

---

**EXERCISE SET 3.3**

10. Show that the polynomial interpolating the following data has degree 3.

$x$	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4

11. a. Show that the cubic polynomials

$$P(x) = 3 - 2(x + 1) + 0(x + 1)(x) + (x + 1)(x)(x - 1)$$

and

$$Q(x) = -1 + 4(x + 2) - 3(x + 2)(x + 1) + (x + 2)(x + 1)(x)$$

both interpolate the data

$x$	-2	-1	0	1	2
$f(x)$	-1	3	1	-1	3

- b. Why does part (a) not violate the uniqueness property of interpolating polynomials?

13. The following data are given for a polynomial  $P(x)$

$x$	0	1	2
$P(x)$	2	-1	4

Determine by hand  $P(x)$

14. The following data are given for a polynomial  $P(x)$

$x$	0	1	2	3
$P(x)$	4	9	15	18

Determine by hand  $P(x)$

16. For a function  $f$ , the Newton divided-difference formula gives the interpolating polynomial

$$P_3(x) = 1 + 4x + 4x(x - 0.25) + \frac{16}{3}x(x - 0.25)(x - 0.5),$$

on the nodes  $x_0 = 0$ ,  $x_1 = 0.25$ ,  $x_2 = 0.5$  and  $x_3 = 0.75$ . Find  $f(0.75)$ .

17. For a function  $f$ , the forward-divided differences are given by

$x_0 = 0.0$	$f[x_0]$		
		$f[x_0, x_1]$	
$x_1 = 0.4$	$f[x_1]$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.7$	$f[x_2] = 6$		

Determine the missing entries in the table.