Cooperative Simultaneous Orbit Determination and Synchronization: A Distributed Factor Graph Approach

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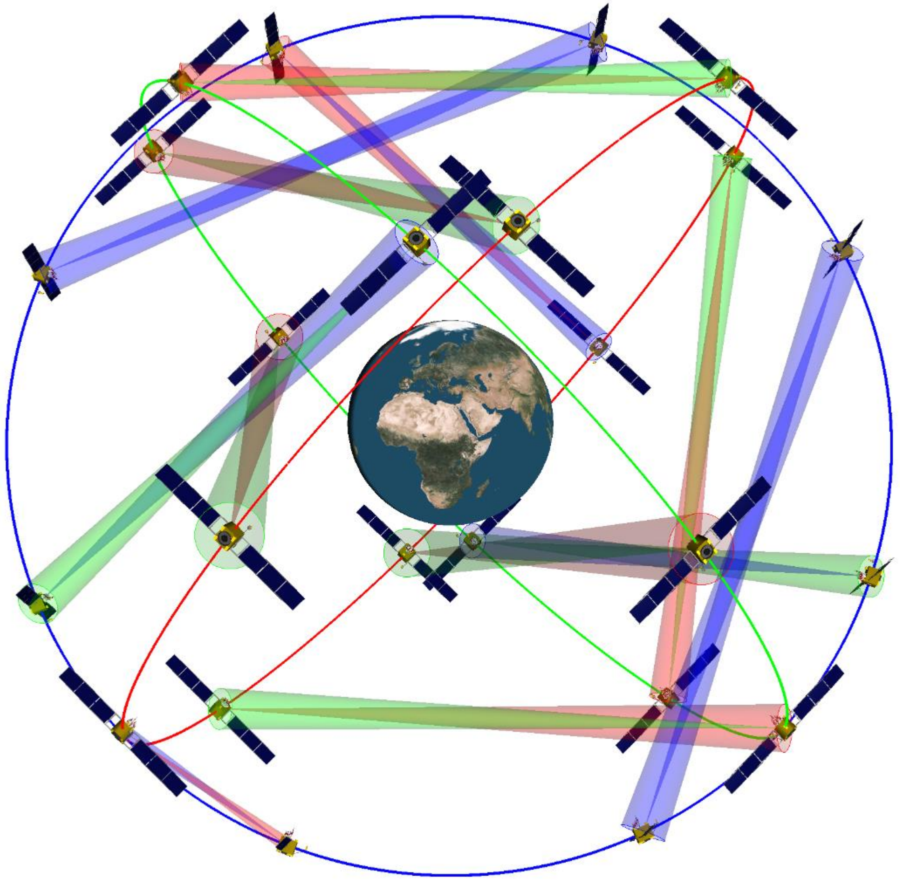
Abstraction

A novel distributed approach based on a factor graph framework, using inter-satellite ranging measurements to achieve simultaneous autonomous orbit determination and time synchronization of networked satellites, is proposed. In order to reduce computation complexity as well as communication overhead considering limited resource constraints onboard, we present a parametric message passing scheme with Gaussian approximation exploiting the geometrical characteristics of inter-satellite measurements. Simulation results show that proposed framework has considerable performance in terms of accuracy and convergence rate.

Introduction

- Satellites of **Global Navigation Satellite Systems(GNSS)** should locate themselves.
- **Autonomous Orbit Determination and Time Synchronization** (AOD&TS) has been proposed years to determine satellites' position and time with minimum or none support from ground station.
- AOD&TS exploits inter-satellite TOA measurements to obtain position and time information, which is an instance of **Cooperative Localization** in Wireless Localization
- **Factor Graph** is a scalable and distributed approach to achieve our algorithm.



System Model

Location-related State Transition Model:

$$\mathbf{x}_i(t_b) = \Phi_i(\mathbf{x}_i(t_a), t_a, t_b) + \boldsymbol{\omega}_i(t_a, t_b), \boldsymbol{\omega}_i \sim N(\mathbf{0}, \mathrm{Q}_i(\mathrm{t}_a, \mathrm{t}_b))$$

when $\boldsymbol{x}_i(t_b)$ is Gaussian at the previous state, $\Phi_i(\boldsymbol{x}_i(t_b), t_a, t_b)$ is estimated to be approximately a Gaussian distribution.

Clock model: can be modeled as a stochastic process and written as a certain order polynomial function

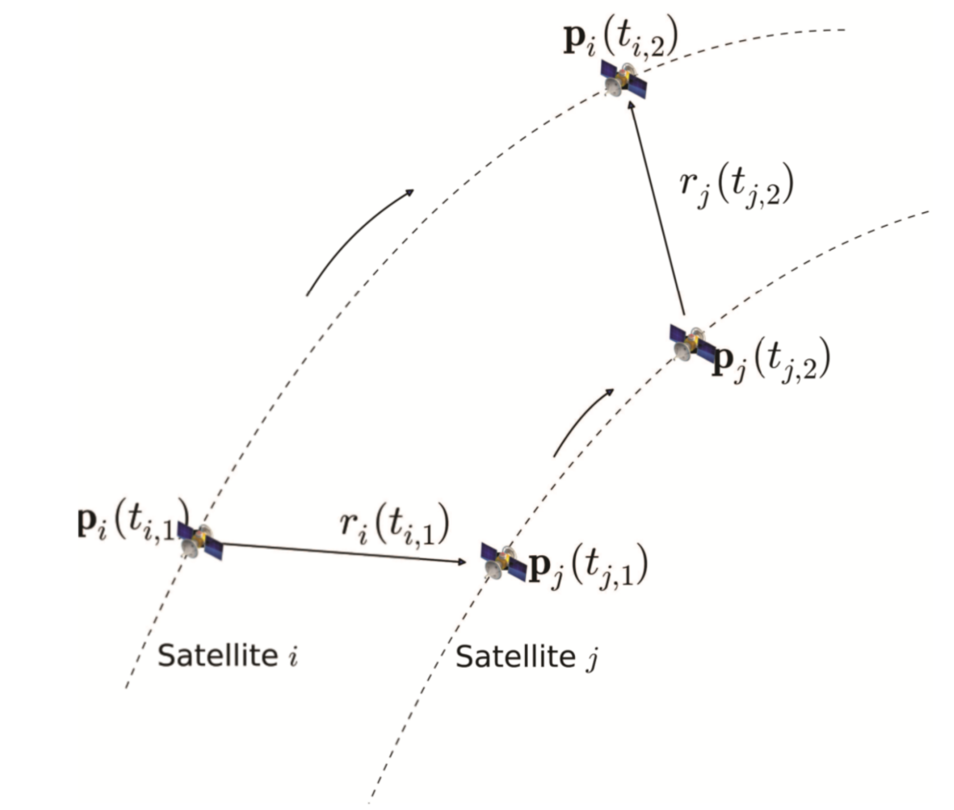
Round-time TOA measurement Model:

$$\varrho_{ij} = \left\|\mathbf{p}_i(t_{i,1}) - \mathbf{p}_j(t_{j,1})\right\| - \Delta_{ij} + v_{ij}$$

$$\varrho_{ji} = \left\|\mathbf{p}_j(t_{j,2}) - \mathbf{p}_i(t_{i,2})\right\| + \Delta_{ij} + v_{ij}$$

Δ_{ij}: Gaussian Distributed Clock difference between satellite i and j.

v_{ij}: Zero-mean Gaussian Distributed noise.



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Statistical Model

For a pair of satellites with inter-satellite measurement, the join pdf of Δ_{ij} and location –related states is

$$p(\mathbf{x}_{i,2},\mathbf{x}_{j,2},\Delta_{ij},\mathbf{x}_{i,1},\mathbf{x}_{j,1}\mid\mathbf{x}_{i,1}^{-},\mathbf{x}_{j,1}^{-},\varrho_{ij},\varrho_{ji},\delta t_i^{-},\delta t_j^{-})$$

$$= p(\mathbf{x}_{i,1}\mid\mathbf{x}_{i,1}^{-})\,p(\mathbf{x}_{j,1}\mid\mathbf{x}_{j,1}^{-})\,p(\Delta_{ij}\mid\delta t_i^{-},\delta t_j^{-})\cdot p(\mathbf{x}_{i,2}\mid\mathbf{x}_{i,1})\,p(\mathbf{x}_{j,1}\mid\varrho_{ij},\mathbf{x}_{i,1},\Delta_{ij})$$

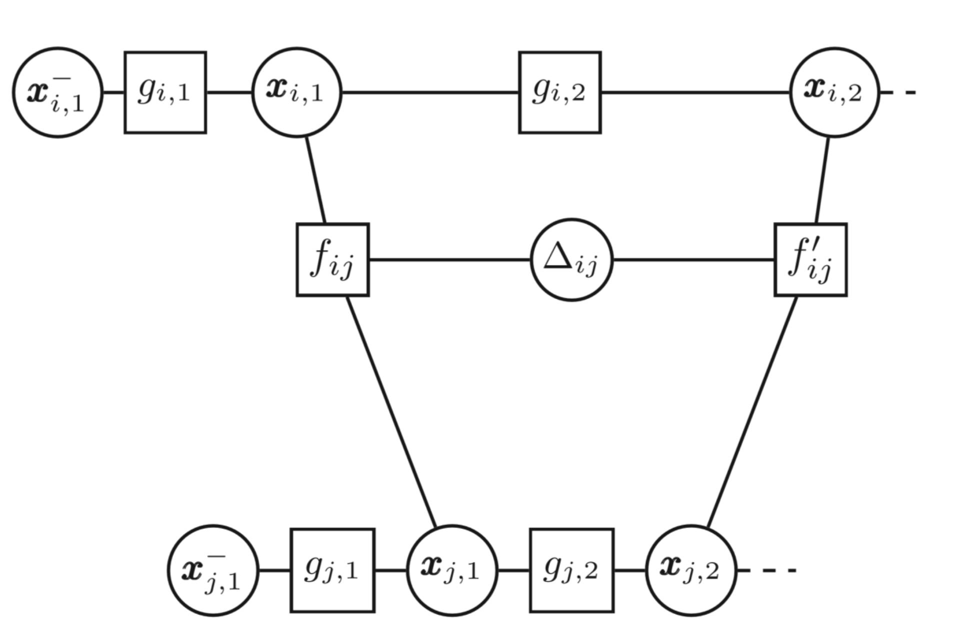
$$\cdot\, p(\mathbf{x}_{j,2}\mid\mathbf{x}_{j,1})\,p(\mathbf{x}_{i,2}\mid\varrho_{ji},\mathbf{x}_{j,2},\Delta_{ij})$$

Specifically, the likelihood of the pseudorange measurements is

$$p(\mathbf{x}_{j,1}\mid\varrho_{ij},\mathbf{x}_{i,1},\Delta_{ij}) = p_c(\mathbf{x}_{j,1};\varrho_{ij}+\Delta_{ij},\ \boldsymbol{\mu}_{\mathbf{p}_{i,1}},\boldsymbol{\Sigma}_{\mathbf{p}_{i,1}}+\sigma_{\varrho}^{2}\,\mathbf{I}+\sigma_{\Delta_{ij}}^{2}\,\mathbf{I})$$

Where[2]

$$p_c(\mathbf{x}_i;\ r,\ \mathbf{p}_j,\boldsymbol{\Sigma}) \propto \exp\left[-\frac{1}{2}\left(\mathbf{p}'-r\frac{\mathbf{p}'}{\|\mathbf{p}'\|}\right)^{T}\boldsymbol{\Sigma}^{-1}\left(\mathbf{p}'-r\frac{\mathbf{p}'}{\|\mathbf{p}'\|}\right)\right]$$



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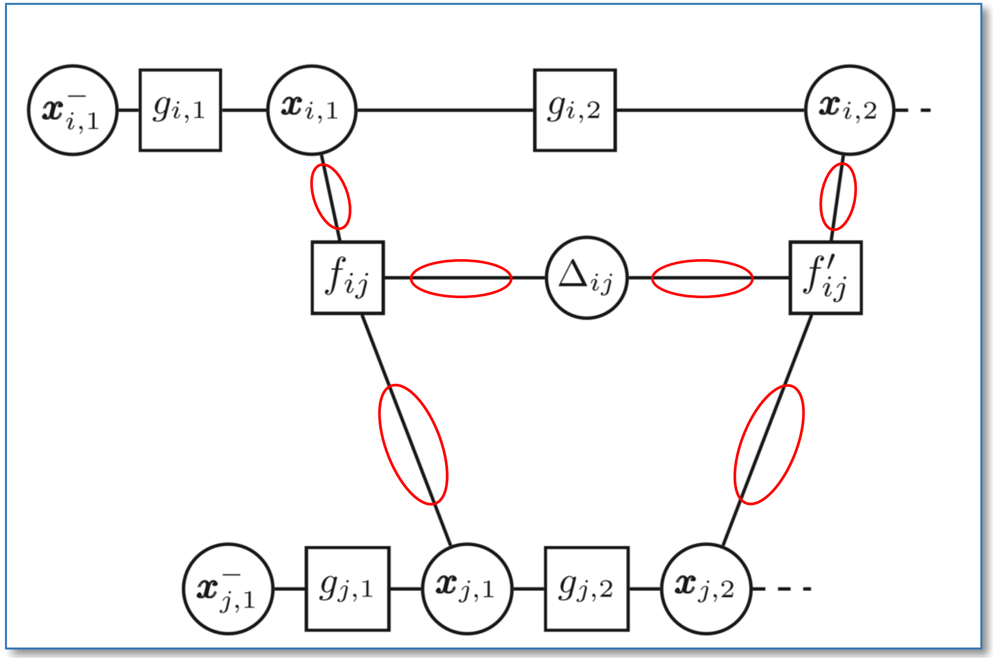
Gaussian Approximation Message Passing(GAMP)

Inspired by **large-scale nature** of spatial geometry of satellites network. The non-Gaussian message can be approximated to Gaussian in a closed form, yet remaining high precision.



Gaussian Approximation Message Passing(GAMP)

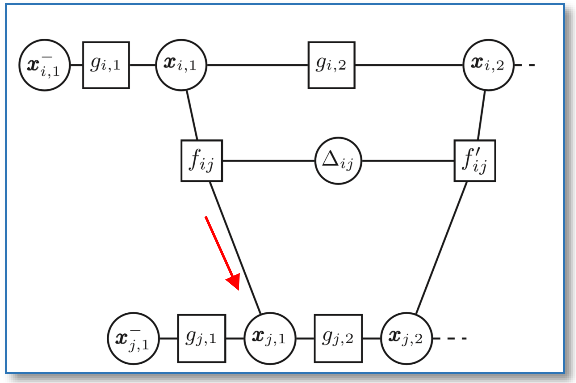
There is large distance between linked satellites(typically ~10^4km), compared with position ambiguity of satellite(typically < 10m). On the tangential plane, ranging measurement can't provide information, implies

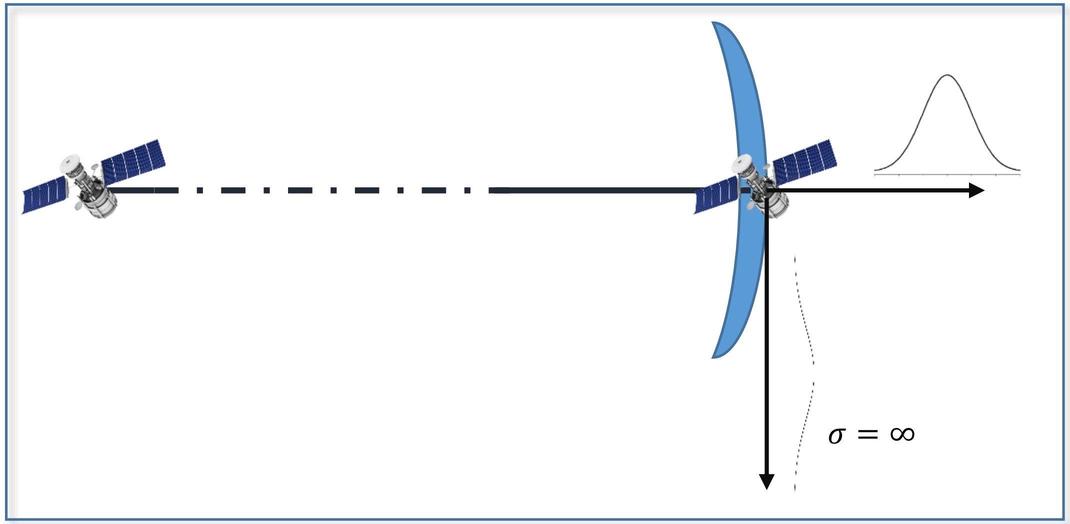
$$p_c(\mathbf{p}_i;\ r,\ \mathbf{p}_j,\boldsymbol{\Sigma}) \propto \exp\left[-\frac{1}{2}\left(\mathbf{p}'-r\frac{\mathbf{p}'}{\|\mathbf{p}'\|}\right)^T \boldsymbol{\Sigma}^{-1}\left(\mathbf{p}'-r\frac{\mathbf{p}'}{\|\mathbf{p}'\|}\right)\right]$$

Can be approximated to Gaussian with the inverse covariance be

$$\boldsymbol{\Sigma}_{\mathbf{p}_i}^{-1} = \begin{bmatrix} \mathbf{q}^T\ \boldsymbol{\Sigma}^{-1}\mathbf{q} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In Rational-Tangential-Normal coordinates, where $\boldsymbol{q}$ is a direction vector.





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GAMP -II

For clock difference likelihood,

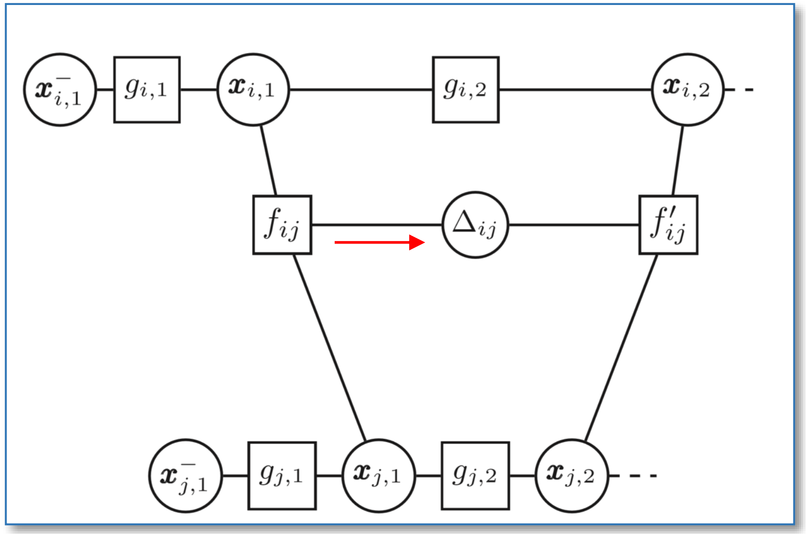
$$\begin{aligned}\Delta_{ij} &= \left\|\mathbf{p}_{i,1} - \mathbf{p}_{j,1}\right\| - \varrho_{ij} - v_{ij} \\ &= \left\|\mathbf{p} + \Delta\mathbf{p}\right\| - \varrho_{ij} - v_{ij}.\end{aligned}$$

$\|\mathbf{p}\|$ is much larger than $\|\Delta\mathbf{p}\|$ (uncertainty),

$$\left\|\mathbf{p} + \Delta\mathbf{p}\right\| \approx \left\|\mathbf{p}\right\|\left(1 + \frac{2\Delta\mathbf{p}^T\mathbf{p} + \Delta\mathbf{p}^T\Delta\mathbf{p}}{2\left\|\mathbf{p}\right\|^2}\right)$$

$$\approx \left\|\mathbf{p}\right\| + \frac{\Delta\mathbf{p}^T\mathbf{p}}{\left\|\mathbf{p}\right\|}$$

is Gaussian approximately.



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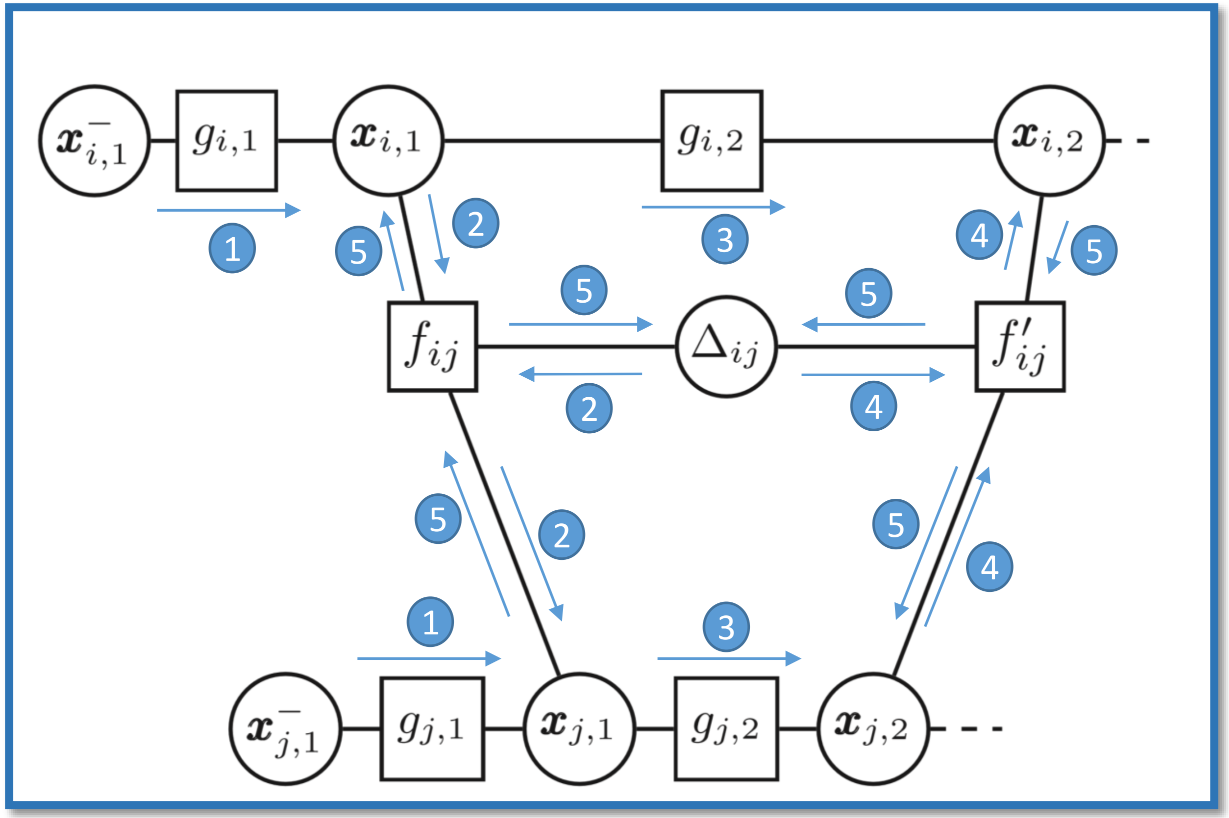
Message passing schedule in a cyclic factor graph

It is necessary to carry out the iterative message passing on this cyclic factor graph.



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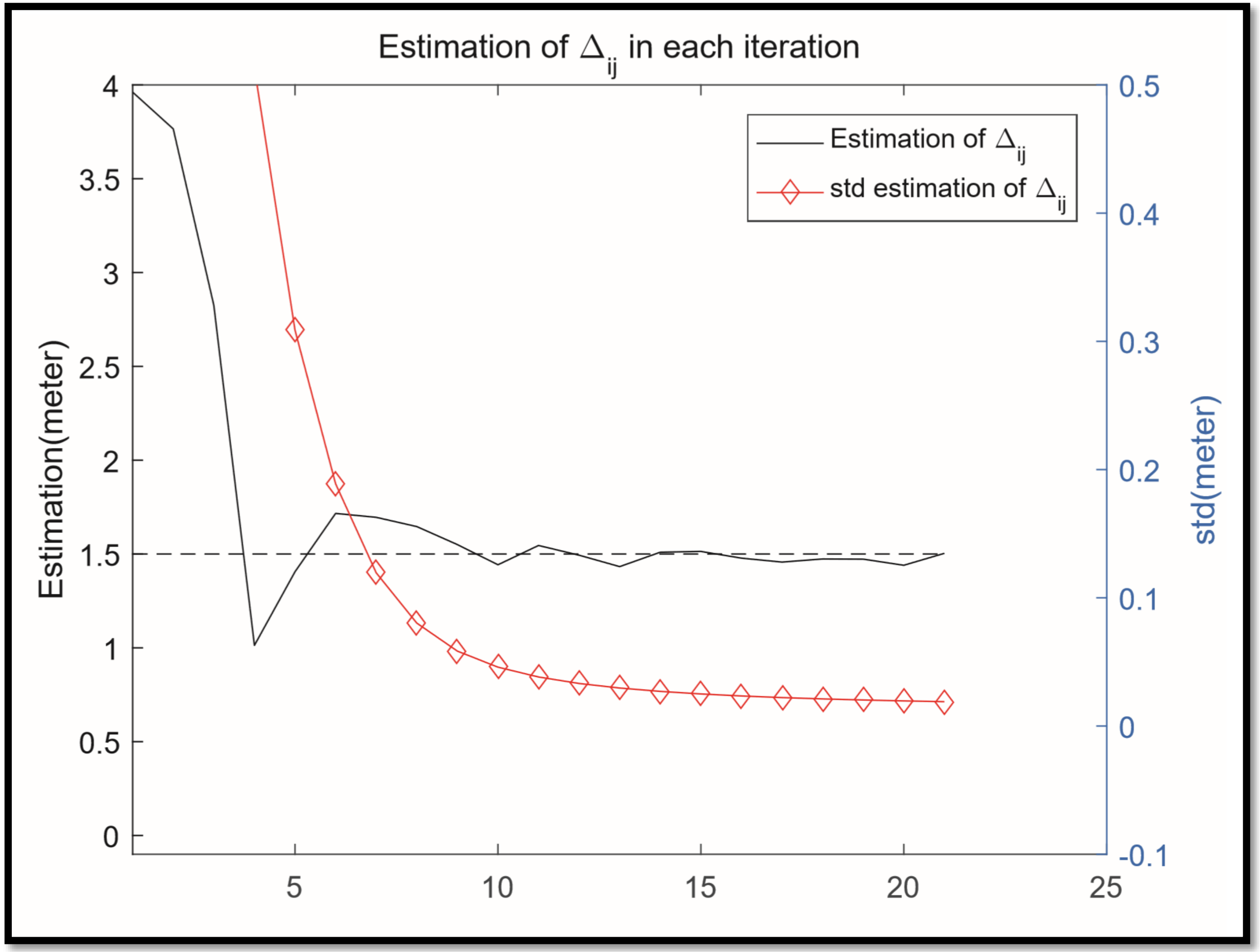
Simulation Results

Simulation settings:

- 30 satellites, with circular orbits at different orbital plane. Orbit altitude: 2×10^4km
- No initial position error.
- Model local clock of satellite $r_i(t) = \alpha_i(t) + \beta_i(t)$, where $\alpha_i(t) \sim \mathcal{N}\left(1, \left(10^{-15}\right)^2\right), \beta_i \sim \mathcal{N}\left(0, \left(10^{-10}s\right)^2\right)$

Compared to Particle-based Approximation Message Passing(PAMP), GAMP is about 34.4 times faster.



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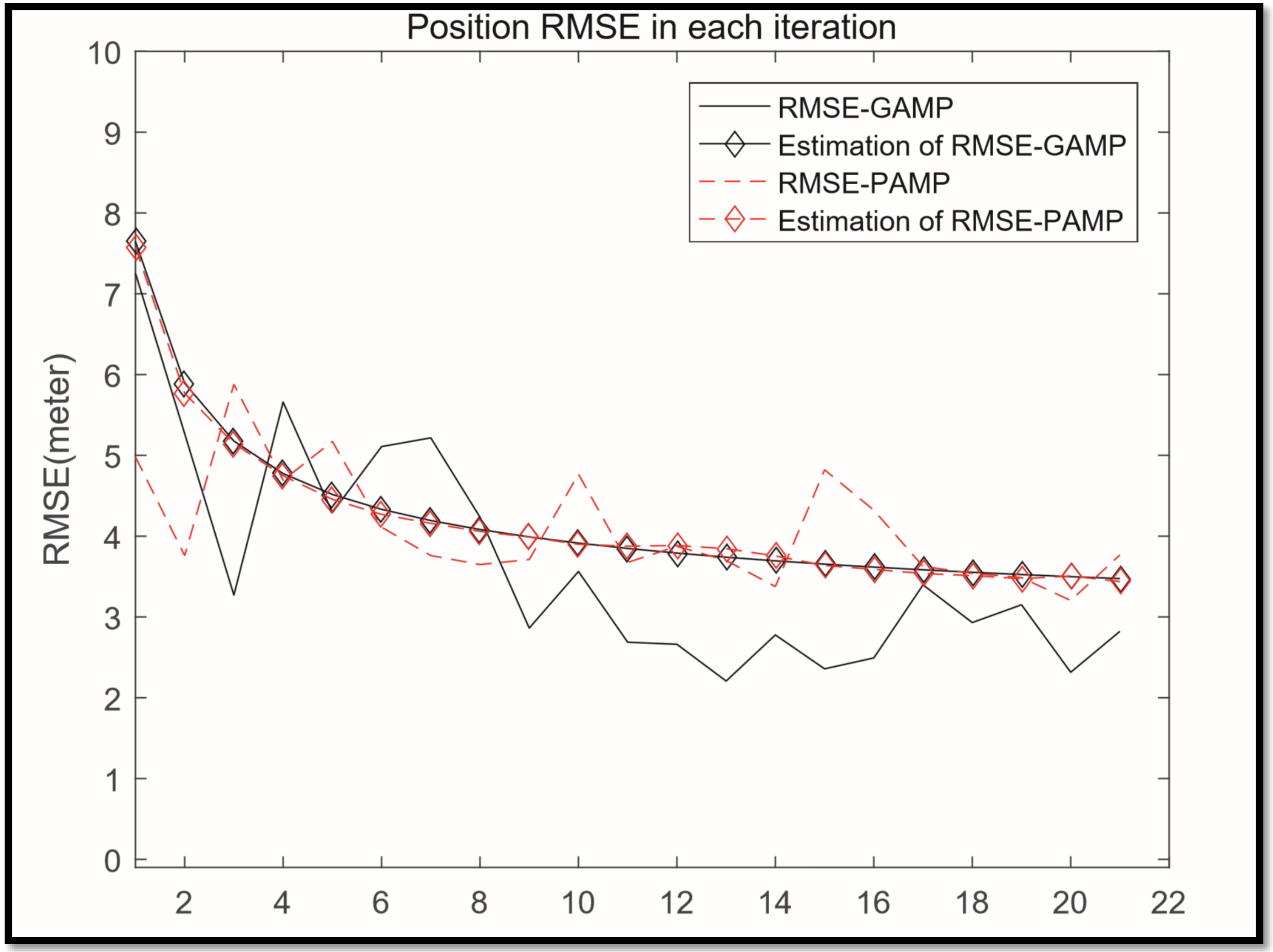

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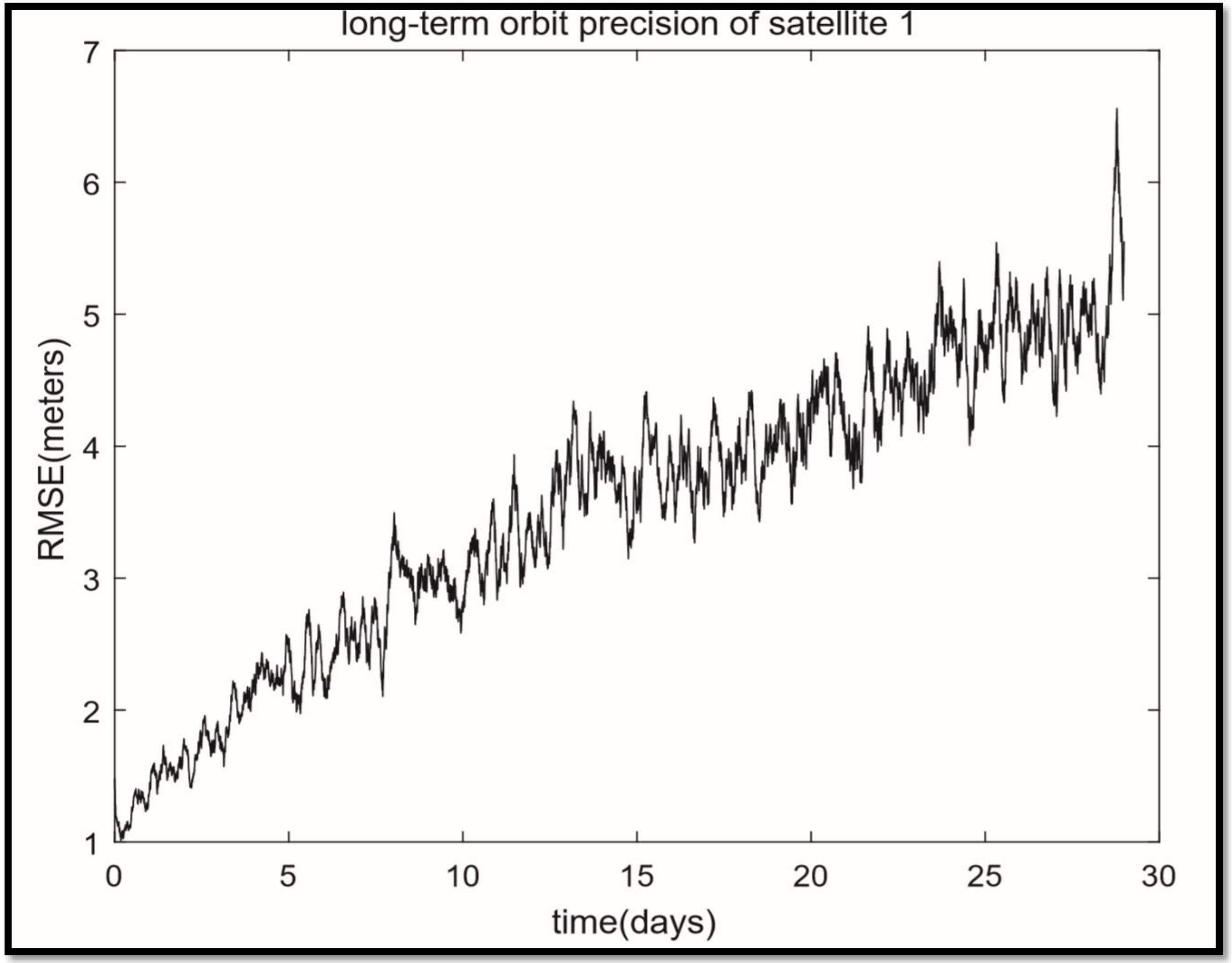

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[2] F. Huang, W. Huang, Y. Wang, Y. Zhou, and K. Lin, "Analysis of Ground Anchor Stations Influence on Autonomous Orbit Determination with Distributed Algorithm," in China Satellite Navigation Conference (CSNC) 2016 Proceedings: Volume III. Springer, Singapore, pp. 75–85.