Intro to Social Network Analysis with **R**

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A step-change in quantitative social science skills

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EUSN Workshop

11 September 2022

Think Formally

A network is not just a metaphor: it is a precise, mathematical construct

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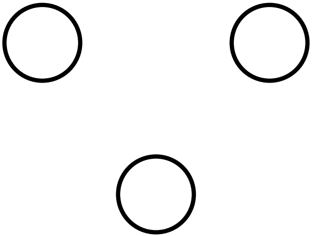

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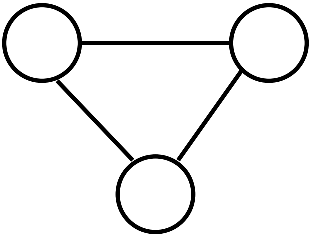
A network is not just a metaphor: it is a precise, mathematical construct of nodes N and edges E



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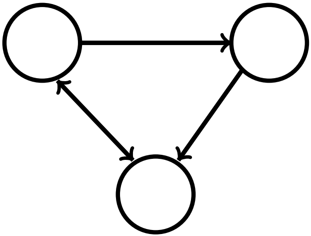

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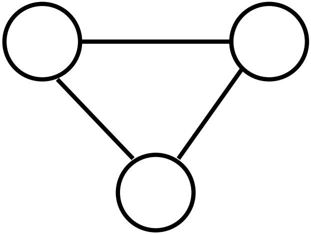

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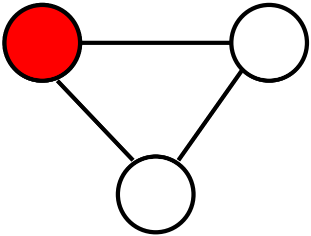

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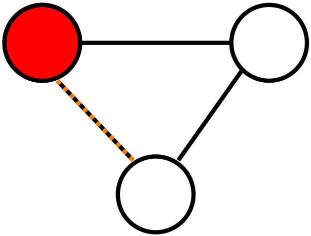
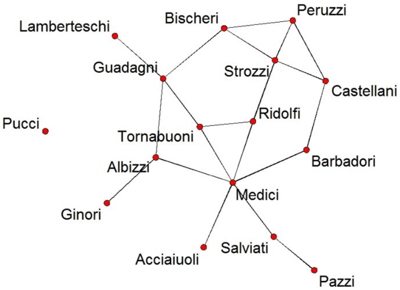


Network Intuition



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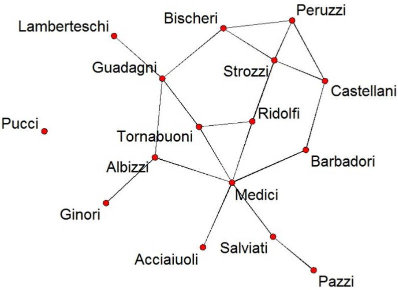

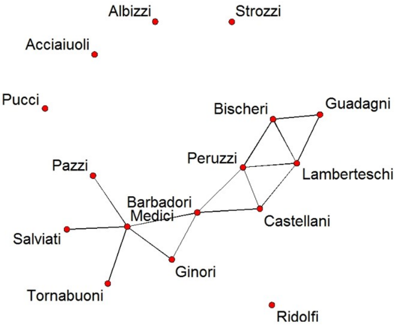

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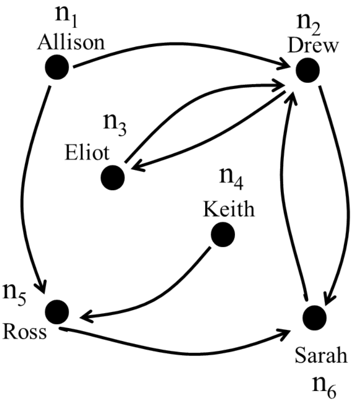
- The whole point is that observations are interdependent
- Want to explicitly model these interdependencies

Data Structures



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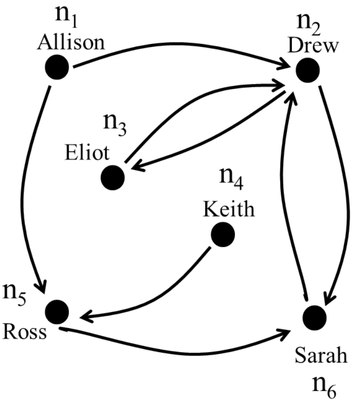

	n_1	n_2	n_3	n_4	n_5	n_6
n_1						
n_2						
n_3						
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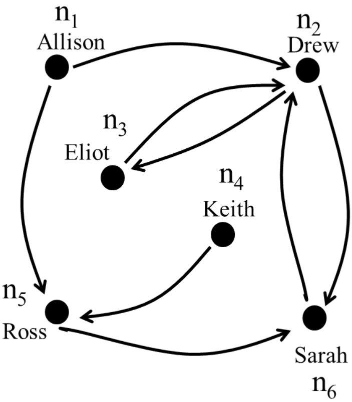

	n_1	n_2	n_3	n_4	n_5	n_6
n_1		1				
n_2						
n_3						
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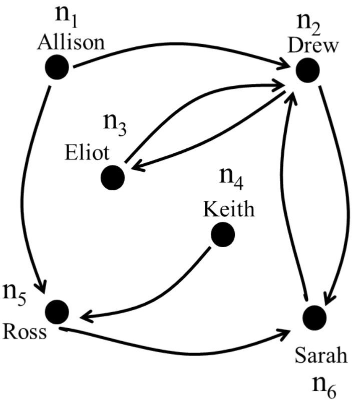

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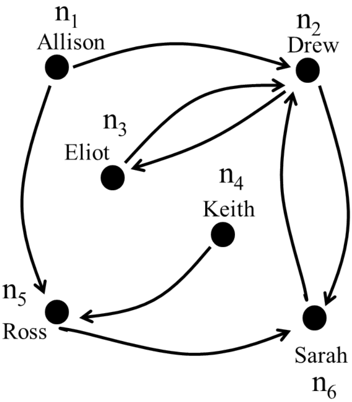

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n_2			1	1		
n_3		1				
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n_6		1				

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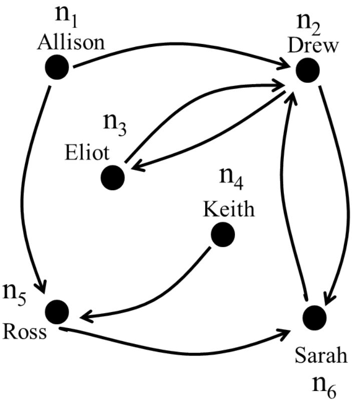

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n_2		-	1	1		
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n_5					-	1
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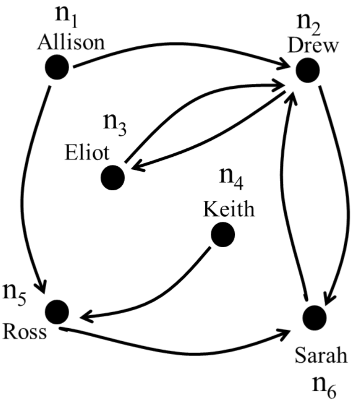

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n_3	0	1	-	0	0	0
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n_6	0	1	0	0	0	-

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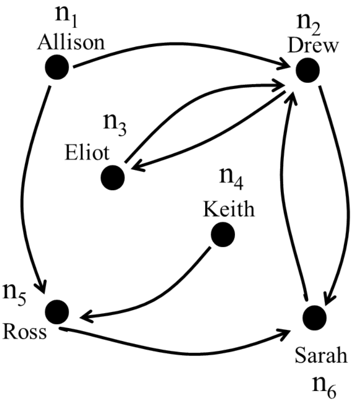

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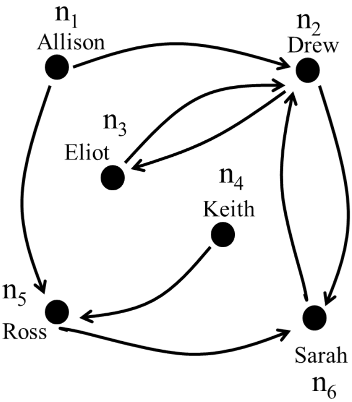

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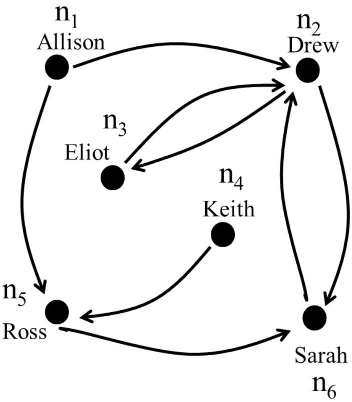

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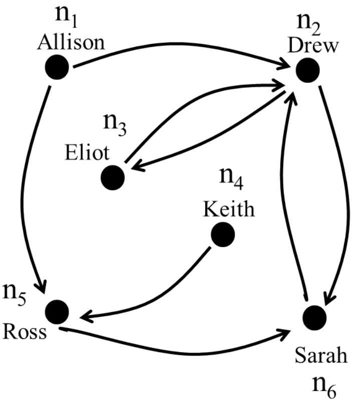

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- Its home on the web: http://www.r-project.org

Basics

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R Intro

Basic SNA Measures

Graph Level Indices

Simulation

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- Everything in **R** is an object – data, functions, everything

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  - Matrices
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  - Network objects

Vectors

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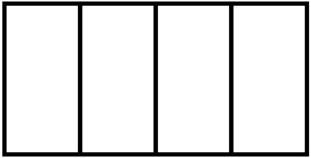

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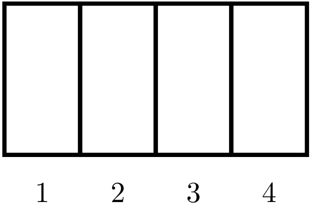

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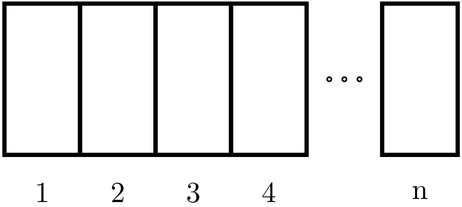

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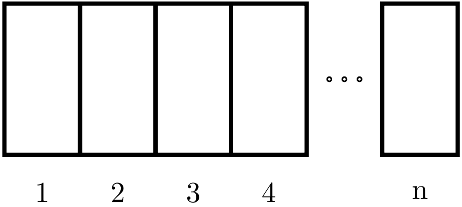

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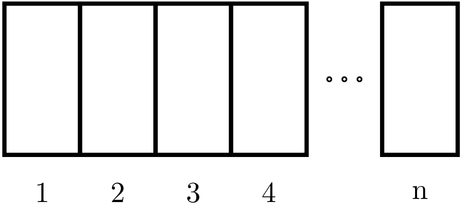

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  - To retrieve all BUT the third value, type `testScoress[-3]`

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- Most functions will accept either a matrix or a data frame

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- Data frames can store data of multiple types
  - Ideal for classical data analysis where you might have a mix of numerical and string data

Working with matrices

id	name	age	sex	handed	lastDocVisit
5012	Danielle	44	F	R	2012
2331	Josh	44	M	R	2008
1989	Mark	40	M	R	2010
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- If this matrix is called `friendSurvey`, the command to retrieve Josh’s age is `friendSurvey[2,3]`

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- Data frames provide another option: dollar sign notation

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- convert between the two using the library `intergraph`

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- We'll focus on the two most used: `Statnet` and `igraph`
- Both have pluses and minuses reflecting the interests of the authors
- `Statnet` is written towards doing exponential random graph models
- `igraph` has better visuals and has routines for community detection
- convert between the two using the library `intergraph`
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- But otherwise, they often don't play well loaded simultaneously
- Different structures and conventions

Network Objects

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  - And network level attributes (`%n%`)

Network Objects

```
Network attributes:
 vertices = 18
 directed = TRUE
 hyper = FALSE
 loops = FALSE
 multiple = FALSE
 bipartite = FALSE
 total edges= 54
   missing edges= 0
   non-missing edges= 54

Vertex attribute names:
   Group vertex.names

Edge attribute names:
   Order
> |
```

Code Time

- Sections 1 and 2

Igraph Objects

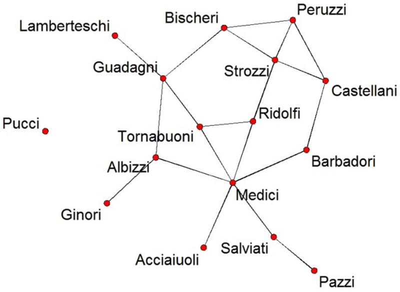
```
net
```

```
## IGRAPH DNW- 17 49 --
## + attr: name (v/c), media (v/c), media.type (v/n), type.label
## | (v/c), audience.size (v/n), type (e/c), weight (e/n)
## + edges (vertex names):
##  [1] s01->s02 s01->s03 s01->s04 s01->s15 s02->s01 s02->s03 s02->s09
##  [8] s02->s10 s03->s01 s03->s04 s03->s05 s03->s08 s03->s10 s03->s11
## [15] s03->s12 s04->s03 s04->s06 s04->s11 s04->s12 s04->s17 s05->s01
## [22] s05->s02 s05->s09 s05->s15 s06->s06 s06->s16 s06->s17 s07->s03
## [29] s07->s08 s07->s10 s07->s14 s08->s03 s08->s07 s08->s09 s09->s10
## [36] s10->s03 s12->s06 s12->s13 s12->s14 s13->s12 s13->s17 s14->s11
## [43] s14->s13 s15->s01 s15->s04 s15->s06 s16->s06 s16->s17 s17->s04
```

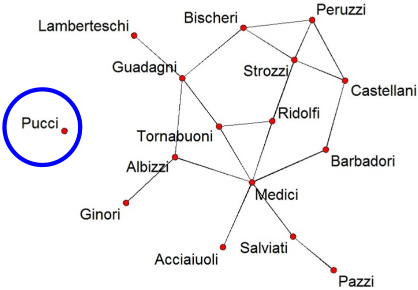
We also have easy access to nodes, edges, and their attributes with:

```
E(net)       # The edges of the "net" object
V(net)       # The vertices of the "net" object
E(net)$type  # Edge attribute "type"
V(net)$media # Vertex attribute "media"
```

Descriptives

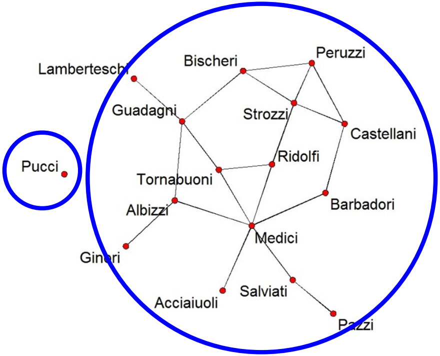


Descriptives



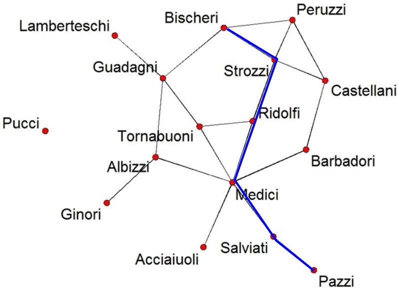
- One isolate

Descriptives



- One isolate
- Two components

Descriptives



- One isolate
- Two components
- Diameter is 5

Networks With **R**

LJasny

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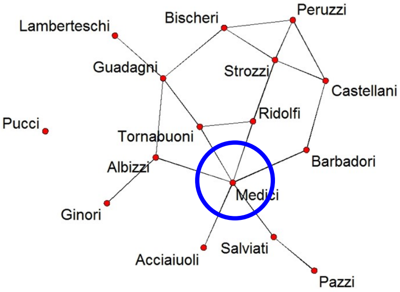
Descriptives



- One isolate
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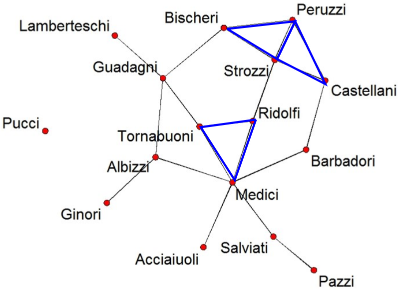
- Medici is most popular

Descriptives



- One isolate
- Two components
- Diameter is 5
- Medici is most popular
- Three triads

Degree

For each node, its degree is

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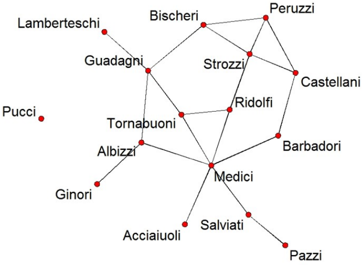

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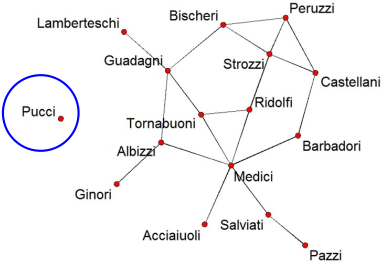

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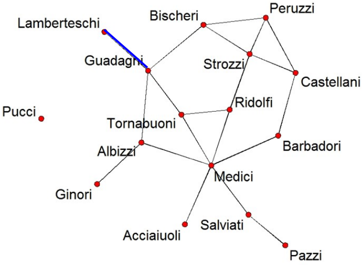

- Pucci has degree 0
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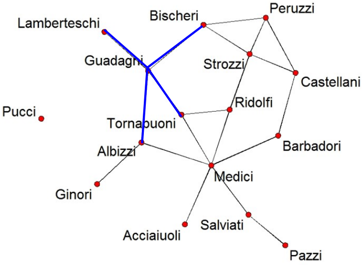

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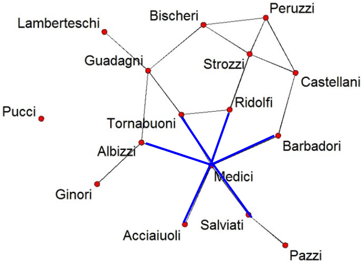
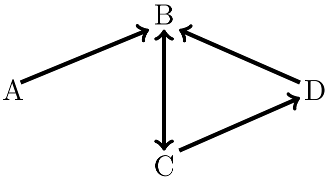


- Pucci has degree 0
- Lamberteschi has degree 1
- Guadagni has degree 4
- Medici has degree 6

Directed Degree

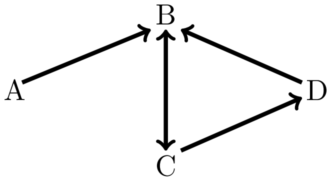
In directed graphs,



Directed Degree

In directed graphs,

- *Indegree* indicates the number of received ties



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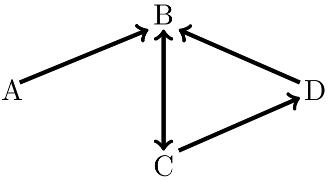

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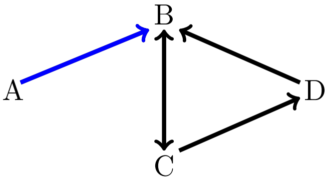

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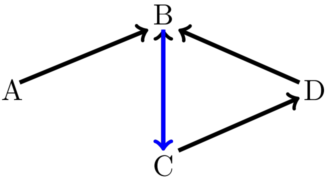

- A has 1 outdegree
- B has 1 outdegree

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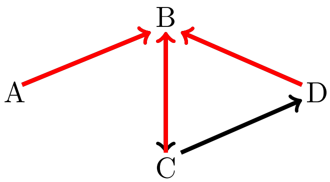

- A has 1 outdegree
- B has 1 outdegree and 3 indegree

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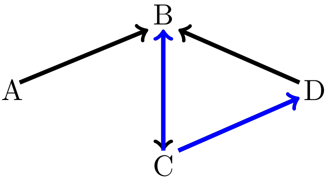

- A has 1 outdegree
- B has 1 outdegree and 3 indegree
- C has 2 outdegree

Directed Degree

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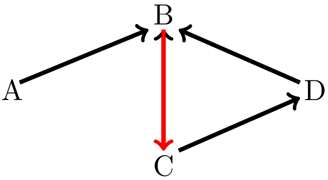

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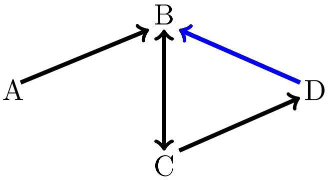

- A has 1 outdegree
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- C has 2 outdegree and 1 indegree
- D has 1 outdegree

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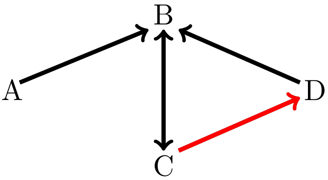

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- C has 2 outdegree and 1 indegree
- D has 1 outdegree and 1 indegree

Betweenness Centrality

Proportion of shortest paths that the given node lies on

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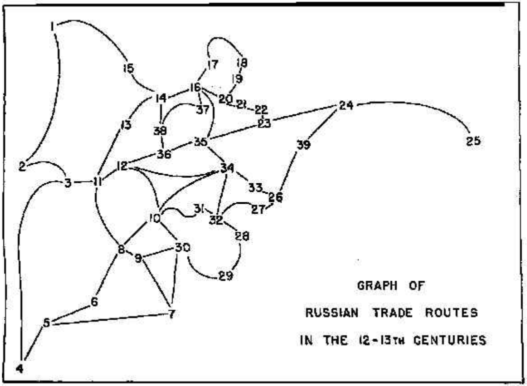
Figure 2. *Graph of Russian trade routes in the 12th - 13th centuries.*



Forrest Pitts 1978 "The River Trade Ntework of Russia, Revisited"

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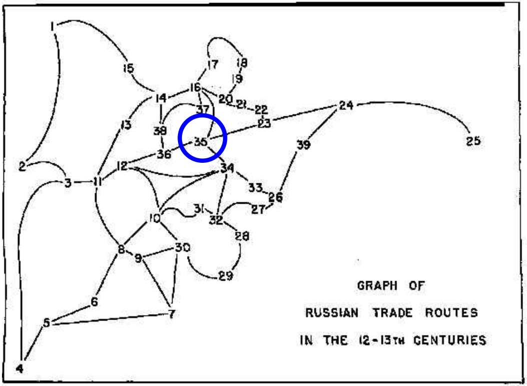

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- Many, many others

Graph Level Indices

Density

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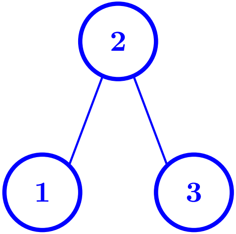

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Graph Level Indices
Simulation

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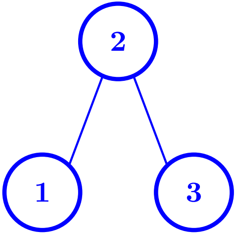
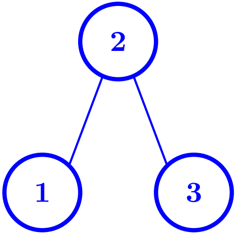


$$= \frac{\frac{2}{3(3-1)}}{2} = \frac{4}{6}$$

Mean Degree

The mean degree, $\bar{d}$, of all nodes in the graph is



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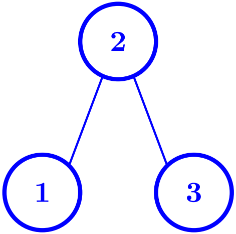
$$\bar{d}(n) = \frac{\sum_{i=1}^{N} d(n_i)}{N} = \frac{2E}{N}$$



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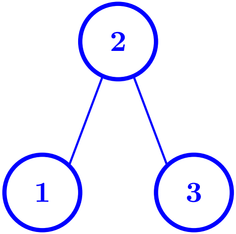
$$\bar{d}(n) = \frac{\sum_{i=1}^{N} d(n_i)}{N} = \frac{2E}{N}$$

$$= \frac{1+2+1}{3} = \frac{4}{3}$$



Size, Density, and Mean Degree

If we hold the mean degree constant, but vary size, what happens to density?

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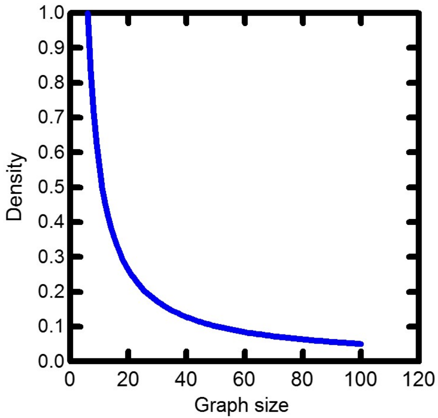

Centralization

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- Extent to which centrality is concentrated on a single vertex

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- Calculated as the sum of the differences between each node's centrality score and the maximum score

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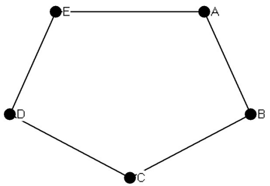
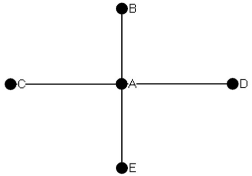
- Calculated as the sum of the differences between each node's centrality score and the maximum score
- Most centralized structure is usually a star network





Dyad Census

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Mutual (M)

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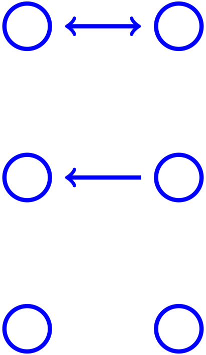
Dyad Census

Mutual (M)

Assymetric (A)

Dyad Census



Mutual (M)

Assymetric (A)

Null (N)

Reciprocity

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  $\frac{M+N}{M+A+N}$

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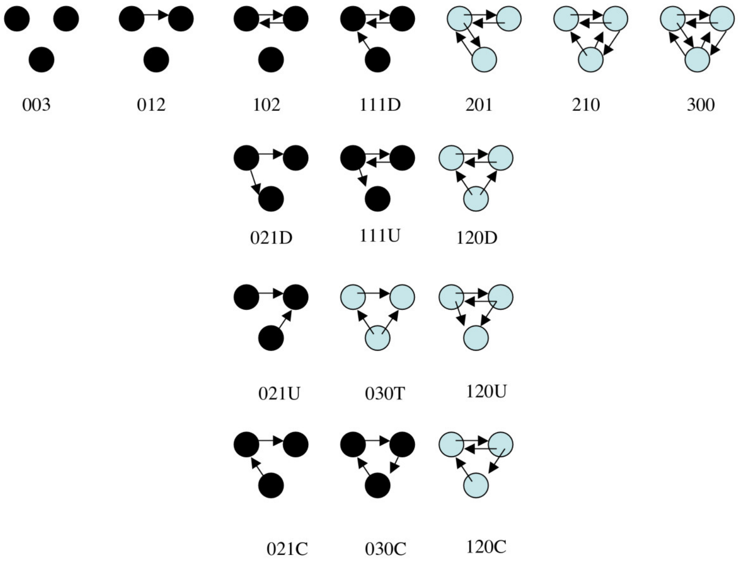
- Dyadic non-null: the proportion of non-null dyads that are reciprocal $\frac{M}{M+A}$
- Edgewise: $\frac{2*M}{2*M+A}$

Triad Census

Triad Census



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Triad Census

16 different triad types

One row per network

Triad Census

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One row per network

i,j cell is the number
of triad type j
in network i

Triad Census

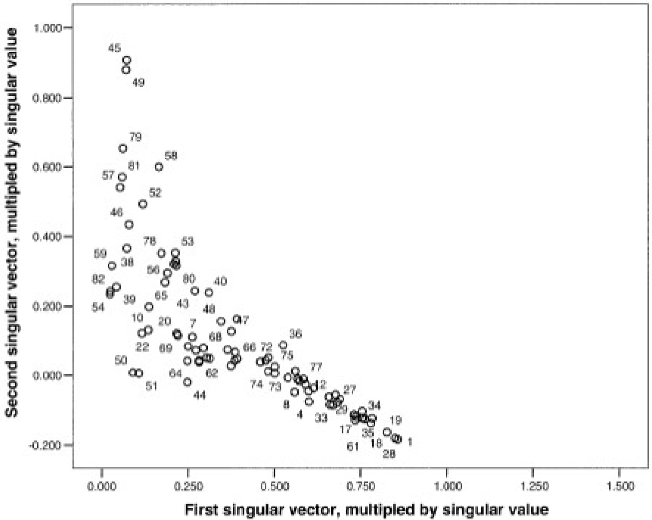
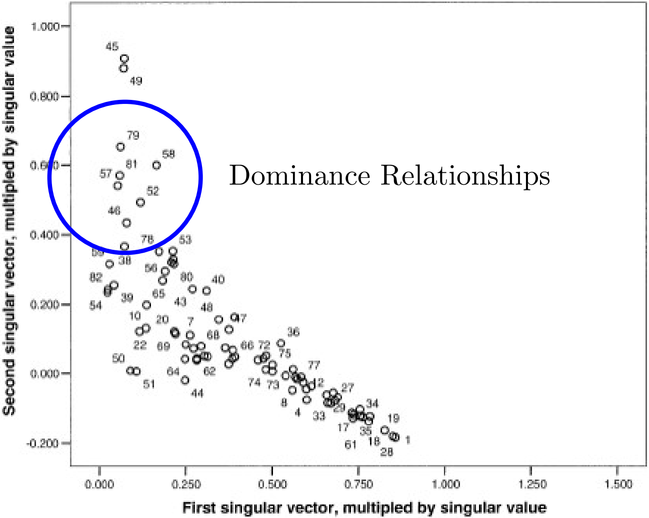


FIGURE 2. Singular value decomposition of triad census array, first two left singular vectors, multiplied by singular values, $N = 82$ networks.

Triad Census



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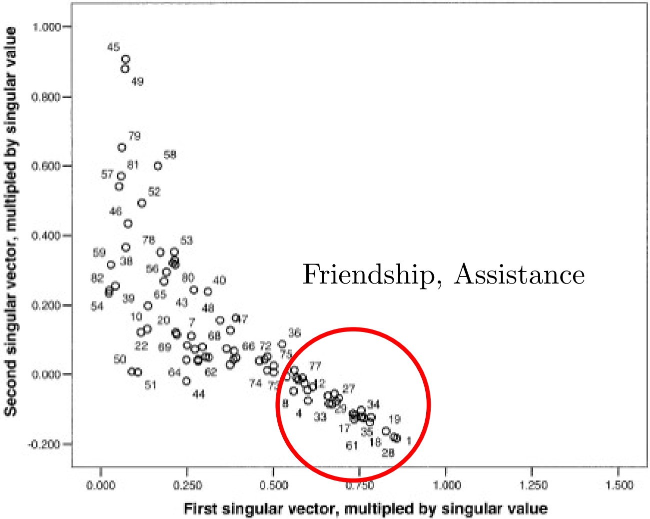

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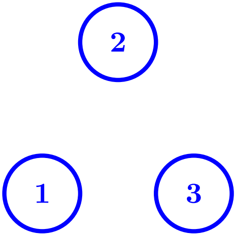

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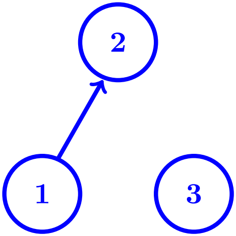

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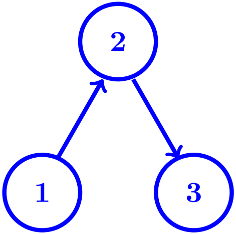

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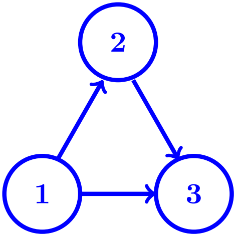
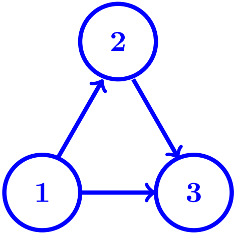
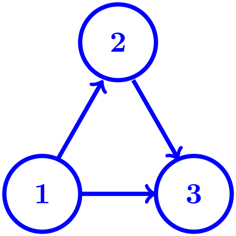


Transitivity



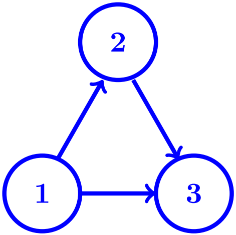
- Usually calculated as the fraction of completed two-paths

Transitivity



- Usually calculated as the fraction of completed two-paths
- Related to Grannovetter's 'forbidden triad'

Transitivity



- Usually calculated as the fraction of completed two-paths
- Related to Grannovetter's 'forbidden triad'
- Can be directed or undirected

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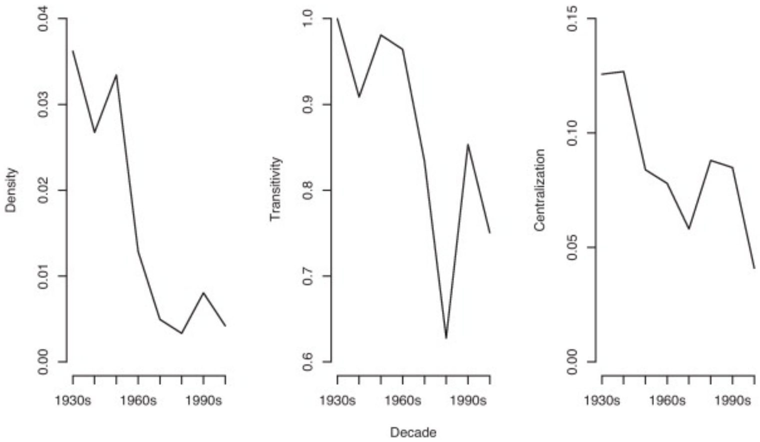
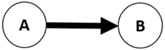


Janet Box-Steffensmeier and Dino Christenson
"The evolution and formation of amicus curiae networks"
Social Networks 2012

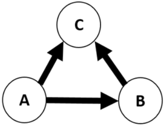
Example

a)



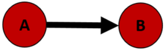
A sends information to B.

c)



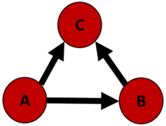
Transitive triad such that A sends information to B and C, and B also sends information to C. The smallest example of a 'chamber.'

b)



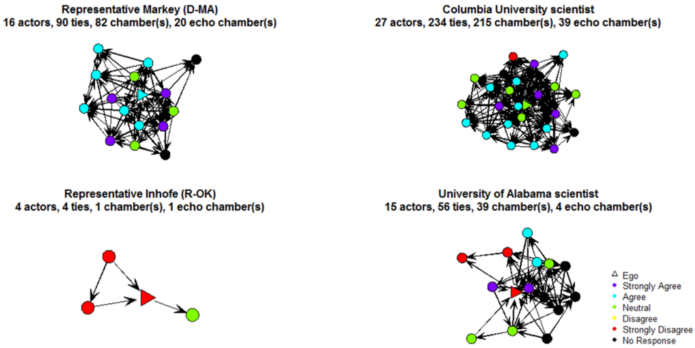
A and B agree so A's information echoes B's understanding.

d)



A transitive triad where each actor already holds the same position – an echo chamber.

Example


Representative Markey (D-MA)
16 actors, 90 ties, 82 chamber(s), 20 echo chamber(s)
Columbia University scientist
27 actors, 234 ties, 215 chamber(s), 39 echo chamber(s)
Representative Inhofe (R-OK)
4 actors, 4 ties, 1 chamber(s), 1 echo chamber(s)
University of Alabama scientist
15 actors, 56 ties, 39 chamber(s), 4 echo chamber(s)
Ego
Strongly Agree
Agree
Neutral
Disagree
Strongly Disagree
No Response

Example


3:42 pm ET
U.S. SENATE
SEN. JAMES INHOFE
R-Oklahoma
C-SPAN2
c-span.org

Code Time

- Section 3-4.7

Forbidden Triad or Structural Hole?

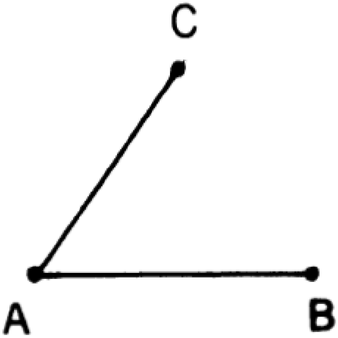


FIG. 1.—Forbidden triad

Forbidden Triad or Structural Hole?

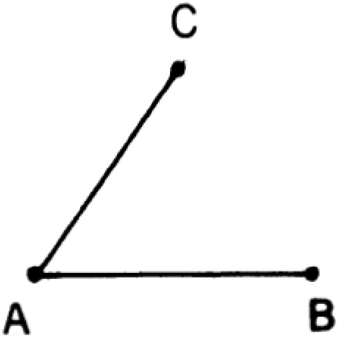


FIG. 1.—Forbidden triad

- Granovetter, Mark S. 1973. "The Strength of Weak Ties"

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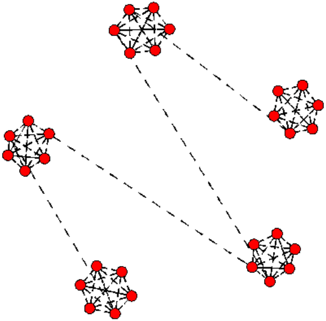
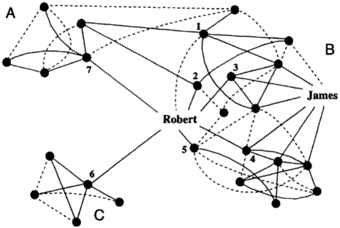
Forbidden Triad or Structural Hole?



- Granovetter, Mark S. 1973. “The Strength of Weak Ties”

Forbidden Triad or Structural Hole?



- Burt, Ronald S. 2004. "Structural Holes: The Social Structure of Competition"

Extensions

Attributes!

Extensions

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- Properties of nodes, edges, or even networks

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- Properties of nodes, edges, or even networks
- Pretty much anything you can measure could be an attribute

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- Extension based on node attributes: Brokerage

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- Pretty much anything you can measure could be an attribute
- Extension based on node attributes: Brokerage
- Extension based on edge attributes: Structural Balance

Brokerage

- Brokerage is a process “by which intermediary actors facilitate transactions between other actors lacking access to or trust in one another” (Marsden 1982)

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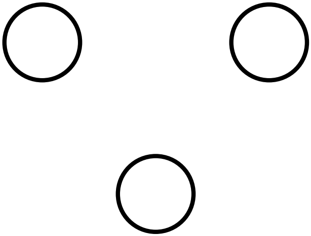
Brokerage

- Brokerage is a process “by which intermediary actors facilitate transactions between other actors lacking access to or trust in one another” (Marsden 1982)
- Brokers play a crucial role in knitting together diverse groups of people, organizations, parties
- Brokers can gain a lot – early access to information, prestige
- But can also be distrusted by everyone

Brokerage: Formal Concept

In a network N with edges E,



Brokerage: Formal Concept

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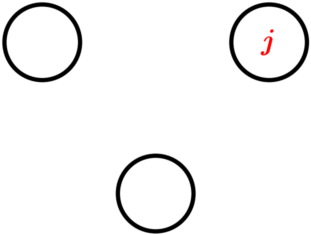
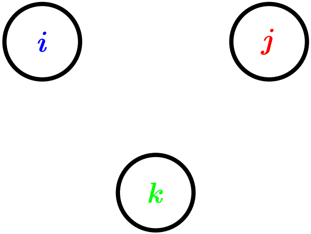
node j brokers



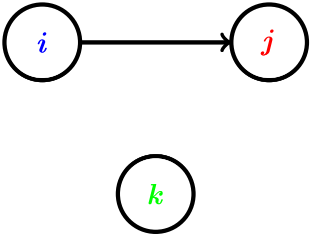
Brokerage: Formal Concept

In a network N with edges E,
node j brokers
nodes i and k



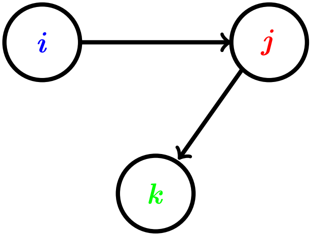
Brokerage: Formal Concept

In a network N with edges E,
node j brokers nodes i and k
if $e_{ij} \in E$



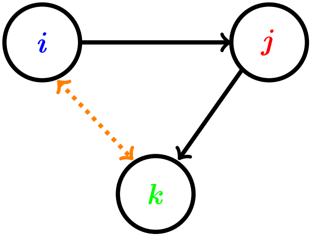
Brokerage: Formal Concept

In a network N with edges E, node j brokers nodes i and k if $e_{ij} \in E$ and $e_{jk} \in E$



Brokerage: Formal Concept

In a network N with edges E,
node j brokers nodes i and k
if $e_{ij} \in E$
and $e_{jk} \in E$
but $e_{ik} \notin E$



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Brokerage: Formal Concept

Gould and Fernandez (1989, 1994)

Brokerage: Formal Concept

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- formalized the concept

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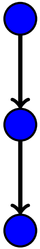
- added a vertex attribute component

Brokerage: Formal Concept

Gould and Fernandez (1989, 1994)

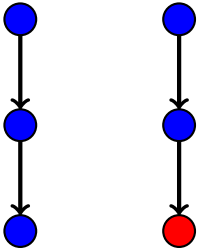
- formalized the concept
- added a vertex attribute component
- compared empirical brokerage counts to counts from random graphs conditioned on the number of edges

Brokerage



Coordinator

Brokerage



Coordinator Representative

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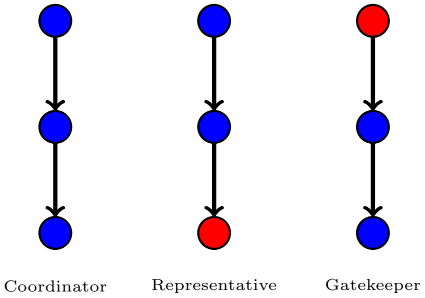
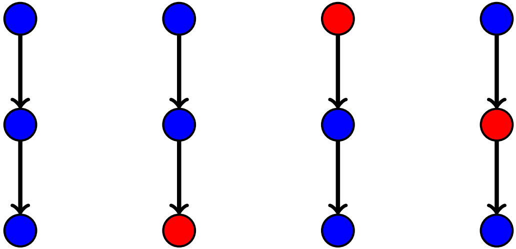
Brokerage

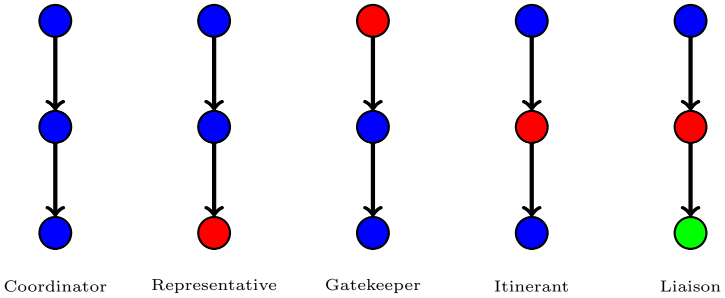


Brokerage



Coordinator Representative Gatekeeper Itinerant

Brokerage


Coordinator
Representative
Gatekeeper
Itinerant
Liaison

Gould and Fernandez' Findings

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- the benefits of brokerage are mediated both by the type of organization (the node sets) and the type of brokerage chain

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- non-governmental organizations were found to have more influence when they held any type of brokerage position

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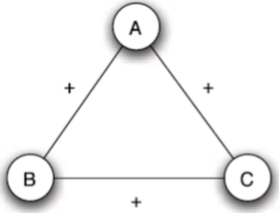
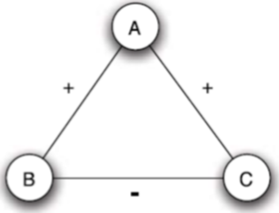
- the benefits of brokerage are mediated both by the type of organization (the node sets) and the type of brokerage chain
- non-governmental organizations were found to have more influence when they held any type of brokerage position
- governmental organizations gained influence only when they held "outsider" brokerage roles in itinerant and liaison chains

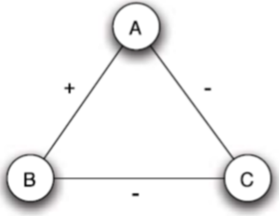
Structural Balance



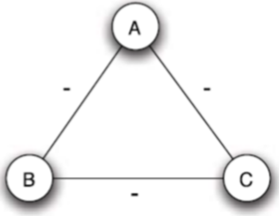
(a) *A, B, and C are mutual friends: balanced.*



(b) *A is friends with B and C, but they don't get along with each other: not balanced.*

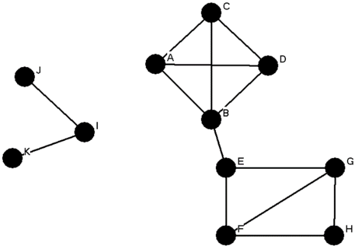


(c) *A and B are friends with C as a mutual enemy: balanced.*

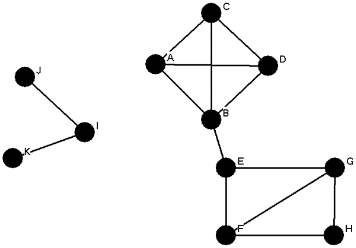


(d) *A, B, and C are mutual enemies: not balanced.*

Subgroups

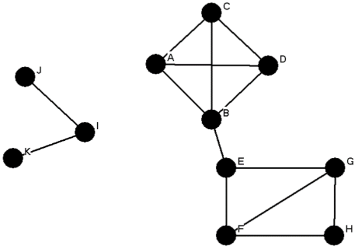


Subgroups



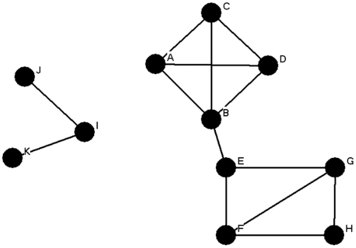
- Component: A maximal connected subgraph

Subgroups

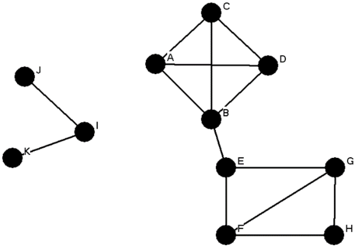


- Component: A maximal connected subgraph
- A subgraph is *maximal* with respect to some property if it has the property, but loses it with the addition of more nodes or edges

Cliques

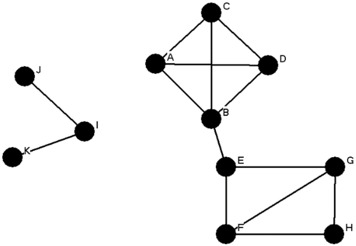


Cliques



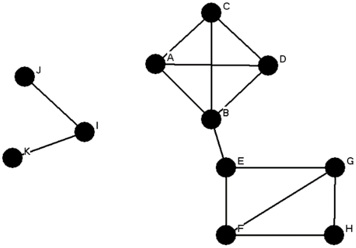
- A maximally complete subgraph of 3 or more

Cliques



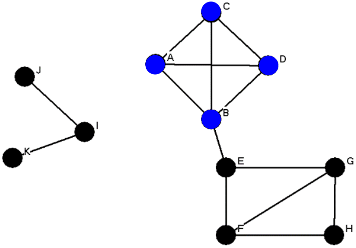
- A maximally complete subgraph of 3 or more
- All nodes are adjacent to all others in the subgraph

Cliques



- A maximally complete subgraph of 3 or more
- All nodes are adjacent to all others in the subgraph
- No nodes can be added that have that property

Cliques



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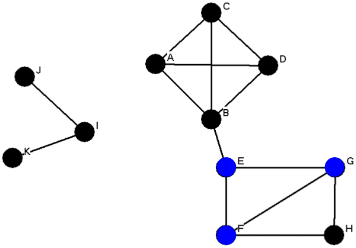

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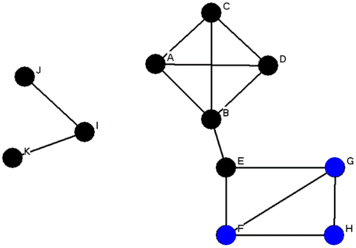

- A maximally complete subgraph of 3 or more
- All nodes are adjacent to all others in the subgraph
- No nodes can be added that have that property

Limitations of Cliques

- A very strict definition
- Often networks contain many small overlapping cliques
- Cliques have no internal structure

Relaxations of Cliques

Subgroups based on reachability or diameter

Relaxations of Cliques

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- n-clique: a maximal subgraph in which the **largest geodesic distance** between any two nodes is no greater than n

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Relaxations of Cliques

Subgroups based on reachability or diameter

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- n-clan: an n-clique in which the geodesic distance between all nodes in the subgraph is no greater than n **for paths within the subgraph**
- n-club: a maximal subgraph of diameter n

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Relaxations of Cliques

Subgroups based on nodal degree

Relaxations of Cliques

Subgroups based on nodal degree

- k-plex: a subgrah of g nodes in whcich each node is adjacent to no fewer than $g - k$ ndoes in the subgraph

Relaxations of Cliques

Subgroups based on nodal degree

- k-plex: a subgrah of g nodes in whcich each node is adjacent to no fewer than $g - k$ ndoes in the subgraph
- k-core: a subgraph in which each node is adjacent to at least k other nodes in the subgroup

What to do with these subgroup definitions?

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- characterize the overall structure of the network

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- characterize the overall structure of the network
- analyze the pattern of co-membership in cliques, etc.

K-Core Example

RESEARCH ARTICLE

The Critical Periphery in the Growth of Social Protests

Pablo Barberá[1]*, Ning Wang[2], Richard Bonneau[3,4], John T. Jost[1,5,6], Jonathan Nagler[6], Joshua Tucker[6], Sandra González-Bailón[7]*

1 Center for Data Science, New York University, New York, New York, 10003, United States of America, **2** Mathematical Institute and Oxford Internet Institute, University of Oxford, Oxford, OX26GG, United Kingdom, **3** Center for Genomics and System Biology, New York University, New York, New York, 10003, United States of America, **4** Simons Center for Data Analysis, Simons Foundation, New York, New York, 10010, United States of America, **5** Department of Psychology, New York University, New York, New York, 10003, United States of America, **6** Department of Politics, New York University, New York, New York, 10012, United States of America, **7** Annenberg School for Communication, University of Pennsylvania, Philadelphia, Pennsylvania, 19104, United States of America

* pablo.barbera@nyu.edu (PB); sgonzalezbailon@asc.upenn.edu (SGB)

K-Core Example

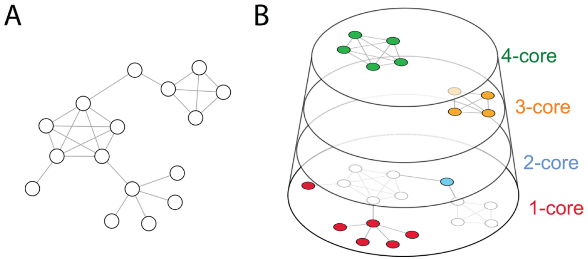

Fig 2. Schematic representation of the *k*-core decomposition for a random network with N = 16 vertices and E = 24 edges. This technique recursively prunes the network to remove nodes with the lowest degree. The coreness of a vertex is *k* if it belongs to the *k*-core but not to the (*k*+1)-core.

K-Core Example

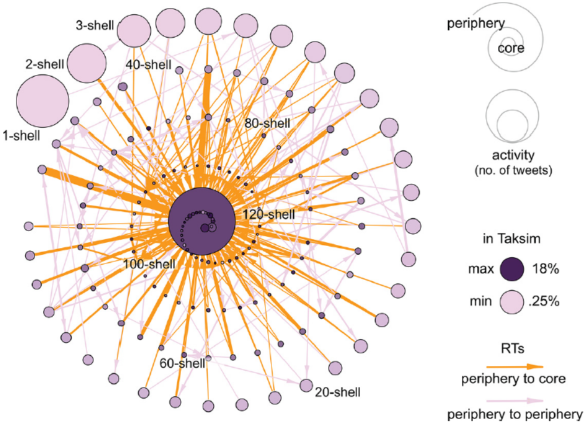

Fig 3. *K*-core decomposition of the network of retweets that emerged during the 2013 Taksim Gezi Park protests in Turkey (see S1 Text). Participants have been grouped in their corresponding *k*-shells, here represented by nodes. Lower *k*-shells contain participants at the periphery of the network; higher *k*-shells contain core participants. Node size is proportional to aggregated activity, measured as total number of protest messages (not just retweets). Arcs indicate retweeting activity, and their width is proportional to normalized strength (arcs with lower strength have been filtered to improve the visualization of the network). The darkness of nodes is proportional to the percentage of participants who reported being in the Taksim Gezi Park (the geographical epicenter of the protests), as indicated by the geographic information attached to their tweets. Most of these participants are at the core of the network where most RTs are also sourced from, thus allowing information to flow from the core to the periphery.

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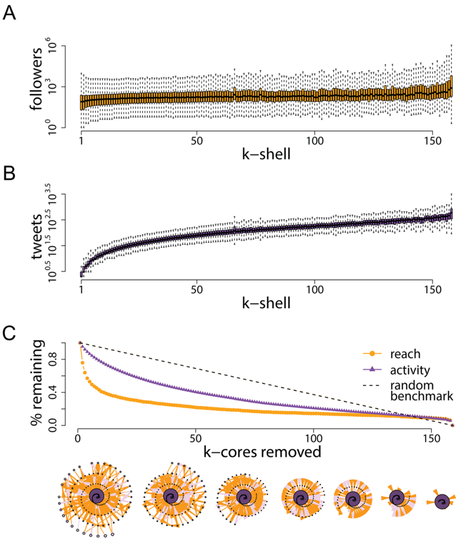
K-Core Example



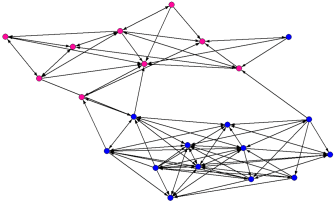
Block Models

Block Models

- Attribute Based

Block Models

- Attribute Based



Block Models

- Attribute Based

```
                      Boys                     Girls
         1 2 4 5 5 7 8 9 9 0 1 2 6   3 4 3 6 7 8 0 1 2
         1 2 1 1 5 1 1 1 9 1 1 1 1   1 4 3 6 7 8 2 2 2
       -------------------------------------------------
 1  1 |     1 1 1   1           1 |     1   1         |
 2  2 |       1 1 1     1   1   1 |                   |
14 14 | 1 1   1 1 1 1   1   1 1 1 |                   |
15 15 | 1   1   1   1   1   1   1 |                   |
 5  5 | 1 1 1       1   1       1 |     1             |
17 17 | 1 1   1 1   1 1 1   1   1 |                   |
18 18 | 1     1 1           1     |                   |
19 19 |     1 1         1       1 |                   |
 9  9 |     1 1                 1 |                   |
10 10 |                           |       1 1       1 |
11 11 |   1   1     1             |                   |
12 12 |   1 1       1 1     1   1 |                 1 |
16 16 |     1 1         1         |                   |
      ------------------------------------------------
13 13 |                           |   1         1 1 1 |
 4  4 | 1                         | 1       1   1 1   |
 3  3 |                           |       1 1         |
 6  6 |                   1       |                   |
 7  7 |                           |     1             |
 8  8 |                           |       1 1   1     |
20 20 |                           | 1 1   1 1 1   1 1 |
21 21 |                           | 1 1     1         |
22 22 |                           | 1     1 1   1     |
       ------------------------------------------------
```

(Row labels at left: Boys, Girls)

Block Models

- Attribute Based
- Relation Based

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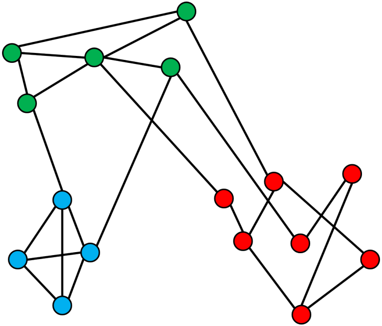

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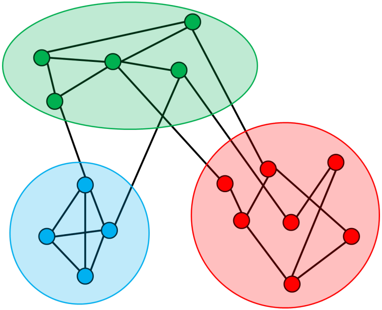

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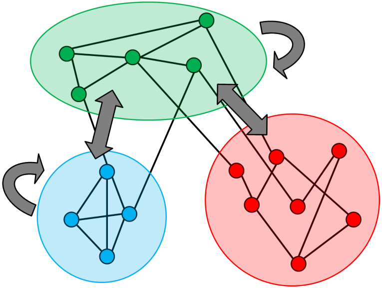

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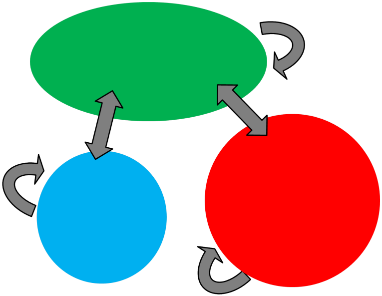

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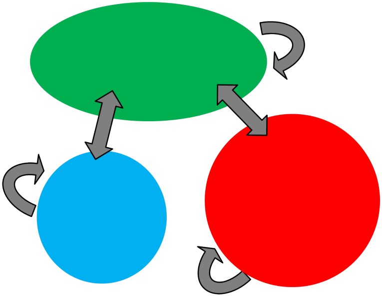
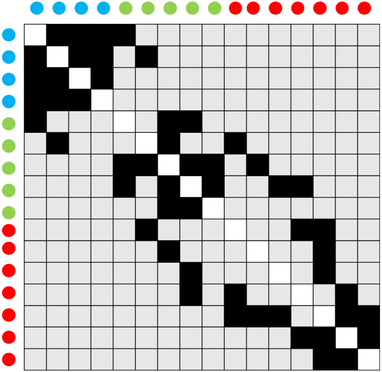
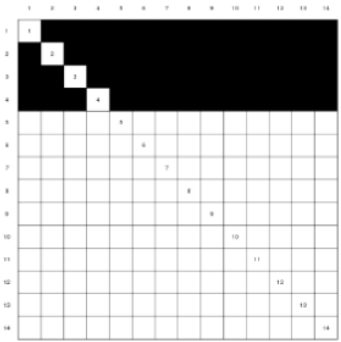




Core-Periphery Models



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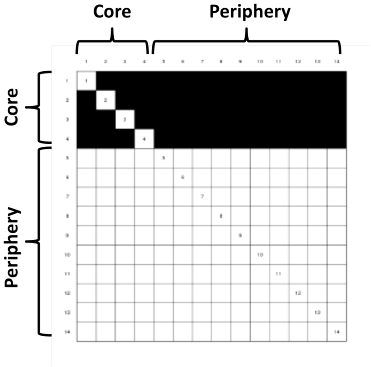

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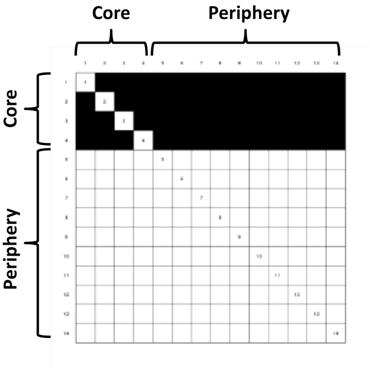
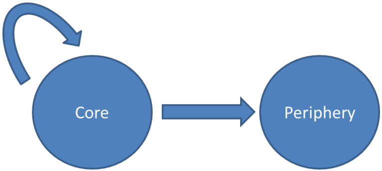
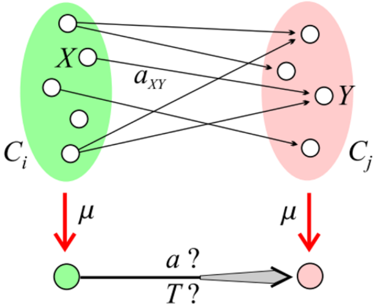




Even more blockmodeling

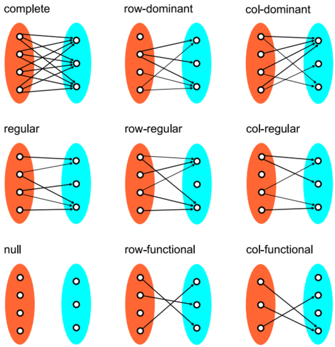
Generalized Blockmodeling

A *blockmodel* consists of structures obtained by identifying all units from the same cluster of the clustering **C**. For an exact definition of a blockmodel we have to be precise also about which blocks produce an arc in the *reduced graph* and which do not, and of what *type*. Some types of connections are presented in the figure on the next slide. The reduced graph can be represented by relational matrix, called also *image matrix*.



Doreian, Batagelj, and Ferligoj *Generalized Blockmodeling*, 2010

Even more blockmodeling



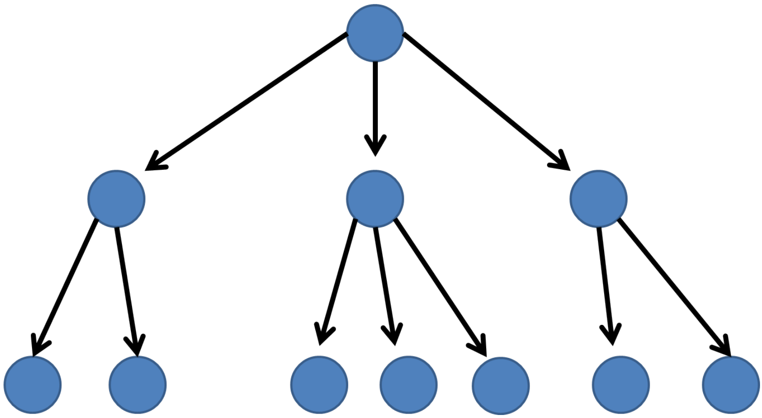
null	nul	all 0 *
complete	com	all 1 *
regular	reg	1-covered rows and columns
row-regular	rre	each row is 1-covered
col-regular	cre	each column is 1 -covered
row-dominant	rdo	∃ all 1 row *
col-dominant	cdo	∃ all 1 column *
row-functional	rfn	∃! one 1 in each row
col-functional	cfn	∃! one 1 in each column
non-null	one	∃ at least one 1

* except this may be diagonal

Doreian, Batagelj, and Ferligoj *Generalized Blockmodeling*, 2010

Structural Equivalence

Structural Equivalence



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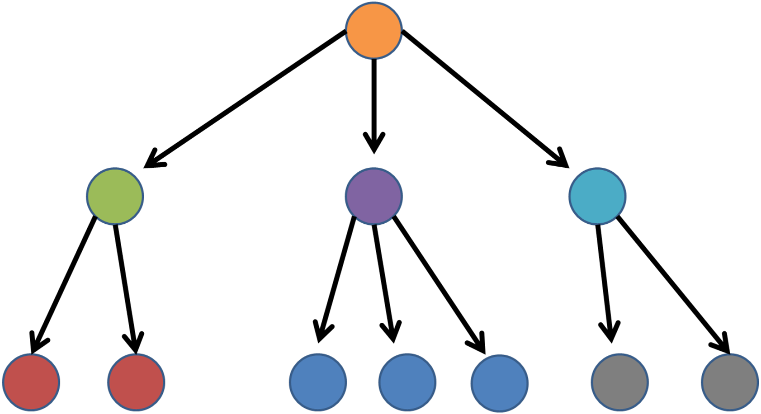
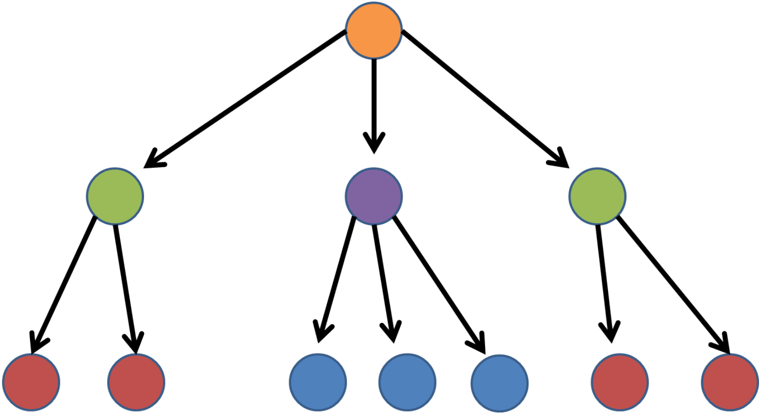
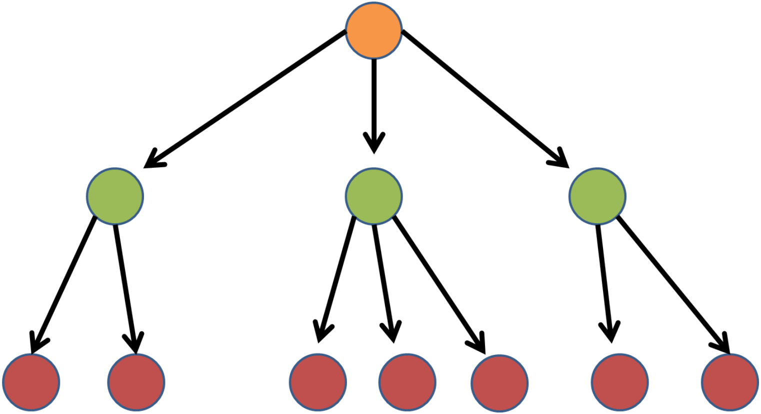


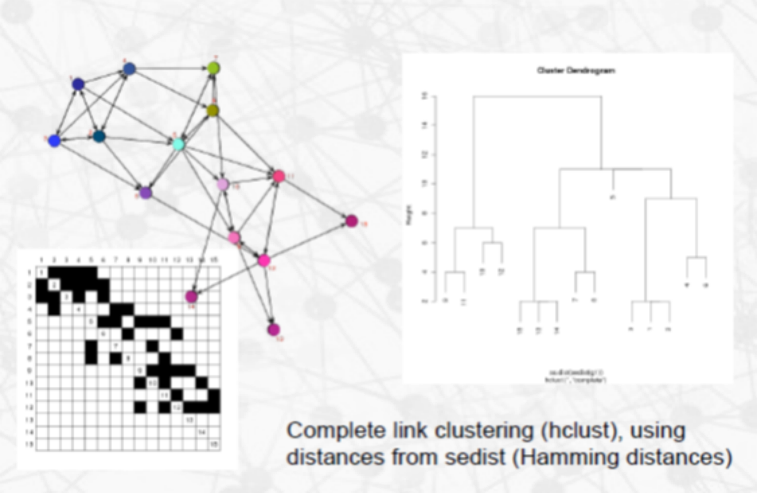
Automorphic Equivalence



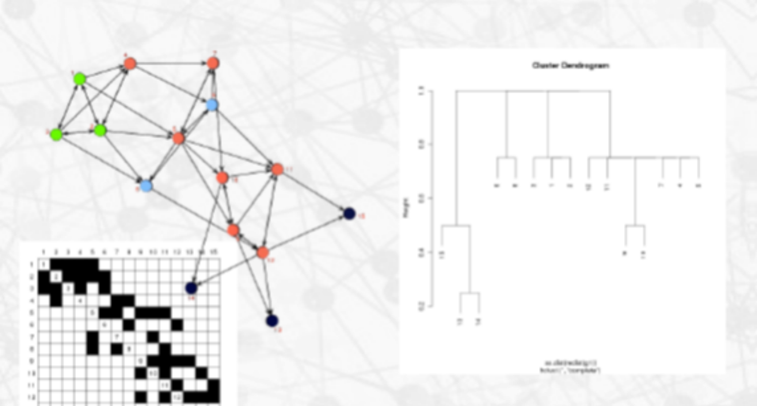
Regular Equivalence



Structural Equivalence Example



Regular Equivalence Example



Complete link clustering (hclust), using distances from redist (CATREGE algorithm)

Community Structure

Community Structure

- Modularity index measures the 'goodness' of an assignment of nodes to subgroups by comparing the number of ties within groups as compared to the number of expected ties

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- M.E.J. Newman "Modularity and community structure in networks" *Proc Natl Acad Sci* 2006

Intro

R Intro

Basic SNA Measures

Graph Level Indices

Simulation

Bernoulli ‘Random’ Graphs

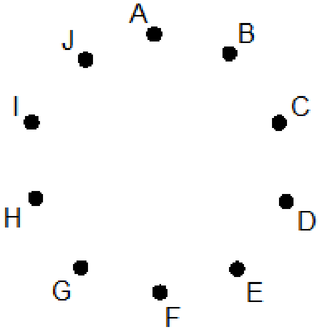
Bernoulli ‘Random’ Graphs

- A probability distribution for a success/failure

Bernoulli 'Random' Graphs

- A probability distribution for a success/failure
- Best known example is the coin flip

Bernoulli ‘Random’ Graphs



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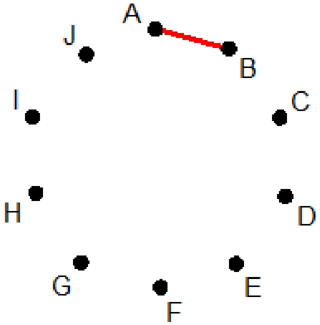

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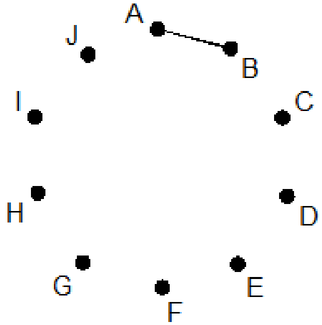

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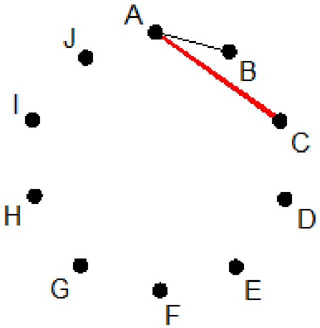

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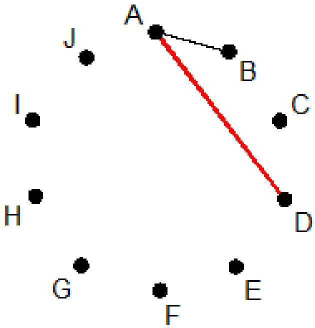

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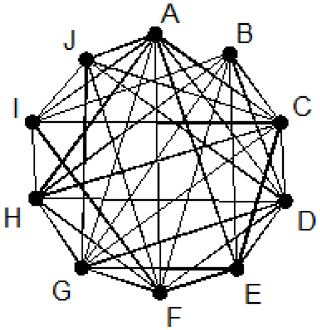

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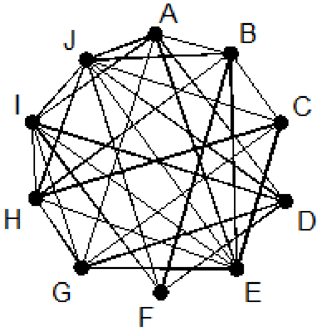

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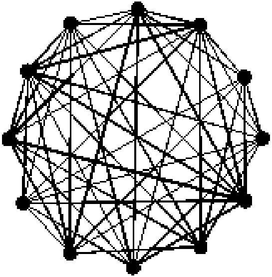

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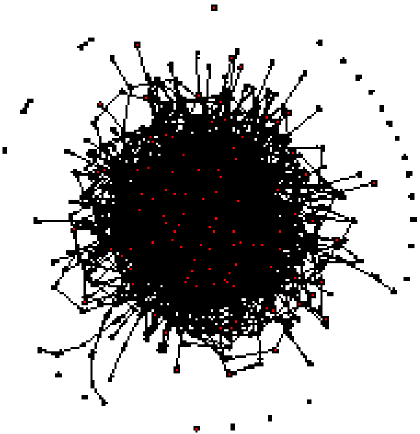

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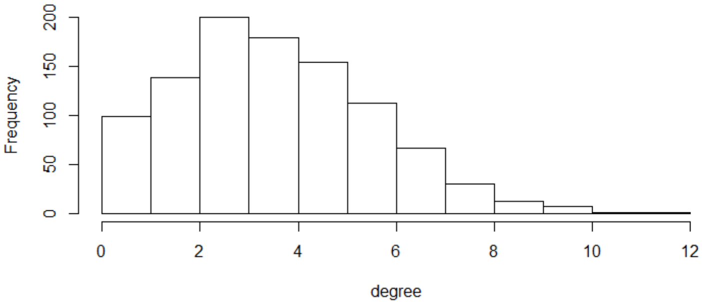

Small Worlds

What were the characteristics of Milgram's small world?

Intro

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Watts-Strogatz Model

Watts-Strogatz Model

- high clustering coefficient

Watts-Strogatz Model

- high clustering coefficient
- low diameter

Clustering Coefficient and Diameter

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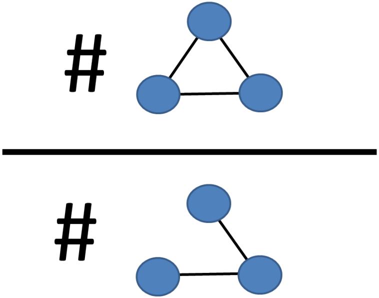
- The ***diameter*** of a graph is the longest shortest path between any pair nodes

Clustering Coefficient and Diameter

- The ***diameter*** of a graph is the longest shortest path between any pair nodes
- The ***clustering coefficient*** of a graph is the number of triangles divided by the number of two-paths

Clustering Coefficient



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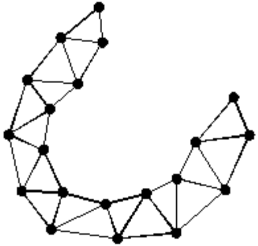
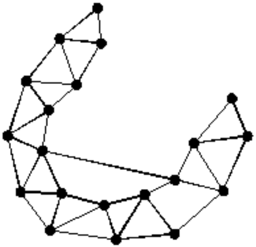
- starts with a lattice structure



Watts-Strogatz Model

- high clustering coefficient
- low diameter
- starts with a lattice structure
- randomly re-wires ties



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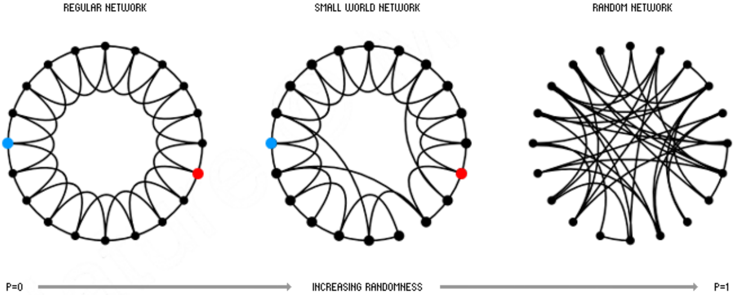

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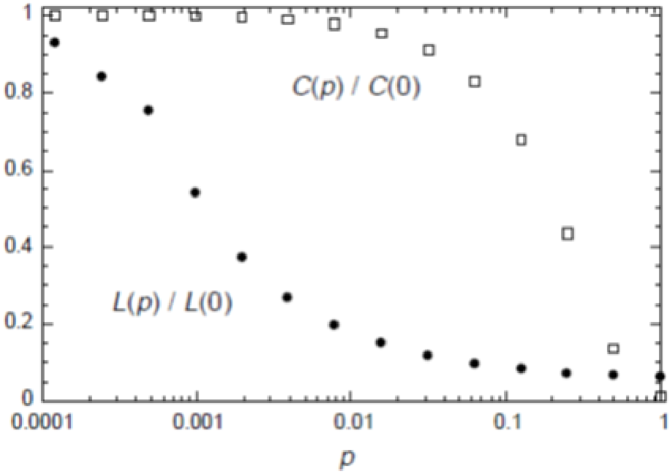

Watts-Strogatz Model

	L_{actual}	L_1	C_{actual}	C_1
Film Actors	3.65	2.99	0.79	0.00027
Power Grid	18.7	12.4	0.08	0.005
C. Elegans	2.65	2.25	0.28	0.05

Code Time

- the rest! whew!