Introduction to Network Statistics

Lorien Jasny[1]

[1]University of Exeter
L.Jasny@exeter.ac.uk

A step-change in quantitative social science skills
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PolNets Workshop
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Contents

Hypothesis Testing

Relating Node level indices to covariates

- Node Level Indices: centrality measures, brokerage, constraint

Relating Node level indices to covariates

- Node Level Indices: centrality measures, brokerage, constraint
- Node Covariates: measures of power, career advancement, gender – really anything you want to study that varies at the node level

Emergent Multi-Organizational Networks (EMON) Dataset

- 7 case studies of EMONs in the context of search and rescue activities from Drabek et. al. (1981)

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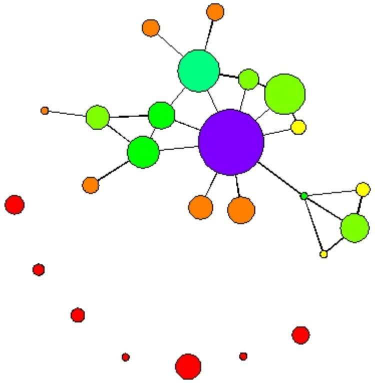
- Ties between organizations are self-reported levels of communication coded from 1 to 4 with 1 as most frequent

Emergent Multi-Organizational Networks (EMON) Dataset

Attribute Data

- Command Rank Score (CRS): mean rank (reversed) for prominence in the command structure
- Decision Rank Score (DRS): mean rank (reversed) for prominence in decision making process
- Paid Staff: number of paid employees
- Volunteer Staff: number of volunteer staff
- Sponsorship: organization type (City, County, State, Federal, or Private)

Correlation between DRS and Degree?



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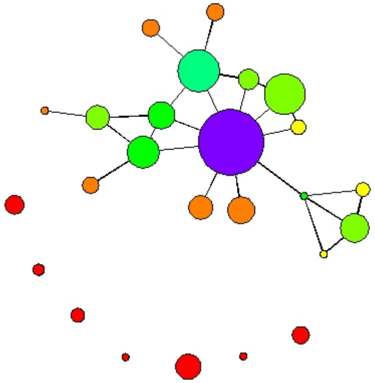
- Subsample of Mutually Reported "Continuous Communication" in Texas EMON



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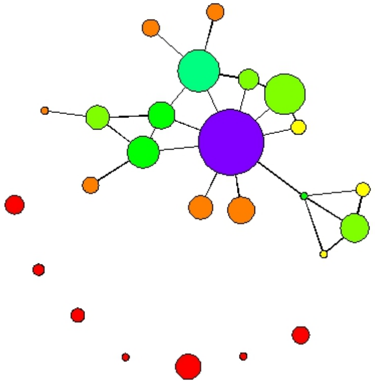

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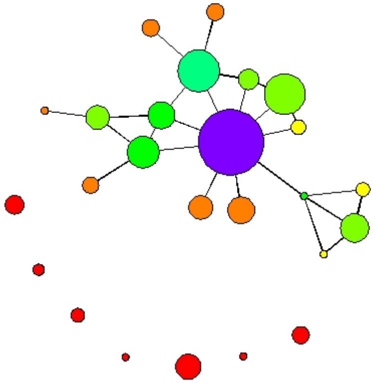

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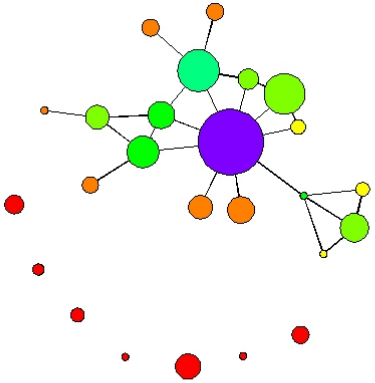
- Empirical corelation $\rho = 0.86$



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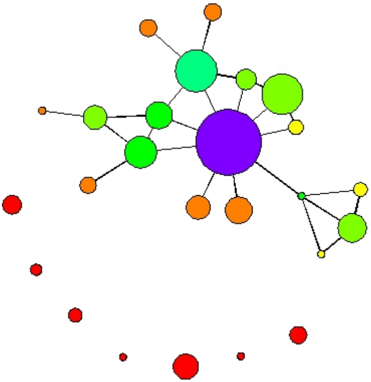

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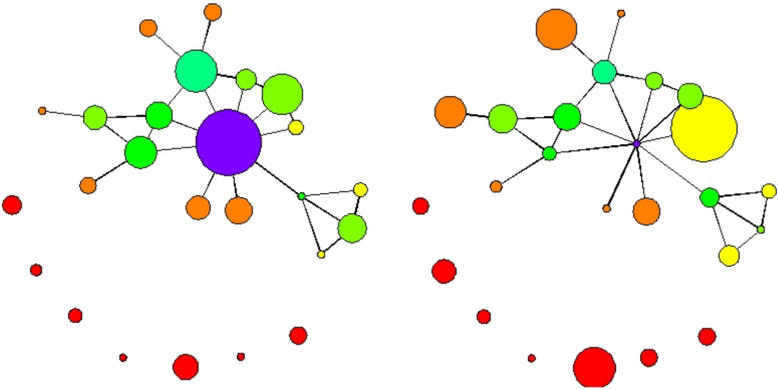

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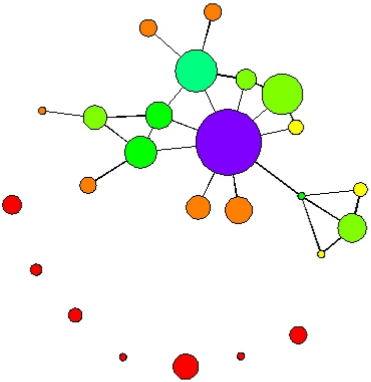

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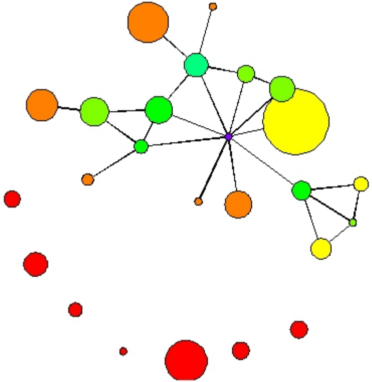

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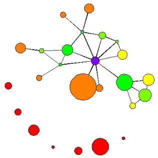

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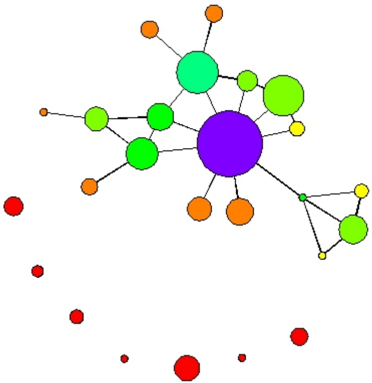

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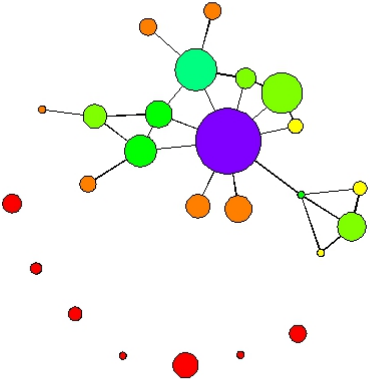

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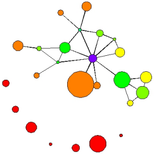

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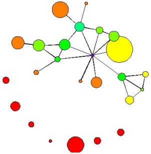

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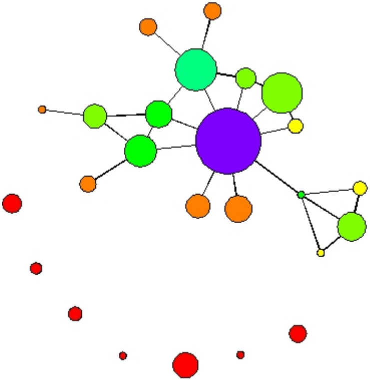

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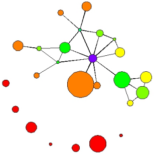

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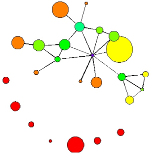

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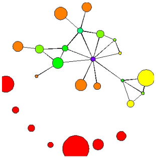

$\rho = -0.39$

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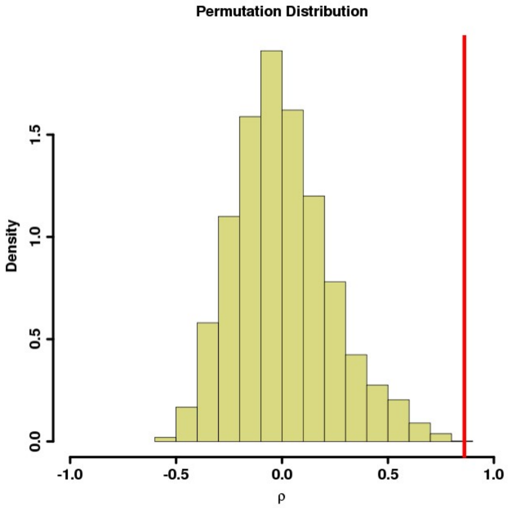

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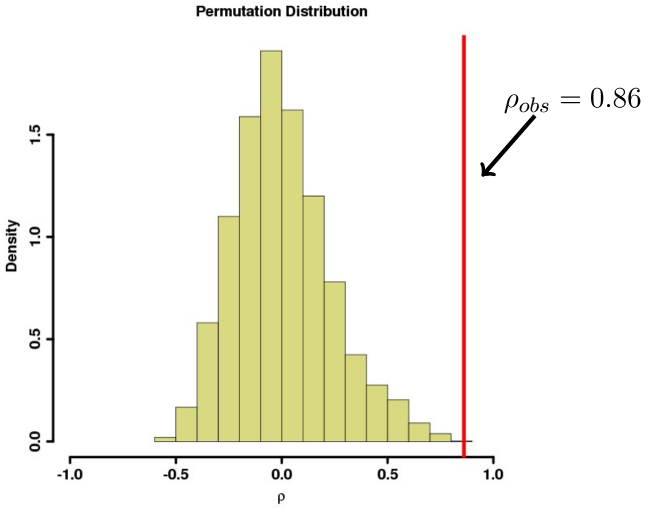

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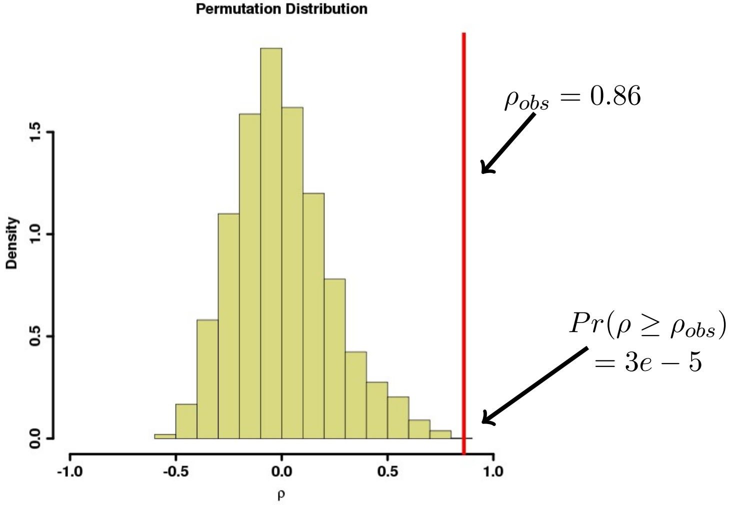

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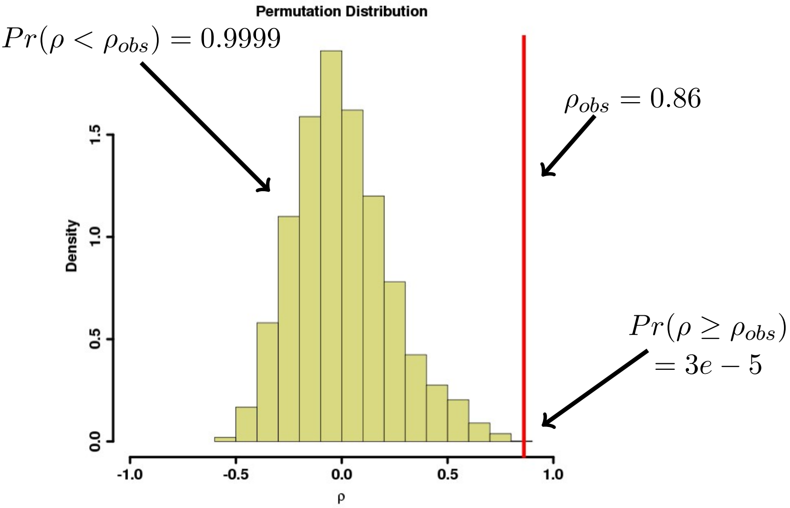

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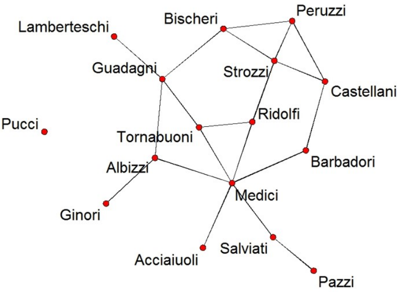
- Big assumption: ***position*** predicts the ***properties of those who hold them***
- Conditioning on NLI values, so dependence in accounted for ***assuming no error in the network***
- NLIs as dependent variables more problematic due to autocorrelation

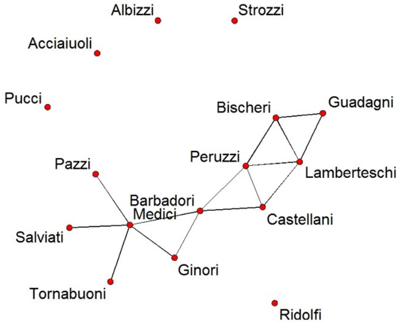
Code Time

Sections 1-2.3

Quadratic Assignment Procedure

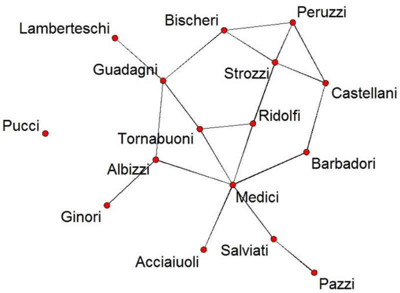


Marriage



Business

Quadratic Assignment Procedure



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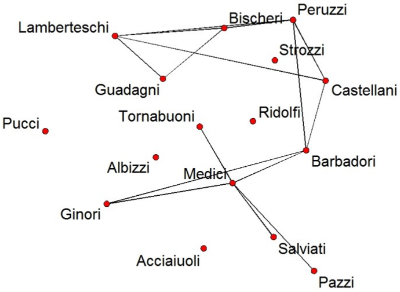

Business

Graph Correlation

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- Simple way of comparing graphs on the same vertex set by element

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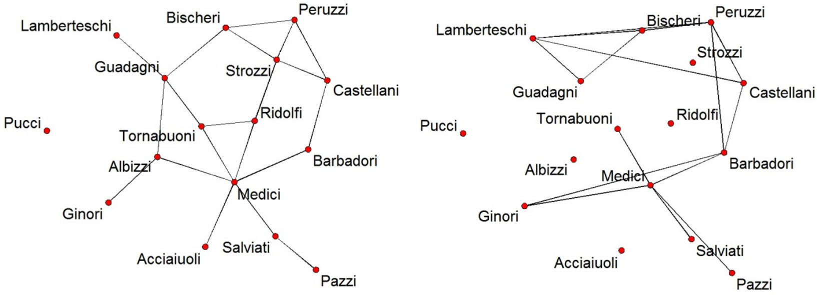
- $gcor\left(\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}, \begin{bmatrix}1 & 1\\2 & 2\end{bmatrix}\right) = cor([1,1,1,0],[1,1,2,2])$

Do business ties coincide with marriages?

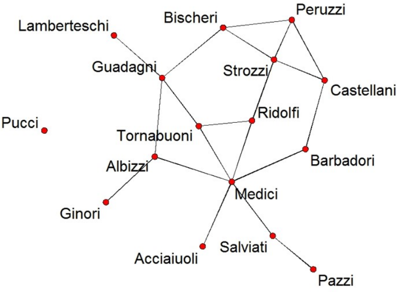


Marriage

Business

$$\rho = 0.372$$

Do business ties coincide with marriages?



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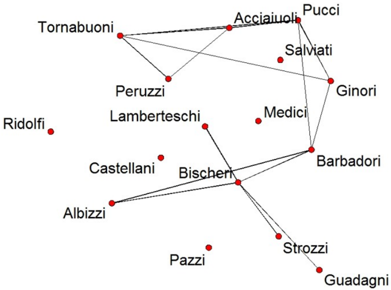

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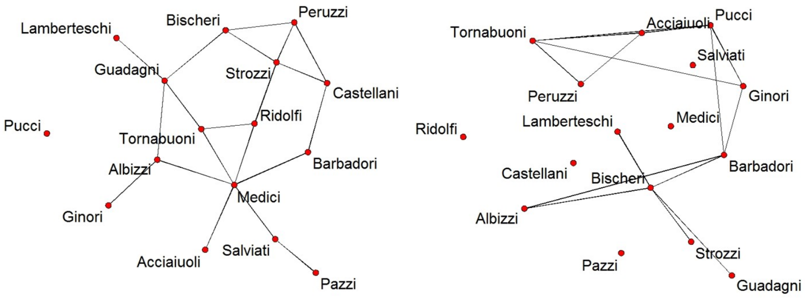
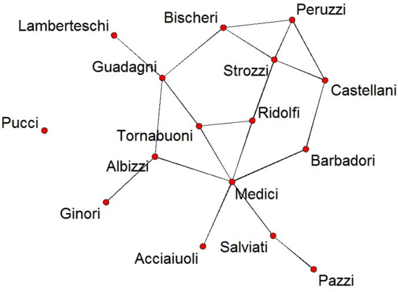
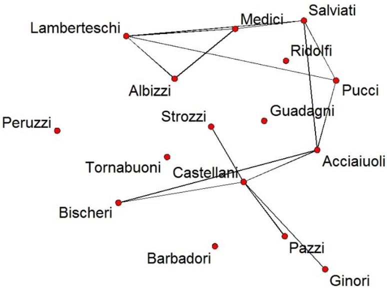


Marriage | Business

$$\rho = 0.169$$

Do business ties coincide with marriages?





Marriage Business

$$\rho = -0.034$$

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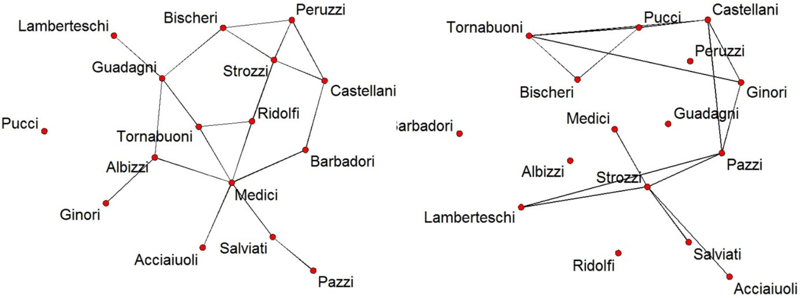
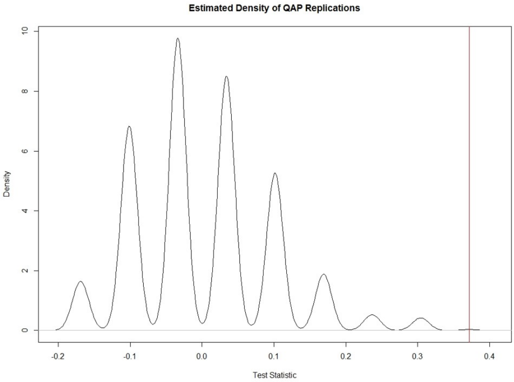


Marriage

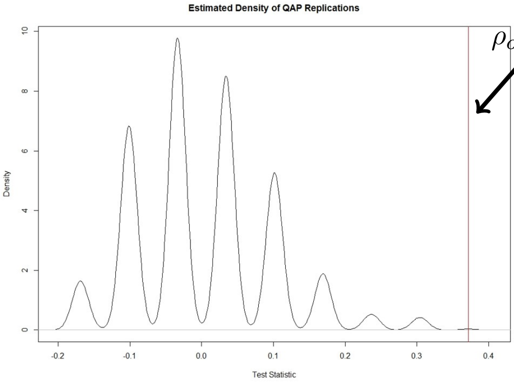
Business

$$\rho = -0.101$$

QAP Test


Estimated Density of QAP Replications
Density
10
8
6
4
2
0
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
Test Statistic

QAP Test



$\rho_{obs} = 0.372$

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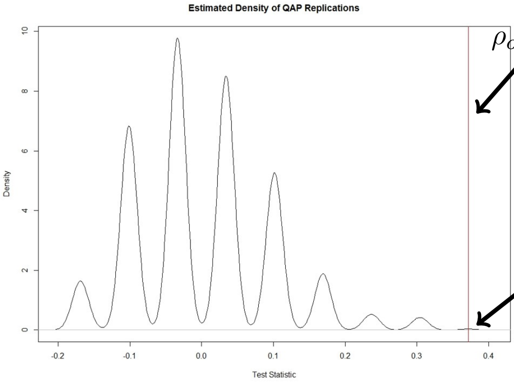


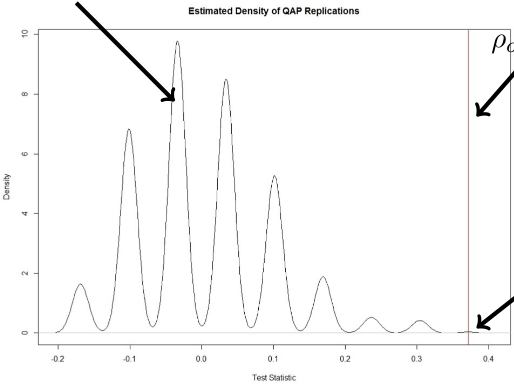
$\rho_{obs} = 0.372$

$Pr(\rho \geq \rho_{obs})$
$= .001$

QAP Test

$Pr(\rho < \rho_{obs}) = 0.999$

$\rho_{obs} = 0.372$



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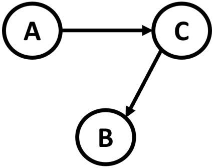
$= .001$

Why can't we use the same permutation test?

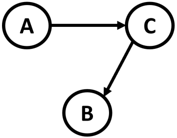
QAP Test



	A	B	C
A	-	0	1
B	0	-	0
C	0	1	-

$$\begin{bmatrix} - \\ 0 \\ 1 \\ 0 \\ - \\ 0 \\ 0 \\ 1 \\ - \end{bmatrix}$$

QAP Test



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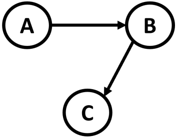

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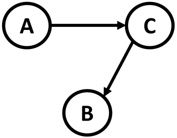

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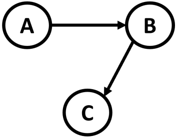

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  - Where $\mathbf{E}$ is the expectation operator (analagous to "mean" or "average")
  - Y_{ij} is the value from i to j on the dependent relation with adjacency matrix Y
  - X_{kij} is the value of the kth predictor for the (i, j) ordered pair, and $\beta_0, \ldots \beta_\rho$ are coefficients

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  - eg. simple adjacency matrix, sender/receiver effects, attribute differences, elements held in common

Code Time

Sections 2.4-2.5

Network Autocorrelation Models

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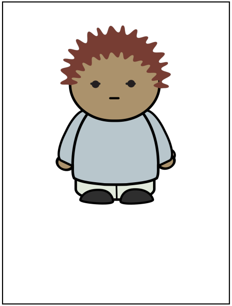
The Classical Regression Model



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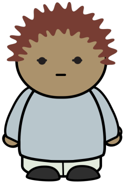
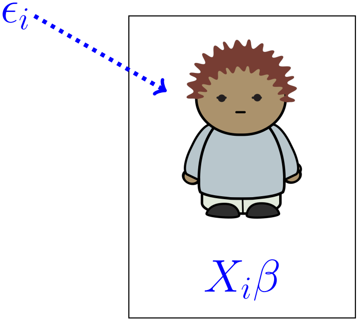


$X_i\beta$

The Classical Regression Model



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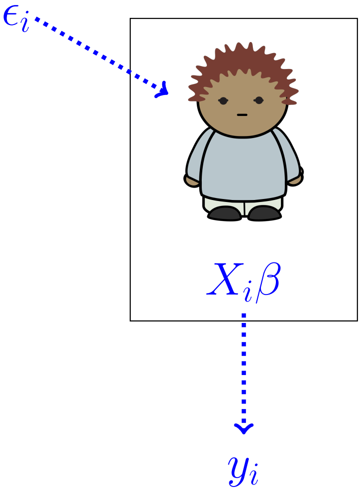
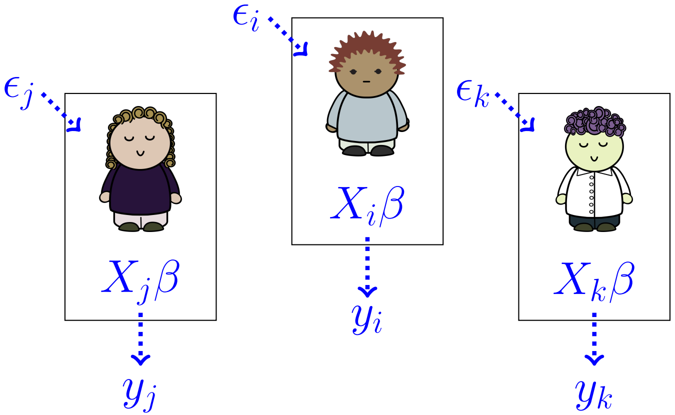


Adding Network AR Effects



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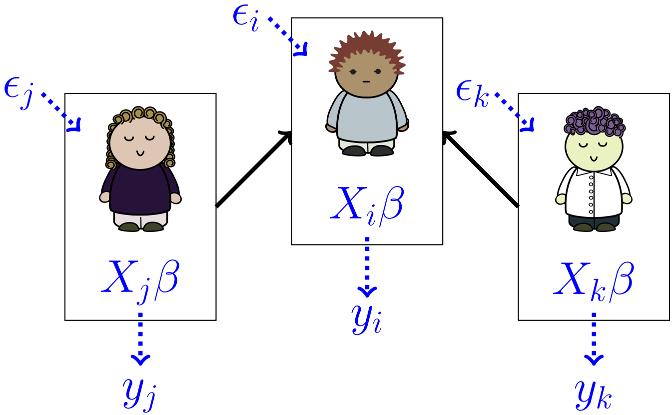

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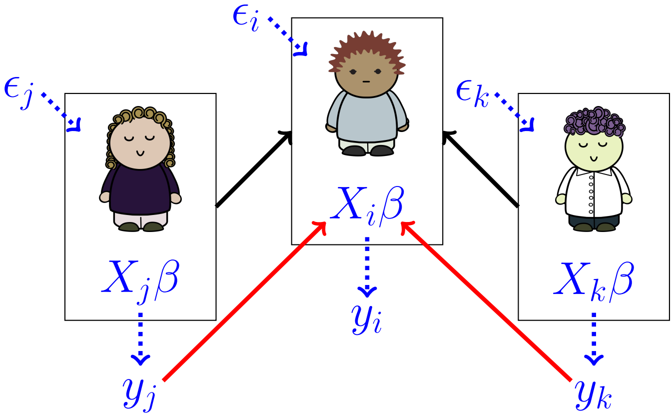

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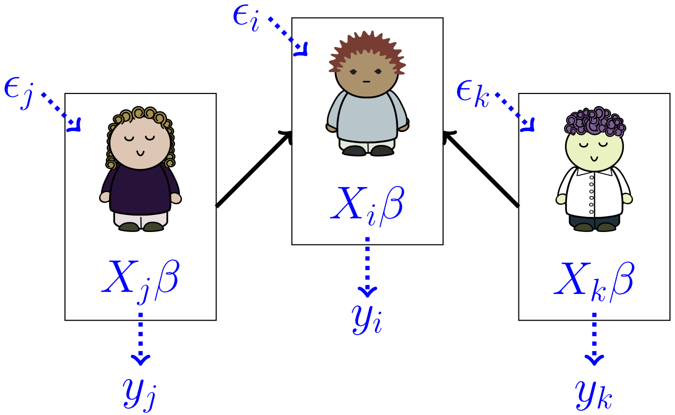

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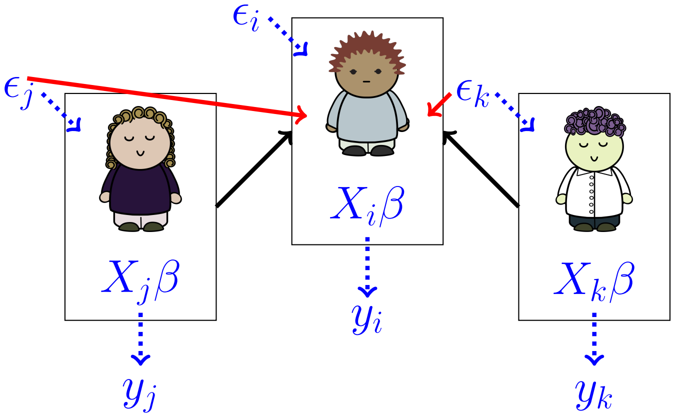
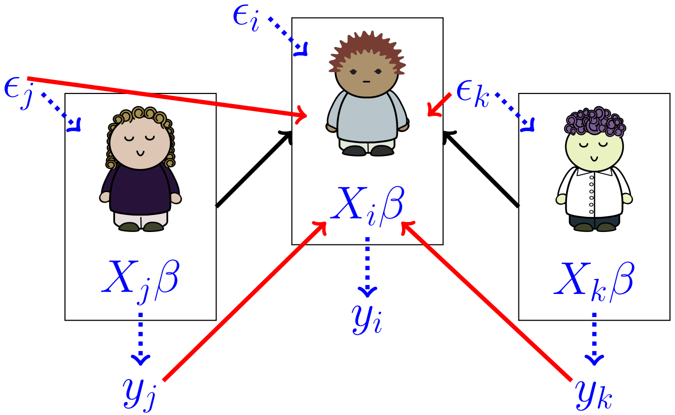
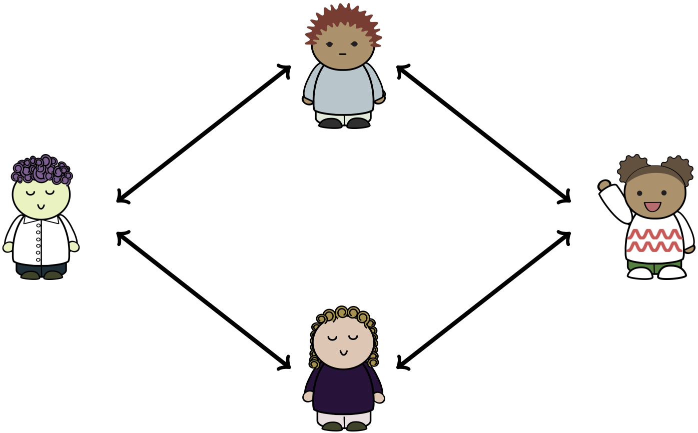


Network ARMA Model



Network 'Resonance'



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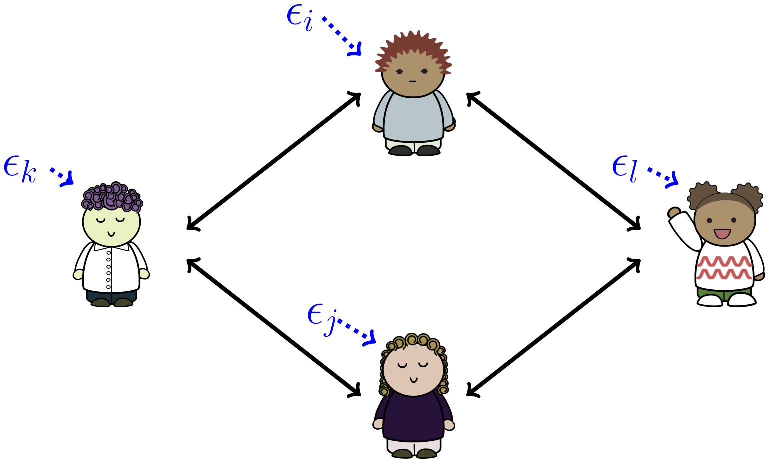

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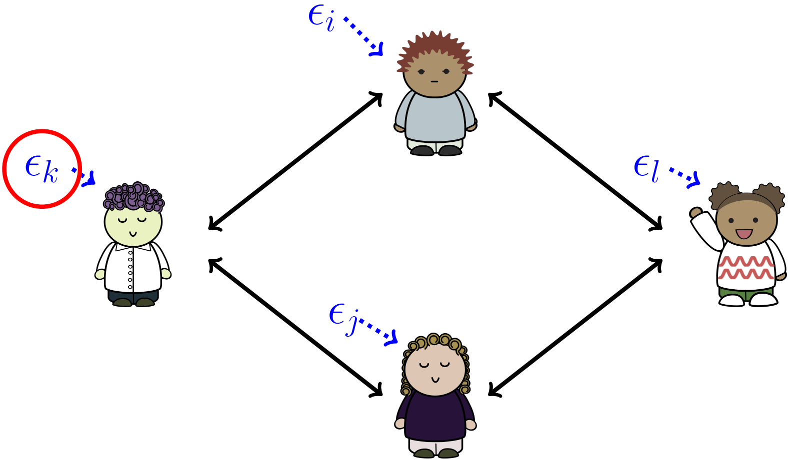

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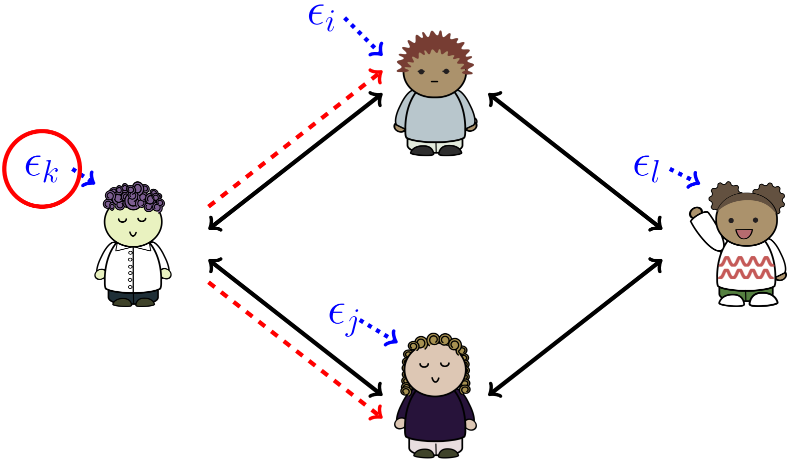

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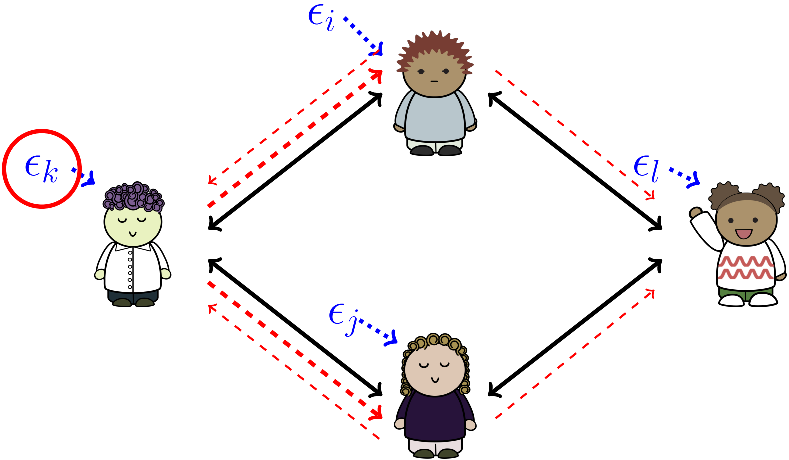

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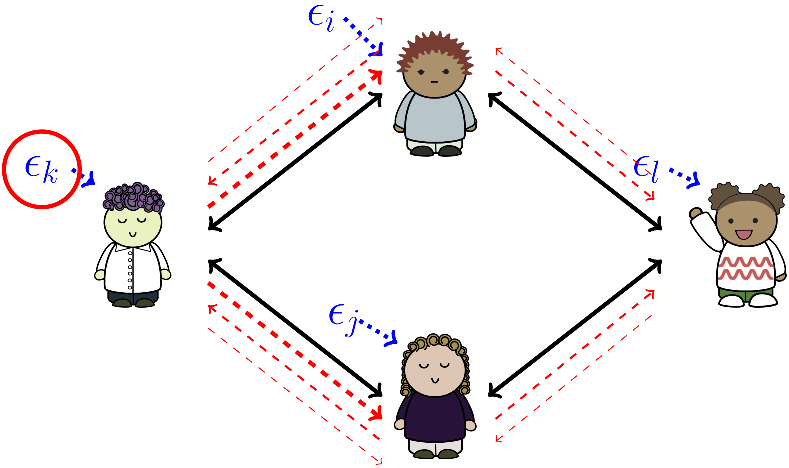

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- Compare models in the usual way (eg AIC, BIC)

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- Many suggestions given by Leenders 2002

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- Can include multiple weight matrices (as a list) for both AR and MA

Leenders 2002

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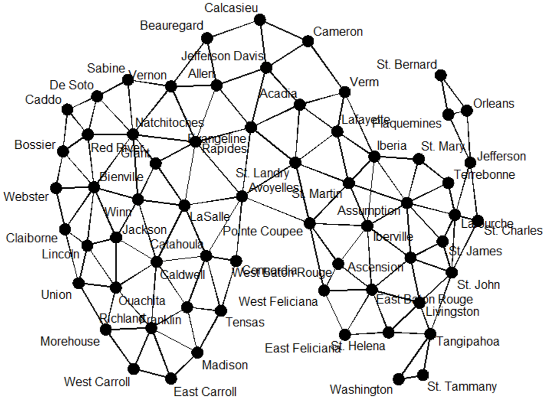

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Leenders 2002

Table 3
Network effects model for the Louisiana voting data

	OLS	$w_{ij}^{[1]}$	$w_{ij}^{[2]}$	$w_{ij}^{[6]}$	$w_{ij}^{[9]}$
ρ	–	0.31* (0.10)	0.07 (0.06)	0.12 (0.25)	0.04 (0.12)
Constant	21.03* (4.40)	13.87* (4.67)	19.83* (4.34)	16.78 (10.06)	19.80* (5.62)
B	0.01 (0.08)	−0.00 (0.07)	0.00 (0.08)	0.01 (0.08)	0.01 (0.08)
C	0.30* (0.04)	0.22* (0.05)	0.28* (0.04)	0.29* (0.05)	0.29 (0.05)
U	−0.11* (0.04)	−0.10* (0.04)	−0.11* (0.04)	−0.11* (0.04)	−0.11* (0.04)
BPE	0.39* (0.06)	0.30* (0.06)	0.37* (0.06)	0.38* (0.06)	0.38* (0.06)

* $P < 0.05$.

Table 4
Network disturbances model for the Louisiana voting data

	$w_{ij}^{[1]}$	$w_{ij}^{[2]}$	$w_{ij}^{[6]}$	$w_{ij}^{[9]}$
ρ	0.69* (0.10)	0.53* (0.13)	0.22 (0.42)	0.74* (0.15)
Constant	26.99* (4.50)	24.98* (4.22)	21.52* (4.30)	24.51* (5.06)
B	−0.11 (0.07)	−0.07 (0.07)	−0.00 (0.08)	−0.09 (0.08)
C	0.37* (0.05)	0.35* (0.04)	0.31* (0.04)	0.38* (0.04)
U	−0.07* (0.03)	0.08* (0.03)	−0.11* (0.04)	−0.10* (0.04)
BPE	0.24* (0.06)	0.30* (0.06)	0.38* (0.06)	0.29* (0.06)

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Leenders 2002

Table 5
Order of W matrices and autocorrelation models according to AIC

	Weight matrix	AIC	Order within model	Overall order
Network effects model	$w_{ij}^{[1]}$	439.12	1	3
	$w_{ij}^{[2]}$	445.52	2	5
	$w_{ij}^{[9]}$	446.78	4	8
	$w_{ij}^{[6]}$	446.44	3	6
Network disturbances model	$w_{ij}^{[1]}$	431.92	1	1
	$w_{ij}^{[2]}$	436.33	2	2
	$w_{ij}^{[9]}$	446.69	4	7
	$w_{ij}^{[6]}$	440.95	3	4
OLS	–	446.82	–	9

Code Time

Section 2.6

Baseline Models

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  - compare observed properties to baseline model
  - useful even when baseline model is not 'realistic'

Types of Baseline Hypotheses

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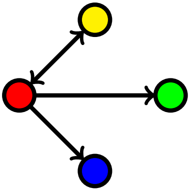

Empirical Network

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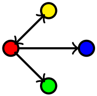

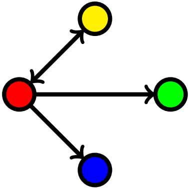

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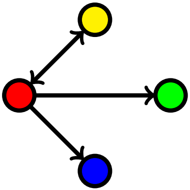

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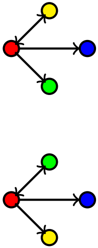

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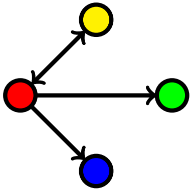

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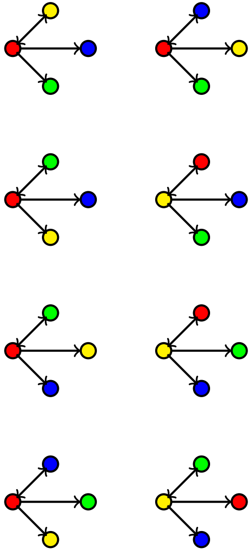

. . . etc

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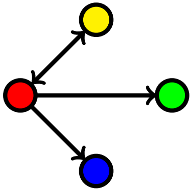

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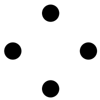

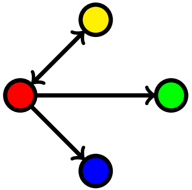

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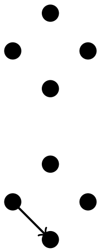

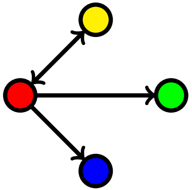

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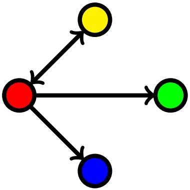

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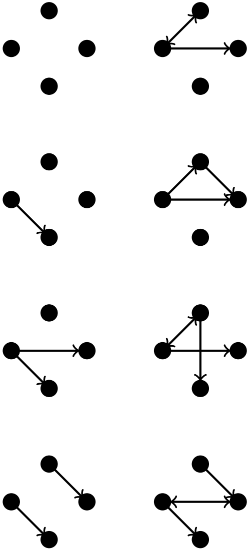

. . . etc

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Example

Transitivity in the Hurricane Frederic EMON

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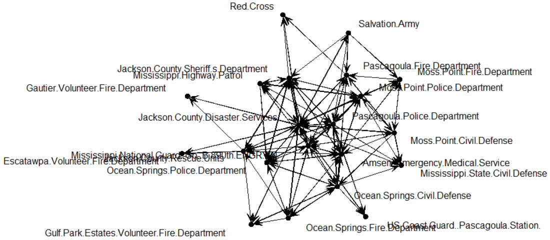
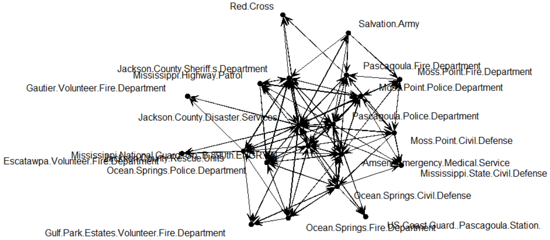


Example

Transitivity in the Hurricane Frederic EMON

- $\rho = 0.475$
- indicates that roughly half the time that $i \to j \to k$, $i \to k$



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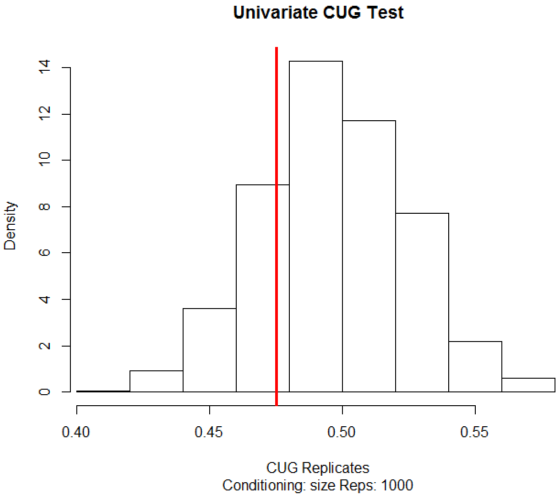
Univariate CUG Test
Density
0
2
4
6
8
10
12
14
0.40
0.45
0.50
0.55
CUG Replicates
Conditioning: size Reps: 1000

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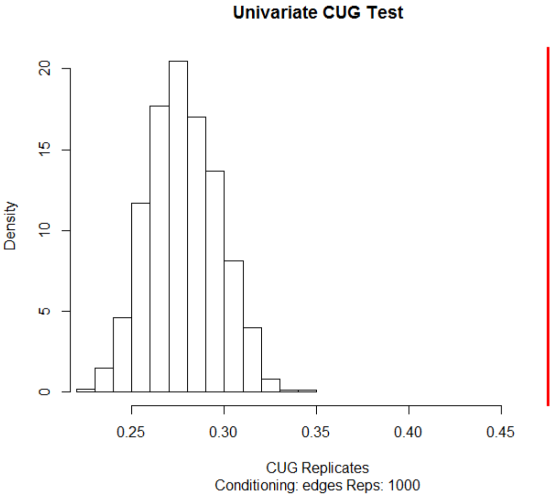
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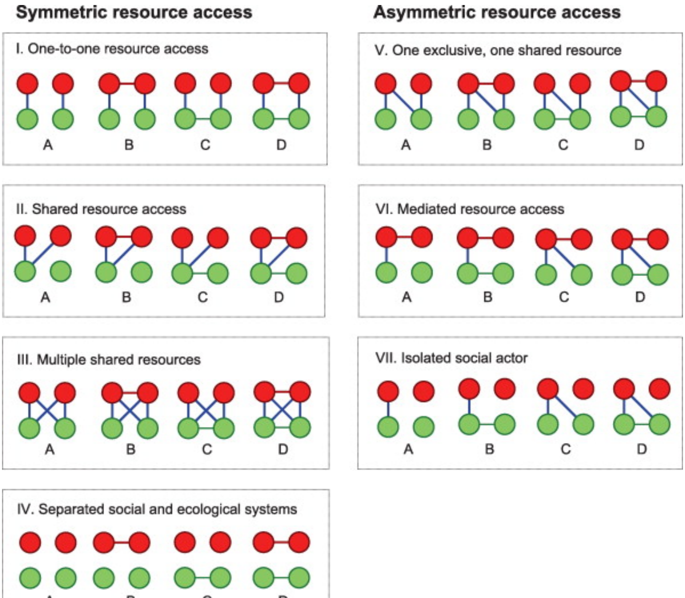
Bodin and Tengo

"Disentangling intangible social–ecological systems"

Bodin and Tengo



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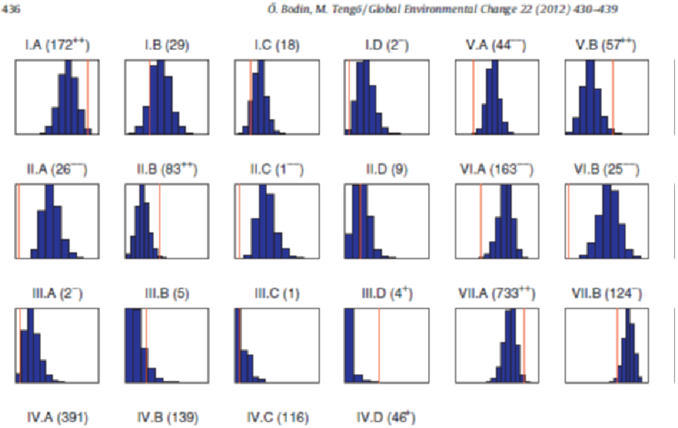

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- CUG tests (network is dependent variable)

Code Time

- the rest! whew!