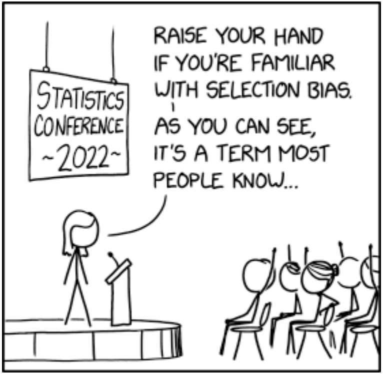
Network Statistics

Lorien Jasny and Örjan Bodin



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    - Permutation tests (eg QAP) and baseline models
    - Network Autocorrelation Models
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- Model choice depends (mostly) on your selection of dependent/independent variables

Relating Node level indices to covariates

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- Node Level Indices: centrality measures, brokerage, constraint
- Node Covariates: measures of power, career advancement, gender – really anything you want to study that varies at the node level

Emergent Multi-Organizational Networks (EMON) Dataset

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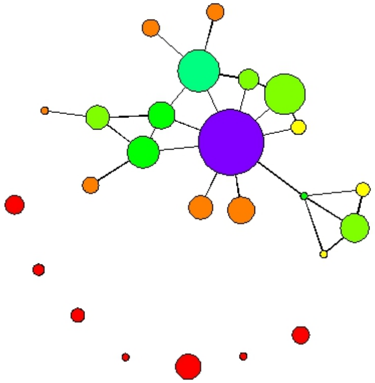
- Ties between organizations are self-reported levels of communication coded from 1 to 4 with 1 as most frequent

Emergent Multi-Organizational Networks (EMON) Dataset

Attribute Data

- Command Rank Score (CRS): mean rank (reversed) for prominence in the command structure
- Decision Rank Score (DRS): mean rank (reversed) for prominence in decision making process
- Paid Staff: number of paid employees
- Volunteer Staff: number of volunteer staff
- Sponsorship: organization type (City, County, State, Federal, or Private)

Correlation between DRS and Degree?



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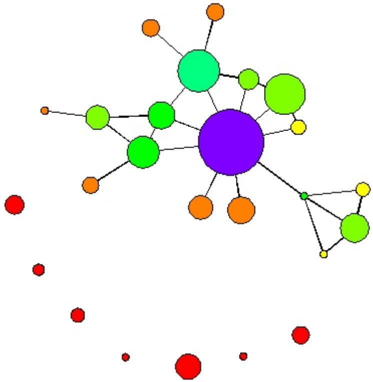
- Subsample of Mutually Reported "Continuous Communication" in Texas EMON



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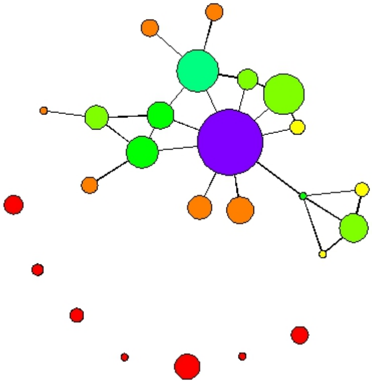

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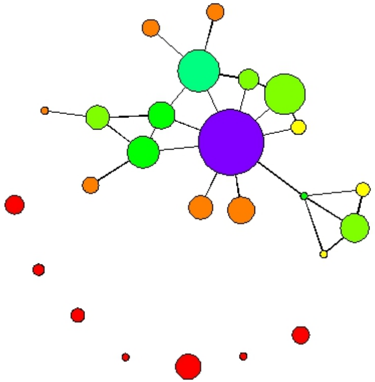

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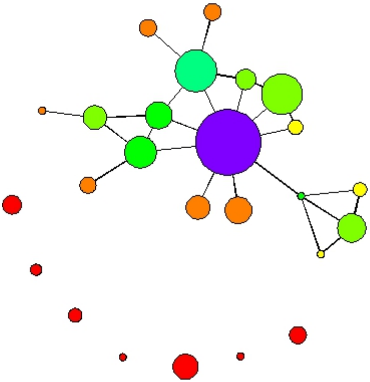
- Empirical corelation $\rho = 0.86$



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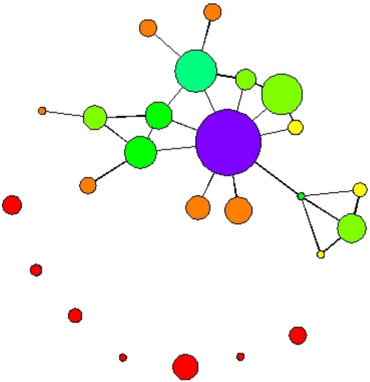

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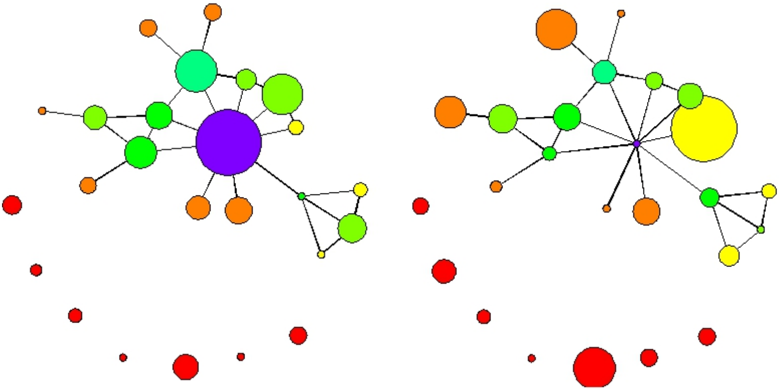

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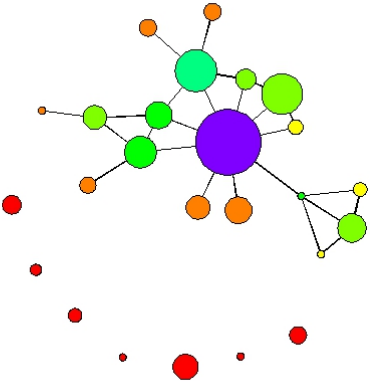

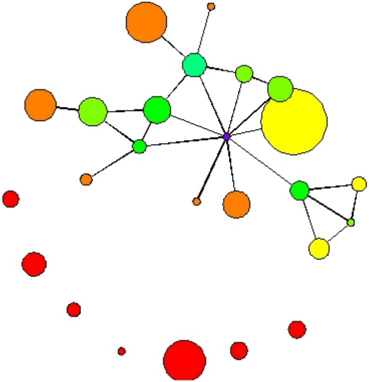

$\rho = 0.86$ $\rho = -0.07$

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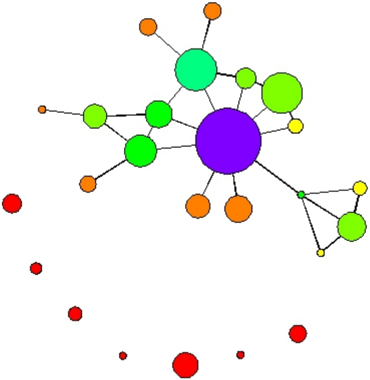

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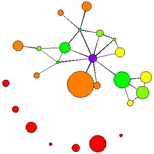

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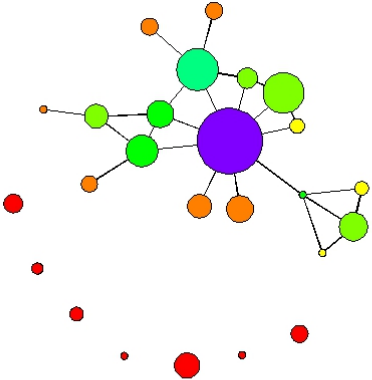

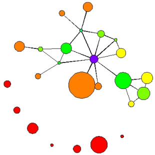

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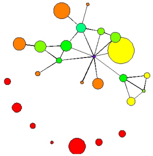

$\rho = -0.12$

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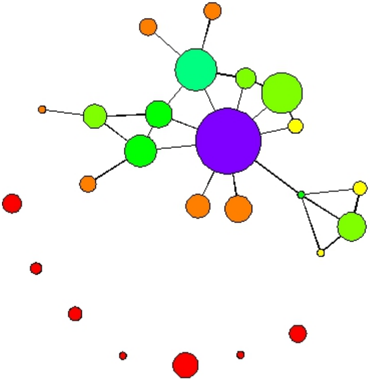

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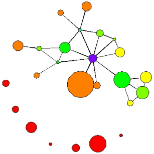

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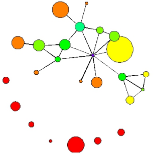

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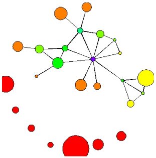

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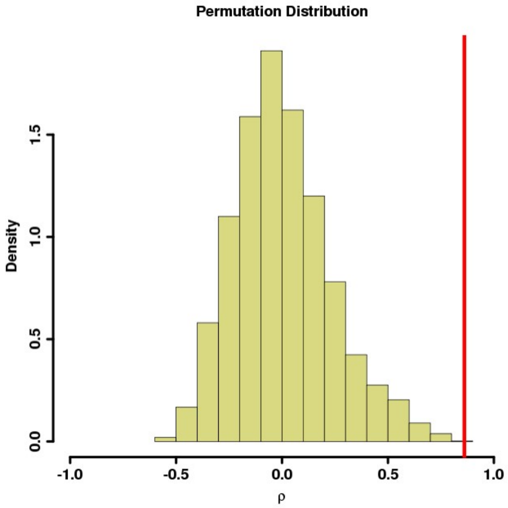

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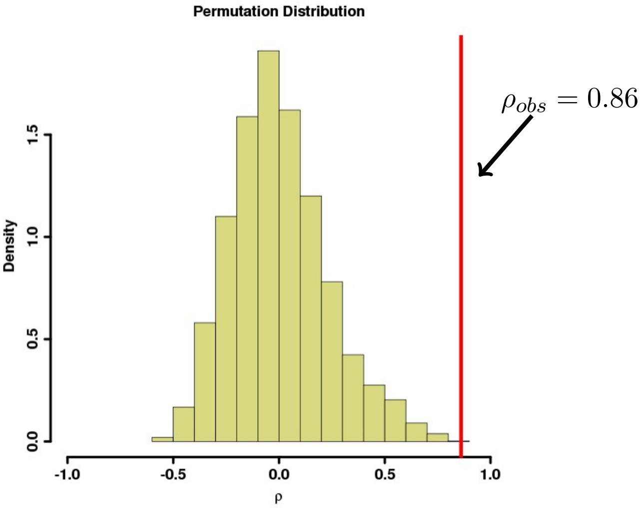

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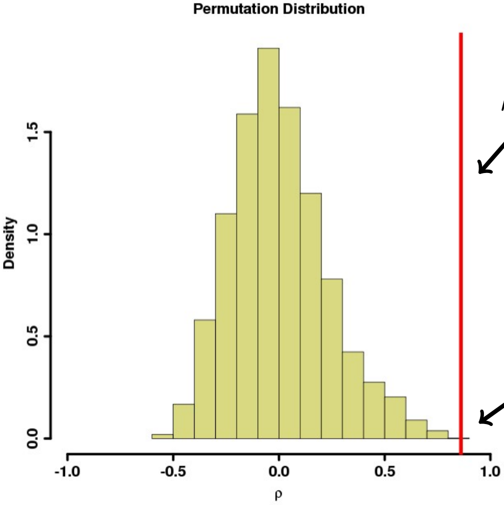
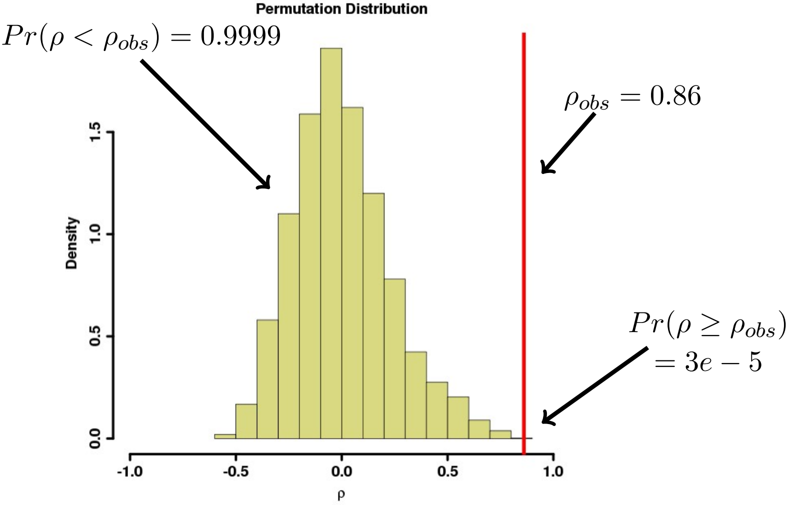


$\rho_{obs} = 0.86$

$Pr(\rho \geq \rho_{obs})$
$= 3e - 5$

Correlation between DRS and Degree?



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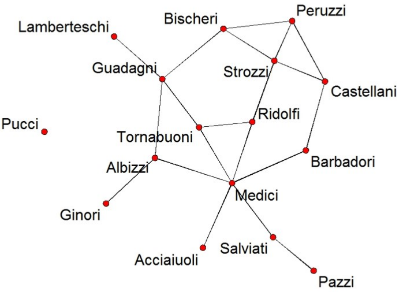
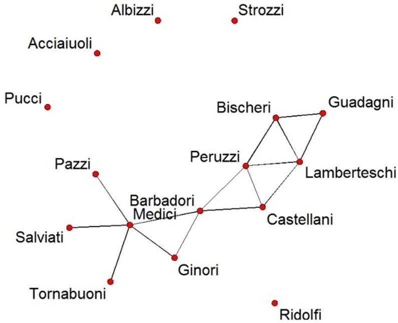
- NLIs as dependent variables more problematic due to autocorrelation

Quadratic Assignment Procedure

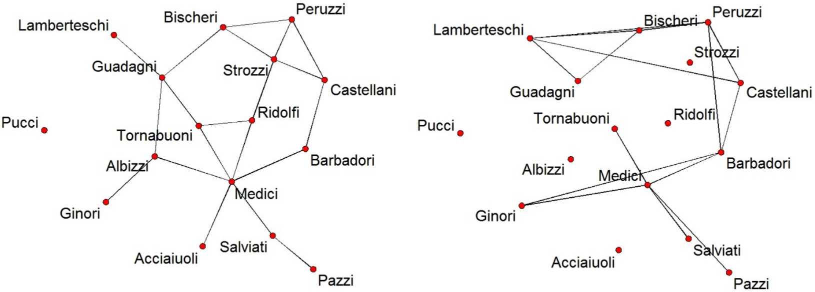


Marriage



Business

Quadratic Assignment Procedure



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Graph Correlation

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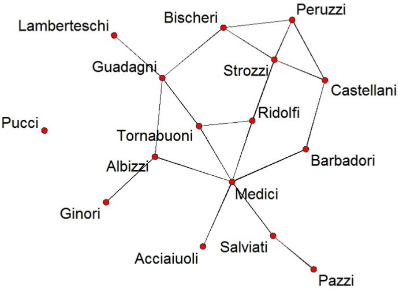
- $gcor\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \right) = cor([1,1,1,0],[1,1,2,2])$

Do business ties coincide with marriages?



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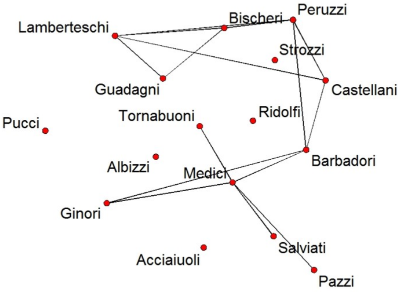

Business

$$\rho = 0.372$$

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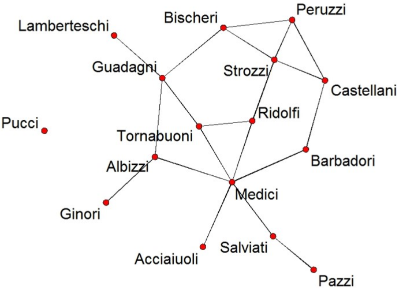

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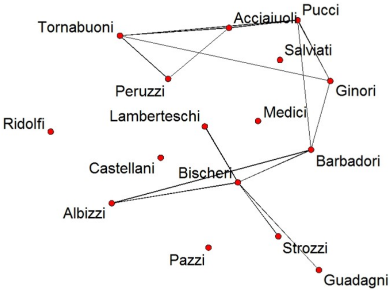

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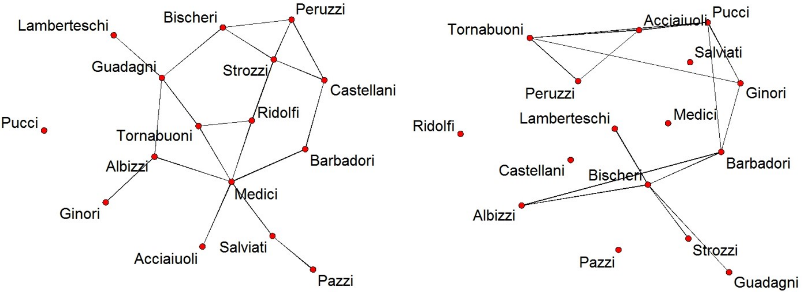

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Business

$$\rho = 0.169$$

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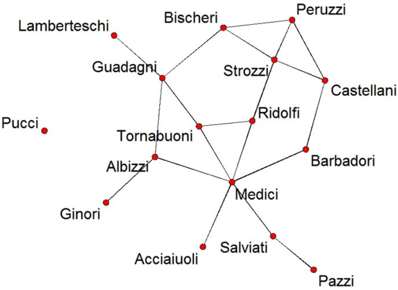

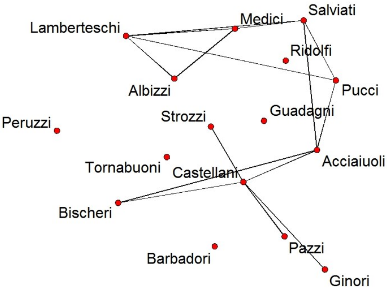

Marriage

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$$\rho = -0.034$$

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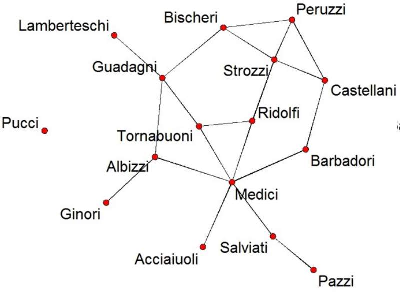
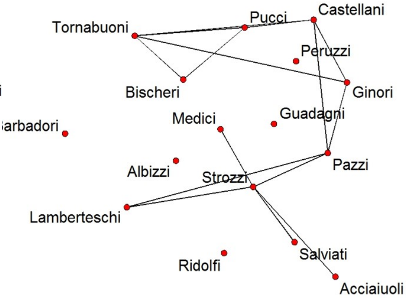


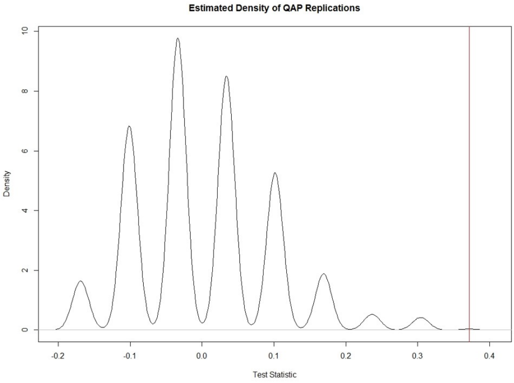
Marriage



Business

$$\rho = -0.101$$

QAP Test


Estimated Density of QAP Replications
Density
Test Statistic
0
2
4
6
8
10
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4

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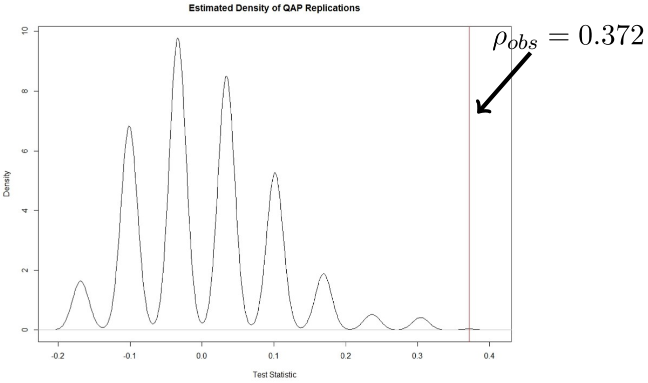
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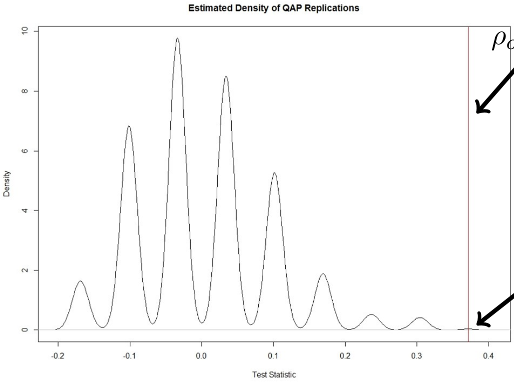

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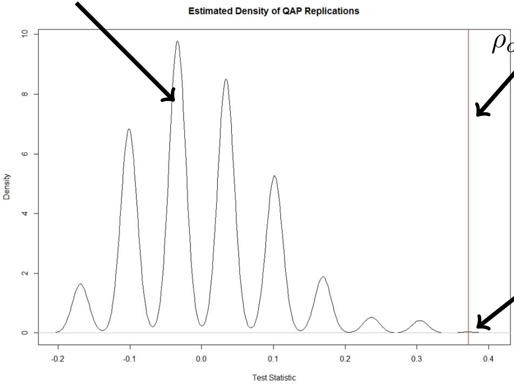
$Pr(\rho \geq \rho_{obs})$
$= .001$

QAP Test

$Pr(\rho < \rho_{obs}) = 0.999$



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  - Where $\mathbf{E}$ is the expectation operator (analagous to "mean" or "average")
  - Y_{ij} is the value from i to j on the dependent relation with adjacency matrix Y
  - X_{kij} is the value of the kth predictor for the (i, j) ordered pair, and $\beta_0, \ldots \beta_\rho$ are coefficients

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- Independent variables also in adjacency matrix form
    - Always takes matrix form, but may be based on vector data
    - eg. simple adjacency matrix, sender/receiver effects, attribute differences, elements held in common

Code Time

Sections 2.4-2.5

Baseline Models

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    - useful even when baseline model is not ‘realistic’

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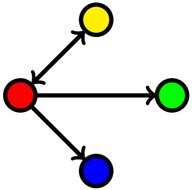

Empirical Network

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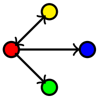

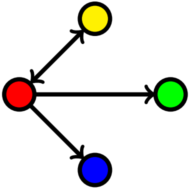

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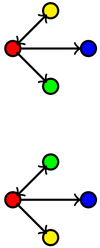

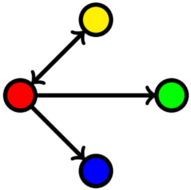

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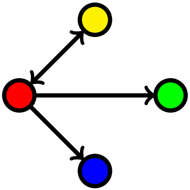

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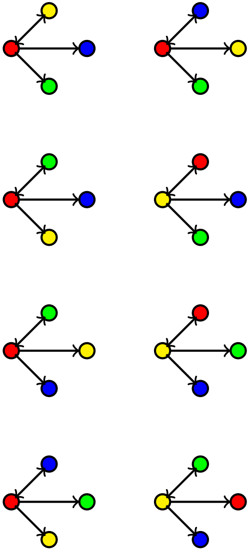

... etc

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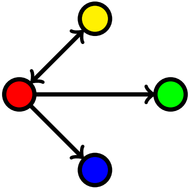

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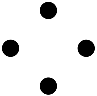

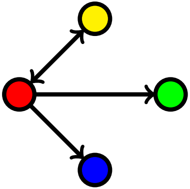

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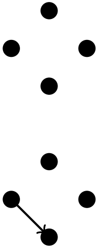

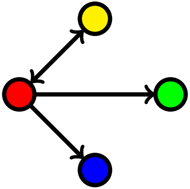

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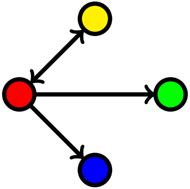
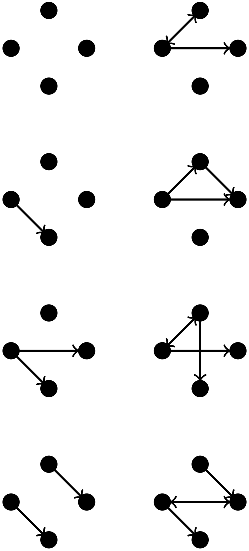


Empirical Network



. . . etc

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- **Number of triangles:** not implemented due to complexity – with ERGM, can condition on the *expected* number of triangles

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- For each simulation, recalculate the test statistic
- Compare empirical value to null distribution, just as in standard statistical testing

Example

Transitivity in the Hurricane Frederic EMON

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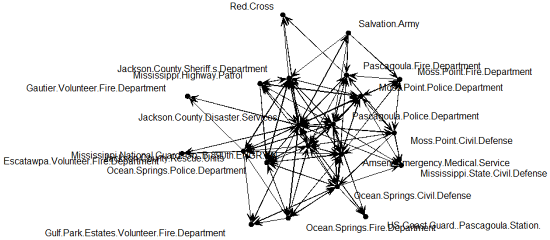
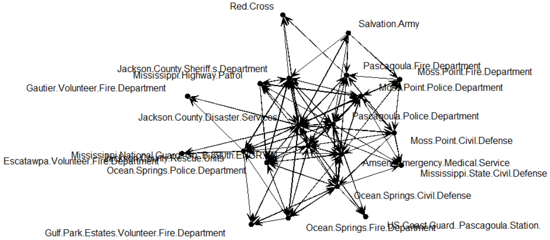


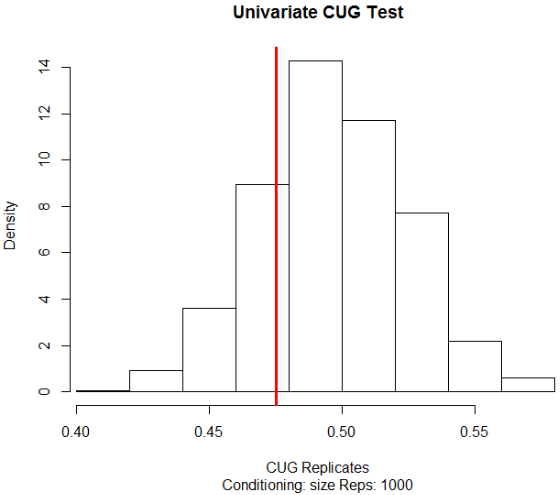
Example

Transitivity in the Hurricane Frederic EMON

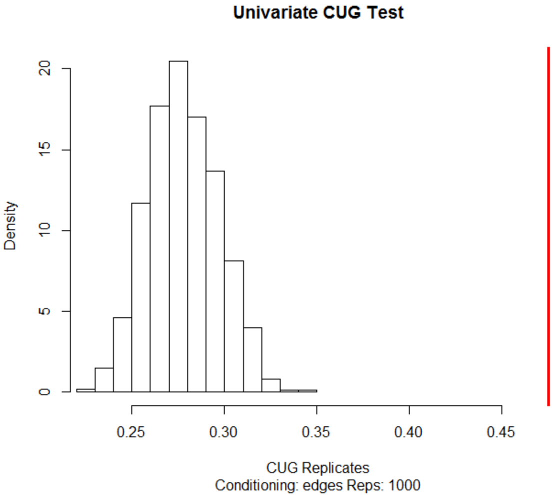
- $\rho = 0.475$
- indicates that roughly half the time that $i \to j \to k$, $i \to k$



Example


Univariate CUG Test
Density
0
2
4
6
8
10
12
14
0.40
0.45
0.50
0.55
CUG Replicates
Conditioning: size Reps: 1000

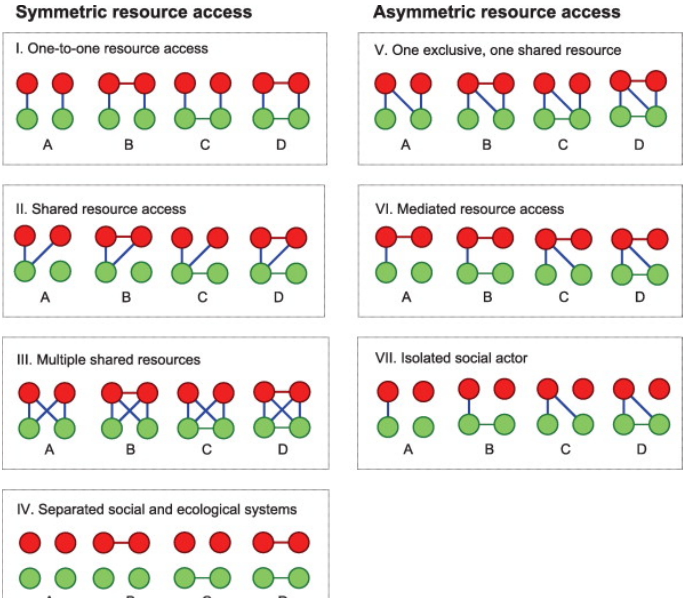
Example


Univariate CUG Test
Density
0
5
10
15
20
0.25
0.30
0.35
0.40
0.45
CUG Replicates
Conditioning: edges Reps: 1000

Bodin and Tengo

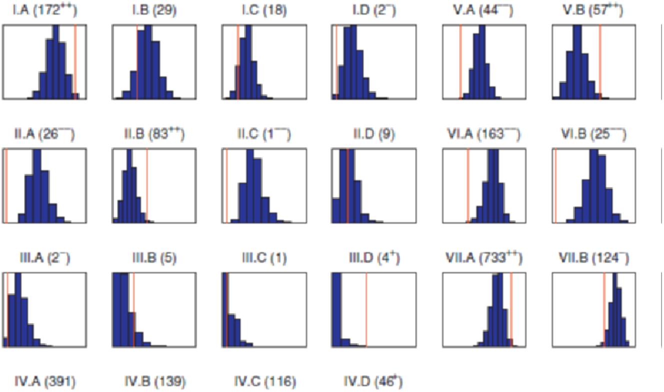
"Disentangling intangible social–ecological systems"

Bodin and Tengo



Bodin and Tengo

436 Ö. Bodin, M. Tengö / Global Environmental Change 22 (2012) 430–439



Caution!

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- Your selection of baseline model controls what hypothesis you're testing
- Changing the model can greatly change the results

Summary

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- QAP regression (edges are the dependent variable)
- Network Autocorrelation Model (vertex attribute is dependent variable)
- CUG tests (network is dependent variable)