Principles Of Mathematical Analysis

数学分析原理

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None

Contents

1 The Real And Complex Number Systems

Introduction

A satisfactory discussion 令人满意的讨论 of the main concepts 概念 of analysis (such as convergence 收敛, continuity, differentiation 微分, and integration) must be based on an accurately 精确地 defined number concept. We shall 助动词, 类似于 **will** not, however, enter into any discussion of the axioms 公理 that govern 统治, 管理 the arithmetic of the integers, but assume familiarity with the rational numbers (i.e., the numbers of the form m/n, where m and n are integers and $n \neq 0$).

The rational number system is inadequate 不足的 for many purposes 目的, 意图, both as a field and as an ordered set. (These terms 术语 will be defined in Secs. 1.6 and 1.12.) For instance, there is no rational p such that $p^2 = 2$. (We shall prove this presently.) This leads to the introduction of so-called "irrational numbers 无理数" which are often written as infinite decimal 无限小数 expansions and are considered to be "approximated 近似, 估计" by the corresponding 对应的 finite decimals. Thus the sequence

$$1, 1.4, 1.41, 1.414, 1.4142, \dots$$

"tends to 趋近于 $\sqrt{2}$." But unless 否则, 表示转折 the irrational number $\sqrt{2}$ has been clearly defined, the question must arise: Just what is it that this sequence "tends to"?

This sort of question can be answered as soon as the so-called "real number system" is constructed 建造, 构造.

Theorem 1.1

We now show that the equation

$$p^2 = 2, \tag{1.1}$$

is not satisfied 满足 by any rational p. If there were such a p, we could write $p = m/n$ where m and n are integers that are not both even. Let us assume 假设 this is done. Then (1.1) implies 表明, 意味着

$$m^2 = 2n^2, \tag{1.2}$$

This shows that m^2 is even. Hence 因此 m is even (if m were 虚拟语气 odd, m^2 would be odd), and so m^2 is divisible by 4. It follows that the right side of (1.2) is divisible by 被 ... 整除 4, so that n^2 is even, which implies that n is even.

The assumption 假定 that (1.1) holds thus leads to 导致 the conclusion that both m and n are even, contrary 相反的 to our choice of m and n. Hence (1.2) is impossible 不可能的 for rational p.

We now examine 研究 this situation a little more closely 更加仔细地. Let A be the set of all positive rationals p such that $p^2 < 2$ and let B consist of 由 ... 组成 all positive rationals p such that $p^2 > 2$. We shall show that A contains no largest number and B contains no smallest.

More explicitly 更加清晰地, for every p in A we can find a rational q in A such that $p < q$, and for every p in B we

can find a rational q in B such that $q < p$. To do this, we associate with each rational $p > 0$ the number
关联, 联系

(1.3)
$$q = p - \frac{p^2 - 2}{p + 2} = \frac{2p + 2}{p + 2}$$

Then

(1.4)
$$q^2 - 2 = \frac{2\left(p^2 - 2\right)}{(p + 2)^2}$$

If p is in A then $p^2 - 2 < 0$, (1.3) shows that $q > p$, and (1.4) shows that $q^2 < 2$. Thus q is in A.
表明

If p is in B then $p^2 - 2 > 0$, (1.3) shows that $0 < q < p$, and (1.4) shows that $q^2 > 2$. Thus q is in B. ♡

Remark 1.2

The purpose of the above discussion has been to show that the rational number system has certain gaps,
... 的目标是
in spite of the fact that between any two rationals there is another: If $r < s$ then $r < (r + s)/2 < s$. The
尽管事实如此
real number system fills these gaps. This is the principal reason for the fundamental role which it plays in
最重要的, 首要的 根本的, 基本的
analysis.

List Of Marks

1. **satisfactory discussion** 令人满意的讨论, P[1]
2. **concepts** 概念, P[1]
3. **convergence** 收敛, P[1]
4. **differentiation** 微分, P[1]
5. **accurately** 精确地, P[1]
6. **shall** 助动词, 类似于 will, P[1]
7. **axioms** 公理, P[1]
8. **govern** 统治, 管理, P[1]
9. **inadequate** 不足的, P[1]
10. **purposes** 目的, 意图, P[1]
11. **terms** 术语, P[1]
12. **irrational numbers** 无理数, P[1]
13. **infinite decimal** 无限小数, P[1]
14. **approximated** 近似, 估计, P[1]
15. **corresponding** 对应的, P[1]
16. **tends to** 趋近于, P[1]
17. **But unless** 否则, 表示转折, P[1]
18. **constructed** 建造, 构造, P[1]
19. **satisfied** 满足, P[1]
20. **assume** 假设, P[1]
21. **implies** 表明, 意味着, P[1]
22. **Hence** 因此, P[1]
23. **were** 虚拟语气, P[1]
24. **divisible by** 被 ... 整除, P[1]
25. **assumption** 假定, P[1]
26. **leads to** 导致, P[1]
27. **contrary** 相反的, P[1]
28. **impossible** 不可能的, P[1]
29. **examine** 研究, P[1]
30. **a little more closely** 更加仔细地, P[1]
31. **consist of** 由 ... 组成, P[1]
32. **More explicitly** 更加清晰地, P[1]
33. **associate** 关联, 联系, P[1]
34. **shows that** 表明, P[1]
35. **purpose of** ... 的目标是, P[2]
36. **in spite of the fact** 尽管事实如此, P[2]
37. **principal** 最重要的, 首要的, P[2]
38. **fundamental** 根本的, 基本的, P[2]