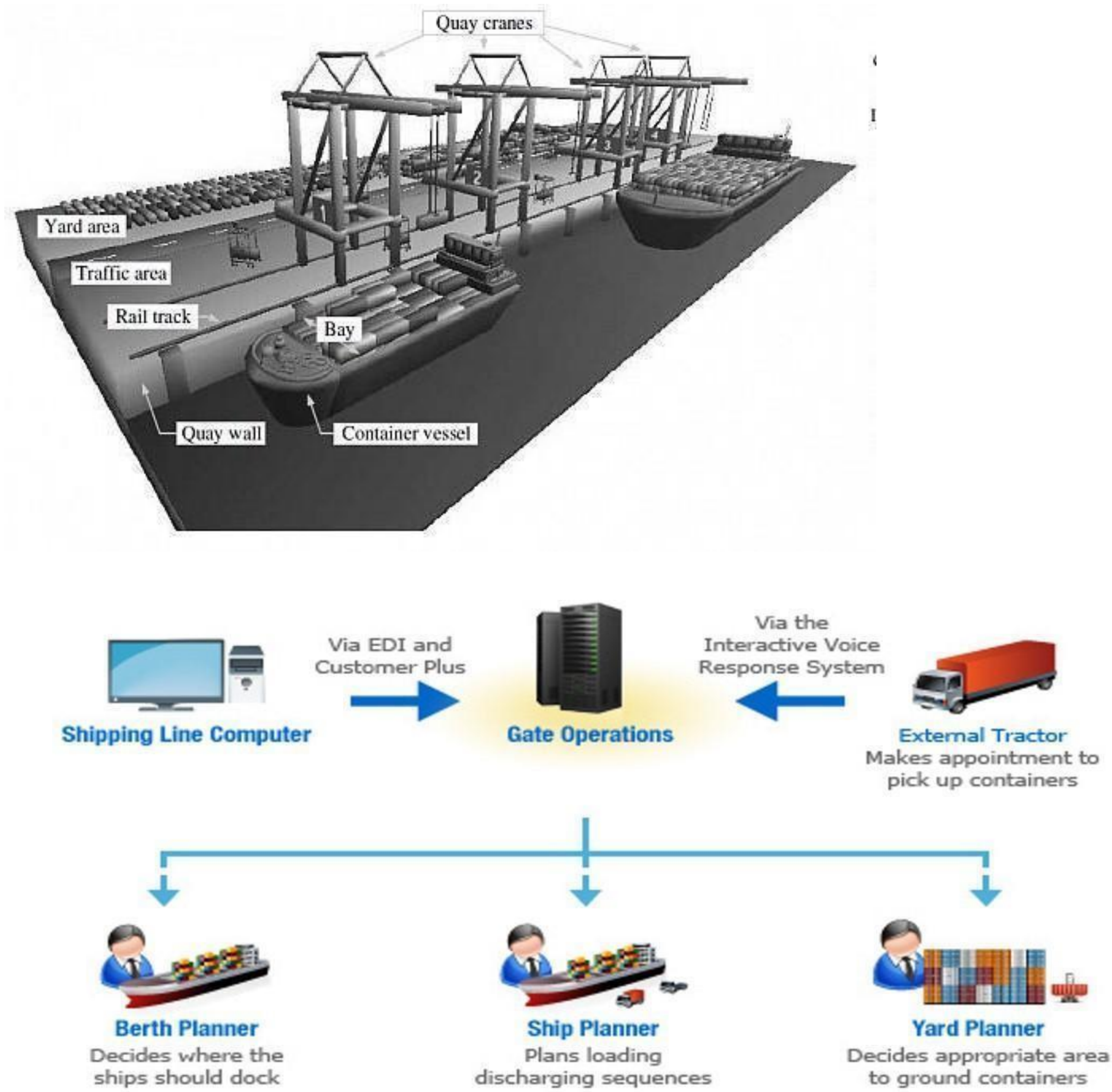

Figure 1.2: Planning at a container terminal

The handling of cargo from the ships takes place at the port terminal. The field of a container terminal can be isolated into different sections or areas. The ships are handle as well as berthed at the quay area. In the transport area, the movement of containers takes place. In the yard area, the containers are stored. The truck movement take place inside the truck area. For efficient transport, the handling time of ships in the container terminal needs to be reduced. The port charges also increase with the amount of time a ship spends at a port. The handling time can be reduced by increasing the number of berths, human resources, quayside cranes and other handling equipment. The berth factor is the most important aspect among these. However, increasing the number of berths is practically infeasible due to natural, economical and infrastructural factors.

So, the optimum use of the available berths is necessary.

1.1 BERTH ALLOCATION PROBLEM(BAP)

The Berth Allocation Problem (BAP) assumes that berth layout of a port is given, along with a set of vessels that are to be served within a considered planned horizon. Each berth in a given port is identified by its unique number, called berth index. Vessels are represented by a set of data, such as: expected arrival time, the size, anticipated handling time, preferred berth in the port, and many others, depending on considered variant of BAP. The goal of BAP is to allocate each vessel to a berth index and a time interval so that the given objective function value is optimized. Objective function can be defined as minimization of the total cost of the allocation, minimization of vessels' waiting times (time that vessels must wait for a berth due to port congestion) and handling times (time used for loading/unloading vessels), minimization of earliness and tardiness (lateness of vessels against their desired departure time), minimization of fuel consumption, maximization of profit, maximization of quay cranes (QC) utilization, etc. BAP is proved to be NP-hard.

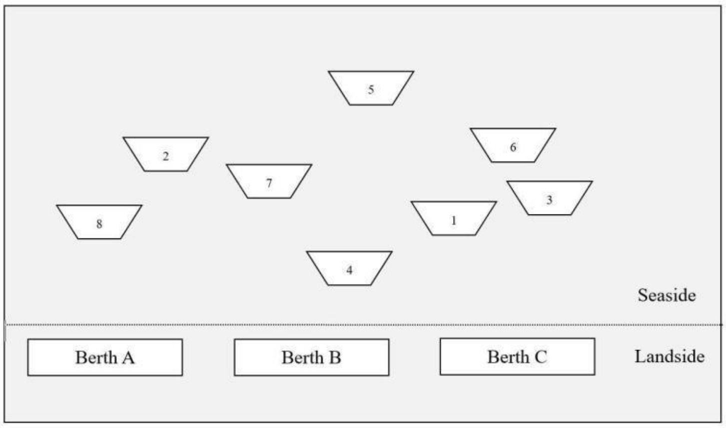

Figure 1.3: Container terminal system for berth planning (Isosceles trapezoids in seaside represent vessels)

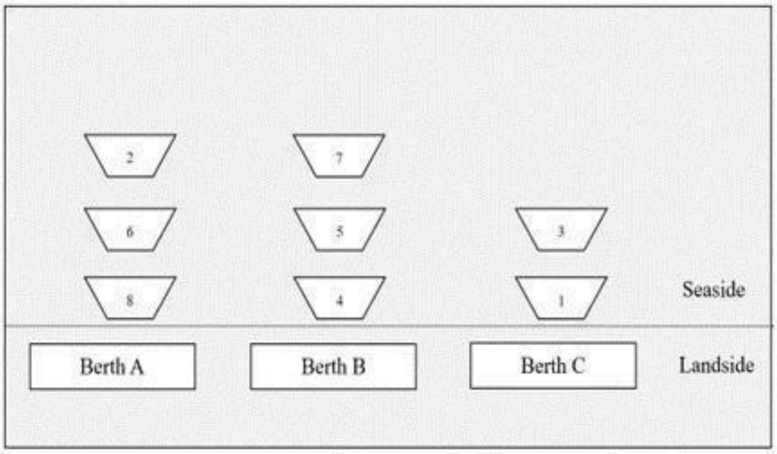

Figure 1.4: Solution indicating vessel assignment across berths and vessel sequencing within each berth

Most of the BAP (Berth Allocation Problem) focuses to reduce the total waiting and cargo handling time of the ships at port. The berth allocation can be static (SBAP) or dynamic (DBAP) depending on the time at which the vessels arrive for the planning period. In the former, the vessels arrive before the planning horizon and in the latter, the ships may arrive before or after the construction of schedule which makes it more realistic. Hybrid, Continuous, Discrete and indented are the different allocation models of berth. In discrete layout, the quay area gets separated as multiple berths and only one ship can be allocated at a time to a berth. For a continuous layout, the ships can be moored to any berth in the quay.

The BAP can be classified into two types based on the arrival of vessels to the terminal: static arrival and dynamic arrival. Static arrival of vessels means that all vessels arrive before scheduling starts, and this guarantees that any schedule will be feasible. Conversely, dynamic arrival means that vessels may arrive before or after the schedule is constructed, and this puts more constraints on the schedule, as not any schedule will be feasible. However, dynamic arrival is more realistic than static. Several styles of BAP models exist based on the layout of the berth.

i. Discrete layout: the quay is divided into a number of berths, and only one vessel can be moored at each berth at a time.

ii. Continuous layout: a vessel can be moored at any location along the quay with the condition of not exceeding the boundaries of the quay.

iii. Hybrid layout: the quay in this layout is partitioned into a set of berths. However, unlike in the normal discrete layout, small vessels in this layout can share one berth simultaneously, and large vessels can take more than one berth too.

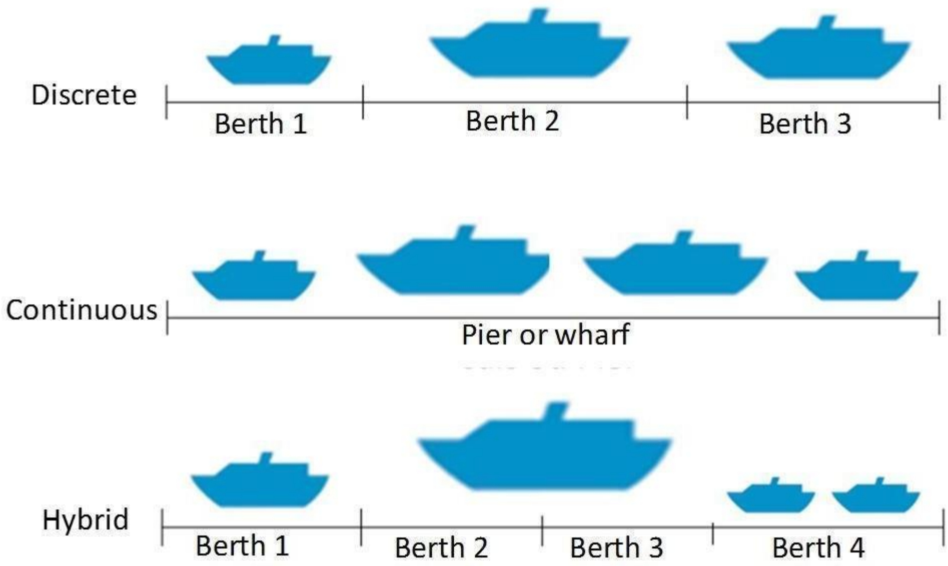

Figure 1.5: Classification of BAP

Models of the BAP also differ in the vessels' handling time parameter; some models consider handling time as a fixed parameter known for each vessel in advance, while other approaches deal with handling time as a berth-dependent parameter. There are different, yet fixed handling times for vessels at different berths. Handling time could also depend on the number of cranes assigned to vessels, work schedules of cranes, or a combination of all of the previous factors.

The operator makes an effort to minimize ship's waiting time by scheduling berth allocation efficiently. In addition, if the expected waiting time of a ship exceeds the criterion, the ship is directed to the external terminal to meet its promised turn-around time. As mentioned before, such re-assignment of ships to the external terminal imposes additional costs; therefore, scientific methods should be applied for the efficient berth allocation scheduling. Some authors have already addressed the berth allocation problem (BAP), minimising the waiting times for berths in order to have a quick turn-around time of a vessel.

For this paper, we consider berths having discrete layout with the objective function of minimising the delaytime in tide affected port. We are proposing a novel Mixed Integer Linear Programming (MILP) model with better computational efficiency than existing models in the above mentioned domain. The rest of the paper is structured as follows: Section 2 presents the literature review. Section 3 provides the problem description. Section 4 introduces the MILP model. Section 5 contains the computational experiments and finally, section 6 includes the conclusion of the paper.

CHAPTER 2

LITERATURE REVIEW

The paper focuses on discrete Berth Allocation Problem, therefore the previous studies on this domain are briefed below. In a discrete layout for a container terminal, the berth allocation choices are made on the allocation of berths individually to a vessel and the exposed time frame in which handling operations take place in the allocated berth.

Imai et al. (1997) proposed a Static Berth Allocation Problem (SBAP) in which the time that a ship spends at a port along with the total dissatisfaction in terms of berthing order in a discrete berth layout is minimized. This multi-objective approach overwrites the First Come-First Serve (FCFS) rule that has been conventionally processed on most of the service ports. However, they don't consider the dynamic arrival of ships. This study has been extended in Imai et al.,(2001) which focuses on Dynamic Berth Allocation Problem (DBAP) in a public berth system that reduces the total sum of ships waiting time and handling time. Obtaining an exact solution was difficult so they also formed a heuristic methodology dependent on the Lagrangian relaxation of DBAP. Imai et al.(2003) made changes to the existing BAP formulation to manage calling vessels with various service priorities. They employed these concepts in Multi-User Container Terminal (MUT) for cost-effective management and by providing unequal service priorities for each ship. Be that as it may, due to the complexity in the solution process, a genetic algorithm (GA) was proposed based on heuristics.

Hansen et al. (2003) reformulated the BAP to allocate a section of the quayside to ships arriving in a container port to reduce the sum total of their waiting and cargo handling time. They extended their formulation to accommodate constraints on size, the due dates of ships and abounded planning horizon. They further provided the computational performance of their model in comparison with the formulation of Imai et al., (2001) with more realistic problem instances. Cordeau et al.(2005) developed a multi-depot vehicle-routing problem with time windows (MDVRPT) formulation for the minimization of the total (weighted) service time for all ships. For discrete cases and continuous cases, a tabu search heuristics with their computational results were provided by them because of its poor computational performance in real size instances.

Imai et al. (2005) proposed BAP in continuous location space (Continuous Berth Allocation Problem-BAPC) to minimize the total service time where the ship's berthing position in the quay decides the handling time of the ship. They also put forward heuristics for the Continuous Berth Allocation Problem (BAPC heuristic) and its results. Imai et al. (2007) addresses the BAP at an indented terminal for mega containerships. They presented a more efficient way for serving mega-ships by providing a linear formulation for the problem and extending it to apply for an indented terminal. Hansen et. al. (2008) studied the minimum cost berth allocation problem (MCBAP) and formulated a mathematical model which was an extension of the Imai et. al. (2001). They proposed a tabu-search algorithm for solving this NP-hard problem.

Nikolic et al., (2017) developed a new mathematical formulation to find an assignment of ships to berth in the way to minimize the sum total of time all ships spend in the system. This is a modified version of Imai et al., (2001) developed to mitigate its weakness. They tested both models with a sample problem example to point out the observed drawbacks.

Golias et al (2014) model is a discrete and dynamic model in which berth length, draft,depth of berth, number of vessels in the quay at the beginningof the time horizon, uncertainty of arrival time andloading and unloading time, and the effect of access channeldepth changes due to the tide are not considered.Sheikholeslami et al. (2014) studied berth planning ofSHRA port, Iran. SHRA port is a shallow port whose berthingarea has been connected to vessels' anchoring area viaan access channel. Low depth of port-arrival channels hasa significant effect on the coast operations. Since dredgingoperation is costly, presenting a model which can optimizecoast operations' cost is valuable.

Sheikholeslami et al. (2019) provided a model in which the constraints in the previous model are considered . As mentioned previously, the special case of SHRA port is its access channel depth changes due to the tide.Each vessel has to wait for its suitable water level to enterthe port.

2.1 SCOPE OF THE BERTH ALLOCATION PROBLEM

The main focus of this study is the discrete berth allocation problem, and henceforth the earlier studies on this domain are now presented. In berth allocation at a container terminal with discrete layout, decisions are made on the allocation of an individual berth to a vessel and the time period during which vessels are subjected to handling operations in the allocated berth.

Dynamic Berth Allocation Problem (DBAP) and its formulation are presented by Imai et al. (2001) in which the berth space is represented as a finite set of berthing points. This problem is characterized by the dynamic arrival of vessels to these berthing points. It was an extension of the Static Berth Allocation Problem (SBAP) studied by Imai et al. (1997), which considers the case where all ships are assumed to arrive at the port before the earliest available time of the berths.

Sheikholeslami (2019) proposed a model with the objective function that minimizes the sum of delay time for the arriving ships at a tide-affected port . The author specifically takes into consideration the SHRA port of Iran for minimizing the delay time in scheduling the vessels

80% of global trade by volume and over 70 % of global trade by value are carried by sea.The huge bulk of the international cargo is transported by the increasing number of containers , tankers and freight vessels.On the other hand , berths are the gift to a nation ;points of strategic location where vessels can be moored and handled. It is not possible to simply increase the number of ports due to geographic, economic, environmental and marine limitations.Hence the most pragmatic and profitable approach is the effective utilisation of the berth spaces. It is imperative to develop new and efficient mathematical models so that we can produce the optimum utilisation of the available berth spaces.In some ports such as Shahid Rajaee (SHRA)port of Iran,the berths and the connecting shipping channels are affected by the tidal rise and fall of water levels. Therefore in these ports, we need to incorporate the tidal factor as well.

CHAPTER 3

PROBLEM DEFINITION & OBJECTIVES

3.1 PROBLEM DEFINITION

The objective of this problem is to allocate the arriving ships to various berths and prepare the schedule of ships at a tide affected port; in a way so as to minimize the total delay time in the operation of the port. Hence in general, it is a minimization problem.

The problem can be formulated as a mathematical model that allocates a set of j vessels on i berths efficiently by considering the tidal effects.In this study we propose a new Mixed Integer Linear Programming (MILP) and compare it with existing literature.

The problem aims at developing a computationally efficient mathematical model for solving the berth allocation problem in a tide affected port with dynamic arrival of ships, which finds a set of ship assignments to minimize the sum of the time they spent waiting for berths and cargo handling, and thereby increases the operational effectiveness and reduces cost, time and energy.

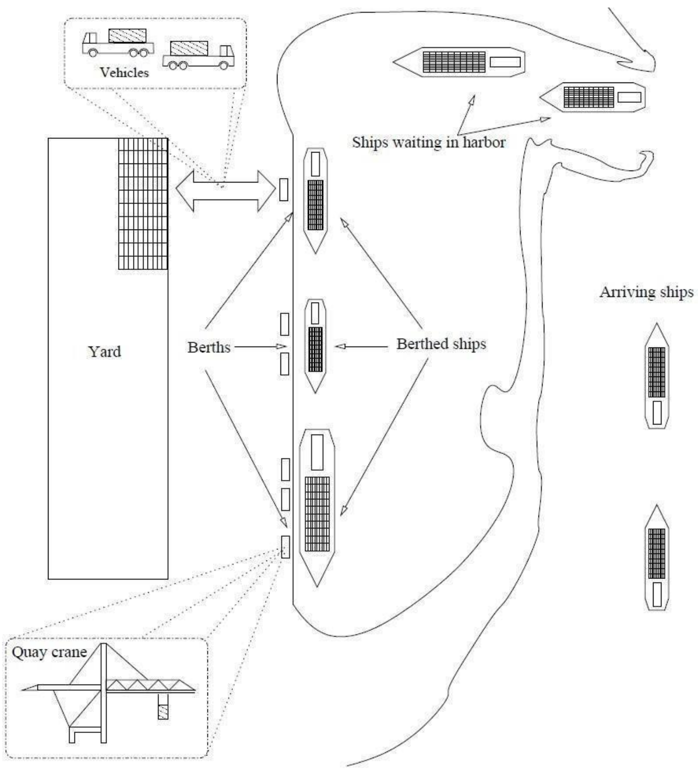

Figure 3.1: DBAP

3.2 TIDAL STUDY

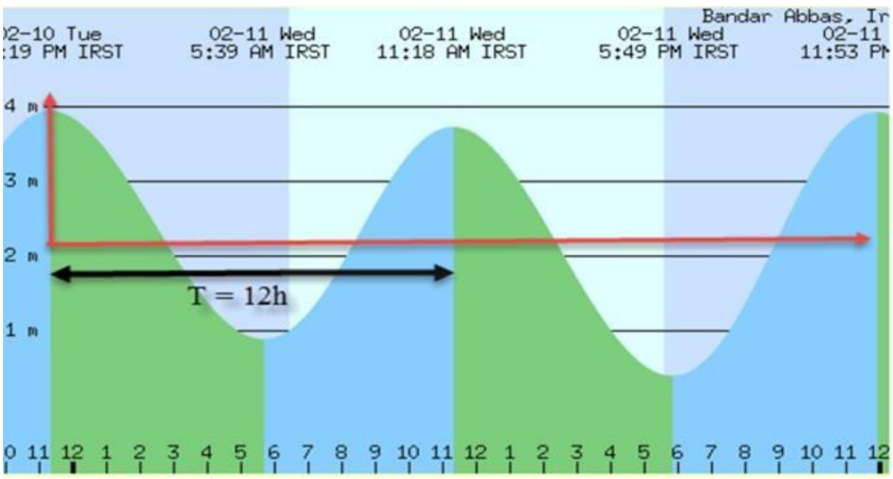

Figure 3.2:Tidal cycle

The port is connected to open sea through a channel which is affected by the tidal cycle . This leads to a berth allocation model, where the vessels are allocated mainly by considering tidal time windows.

, the first 24 h of access channel water level changes is shown in Fig.3.3 As shown in the figure, the plot is approximately like trigonometric functions. As can be seen, there is a 12-h gap between two subsequent ebbs or flows, so it can be said that this plot is a periodic function with a period of 12.

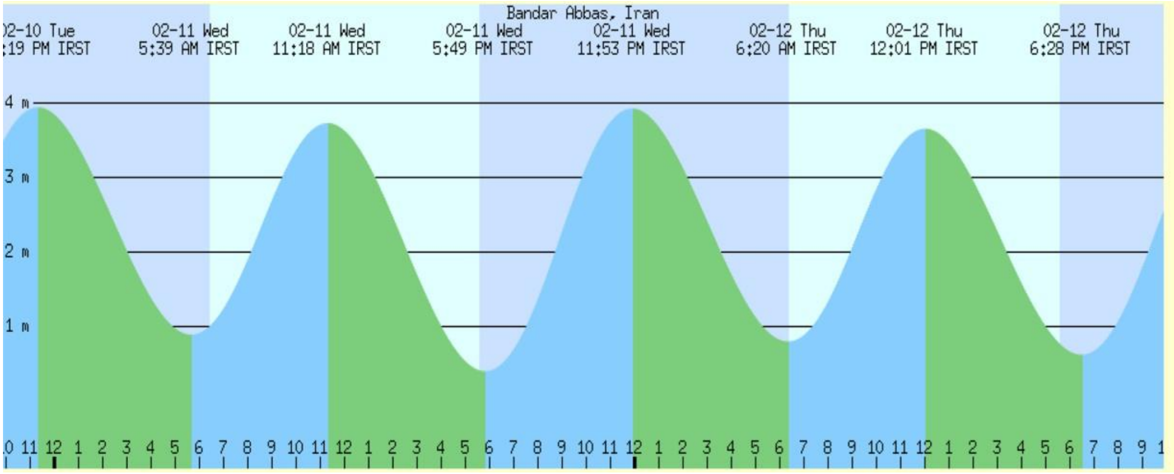

fig 3.3:recorded tidal data

Tide recorded data at http://tides .mobilegeographics.com/ were used for the purpose of this study. At the URL above,tide data are displayed like Fig 3. graphs. Exact tide data were estimated from these hydrographical plots. For example,Fig. 3 shows water depth at Bandar-Abbas port, Iran, in comparison with sea level on February 11, 2009. Vertical Axis represents water depth, and horizontal axis represents day hours. As can be seen, water depth at 11:30 A.M. isabout 3.7 m.

From the eqn $D_t=D_b+\Delta D_t$ a is constant and equals to half of the maximum amplitude of water level, T is the period, and b is the equation constant which equals to the minimum difference of water level with sea level plus half of the swing amplitude. For example, on February 11, 2009,minimum and maximum water levels were about 0.7 and3.95 ms, respectively. Time intervals were 15 min, and thus,a = 3.95−0.7/2, b = 2.10, T = 12 × 4 = 48.Db is assumed to be the ratio of the access channel depth over sea depth and Dt is assumed to be the access channel depth at the moment .

Alpha matrix is the main parameter which determines whether a vessel will be able to pass through the access channel at a particular time.Developed a method to generate the alpha matrix with ship designation across the rows and tidal slots across the column. Value 1 indicates the ship can pass through the channel and vice versa .

3.3 OBJECTIVES

1. To develop a mathematical model which minimizes the total delay ships spend during loading and unloading of containers.

2. To solve the berth allocation problem with lesser computational effort.
3. To apply operations research techniques in port operations.
4. To help the decision-makers to increase the service level of port operations

3.4 METHODOLOGY

1. Propose a Mixed Integer Linear Programming (MILP) model
2. Generate datasets to test the effectiveness of the model
3. Simulation study is to be conducted using the software IBM ILOG CPLEX Optimization Studio 12.9 installed on a Desktop with the latest configuration.
4. Computational study with sample problem instances is conducted to evaluate the performance of the proposed model.

CHAPTER 4

ANALYSIS OF THE EXISTING MODEL

Several models have been proposed based on BAP. Among these, the model " A Dynamic and Discrete Berth Allocation Problem in Container Terminals Considering Tide Effects " by Sheikholeslami et al.(2019) is the most studied and applied in port operations. . The findings of Sheikholeslami et al.(2019) were analysed and verified.During the analysis, a drawback of Sheikholeslami et al.(2019) was observed.

4.1 Existing Model

The Dynamic Berth Allocation Problem for a Tide affected Port : Sheikholeslami et al.(2019)

The formulation of DBAP is shown:

Minimize $\square\square\square \sum_{\square\in\square} (\square\square_{\square} - \square_{\square})$ (1)

Subjected to the constraints

$$\sum_{\square\in\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (2)$$

$$\square_{\square} + \sum_{\square\neq\square\in\square\square\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (3)$$

$$\square_{\square} + \sum_{\square\neq\square\in\square\square\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (4)$$

$$f_a + f_b \leq 3 - x_{ia} - x_{ib} = 1 \qquad \forall\, i \in I, a, b \in J, a \neq b \qquad (5)$$

$$e_a + e_b \leq 3 - x_{ia} - x_{ib} = 1 \qquad \forall\, i \in I, a, b \in J, a \neq b \qquad (6)$$

$$y_{ab} - 1 \leq x_{ia} - x_{ib} \leq 1 - y_{ab} \qquad \forall\, i \in I, a, b \in J, a \neq b \qquad (7)$$

$$w_{ji} \leq \alpha_{jt} \qquad \forall\, j \in J, \quad \forall\, t \in T \qquad (8)$$

$$q_{jt} \leq \alpha_{jt} \qquad \forall\, j \in J, \quad \forall\, t \in T \qquad (9)$$

$$\sum_{\square\in\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (10)$$

$$\sum_{\square\in\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (11)$$

$$\square\square_{\square} = \sum_{\square\in\square} \square\ \square\ \square_{\square\square} \qquad \forall \square \in \square \qquad (12)$$

$$\square\square_{\square} = \sum_{\square\in\square} \square\ \square\ \square_{\square\square} \qquad \forall \square \in \square \qquad (13)$$

$$St_j \geq A_j \qquad \forall \square \in \square \qquad (14)$$

$$Ft_j \geq St_j + H_j \qquad \forall \square \in \square \qquad (15)$$

$$St_b \geq Ft_a - M\,(1\text{-}y_{ab}) \qquad a,b \in J,\ a \neq b \qquad (16)$$

$$(L_i\text{-}l_j)x_{ij} \geq 0 \qquad \forall\, i \in I, j \in J, \qquad (17)$$

$$(D_i\text{-}d_j)x_{ij} \geq 0 \qquad \forall\, i \in I, j \in J, \qquad (18)$$

$$V_i \ \text{x}\ x_{ij} \leq St_j \qquad \forall\, i \in I, j \in J, \qquad (19)$$

Objective function #1 is the minimization of the total delay time of vessels. Constraint set #2 ensures that each berth only services one vessel. Constraint set #3 guarantees that each vessel is serviced as the first vessel or after the other vessels.Constraint set #4 causes each vessel to be serviced as the last vessel or after the other vessels.Constraint sets #5 and #6 declare that in each berth only one vessel can be serviced as the first vessel and one vessel can be serviced as the last vessel.

Constraint set #7 assures that a vessel can be serviced after another vessel if and only if the two vessels are allocated to the same berth. Constraint sets #8 and #9 guarantee appropriate access channel depth for vessels at mooring or exciting times. Constraint #10 and #11 ensure that each vessel moors and leaves the port only once. In constraint set #12, mooring time of each vessel is computed.

Constraint set #13 is about the departure time of each vessel. Constraint set #14 guarantees that no vessel moors before its arrival at the port. Constraint set #15 assures vessels departure from the berth after finishing loading and unloading operations. Unlike the available models in the literature, in the proposed model, all the vessels cannot leave the port immediately after the loading and unloading operations. This is because of inappropriate access channel depth at the end of the loading and unloading operations. In this case, vessels have to wait for the access channel water level to reach an appropriate level. Constraint set #16 guarantees that no time disorder in mooring scheduling of vessels at a berth happens. Constraint sets #17 and #18 ensure enough berth length and draft depth, respectively. Constraint set #19 shows that the servicing time of a vessel at a berth starts when the berth is vacant at the beginning of the time horizon. Relative mooring position of vessels is computed by constraint set #20.

4.1.1 Sample Problem Instance

Let us consider the following problem instances. Suppose that we have 5 ships and 3 berths with the data given in tables 4.1.1 and 4.1.2

Parameter	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Length	340	274	184	321	236
Arrival	80	123	310	408	118
Handling	16	19	11	19	19
Departure	114	162	329	437	155
Draft	14	12	9	13	11

Table 4.1 vessels data

Parameter	Berth 1	Berth 2	Berth 3
Length	240	230	353
Draft	11	13	19
Available time	118	129	172

Table 4.2 Berths data

4.1.2 Solution

Using the above mentioned datas optimization is done by IBM CPLEX and results are decoded and analysed manually . The time taken by the optimization studio for the existing model was 0.46 seconds with an objective value 1.9000000000e+02 .

Berth 1	Berth 2	Berth3
		Vessel 5
		Vessel 2
		Vessel 1
		Vessel 3
		Vessel 4

Table 4.3 Vessel allocation

4.1.3 Limitations

Results from CPLEX were manually taken into consideration and prepared a gantt chart for the manual analysis and found some shortcomings

1. The existing mathematical model assigned ships to a single berth,not utilizing the full berth spaces .
2. It leads to the idle time of available resources .
3. Wastage of resource
4. Time consumed for the whole schedule will be greater since only one berth is used

CHAPTER 5

MODEL DEVELOPMENT

The BAP problem under study is modeled using a newly proposed mixed-integer linear programming (MILP) model. The MILP model is explained below. For the proposed formulations, the assumptions taken where the following,

- At a specified time, one berth can handle only one vessel.
- Tidal factors like water depth at the access channel are taken into consideration.
- The allocated berth determines the handling time of a vessel since vessel handling infrastructure is not the same for all berths. That is they are not identical berths.
- Vessel handling actions are not affected by maintenance activities.
- Arrival time of vessels is known to the operations manager.

- *Sets*
- $I = \{1, 2\ldots, K\}$ Berths' set
- $J = \{1, 2\ldots, N\}$ Vessels' set
-
- *Parameters*
- A_j Vessel j estimated arrival time
- H_j Vessel j estimated loading and unloading time
- l_j The length of vessel j including vessel safe distance for mooring
- d_j Draft of vessel j
- R_j Scheduled departure time of vessel j
- L_i The length of berth i
- D_i The depth of berth i
- α_{jt} Binary matrix, 1 if vessel j at time t can go through the access channel, otherwise 0.
- M A very large number
- v_i The time when berth i become empty at the beginning of the time horizon
- Y_i Coordinates of the middle of berth i
-
- *Decision variables*
- St_j Mooring time of vessel j
- Ft_j Departure time of vessel j

- x_{ij} Binary variable, 1 if vessel j moors at berth i, otherwise 0
- f_j 1 if vessel j is the first vessel that moors at its allocated berth,otherwise 0
- e_j 1 if vessel j is the last vessel that moors at its allocated berth, otherwise 0
- y_{ab} If $x_{ia} = x_{ib}$ and vessel b is serviced immediately after vessel a 1,otherwise 0
- w_{jt} Binary variable, 1 if vessel j moors at time t it is set, otherwise 0
- q_{jt} Binary variable, 1 if vessel j departs the port at time t it is set, otherwise 0

- αjt -the binary matrix which provides a value 1 if the channel draft is greater than that of a vessel and 0 if passage is not possible.Alpha matrix makes this work one of a kind.
- xij - binary variable takes the value 1 if the vessel j moors at the berth i ie. allocation.

● □□ - - if the value is 1 ,then it denotes that vessel j is the first servicing vessel for the berth it was assigned (helps to identify the first vessel that will be moored to its allocated berth).
● □□ - if the value is 1 ,then it denotes that vessel j is the last servicing vessel for the berth it was assigned.
● □□□ -A binary variable which denotes the order of service at the berth. □_□□ = 1 means vessel b need to be serviced only after □ ie .sequence
● wjt and □□□ - binary variable showing the mooring and departure of vessel related to time . Value 1 denotes that the vessel j would be moored at time t .(same for departure)

Proposed model

objective function $\square\square\square \sum_{\square\in\square} (\square\square_{\square} - \square_{\square})$ (1)

subjected to

$$\sum_{\square\in\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (2)$$

$$\square_{\square} + \sum_{\square\neq\square\in\square\square\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (3)$$

$$\square_{\square} + \sum_{\square\neq\square\in\square\square\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (4)$$

$$f_a + f_b + x_{ia} - x_{ib} \leq 3 \qquad \forall\, i \in I, a,b \in J,\ a \neq b \qquad (5)$$

$$e_a + e_b + x_{ia} - x_{ib} \leq 3 \qquad \forall\, i \in I, a,b \in J,\ a \neq b \qquad (6)$$

$$y_{ab} - 1 \leq x_{ia} - x_{ib} \leq 1 - y_{ab} \qquad \forall\, i \in I, a,b \in J,\ a \neq b \qquad (7)$$

$$w_{ji} \leq \alpha_{jt} K \qquad \forall\ j \in J, \quad \forall\ t \in T \qquad (8)$$

$$q_{jt} \leq \alpha_{jt} \qquad \forall\ j \in J, \quad \forall\ t \in T \qquad (9)$$

$$\sum_{\square\in\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (10)$$

$$\sum_{\square\in\square} \square_{\square\square} = 1 \qquad \forall \square \in \square \qquad (11)$$

$$\square\square_{\square} = \sum_{\square\in\square} \square\ \square\ \square_{\square\square} \qquad \forall \square \in \square \qquad (12)$$

$$\square\square_{\square} = \sum_{\square\in\square} \square\ \square\ \square_{\square\square} \qquad \forall \square \in \square \qquad (13)$$

$$St_j \geq A_j \qquad \forall \square \in \square \qquad (14)$$

$$Ft_j \geq St_j + H_j \qquad \forall \square \in \square \qquad (15)$$

$$St_b \geq Ft_a - M\,(1\text{-}y_{ab}) \qquad a,b \in J,\ a \neq b \qquad (16)$$

$$(L_i\text{-}l_j)x_{ij} \geq 0 \qquad \forall\, i \in I, j \in J, \qquad (17)$$

$$(D_i\text{-}d_j)x_{ij} \geq 0 \qquad \forall\, i \in I, j \in J, \qquad (18)$$

$$V_i \ \ \mathrm{x}\ x_{ij} \leq St_j \qquad \forall\, i \in I, j \in J, \qquad (19)$$

Objective function #1 is the minimization of total delay time of vessels. Constraint set #2 ensures that each berth only services one vessel. Constraint set #3 guarantees that each vessel is served as the first vessel or after the other vessels. Constraint set #4 causes each vessel to be serviced as the last vessel or after the other vessels. Constraint sets #5 and #6 declare that in each berth only one vessel can be serviced as the first vessel and one vessel can be serviced as the last vessel.

Constraint set #7 assures that a vessel can be serviced after another vessel if and only if the two vessels are allocated to the same berth. Constraint sets #8 and #9 guarantee appropriate access channel depth for vessels at mooring or exciting time. Constraint #10 and #11 ensure that each vessel moors and leaves the port only once. In constraint set #12, mooring time of each vessel is computed. Constraint set #13 is about the departure time of each vessel. Constraint set #14 guarantees that no vessel moors before its arrival at the port. Constraint set #15 assures vessels departure from the berth after finishing loading and unloading operations.

Unlike the available models in the literature, in the proposed model, all the vessels cannot leave the port immediately after the loading and unloading operations. This is because of inappropriate access channel depth at the end of the loading and unloading operations. In this case, vessels have to wait for access channel water level to reach an appropriate level. Constraint set #16 guarantees that no time disorder in mooring scheduling of vessels at a berths happen. Constraint sets #17 and #18 ensure enough berth length and draft depth, respectively. Constraint set #19 shows that serving time of a vessel at a berth starts when the berth is vacant at the beginning of the time horizon.

5.1 Numerical illustration of the proposed DBAP model

Below given are the parameter values of sample problem instance with 5 ships and 3 berths

No: of vessels (j) 5 No: of berths (i) 3

vessel draft (l_j)	Arrival time (A_j)	Handling time(H_j)	Departure Time (R_j)	Vessel draft (d_j)
340	80	16	114	14
274	123	19	162	12
184	310	11	329	9
321	408	19	437	13
236	118	19	155	11

table 5.1 Vessel data 5*3

Berth length (L_i)	Berth draft(D_i)	Availability time (v_i)
240	11	118
230	13	129
353	19	172

table 5.2 Berth data 5*3

5.2 Solution

Solution obtained from the IBM CPLEX ILOG 12.9 solver in the case of 5 ships and 3 berths:

Objective = 1.0100000000e+02 Solution time = 0.25 sec.

Variable Name	Solution Value
Ft1	216.000000
Ft2	197.000000
Ft3	321.000000
Ft4	427.000000
Ft5	137.000000
x3,1	1.000000
x3,2	1.000000
x2,3	1.000000
x3,4	1.000000
x1,5	1.000000
y2,1	1.000000
f2	1.000000
f3	1.000000

y1,4	1.000000
f5	1.000000
ea3	1.000000
ea4	1.000000
ea5	1.000000
w1,200	1.000000
w2,172	1.000000
w3,310	1.000000
w5,118	1.000000
q1,216	1.000000
q2,197	1.000000
q3,321	1.000000
q4,427	1.000000
q5,137	1.000000
St1	200.000000
St2	172.000000
St3	310.000000
St5	118.000000

All other variables in the range 1-4976 are 0.

5.3 Result Analysis and comparison

The mathematical model proposed by Sheikholeslami et al.is referred to as the Old model .Results obtained from optimization studio undergoes a manual analysis and a comparison have been carried out on the same generated data set .

Analysis concluded that the shortcoming found in Sheikholeslami et al.s mathematical model was rectified . In the proposed model the allocation was done in multiple available berths .

Berth 1	Berth 2	Berth 3
		Vessel 5
		Vessel 2
		Vessel 1
		Vessel 3
		Vessel 4

Table 5.3 Old Allocation

Berth 1	Berth 2	Berth3
Vessel 5	Vessel 3	Vessel 2
		Vessel 1
		Vessel 4

Table 5.4 New Allocation

CHAPTER 6

COMPUTATIONAL EXPERIMENTATION

The computational experiment of the new MILP model with the existing model is presented here. For the evaluation of computational experiments,realistic instances are randomly generated by following existing literature. The generation algorithm of problem instances and the computational study of the proposed MILP model with the existing model is described below.

6.1 GENERATION OF PROBLEM INSTANCES

The systematic models in dynamic berth allocation problems can be tested computationally using existing data generation techniques.The data sets were generated using computer programming for ship and berth in accordance with existing literature. The vessel data (length, arrival, handling time, departure, and drafts) and berth data (length, drafts and available time) are generated for the scheduling horizon.The data generation method used is Random generation within the limits of scientific formulae.

6.2 COMPUTATIONAL PERFORMANCE ON IBM ILOG CPLEX 20.1.0

The Old existing model and proposed mathematical models were coded in CPP and stored in a Linear Programming(LP) file for corresponding datasets. They are then executed on a laptop equipped with Intel Core(TM) i7 7500U 8th gen 2.70 GHz CPU and 8 GB RAM. The commercial software IBM ILOG CPLEX 20.1.0 was used to solve the integer linear programming model for different problem instances. The time limit for computation of each model was set at 3600 seconds i.e., 1 hour. Once the CPLEX solution-finding process reached the time limit, it aborted and outputted the incumbent best solution .

```
Clique table members: 2750.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 8 threads.
Root relaxation solution time = 0.00 sec. (1.92 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt     Gap

*     0+    0                         709.0000       67.0000           90.55%
      0     0       67.0000    25     709.0000       67.0000       39   90.55%
      0     0      103.6801    32     709.0000      Cuts: 26       77   85.38%
      0     0      136.5758    32     709.0000      Cuts: 37       94   80.74%
      0     0      136.5758    32     709.0000      Cuts: 12       98   80.74%
      0     0      142.3897    38     709.0000      Cuts: 10      148   79.92%
      0     0      143.8529    36     709.0000      Cuts: 10      159   79.71%
Detecting symmetries...
*     0+    0                         190.0000      143.8529           24.29%
      0     0        cutoff           190.0000      190.0000      164    0.00%
Elapsed time = 0.14 sec. (99.76 ticks, tree = 0.01 MB, solutions = 2)

Clique cuts applied:  5
Cover cuts applied:  2
Implied bound cuts applied:  13
Mixed integer rounding cuts applied:  1
Zero-half cuts applied:  3
Multi commodity flow cuts applied:  2
Gomory fractional cuts applied:  6

Root node processing (before b&c):
  Real time             =    0.14 sec. (99.98 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    0.14 sec. (99.98 ticks)

Solution pool: 2 solutions saved.

MIP - Integer optimal solution:  Objective =  1.9000000000e+02
Solution time =    0.14 sec.  Iterations = 164  Nodes = 0
Deterministic time = 99.99 ticks  (714.22 ticks/sec)
```

figure 6.1 Interface of IBM ILOG CPLEX 20.1.0

6.3 RESULTS

The derived results are then tabulated to derive conclusions regarding the efficiency of the Proposed model . The tabulated results were given in the table 6.3.1 where **f_j represents the first ship serviced in its allocated berth , e_j represents the final vessel serviced in its allocated berth and the objective function values.**

			OLD			NEW		
No. of ships	No. of berths	Prob No	f_j	e_j	Obj	f_j	e_j	Obj
		1	4	5	3	1	5	3
		2	1	4	-35	1	4	-35
		3	3	4	-52	1,3	1,4	-52
		4	1	3	-30	1,5	1,3	-30
	2	5	1	4	-52	1,4	2,4	-52
		1	4	3	75	4,5	1,3	6
		2	2	4	-24	1,2	1,4	-24
		3	4	1	-33	3,4,5	1,2,4	-57
		4	1	2	-43	1,3	2,4	-57
5	3	5	3	2	52	2,3,4	1,2,4	17

Table 6.1 : computational result 1

The above table 6.1 represents the results of a problem instance for 5 ships and 2 berths ,5 ships and 3 berths .From that table we can observe that the objective function of new model is same or less than the old model.In small case instances,the improvement in the objective function is less.

			OLD	NEW	
No. of ships	No. of berths	Prob No	execution time	execution time	% REDUCTION IN EXECUTION TIME
		1	0.13	0.11	15.38
		2	0.14	0.14	0.00
		3	0.09	0.08	11.11
		4	0.11	0.09	18.18
	2	5	0.06	0.05	16.67
		1	0.09	0.06	33.33
		2	0.13	0.13	0.00
		3	0.14	0.09	35.71
		4	0.08	0.06	25.00
5	3	5	0.16	0.08	50.00

Table 6.2 Computational result 2

From table 6.2,the execution time required is less for the new model,with a time difference ranging from 0-0.16 than the old model.

			OLD			NEW		
No. of ships	No. of berths	Prob No	f_j	e_j	Obj	f_j	e_j	Obj
		1	4	6	-69	4	6	-69
		2	5	1	-48	2,5	1,2	-48
		3	6	2	-54	3,6	2,5	-67
		4	2	1	-47	2,4	1,6	-62
	2	5	1	2	-66	1	2	-66
		1	2	5	43	1,2	2,5	43
		2	3	4	-63	3,6	1,4	-74
		3	2	3	-37	1,2	3,4	-47
		4	1	6	48	1,3,5	3,5,6	-45
6	3	5	5	4	162	2,5	2,4	-66

Table 6.3 : Computational result 3

The following table 6.3 represents the results of a problem instance for 6 ships and 2 berths,6 ships and 3 berths.We can observe that the objective function of the new model is same or less than the old model.

			OLD	NEW	
No. of ships	No. of berths	Prob No	execution time	execution time	% REDUCTION IN EXECUTION TIME
		1	0.21	0.17	19.05
		2	0.23	0.17	26.09
		3	0.22	0.09	59.09
		4	0.16	0.09	43.75
	2	5	0.16	0.14	12.50
		1	0.11	0.09	18.18
		2	0.2	0.09	55.00
		3	0.36	0.11	69.44
		4	0.34	0.16	52.94
6	3	5	0.73	0.13	82.19

Table 6.4 : Computational results 4

From table 6.4,the execution time required is less for the new model,with a time difference ranging from 0.4-0.60 than the old model.

			OLD			NEW		
No. of ships	No. of berths	Prob No	f_j	e_j	Obj	f_j	e_j	Obj
		1	3	10	-15	3,8,11	1,10,14	-99
		2	10	6	722	3,10,14	1,4,8	592
		3	14	8	-79	2,10,14	6,10,12	-111
		4	6	2	11	3,6,14	2,7,9	-133
	2	5	9	8	-30	3,9,13	8,13,14	-79
		1	3	10	-15	3,8,11	1,10,14	-99
		2	10	6	722	3,10,12	1,4,6	592
		3	14	8	-79	2,10,14	6,8,12	-111
		4	6	2	11	3,6,14	2,7,9	-133
14	3	5	9	8	-30	3,9,13	8,13,14	-79

Table 6.5 : Computational results 5

The above table 6.5 represents the results of a problem instance for 14 ships and 2 berths ,14 ships and 3 berths .From that table we can observe that the objective function of new model is less than the old model,i.e new model outdid old model in all problem instances.

No. of ships	No. of berths	Prob No	OLD	NEW	% REDUCTION IN EXECUTION TIME
		1	6.52	1.39	78.68
		2	639.84	175.92	72.51
		3	3.92	1.31	66.58
		4	32.69	0.34	98.96
	2	5	2.34	0.86	63.25
		1	6.16	1.06	82.79
		2	622.42	154.55	75.17
		3	3.31	1	69.79
		4	31.69	0.36	98.86
14	3	5	1.92	0.66	65.63

Table 6.6 : Computational results 6

From table 6.6,the execution time required is less for the new model,with much lesser time deviation than the old model.

			OLD			NEW		
No. of ships	No. of berths	Prob No	f_j	e_j	Obj	f_j	e_j	Obj
		1	5	7	75	3,7	2,9	26
		2	16	19	203	3,7	2,9	123
		3	6	10	-90.1	3,15	5,11	-152
		4	18	6	218	4,15	6,17	26
	2	5	12	18	-24	4,11	9,13	-102
		1	10	2	134	6,7,16	3,11,15	-175
		2	17	2	105	14,15,17	2,7,10	-108
		3	10	20	-171	12,16,19	8,10,11	-130
		4	17	10	-107	3,4,6	7,18,20	-200
20	3	5	15	9	300	1,4,9	14,17,19	-32

Table 6.7 : Computational results 7

The following table 6.7 represents the results of a problem instance for 20 ships and 2 berths ,20 ships and 3 berths .From the above table we can observe that the objective function of the new model is less than the old model,i.e new model outdid the old model in all problem instances.

No. of ships	No. of berths	Prob No	OLD	NEW	% REDUCTION IN EXECUTION TIME
		1	3600	17.09	99.53
		2	55.92	16.88	69.81
		3	36.69	15.08	58.90
		4	282.58	18.7	93.38
	2	5	44.2	11.91	73.05
		1	225.97	15.34	93.21
		2	1175.45	11.63	99.01
		3	40.53	8.8	78.29
		4	34.58	0.59	98.29
20	3	5	1255.69	11.41	99.09

Table 6.8 : Computational results 8

From table 6.8,the execution time required is less for the new model,with a time difference by a huge margin.

			OLD			NEW		
No. of ships	No. of berths	Prob No	f_j	e_j	Obj	f_j	e_j	Obj
		1	12	6	210	2,4,6,17,24	5,7,11,18,21	80
		2	10	9	234	4,10,12,17	9,14,20,24	-130
		3	4	6	-153	4,10,21,24	1,5,6,21	-298
		4	6	25	340	6,10,16,18,24	7,12,13,23,25	-108
25	5	5	23	22	1109	12,14,20,23	7,22,23,25	-151

Table 6.9 : Computational results 9

The following table 6..9 represents the results of a problem instance for 25 ships and 5 berths.From the following table we can observe that the objective function of new model is less than the old model by a huge margin.

No. of ships	No. of berths	Prob No	OLD	NEW	% REDUCTION IN EXECUTION TIME
		1	3625	96.11	97.35
		2	3675	55.72	98.48
		3	262.67	30.98	88.21
		4	3604	65.69	98.18
25	5	5	3620	74.22	97.95

Table 6.10 : computational result 10

From the table 6.10,the execution time required is less for the new model,this time by very high margin.

6.4 ANALYSIS

In order to get a clearer idea on the stark difference in the computational efficiencies of both models, the results are tabulated and an analysis is done. From the tables above (table 6.1,6.3,6.5,6.7,6.9) we found that the Old model was only assigning vessels to a single berths ie the column f_j shows only a vessel that means only a vessel is considered as the first vessel serviced even if there are other available berths . The column e_j also has a single vessel that shows only a vessel is considered as the last vessel being serviced in that whole schedule that means only one berth is made use of during the course . whereas the New proposed model shows multiple ships in the f_j and e_j column that means multiple ships were considered as first vessels being serviced and last vessel being serviced so it only happens if multiple berths are made use of .

Coming to the analysis of execution time reduction tables 6.2 ,6.4,6.6,6.8 an average of 5 instances were taken from Old model and New model for 5,6,10,12,14 and 20 ships .The percentage time reduction achieved via New model is of 21.24% for the least number of ships (5 ships) and it reaches to 98.11% when the number of ships become 20.This shows that the New Model is more efficient in case of execution time.

Time of Execution			
Number of ships	Existing	Proposed	%Reduction in execution time
5	0.113	0.089	21.24
6	0.265	0.124	53.21
10	1.8845	0.53	71.88
12	5.583	1.4725	73.63
14	135.081	33.745	75.02
20	675.161	12.743	98.11

Table 6.11 : Analysis of execution time

6.5 GRAPHICAL VISUALIZATION

The following are the graphical representations for the 5,14,20 ships problem instances respectively. The two models are given along the X axis with multiple problem instances, the objective function along the Y axis. The graphical representation would enable an easy interpretation of the extent of reduction in objective function .

Figure 6.2: Comparison of models for 5 ships and 3 berths(objective function)

Figure 6.3: Comparison of models for 14 ships and 3 berths(objective function)

Figure 6.4 : Comparison of models for 20 ships and 2 berths(objective function)

The succeeding are the graphical representations for the 5,14,20 ships problem instances respectively. The two models are given along the X axis with multiple problem instances, the execution time along the Y axis. The graphical representation would enable an easy interpretation of the extent of reduction in objective function .

Figure 6.5: Comparison of models for 5 ships and 3 berths(execution time)

Figure 6.6: Comparison of models for 14 ships and 3 berths(execution time)

Figure 6.7: Comparison of models for 20 ships and 2 berths(execution time)

CHAPTER 7

CONCLUSION

The Berth Allocation Problem (BAP) involves decisions on how to allocate the berth space and to sequence maritime vessels that are to be loaded and unloaded at a container terminal involved in the maritime logistics. As the berth is a critical resource in a container terminal, an effective use of it is highly essential to have efficient berthing and servicing of vessels and to optimize the associated costs. Dynamic Berth Allocation Problem (DBAP) aims to minimize the sum of total service time of ships in a container terminal.

With the expanding globalisation and associated global trade ; the shipping industry is demanding increased efficiency and transports against limited ports, geographic , environmental and economic restrictions.

With the marine logistics industry being the most traffic intensive and a bottleneck ,it is important to optimise the docking allocation to reduce the congestion and delay at ports.

Research has to be put into improving the berth allocation problem.

The model has been successful in allocating vessels to multiple berths with the least delay time in tide affected ports .

The mathematical model we've proposed has upgraded the capabilities of the existing one and will immensely help in the scheduling of ships in international marine terminals. .

This model would also be an asset to the port operations administration in the effective allocation of berths in tide affected ports .This project will also aid in further research in this field.Future work can include the implementation of the proposed formulation approaches in other variants of the berth allocation problems such as given in Imai et al. (2003, 2008), Cordeau et al. (2005), and in problems with continuous berth layouts. Future work can also look at the development of efficient lower bounds in order to evaluate heuristic solutions in very large-sized problem instances.

REFERENCES

Imai, A., Nishimura, E., Papadimitriou, S. (2001). The dynamic berth allocation problem for a container port. Transportation Research Part B: Methodological, 35(4), 401–417.

Miloˇs Nikolic, D. T. (2017). New mathematical formulations of the dynamic berth allocation problem. 3rd Logistics International Conference Belgrade,Serbia 2017.

Imai, A., Nishimura, E., Papadimitriou, S. (2003). Berth allocation with service priority. Transportation Research Part B: Methodological, 37(5), 437–457.

Imai, A., Sun, X., Nishimura, E., Papadimitriou, S. (2005). Berth allocation in a container port: Using a continuous location space approach. Transportation Research Part B: Methodological, 39(3), 199–221.

Bierwirth, C., Meisel, F. (2010). A survey of berth allocation and quay crane scheduling problems in container terminals. European Journal of Operational Research, 202, 615–627.

Sheikholeslami, A., Mardani, M., Ayazi, E., & Aref Kahani, H. (2019). A Dynamic and Discrete Berth Allocation Problem in Container Terminals Considering Tide Effects. Iranian Journal of Science and Technology, Transactions of Civil Engineering. doi:10.1007/s40996-019-00239-1

Golias M, Portal I, Konur D, Kaisar E, Kolomvos G (2014) Robust berth scheduling at marine container terminals via hierarchical optimization Comput Oper Res 41:412–422.

Lv, X., Jin, J. G., & Hu, H. (2020). Berth allocation recovery for container transshipment terminals. Maritime Policy & Management, 1-17.

Correcher, J. F., Van den Bossche, T., Alvarez-Valdes, R., & Berghe, G. V. (2019). The berth allocation problem in terminals with irregular layouts. European Journal of Operational Research, 272(3), 1096-1108.

Lalla-Ruiz, E., Expósito-Izquierdo, C., Melián-Batista, B., & Moreno-Vega, J. M. (2016). A set-partitioning-based model for the berth allocation problem under time-dependent limitations. European Journal of Operational Research, 250(3), 1001-1012.

ANNEXURE-1

DERIVED MATHEMATICAL MODEL FOR THE CASE EXAMPLE - NEW MODEL

Given below are the representations of the MILP Model 1 in the expanded form:

- A_j Vessel j estimated arrival time
- H_j Vessel j estimated loading and unloading time
- l_j The length of vessel j including vessel safe distance for mooring
- d_j Draft of vessel j
- R_j Scheduled departure time of vessel j
- L_i The length of berth i
- D_i The depth of berth i
- α_{jt} Binary matrix, 1 if vessel j at time t can go through the access channel, otherwise 0.
- M A very large number
- v_i The time when berth i become empty at the beginning of the time horizon
- Y_i Coordinates of the middle of berth i
-
- *Decision variables*
- St_j Mooring time of vessel j
- Ft_j Departure time of vessel j
- x_{ij} Binary variable, 1 if vessel j moors at berth i, otherwise 0
- f_j 1 if vessel j is the first vessel that moors at its allocated berth,otherwise 0
- e_j 1 if vessel j is the last vessel that moors at its allocated berth, otherwise 0
- y_{ab} If $x_{ia} = x_{ib}$ and vessel b is serviced immediately after vessel a 1,otherwise 0
- w_{jt} Binary variable, 1 if vessel j moors at time t it is set, otherwise 0
- q_{jt} Binary variable, 1 if vessel j departs the port at time t it is set, otherwise 0

- αjt -the binary matrix which provides a value 1 if the channel draft is greater than that of a vessel and 0 if passage is not possible.Alpha matrix makes this work one of a kind.
- xij - binary variable takes the value 1 if the vessel j moors at the berth i ie. allocation.

● □□ - - if the value is 1 ,then it denotes that vessel j is the first servicing vessel for the berth it was assigned (helps to identify the first vessel that will be moored to its allocated berth).
● □□ - if the value is 1 ,then it denotes that vessel j is the last servicing vessel for the berth it was assigned.
● □□□ -A binary variable which denotes the order of service at the berth. □_□□ = 1 means vessel b need to be serviced only after □ ie .sequence
● wjt and □□□ - binary variable showing the mooring and departure of vessel related to time . Value 1 denotes that the vessel j would be moored at time t .(same for departure)
●

minimize

+Ft1+Ft2+Ft3-1212

subject to

+x1,1+x2,1=1

+x1,2+x2,2=1

+x1,3+x2,3=1

+f1+y2,1+y3,1=1

+f2+y1,2+y3,2=1

```
+f3+y1,3+y2,3=1

+ea1+y1,2+y1,3=1
+ea2+y2,1+y2,3=1
+ea3+y3,1+y3,2=1
+f1+f2+x1,1+x1,2<=3
+f1+f3+x1,1+x1,3<=3
+f2+f1+x1,2+x1,1<=3
+f2+f3+x1,2+x1,3<=3
+f3+f1+x1,3+x1,1<=3
+f3+f2+x1,3+x1,2<=3
+f1+f2+x2,1+x2,2<=3
+f1+f3+x2,1+x2,3<=3
+f2+f1+x2,2+x2,1<=3
+f2+f3+x2,2+x2,3<=3
+f3+f1+x2,3+x2,1<=3
+f3+f2+x2,3+x2,2<=3
+ea1+ea2+x1,1+x1,2<=3
+ea1+ea3+x1,1+x1,3<=3
+ea2+ea1+x1,2+x1,1<=3
+ea2+ea3+x1,2+x1,3<=3
+ea3+ea1+x1,3+x1,1<=3
+ea3+ea2+x1,3+x1,2<=3
+ea1+ea2+x2,1+x2,2<=3
+ea1+ea3+x2,1+x2,3<=3
+ea2+ea1+x2,2+x2,1<=3
+ea2+ea3+x2,2+x2,3<=3
+ea3+ea1+x2,3+x2,1<=3
+ea3+ea2+x2,3+x2,2<=3
+y1,2-x1,1+x1,2<=1
+y1,3-x1,1+x1,3<=1
+y2,1-x1,2+x1,1<=1
+y2,3-x1,2+x1,3<=1
+y3,1-x1,3+x1,1<=1
+y3,2-x1,3+x1,2<=1
+y1,2-x2,1+x2,2<=1
+y1,3-x2,1+x2,3<=1
+y2,1-x2,2+x2,1<=1
+y2,3-x2,2+x2,3<=1
+y3,1-x2,3+x2,1<=1
+y3,2-x2,3+x2,2<=1
```

+x1,1-x1,2+y1,2<=1
+x1,1-x1,3+y1,3<=1
+x1,2-x1,1+y2,1<=1
+x1,2-x1,3+y2,3<=1
+x1,3-x1,1+y3,1<=1
+x1,3-x1,2+y3,2<=1
+x2,1-x2,2+y1,2<=1
+x2,1-x2,3+y1,3<=1
+x2,2-x2,1+y2,1<=1
+x2,2-x2,3+y2,3<=1
+x2,3-x2,1+y3,1<=1
+x2,3-x2,2+y3,2<=1
+w1,50<=0
+w1,51<=0
+w1,52<=0
+w1,53<=1
+w1,54<=1
+w1,55<=1
+w1,56<=1
+w1,57<=1
+w1,58<=1
+w1,59<=1
+w1,60<=1
+w1,61<=1
+w1,118<=1
+w1,119<=1
+w1,120<=1
+w1,121<=1
+w1,122<=1
+w1,123<=1
+w1,124<=1
+w1,125<=1
+w1,126<=1
+w1,127<=1
+w1,128<=1
+w1,129<=1
+w1,130<=1
+w1,131<=1
+w1,132<=1
+w1,133<=1

+w1,134<=1

+w1,135<=1

+w1,136<=1

+q1,462<=1

+q1,463<=1

+q1,464<=1

+q1,465<=1

+q1,466<=1

+q1,467<=1

+q1,468<=1

+q1,469<=1

+q1,470<=1

+q1,471<=1

+q1,472<=1

+q1,473<=1

+q1,474<=1

+q1,475<=1

+q1,476<=1

+q1,578<=0

+q1,579<=0

+q1,580<=0

+q1,581<=1

+q1,582<=1

+q1,583<=1

+q1,584<=1

+q1,585<=1

+q1,586<=1

+q1,587<=1

+q1,588<=1

+q1,589<=1

+q1,590<=1

+q1,591<=1

+q2,152<=1

+q2,153<=1

+q2,154<=1

+q2,155<=1

+q2,156<=1

+q2,157<=1

+q2,158<=1

+q2,159<=1

+q2,160<=1

+q2,161<=1

+q2,162<=1

+q2,163<=1

+q2,164<=1

+w1,1+w1,2+w1,3+w1,4+w1,5+w1,6+w1,7+w1,8+w1,9+w1,10+w1,11+w1,12+w1,13+w
1,14+w1,15+w1,16+w1,17+w1,18+w1,19+w1,20+w1,21+w1,22+w1,23+w1,24+w1,25+w
1,26+w1,27+w1,28+w1,29+w1,30+w1,31+w1,32+w1,33+w1,34+w1,35+w1,36+w1,37+w
1,38+w1,39+w1,40+w1,41+w1,42+w1,43+w1,44+w1,45+w1,46+w1,47+w1,48+w1,49+w
1,50+w1,51+w1,52+w1,53+w1,54+w1,55+w1,56+w1,57+w1,58+w1,59+w1,60+w1,61+w
1,62+w1,63+w1,64+w1,65+w1,66+w1,67+w1,68+w1,69+w1,70+w1,71+w1,72+w1,73+w
1,74+w1,75+w1,76+w1,77+w1,78+w1,79+w1,80+w1,81+w1,82+w1,83+w1,84+w1,85+w
1,86+w1,87+w1,88+w1,89+w1,90+w1,91+w1,92+w1,93+w1,94+w1,95+w1,96+w1,97+w
1,98+w1,99+w1,100+w1,101+w1,102+w1,103+w1,104+w1,105+w1,106+w1,107+w1,108
+w1,109+w1,110+w1,111+w1,112+w1,113+w1,114+w1,115+w1,116+w1,117+w1,118+w
1,119+w1,120+w1,121+w1,122+w1,123+w1,124+w1,125+w1,126+w1,127+w1,128+w1,1
29+w1,130+w1,131+w1,132+w1,133+w1,134+w1,135+w1,136+w1,137+w1,138+w1,139
+w1,140+w1,141+w1,142+w1,143+w1,144+w1,145+w1,146+w1,147+w1,148+w1,149+w
1,150+w1,151+w1,152+w1,153+w1,154+w1,155+w1,156+w1,157+w1,158+w1,159+w1,1
60+w1,161+w1,162+w1,163+w1,164+w1,165+w1,166+w1,167+w1,168+w1,169+w1,170
+w1,171+w1,172+w1,173+w1,174+w1,175+w1,176+w1,177+w1,178+w1,179+w1,180+w
1,181+w1,182+w1,183+w1,184+w1,185+w1,186+w1,187+w1,188+w1,189+w1,190+w1,1
91+w1,192+w1,193+w1,194+w1,195+w1,196+w1,197+w1,198+w1,199+w1,200+w1,201
+w1,202+w1,203+w1,204+w1,205+w1,206+w1,207+w1,208+w1,209+w1,210+w1,211+w
1,212+w1,213+w1,214+w1,215+w1,216+w1,217+w1,218+w1,219+w1,220+w1,221+w1,2
22+w1,223+w1,224+w1,225+w1,226+w1,227+w1,228+w1,229+w1,230+w1,231+w1,232
+w1,233+w1,234+w1,235+w1,236+w1,237+w1,238+w1,239+w1,240+w1,241+w1,242+w
1,243+w1,244+w1,245+w1,246+w1,247+w1,248+w1,249+w1,250+w1,251+w1,252+w1,2
53+w1,254+w1,255+w1,256+w1,257+w1,258+w1,259+w1,260+w1,261+w1,262+w1,263
+w1,264+w1,265+w1,266+w1,267+w1,268+w1,269+w1,270+w1,271+w1,272+w1,273+w
1,274+w1,275+w1,276+w1,277+w1,278+w1,279+w1,280+w1,281+w1,282+w1,283+w1,2
84+w1,285+w1,286+w1,287+w1,288+w1,289+w1,290+w1,291+w1,292+w1,293+w1,294
+w1,295+w1,296+w1,297+w1,298+w1,299+w1,300+w1,301+w1,302+w1,303+w1,304+w
1,305+w1,306+w1,307+w1,308+w1,309+w1,310+w1,311+w1,312+w1,313+w1,314+w1,3
15+w1,316+w1,317+w1,318+w1,319+w1,320+w1,321+w1,322+w1,323+w1,324+w1,325
+w1,326+w1,327+w1,328+w1,329+w1,330+w1,331+w1,332+w1,333+w1,334+w1,335+w
1,336+w1,337+w1,338+w1,339+w1,340+w1,341+w1,342+w1,343+w1,344+w1,345+w1,3
46+w1,347+w1,348+w1,349+w1,350+w1,351+w1,352+w1,353+w1,354+w1,355+w1,356
+w1,357+w1,358+w1,359+w1,360+w1,361+w1,362+w1,363+w1,364+w1,365+w1,366+w
1,367+w1,368+w1,369+w1,370+w1,371+w1,372+w1,373+w1,374+w1,375+w1,376+w1,3
77+w1,378+w1,379+w1,380+w1,381+w1,382+w1,383+w1,384+w1,385+w1,386+w1,387
+w1,388+w1,389+w1,390+w1,391+w1,392+w1,393+w1,394+w1,395+w1,396+w1,397+w
1,398+w1,399+w1,400+w1,401+w1,402+w1,403+w1,404+w1,405+w1,406+w1,407+w1,4
08+w1,409+w1,410+w1,411+w1,412+w1,413+w1,414+w1,415+w1,416+w1,417+w1,418
+w1,419+w1,420+w1,421+w1,422+w1,423+w1,424+w1,425+w1,426+w1,427+w1,428+w
1,429+w1,430+w1,431+w1,432+w1,433+w1,434+w1,435+w1,436+w1,437+w1,438+w1,4
39+w1,440+w1,441+w1,442+w1,443+w1,444+w1,445+w1,446+w1,447+w1,448+w1,449
+w1,450+w1,451+w1,452+w1,453+w1,454+w1,455+w1,456+w1,457+w1,458+w1,459+w
1,460+w1,461+w1,462+w1,463+w1,464+w1,465+w1,466+w1,467+w1,468+w1,469+w1,4

70+w1,471+w1,472+w1,473+w1,474+w1,475+w1,476+w1,477+w1,478+w1,479+w1,480+w1,481+w1,482+w1,483+w1,484+w1,485+w1,486+w1,487+w1,488+w1,489+w1,490+w1,491+w1,492+w1,493+w1,494+w1,495+w1,496+w1,497+w1,498+w1,499+w1,500+w1,501+w1,502+w1,503+w1,504+w1,505+w1,506+w1,507+w1,508+w1,509+w1,510+w1,511+w1,512+w1,513+w1,514+w1,515+w1,516+w1,517+w1,518+w1,519+w1,520+w1,521+w1,522+w1,523+w1,524+w1,525+w1,526+w1,527+w1,528+w1,529+w1,530+w1,531+w1,532+w1,533+w1,534+w1,535+w1,536+w1,537+w1,538+w1,539+w1,540+w1,541+w1,542+w1,543+w1,544+w1,545+w1,546+w1,547+w1,548+w1,549+w1,550+w1,551+w1,552+w1,553+w1,554+w1,555+w1,556+w1,557+w1,558+w1,559+w1,560+w1,561+w1,562+w1,563+w1,564+w1,565+w1,566+w1,567+w1,568+w1,569+w1,570+w1,571+w1,572+w1,573+w1,574+w1,575+w1,576+w1,577+w1,578+w1,579+w1,580+w1,581+w1,582+w1,583+w1,584+w1,585+w1,586+w1,587+w1,588+w1,589+w1,590+w1,591+w1,592+w1,593+w1,594+w1,595+w1,596+w1,597+w1,598+w1,599+w1,600+w1,601+w1,602+w1,603+w1,604+w1,605+w1,606+w1,607+w1,608+w1,609=1

+1w1,1+2w1,2+3w1,3+4w1,4+5w1,5+6w1,6+7w1,7+8w1,8+9w1,9+10w1,10+11w1,11+12w1,12+13w1,13+14w1,14+15w1,15+16w1,16+17w1,17+18w1,18+19w1,19+20w1,20+21w1,21+22w1,22+23w1,23+24w1,24+25w1,25+26w1,26+27w1,27+28w1,28+29w1,29+30w1,30+31w1,31+32w1,32+33w1,33+34w1,34+35w1,35+36w1,36+37w1,37+38w1,38+39w1,39+40w1,40+41w1,41+42w1,42+43w1,43+44w1,44+45w1,45+46w1,46+47w1,47+48w1,48+49w1,49+50w1,50+51w1,51+52w1,52+53w1,53+54w1,54+55w1,55+56w1,56+57w1,57+58w1,58+59w1,59+60w1,60+61w1,61+62w1,62+63w1,63+64w1,64+65w1,65+66w1,66+67w1,67+68w1,68+69w1,69+70w1,70+71w1,71+72w1,72+73w1,73+74w1,74+75w1,75+76w1,76+77w1,77+78w1,78+79w1,79+80w1,80+81w1,81+82w1,82+83w1,83+84w1,84+85w1,85+86w1,86+87w1,87+88w1,88+89w1,89+90w1,90+91w1,91+92w1,92+93w1,93+94w1,94+95w1,95+96w1,96+97w1,97+98w1,98+99w1,99+100w1,100+101w1,101+102w1,102+103w1,103+104w1,104+105w1,105+106w1,106+107w1,107+108w1,108+109w1,109+110w1,110+111w1,111+112w1,112+113w1,113+114w1,114+115w1,115+116w1,116+117w1,117+118w1,118+119w1,119+120w1,120+121w1,121+122w1,122+123w1,123+124w1,124+125w1,125+126w1,126+127w1,127+128w1,128+129w1,129+130w1,130+131w1,131+132w1,132+133w1,133+134w1,134+135w1,135+136w1,136+137w1,137+138w1,138+139w1,139+140w1,140+141w1,141+142w1,142+143w1,143+144w1,144+145w1,145+146w1,146+147w1,147+148w1,148+149w1,149+150w1,150+151w1,151+152w1,152+153w1,153+154w1,154+155w1,155+156w1,156+157w1,157+158w1,158+159w1,159+160w1,160+161w1,161+162w1,162+163w1,163+164w1,164+165w1,165+166w1,166+167w1,167+168w1,168+169w1,169+170w1,170+171w1,171+172w1,172+173w1,173+174w1,174+175w1,175+176w1,176+177w1,177+178w1,178+179w1,179+180w1,180+181w1,181+182w1,182+183w1,183+184w1,184+185w1,185+186w1,186+187w1,187+188w1,188+189w1,189+190w1,190+191w1,191+192w1,192+193w1,193+194w1,194+195w1,195+196w1,196+197w1,197+198w1,198+199w1,199+200w1,200+201w1,201+202w1,202+203w1,203+204w1,204+205w1,205+206w1,206+207w1,207+208w1,208+209w1,209+210w1,210+211w1,211+212w1,212+213w1,213+214w1,214+215w1,215+216w1,216+217w1,217+218w1,218+219w1,219+220w1,220+221w1,221+222w1,222+223w1,223+224w1,224+225w1,225+226w1,226+227w1,227+228w1,228+229w1,229+230w1,230+231w1,231+232w1,232+233w1,233+234w1,234+235w1,235+236w1,236+237w1,237+238w1,238+239w1,239+240w1,240+241w1,241+242w1,242+243w1,243+244w1,244+245w1,245+246w1,246+247w1,247+248w1,248+249w1,249+250w1,250+251w1,251+252w1,252+253

w1,253+254w1,254+255w1,255+256w1,256+257w1,257+258w1,258+259w1,259+260w1
,260+261w1,261+262w1,262+263w1,263+264w1,264+265w1,265+266w1,266+267w1,26
7+268w1,268+269w1,269+270w1,270+271w1,271+272w1,272+273w1,273+274w1,274+
275w1,275+276w1,276+277w1,277+278w1,278+279w1,279+280w1,280+281w1,281+28
2w1,282+283w1,283+284w1,284+285w1,285+286w1,286+287w1,287+288w1,288+289w
1,289+290w1,290+291w1,291+292w1,292+293w1,293+294w1,294+295w1,295+296w1,2
96+297w1,297+298w1,298+299w1,299+300w1,300+301w1,301+302w1,302+303w1,303
+304w1,304+305w1,305+306w1,306+307w1,307+308w1,308+309w1,309+310w1,310+3
11w1,311+312w1,312+313w1,313+314w1,314+315w1,315+316w1,316+317w1,317+318
w1,318+319w1,319+320w1,320+321w1,321+322w1,322+323w1,323+324w1,324+325w1
,325+326w1,326+327w1,327+328w1,328+329w1,329+330w1,330+331w1,331+332w1,33
2+333w1,333+334w1,334+335w1,335+336w1,336+337w1,337+338w1,338+339w1,339+
340w1,340+341w1,341+342w1,342+343w1,343+344w1,344+345w1,345+346w1,346+34
7w1,347+348w1,348+349w1,349+350w1,350+351w1,351+352w1,352+353w1,353+354w
1,354+355w1,355+356w1,356+357w1,357+358w1,358+359w1,359+360w1,360+361w1,3
61+362w1,362+363w1,363+364w1,364+365w1,365+366w1,366+367w1,367+368w1,368
+369w1,369+370w1,370+371w1,371+372w1,372+373w1,373+374w1,374+375w1,375+3
76w1,376+377w1,377+378w1,378+379w1,379+380w1,380+381w1,381+382w1,382+383
w1,383+384w1,384+385w1,385+386w1,386+387w1,387+388w1,388+389w1,389+390w1
,390+391w1,391+392w1,392+393w1,393+394w1,394+395w1,395+396w1,396+397w1,39
7+398w1,398+399w1,399+400w1,400+401w1,401+402w1,402+403w1,403+404w1,404+
405w1,405+406w1,406+407w1,407+408w1,408+409w1,409+410w1,410+411w1,411+41
2w1,412+413w1,413+414w1,414+415w1,415+416w1,416+417w1,417+418w1,418+419w
1,419+420w1,420+421w1,421+422w1,422+423w1,423+424w1,424+425w1,425+426w1,4
26+427w1,427+428w1,428+429w1,429+430w1,430+431w1,431+432w1,432+433w1,433
+434w1,434+435w1,435+436w1,436+437w1,437+438w1,438+439w1,439+440w1,440+4
41w1,441+442w1,442+443w1,443+444w1,444+445w1,445+446w1,446+447w1,447+448
w1,448+449w1,449+450w1,450+451w1,451+452w1,452+453w1,453+454w1,454+455w1
,455+456w1,456+457w1,457+458w1,458+459w1,459+460w1,460+461w1,461+462w1,46
2+463w1,463+464w1,464+465w1,465+466w1,466+467w1,467+468w1,468+469w1,469+
470w1,470+471w1,471+472w1,472+473w1,473+474w1,474+475w1,475+476w1,476+47
7w1,477+478w1,478+479w1,479+480w1,480+481w1,481+482w1,482+483w1,483+484w
1,484+485w1,485+486w1,486+487w1,487+488w1,488+489w1,489+490w1,490+491w1,4
91+492w1,492+493w1,493+494w1,494+495w1,495+496w1,496+497w1,497+498w1,498
+499w1,499+500w1,500+501w1,501+502w1,502+503w1,503+504w1,504+505w1,505+5
06w1,506+507w1,507+508w1,508+509w1,509+510w1,510+511w1,511+512w1,512+513
w1,513+514w1,514+515w1,515+516w1,516+517w1,517+518w1,518+519w1,519+520w1
,520+521w1,521+522w1,522+523w1,523+524w1,524+525w1,525+526w1,526+527w1,52
7+528w1,528+529w1,529+530w1,530+531w1,531+532w1,532+533w1,533+534w1,534+
535w1,535+536w1,536+537w1,537+538w1,538+539w1,539+540w1,540+541w1,541+54
2w1,542+543w1,543+544w1,544+545w1,545+546w1,546+547w1,547+548w1,548+549w
1,549+550w1,550+551w1,551+552w1,552+553w1,553+554w1,554+555w1,555+556w1,5
56+557w1,557+558w1,558+559w1,559+560w1,560+561w1,561+562w1,562+563w1,563
+564w1,564+565w1,565+566w1,566+567w1,567+568w1,568+569w1,569+570w1,570+5
71w1,571+572w1,572+573w1,573+574w1,574+575w1,575+576w1,576+577w1,577+578
w1,578+579w1,579+580w1,580+581w1,581+582w1,582+583w1,583+584w1,584+585w1
,585+586w1,586+587w1,587+588w1,588+589w1,589+590w1,590+591w1,591+592w1,59
2+593w1,593+594w1,594+595w1,595+596w1,596+597w1,597+598w1,598+599w1,599+

600w1,600+601w1,601+602w1,602+603w1,603+604w1,604+605w1,605+606w1,606+607w1,607+608w1,608+609w1,609-St1=0

+1q1,1+2q1,2+3q1,3+4q1,4+5q1,5+6q1,6+7q1,7+8q1,8+9q1,9+10q1,10+11q1,11+12q1,12+13q1,13+14q1,14+15q1,15+16q1,16+17q1,17+18q1,18+19q1,19+20q1,20+21q1,21+22q1,22+23q1,23+24q1,24+25q1,25+26q1,26+27q1,27+28q1,28+29q1,29+30q1,30+31q1,31+32q1,32+33q1,33+34q1,34+35q1,35+36q1,36+37q1,37+38q1,38+39q1,39+40q1,40+41q1,41+42q1,42+43q1,43+44q1,44+45q1,45+46q1,46+47q1,47+48q1,48+49q1,49+50q1,50+51q1,51+52q1,52+53q1,53+54q1,54+55q1,55+56q1,56+57q1,57+58q1,58+59q1,59+60q1,60+61q1,61+62q1,62+63q1,63+64q1,64+65q1,65+66q1,66+67q1,67+68q1,68+69q1,69+70q1,70+71q1,71+72q1,72+73q1,73+74q1,74+75q1,75+76q1,76+77q1,77+78q1,78+79q1,79+80q1,80+81q1,81+82q1,82+83q1,83+84q1,84+85q1,85+86q1,86+87q1,87+88q1,88+89q1,89+90q1,90+91q1,91+92q1,92+93q1,93+94q1,94+95q1,95+96q1,96+97q1,97+98q1,98+99q1,99+100q1,100+101q1,101+102q1,102+103q1,103+104q1,104+105q1,105+106q1,106+107q1,107+108q1,108+109q1,109+110q1,110+111q1,111+112q1,112+113q1,113+114q1,114+115q1,115+116q1,116+117q1,117+118q1,118+119q1,119+120q1,120+121q1,121+122q1,122+123q1,123+124q1,124+125q1,125+126q1,126+127q1,127+128q1,128+129q1,129+130q1,130+131q1,131+132q1,132+133q1,133+134q1,134+135q1,135+136q1,136+137q1,137+138q1,138+139q1,139+140q1,140+141q1,141+142q1,142+143q1,143+144q1,144+145q1,145+146q1,146+147q1,147+148q1,148+149q1,149+150q1,150+151q1,151+152q1,152+153q1,153+154q1,154+155q1,155+156q1,156+157q1,157+158q1,158+159q1,159+160q1,160+161q1,161+162q1,162+163q1,163+164q1,164+165q1,165+166q1,166+167q1,167+168q1,168+169q1,169+170q1,170+171q1,171+172q1,172+173q1,173+174q1,174+175q1,175+176q1,176+177q1,177+178q1,178+179q1,179+180q1,180+181q1,181+182q1,182+183q1,183+184q1,184+185q1,185+186q1,186+187q1,187+188q1,188+189q1,189+190q1,190+191q1,191+192q1,192+193q1,193+194q1,194+195q1,195+196q1,196+197q1,197+198q1,198+199q1,199+200q1,200+201q1,201+202q1,202+203q1,203+204q1,204+205q1,205+206q1,206+207q1,207+208q1,208+209q1,209+210q1,210+211q1,211+212q1,212+213q1,213+214q1,214+215q1,215+216q1,216+217q1,217+218q1,218+219q1,219+220q1,220+221q1,221+222q1,222+223q1,223+224q1,224+225q1,225+226q1,226+227q1,227+228q1,228+229q1,229+230q1,230+231q1,231+232q1,232+233q1,233+234q1,234+235q1,235+236q1,236+237q1,237+238q1,238+239q1,239+240q1,240+241q1,241+242q1,242+243q1,243+244q1,244+245q1,245+246q1,246+247q1,247+248q1,248+249q1,249+250q1,250+251q1,251+252q1,252+253q1,253+254q1,254+255q1,255+256q1,256+257q1,257+258q1,258+259q1,259+260q1,260+261q1,261+262q1,262+263q1,263+264q1,264+265q1,265+266q1,266+267q1,267+268q1,268+269q1,269+270q1,270+271q1,271+272q1,272+273q1,273+274q1,274+275q1,275+276q1,276+277q1,277+278q1,278+279q1,279+280q1,280+281q1,281+282q1,282+283q1,283+284q1,284+285q1,285+286q1,286+287q1,287+288q1,288+289q1,289+290q1,290+291q1,291+292q1,292+293q1,293+294q1,294+295q1,295+296q1,296+297q1,297+298q1,298+299q1,299+300q1,300+301q1,301+302q1,302+303q1,303+304q1,304+305q1,305+306q1,306+307q1,307+308q1,308+309q1,309+310q1,310+311q1,311+312q1,312+313q1,313+314q1,314+315q1,315+316q1,316+317q1,317+318q1,318+319q1,319+320q1,320+321q1,321+322q1,322+323q1,323+324q1,324+325q1,325+326q1,326+327q1,327+328q1,328+329q1,329+330q1,330+331q1,331+332q1,332+333q1,333+334q1,334+335q1,335+336q1,336+337q1,337+338q1,338+339q1,339+340q1,340+341q1,341+342q1,342+343q1,343+344q1,344+345q1,345+346q1,346+347q1,347+348q1,348+349q1,349+350q1,350+351q1,351+352q1,352+353q1,353+354q1,354+355q1,355+356q1,356+357q1,357+358q1,358+359q1,359+360q1,360+361q1,361+362q1,362+363q1, 363+364q1,364+365q1,365+366q1,366+367q1,367+368q1,368+369q1,369+370q1,370+371q1,371+372q1,372+373q1,373+374q1,374+375q1,375+376q1,376+377q1,377+378q1,378+379q1,379+380q1,380+381q1,381+382q1,382+383q1,383+384q1,384+385q1,385+386q1,386+387q1,387+388q1,388+389q1,389+390q1,390+391q1,391+392q1,392+393q1,393+394q1,394+395q1,395+396q1,396+397q1,397+398q1,398+399q1,399+400q1,400+401q1,401+402q1,402+403q1,403+404q1,404+405q1,405+406q1,406+407q1,407+408q1,408+409q1,409+410q1,410+411q1,411+412q1,412+413q1,413+414q1,414+415q1,415+416q1,416+417q1,417+418q1,418+419q1,419+420q1,420+421q1,421+422q1,422+423q1,423+424q1,424+425q1,425+426q1,426+427q1,427+428q1,428+429q1,429+430q1,430+431q1,431+432q1,432+433q1,433+434q1,434+435q1,435+436q1,436+437q1,437+438q1,438+439q1,439+440q1,440+441q1,441+442q1,442+443q1,443+444q1,444+445q1,445+446q1,446+447q1,447+448q1,448+449q1,449+450q1,450+451q1,451+452q1,452+453q1,453+454q1,454+455q1,455+456q1,456+457q1,457+458q1,458+459q1,459+460q1,460+461q1,461+462q1,462+463q1,463+464q1,464+465q1,465+466q1,466+467q1,467+468q1,468+469q1,469+470q1,470+471q1,471+472q1,472+473q1,473+474q1,474+475q1,475+476q1,476+477q1,477+478q1,478+479q1,479+480q1,480+481q1,481+482q1,482+483q1,483+484q1,484+485q1,485+486q1,486+487q1,487+488q1,488+489q1,489+490q1,490+491q1,491+492q1,492+493q1,493+494q1,494+495q1,495+496q1,496+497q1,497+498q1,498+499q1,499+500q1,500+501q1,501+502q1,502+503q1,503+504q1,504+505q1,505+506q1,506+507q1,507+508q1,508+509q1,509+510q1,510+511q1,511+512q1,512+513q1,513+514q1,514+515q1,515+516q1,516+517q1,517+518q1,518+519q1,519+520q1,520+521q1,521+522q1,522+523q1,523+524q1,524+525q1,525+526q1,526+527q1,527+528q1,528+529q1,529+530q1,530+531q1,531+532q1,532+533q1,533+534q1,534+535q1,535+536q1,536+537q1,537+538q1,538+539q1,539+540q1,540+541q1,541+542q1,542+543q1,543+544q1,544+545q1,545+546q1,546+547q1,547+548q1,548+549q1,549+550q1,550+551q1,551+552q1,552+553q1,553+554q1,554+555q1,555+556q1,556+557q1,557+558q1,558+559q1,559+560q1,560+561q1,561+562q1,562+563q1,563+564q1,564+565q1,565+566q1,566+567q1,567+568q1,568+569q1,569+570q1,570+571q1,571+572q1,572+573q1,573+574q1,574+575q1,575+576q1,576+577q1,577+578q1,578+579q1,579+580q1,580+581q1,581+582q1,582+583q1,583+584q1,584+585q1,585+586q1,586+587q1,587+588q1,588+589q1,589+590q1,590+591q1,591+592q1,592+593q1,593+594q1,594+595q1,595+596q1,596+597q1,597+598q1,598+599q1,599+600q1,600+601q1,601+602q1,602+603q1,603+604q1,604+605q1,605+606q1,606+607q1,607+608q1,608+609q1,609-Ft1=0

+1q2,1+2q2,2+3q2,3+4q2,4+5q2,5+6q2,6+7q2,7+8q2,8+9q2,9+10q2,10+11q2,11+12q2,12+13q2,13+14q2,14+15q2,15+16q2,16+17q2,17+18q2,18+19q2,19+20q2,20+21q2,21+22q2,22+23q2,23+24q2,24+25q2,25+26q2,26+27q2,27+28q2,28+29q2,29+30q2,30+31q2,31+32q2,32+33q2,33+34q2,34+35q2,35+36q2,36+37q2,37+38q2,38+39q2,39+40q2,40+41q2,41+42q2,42+43q2,43+44q2,44+45q2,45+46q2,46+47q2,47+48q2,48+49q2,49+50q2,50+51q2,51+52q2,52+53q2,53+54q2,54+55q2,55+56q2,56+57q2,57+58q2,58+59q2,59+60q2,60+61q2,61+62q2,62+63q2,63+64q2,64+65q2,65+66q2,66+67q2,67+68q2,68+69q2,69+70q2,70+71q2,71+72q2,72+73q2,73+74q2,74+75q2,75+76q2,76+77q2,77+78q2,78+79q2,79+80q2,80+81q2,81+82q2,82+83q2,83+84q2,84+85q2,85+86q2,86+87q2,87+88q2,88+89q2,89+90q2,90+91q2,91+92q2,92+93q2,93+94q2,94+95q2,95+96q2,96+97q2,97+98q2,98+99q2,99+100q2,100+101q2,101+102q2,102+103q2,103+104q2,104+105q2,105+106q2,106+107q2,107+108q2,108+109q2,109+110q2,110+111q2,111+112q2,112+113q2,113+114q2,114+115q2,115+116q2,116+117q2,117+118q2,118+119q2,119+120q2,120+121q2,121+122q2,122+123q2,123+124q2,124+125q2,125+126q2,126+127q2,127+128q2,128+129

q2,129+130q2,130+131q2,131+132q2,132+133q2,133+134q2,134+135q2,135+136q2,136
+137q2,137+138q2,138+139q2,139+140q2,140+141q2,141+142q2,142+143q2,143+144q
2,144+145q2,145+146q2,146+147q2,147+148q2,148+149q2,149+150q2,150+151q2,151+
152q2,152+153q2,153+154q2,154+155q2,155+156q2,156+157q2,157+158q2,158+159q2,
159+160q2,160+161q2,161+162q2,162+163q2,163+164q2,164+165q2,165+166q2,166+1
67q2,167+168q2,168+169q2,169+170q2,170+171q2,171+172q2,172+173q2,173+174q2,1
74+175q2,175+176q2,176+177q2,177+178q2,178+179q2,179+180q2,180+181q2,181+18
2q2,182+183q2,183+184q2,184+185q2,185+186q2,186+187q2,187+188q2,188+189q2,18
9+190q2,190+191q2,191+192q2,192+193q2,193+194q2,194+195q2,195+196q2,196+197
q2,197+198q2,198+199q2,199+200q2,200+201q2,201+202q2,202+203q2,203+204q2,204
+205q2,205+206q2,206+207q2,207+208q2,208+209q2,209+210q2,210+211q2,211+212q
2,212+213q2,213+214q2,214+215q2,215+216q2,216+217q2,217+218q2,218+219q2,219+
220q2,220+221q2,221+222q2,222+223q2,223+224q2,224+225q2,225+226q2,226+227q2,
227+228q2,228+229q2,229+230q2,230+231q2,231+232q2,232+233q2,233+234q2,234+2
35q2,235+236q2,236+237q2,237+238q2,238+239q2,239+240q2,240+241q2,241+242q2,2
42+243q2,243+244q2,244+245q2,245+246q2,246+247q2,247+248q2,248+249q2,249+25
0q2,250+251q2,251+252q2,252+253q2,253+254q2,254+255q2,255+256q2,256+257q2,25
7+258q2,258+259q2,259+260q2,260+261q2,261+262q2,262+263q2,263+264q2,264+265
q2,265+266q2,266+267q2,267+268q2,268+269q2,269+270q2,270+271q2,271+272q2,272
+273q2,273+274q2,274+275q2,275+276q2,276+277q2,277+278q2,278+279q2,279+280q
2,280+281q2,281+282q2,282+283q2,283+284q2,284+285q2,285+286q2,286+287q2,287+
288q2,288+289q2,289+290q2,290+291q2,291+292q2,292+293q2,293+294q2,294+295q2,
295+296q2,296+297q2,297+298q2,298+299q2,299+300q2,300+301q2,301+302q2,302+3
03q2,303+304q2,304+305q2,305+306q2,306+307q2,307+308q2,308+309q2,309+310q2,3
10+311q2,311+312q2,312+313q2,313+314q2,314+315q2,315+316q2,316+317q2,317+31
8q2,318+319q2,319+320q2,320+321q2,321+322q2,322+323q2,323+324q2,324+325q2,32
5+326q2,326+327q2,327+328q2,328+329q2,329+330q2,330+331q2,331+332q2,332+333
q2,333+334q2,334+335q2,335+336q2,336+337q2,337+338q2,338+339q2,339+340q2,340
+341q2,341+342q2,342+343q2,343+344q2,344+345q2,345+346q2,346+347q2,347+348q
2,348+349q2,349+350q2,350+351q2,351+352q2,352+353q2,353+354q2,354+355q2,355+
356q2,356+357q2,357+358q2,358+359q2,359+360q2,360+361q2,361+362q2,362+363q2,
363+364q2,364+365q2,365+366q2,366+367q2,367+368q2,368+369q2,369+370q2,370+3
71q2,371+372q2,372+373q2,373+374q2,374+375q2,375+376q2,376+377q2,377+378q2,3
78+379q2,379+380q2,380+381q2,381+382q2,382+383q2,383+384q2,384+385q2,385+38
6q2,386+387q2,387+388q2,388+389q2,389+390q2,390+391q2,391+392q2,392+393q2,39
3+394q2,394+395q2,395+396q2,396+397q2,397+398q2,398+399q2,399+400q2,400+401
q2,401+402q2,402+403q2,403+404q2,404+405q2,405+406q2,406+407q2,407+408q2,408
+409q2,409+410q2,410+411q2,411+412q2,412+413q2,413+414q2,414+415q2,415+416q
2,416+417q2,417+418q2,418+419q2,419+420q2,420+421q2,421+422q2,422+423q2,423+
424q2,424+425q2,425+426q2,426+427q2,427+428q2,428+429q2,429+430q2,430+431q2,
431+432q2,432+433q2,433+434q2,434+435q2,435+436q2,436+437q2,437+438q2,438+4
39q2,439+440q2,440+441q2,441+442q2,442+443q2,443+444q2,444+445q2,445+446q2,4
46+447q2,447+448q2,448+449q2,449+450q2,450+451q2,451+452q2,452+453q2,453+45
4q2,454+455q2,455+456q2,456+457q2,457+458q2,458+459q2,459+460q2,460+461q2,46
1+462q2,462+463q2,463+464q2,464+465q2,465+466q2,466+467q2,467+468q2,468+469
q2,469+470q2,470+471q2,471+472q2,472+473q2,473+474q2,474+475q2,475+476q2,476
+477q2,477+478q2,478+479q2,479+480q2,480+481q2,481+482q2,482+483q2,483+484q

2,484+485q2,485+486q2,486+487q2,487+488q2,488+489q2,489+490q2,490+491q2,491+492q2,492+493q2,493+494q2,494+495q2,495+496q2,496+497q2,497+498q2,498+499q2,499+500q2,500+501q2,501+502q2,502+503q2,503+504q2,504+505q2,505+506q2,506+507q2,507+508q2,508+509q2,509+510q2,510+511q2,511+512q2,512+513q2,513+514q2,514+515q2,515+516q2,516+517q2,517+518q2,518+519q2,519+520q2,520+521q2,521+522q2,522+523q2,523+524q2,524+525q2,525+526q2,526+527q2,527+528q2,528+529q2,529+530q2,530+531q2,531+532q2,532+533q2,533+534q2,534+535q2,535+536q2,536+537q2,537+538q2,538+539q2,539+540q2,540+541q2,541+542q2,542+543q2,543+544q2,544+545q2,545+546q2,546+547q2,547+548q2,548+549q2,549+550q2,550+551q2,551+552q2,552+553q2,553+554q2,554+555q2,555+556q2,556+557q2,557+558q2,558+559q2,559+560q2,560+561q2,561+562q2,562+563q2,563+564q2,564+565q2,565+566q2,566+567q2,567+568q2,568+569q2,569+570q2,570+571q2,571+572q2,572+573q2,573+574q2,574+575q2,575+576q2,576+577q2,577+578q2,578+579q2,579+580q2,580+581q2,581+582q2,582+583q2,583+584q2,584+585q2,585+586q2,586+587q2,587+588q2,588+589q2,589+590q2,590+591q2,591+592q2,592+593q2,593+594q2,594+595q2,595+596q2,596+597q2,597+598q2,598+599q2,599+600q2,600+601q2,601+602q2,602+603q2,603+604q2,604+605q2,605+606q2,606+607q2,607+608q2,608+609q2,609-Ft2=0

+1w3,1+2w3,2+3w3,3+4w3,4+5w3,5+6w3,6+7w3,7+8w3,8+9w3,9+10w3,10+11w3,11+12w3,12+13w3,13+14w3,14+15w3,15+16w3,16+17w3,17+18w3,18+19w3,19+20w3,20+21w3,21+22w3,22+23w3,23+24w3,24+25w3,25+26w3,26+27w3,27+28w3,28+29w3,29+30w3,30+31w3,31+32w3,32+33w3,33+34w3,34+35w3,35+36w3,36+37w3,37+38w3,38+39w3,39+40w3,40+41w3,41+42w3,42+43w3,43+44w3,44+45w3,45+46w3,46+47w3,47+48w3,48+49w3,49+50w3,50+51w3,51+52w3,52+53w3,53+54w3,54+55w3,55+56w3,56+57w3,57+58w3,58+59w3,59+60w3,60+61w3,61+62w3,62+63w3,63+64w3,64+65w3,65+66w3,66+67w3,67+68w3,68+69w3,69+70w3,70+71w3,71+72w3,72+73w3,73+74w3,74+75w3,75+76w3,76+77w3,77+78w3,78+79w3,79+80w3,80+81w3,81+82w3,82+83w3,83+84w3,84+85w3,85+86w3,86+87w3,87+88w3,88+89w3,89+90w3,90+91w3,91+92w3,92+93w3,93+94w3,94+95w3,95+96w3,96+97w3,97+98w3,98+99w3,99+100w3,100+101w3,101+102w3,102+103w3,103+104w3,104+105w3,105+106w3,106+107w3,107+108w3,108+109w3,109+110w3,110+111w3,111+112w3,112+113w3,113+114w3,114+115w3,115+116w3,116+117w3,117+118w3,118+119w3,119+120w3,120+121w3,121+122w3,122+123w3,123+124w3,124+125w3,125+126w3,126+127w3,127+128w3,128+129w3,129+130w3,130+131w3,131+132w3,132+133w3,133+134w3,134+135w3,135+136w3,136+137w3,137+138w3,138+139w3,139+140w3,140+141w3,141+142w3,142+143w3,143+144w3,144+145w3,145+146w3,146+147w3,147+148w3,148+149w3,149+150w3,150+151w3,151+152w3,152+153w3,153+154w3,154+155w3,155+156w3,156+157w3,157+158w3,158+159w3,159+160w3,160+161w3,161+162w3,162+163w3,163+164w3,164+165w3,165+166w3,166+167w3,167+168w3,168+169w3,169+170w3,170+171w3,171+172w3,172+173w3,173+174w3,174+175w3,175+176w3,176+177w3,177+178w3,178+179w3,179+180w3,180+181w3,181+182w3,182+183w3,183+184w3,184+185w3,185+186w3,186+187w3,187+188w3,188+189w3,189+190w3,190+191w3,191+192w3,192+193w3,193+194w3,194+195w3,195+196w3,196+197w3,197+198w3,198+199w3,199+200w3,200+201w3,201+202w3,202+203w3,203+204w3,204+205w3,205+206w3,206+207w3,207+208w3,208+209w3,209+210w3,210+211w3,211+212w3,212+213w3,213+214w3,214+215w3,215+216w3,216+217w3,217+218w3,218+219w3,219+220w3,220+221w3,221+222w3,222+223w3,223+224w3,224+225w3,225+226w3,226+227w3,227+228w3,228+229w3,229+230w3,230+231w3,231+232w3,232+233w3,233+234w3,234+235w3,235+236w3,236+237w3,237+238w3,238+239w3,239+240w3,240+241w3,241+242w3,242+243w3,243+244w3,244+245w3,245+2

46w3,246+247w3,247+248w3,248+249w3,249+250w3,250+251w3,251+252w3,252+253
w3,253+254w3,254+255w3,255+256w3,256+257w3,257+258w3,258+259w3,259+260w3
,260+261w3,261+262w3,262+263w3,263+264w3,264+265w3,265+266w3,266+267w3,26
7+268w3,268+269w3,269+270w3,270+271w3,271+272w3,272+273w3,273+274w3,274+
275w3,275+276w3,276+277w3,277+278w3,278+279w3,279+280w3,280+281w3,281+28
2w3,282+283w3,283+284w3,284+285w3,285+286w3,286+287w3,287+288w3,288+289w
3,289+290w3,290+291w3,291+292w3,292+293w3,293+294w3,294+295w3,295+296w3,2
96+297w3,297+298w3,298+299w3,299+300w3,300+301w3,301+302w3,302+303w3,303
+304w3,304+305w3,305+306w3,306+307w3,307+308w3,308+309w3,309+310w3,310+3
11w3,311+312w3,312+313w3,313+314w3,314+315w3,315+316w3,316+317w3,317+318
w3,318+319w3,319+320w3,320+321w3,321+322w3,322+323w3,323+324w3,324+325w3
,325+326w3,326+327w3,327+328w3,328+329w3,329+330w3,330+331w3,331+332w3,33
2+333w3,333+334w3,334+335w3,335+336w3,336+337w3,337+338w3,338+339w3,339+
340w3,340+341w3,341+342w3,342+343w3,343+344w3,344+345w3,345+346w3,346+34
7w3,347+348w3,348+349w3,349+350w3,350+351w3,351+352w3,352+353w3,353+354w
3,354+355w3,355+356w3,356+357w3,357+358w3,358+359w3,359+360w3,360+361w3,3
61+362w3,362+363w3,363+364w3,364+365w3,365+366w3,366+367w3,367+368w3,368
+369w3,369+370w3,370+371w3,371+372w3,372+373w3,373+374w3,374+375w3,375+3
76w3,376+377w3,377+378w3,378+379w3,379+380w3,380+381w3,381+382w3,382+383
w3,383+384w3,384+385w3,385+386w3,386+387w3,387+388w3,388+389w3,389+390w3
,390+391w3,391+392w3,392+393w3,393+394w3,394+395w3,395+396w3,396+397w3,39
7+398w3,398+399w3,399+400w3,400+401w3,401+402w3,402+403w3,403+404w3,404+
405w3,405+406w3,406+407w3,407+408w3,408+409w3,409+410w3,410+411w3,411+41
2w3,412+413w3,413+414w3,414+415w3,415+416w3,416+417w3,417+418w3,418+419w
3,419+420w3,420+421w3,421+422w3,422+423w3,423+424w3,424+425w3,425+426w3,4
26+427w3,427+428w3,428+429w3,429+430w3,430+431w3,431+432w3,432+433w3,433
+434w3,434+435w3,435+436w3,436+437w3,437+438w3,438+439w3,439+440w3,440+4
41w3,441+442w3,442+443w3,443+444w3,444+445w3,445+446w3,446+447w3,447+448
w3,448+449w3,449+450w3,450+451w3,451+452w3,452+453w3,453+454w3,454+455w3
,455+456w3,456+457w3,457+458w3,458+459w3,459+460w3,460+461w3,461+462w3,46
2+463w3,463+464w3,464+465w3,465+466w3,466+467w3,467+468w3,468+469w3,469+
470w3,470+471w3,471+472w3,472+473w3,473+474w3,474+475w3,475+476w3,476+47
7w3,477+478w3,478+479w3,479+480w3,480+481w3,481+482w3,482+483w3,483+484w
3,484+485w3,485+486w3,486+487w3,487+488w3,488+489w3,489+490w3,490+491w3,4
91+492w3,492+493w3,493+494w3,494+495w3,495+496w3,496+497w3,497+498w3,498
+499w3,499+500w3,500+501w3,501+502w3,502+503w3,503+504w3,504+505w3,505+5
06w3,506+507w3,507+508w3,508+509w3,509+510w3,510+511w3,511+512w3,512+513
w3,513+514w3,514+515w3,515+516w3,516+517w3,517+518w3,518+519w3,519+520w3
,520+521w3,521+522w3,522+523w3,523+524w3,524+525w3,525+526w3,526+527w3,52
7+528w3,528+529w3,529+530w3,530+531w3,531+532w3,532+533w3,533+534w3,534+
535w3,535+536w3,536+537w3,537+538w3,538+539w3,539+540w3,540+541w3,541+54
2w3,542+543w3,543+544w3,544+545w3,545+546w3,546+547w3,547+548w3,548+549w
3,549+550w3,550+551w3,551+552w3,552+553w3,553+554w3,554+555w3,555+556w3,5
56+557w3,557+558w3,558+559w3,559+560w3,560+561w3,561+562w3,562+563w3,563
+564w3,564+565w3,565+566w3,566+567w3,567+568w3,568+569w3,569+570w3,570+5
71w3,571+572w3,572+573w3,573+574w3,574+575w3,575+576w3,576+577w3,577+578
w3,578+579w3,579+580w3,580+581w3,581+582w3,582+583w3,583+584w3,584+585w3

,585+586w3,586+587w3,587+588w3,588+589w3,589+590w3,590+591w3,591+592w3,592+593w3,593+594w3,594+595w3,595+596w3,596+597w3,597+598w3,598+599w3,599+600w3,600+601w3,601+602w3,602+603w3,603+604w3,604+605w3,605+606w3,606+607w3,607+608w3,608+609w3,609-St3=0

St1>=558

St2>=86

St3>=495

+Ft1-St1>=20

+Ft2-St2>=20

+Ft3-St3>=11

+Ft1-St2+10000y1,2<=10000

+Ft1-St3+10000y1,3<=10000

+Ft2-St1+10000y2,1<=10000

+Ft2-St3+10000y2,3<=10000

+Ft3-St1+10000y3,1<=10000

+Ft3-St2+10000y3,2<=10000

256x1,1-262x1,1>=0

256x1,2-331x1,2>=0

256x1,3-93x1,3>=0

350x2,1-262x2,1>=0

350x2,2-331x2,2>=0

350x2,3-93x2,3>=0

13x1,1-12x1,1>=0

13x1,2-14x1,2>=0

13x1,3-7x1,3>=0

18x2,1-12x2,1>=0

18x2,2-14x2,2>=0

18x2,3-7x2,3>=0

7x1,1-St1<=0

7x1,2-St2<=0

7x1,3-St3<=0

62x2,1-St1<=0

62x2,2-St2<=0

62x2,3-St3<=0

ALL VARIABLES>=0

binaries
x1,1
x1,2

x1,3
x2,1
x2,2
x2,3
f1
f2
f3
ea1
ea2
ea3
y1,2
y1,3
y2,1
y2,3
y3,1
y3,2
w1,1
w1,2
w1,3
w1,4
w1,5
w1,6
w1,7
w1,8
w1,9
w1,10
w1,11
w1,12
w1,13
w1,14
w1,15
w1,16
w1,17
w1,18
w1,19
w1,20
w1,21
w1,22
w1,23
w1,24
w1,25

w1,26
w1,27
w1,28
w1,29
w1,30
w1,31
w1,32
w1,33
w1,34
w1,35
w1,36
w1,37
w1,38
w1,39
w1,40
w1,41
w1,42
w1,43
w1,44
w1,45
w1,46
w1,47
w1,48
w1,49
w1,50
w1,51
w1,52
w1,53
w1,54
w1,55
w1,56
w1,57
w1,58
w1,59
w1,60
w1,61
w1,62
w1,63
w1,64
w1,65
w1,66

w1,67
w1,68
w1,69
w1,70
w1,71
w1,72
w1,73
w1,74
w1,75
w1,76
w1,77
w1,78
w1,79
w1,80
w1,81
w1,82
w1,83
w1,84
w1,85
w1,86
w1,87
w1,88
w1,89
w1,90
w1,91
w1,92
w1,93
w1,94
w1,95
w1,96
w1,97
w1,98
w1,99
w1,100
w1,101
w1,102
w1,103
w1,104
w1,105
w1,106
w1,107

w1,108
w1,109
w1,110
w1,111
w1,112
w1,113
w1,114
w1,115
w1,116
w1,117
w1,118
w1,119
w1,120
w1,121
w1,122
w1,123
w1,124
w1,125
w1,126
w1,127
w1,128
w1,129
w1,130
w1,131
w1,132
w1,133
w1,134
w1,135
w1,136
w1,137
w1,138
w1,139
w1,140
w1,141
w1,142
w1,143
w1,144
w1,145
w1,146
w1,147
w1,148

w1,149
w1,150
w1,151
w1,152
w1,153
w1,154
w1,155
w1,156
w1,157
w1,158
w1,159
w1,160
w1,161
w1,162
w1,163
w1,164
w1,165
w1,166
w1,167
w1,168
w1,169
w1,170
w1,171
w1,172
w1,173
w1,174
w1,175
w1,176
w1,177
w1,178
w1,179
w1,180
w1,181
w1,182
w1,183
w1,184
w1,185
w1,186
w1,187
w1,188
w1,189

w1,190
w1,191
w1,192
w1,193
w1,194
w1,195
w1,196
w1,197
w1,198
w1,199
w1,200
w1,201
w1,202
w1,203
w1,204
w1,205
w1,206
w1,207
w1,208
w1,209
w1,210
w1,211
w1,212
w1,213
w1,214
w1,215
w1,216
w1,217
w1,218
w1,219
w1,220
w1,221
w1,222
w1,223
w1,224
w1,225
w1,226
w1,227
w1,228
w1,229
w1,230

w1,231
w1,232
w1,233
w1,234
w1,235
w1,236
w1,237
w1,238
w1,239
w1,240
w1,241
w1,242
w1,243
w1,244
w1,245

w1,244
w1,245
w1,246
w1,247
w1,248
w1,249
w1,250
w1,251
w1,252
w1,253
w1,254
w1,255
w1,256
w1,257
w1,258
w1,259
w1,260
w1,261
w1,262
w1,263
w1,264
w1,265
w1,266
w1,267

w1,268
w1,269
w1,270
w1,271
w1,272
w1,273
w1,274
w1,275
w1,276
w1,277
w1,278
w1,279
w1,280
w1,281
w1,282
w1,283
w1,284
w1,285
w1,286
w1,287
w1,288
w1,289
w1,290
w1,291
w1,292
w1,293
w1,294
w1,295
w1,296
w1,297
w1,298
w1,299
w1,300
w1,301
q2,593
q2,594
q2,595
q2,596
q2,597
q2,598
q2,599

q2,600
q2,601
q2,602
q2,603
q2,604
q2,605
q2,606
q2,607
q2,608
q2,609
q3,608
q3,609

end